Wave Hydro Dynamics

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Module No. # 06
Wave Theories and Testing Facilities
Lecture No. # 01
Finite Amplitude Wave Theories

Having seen, the aspects of small amplitude theory, where in the basic assumption was that the amplitude of the wave is much less compared to the wavelength, which can be most cases for the waves, which are in deeper waters. But when the waves travel propagate in the coastal waters, then you will not you cannot really, have linear waves. That is, when I say linear waves crest and the trough portion should be equal. That is, we define amplitude as equal to wave height by 2.

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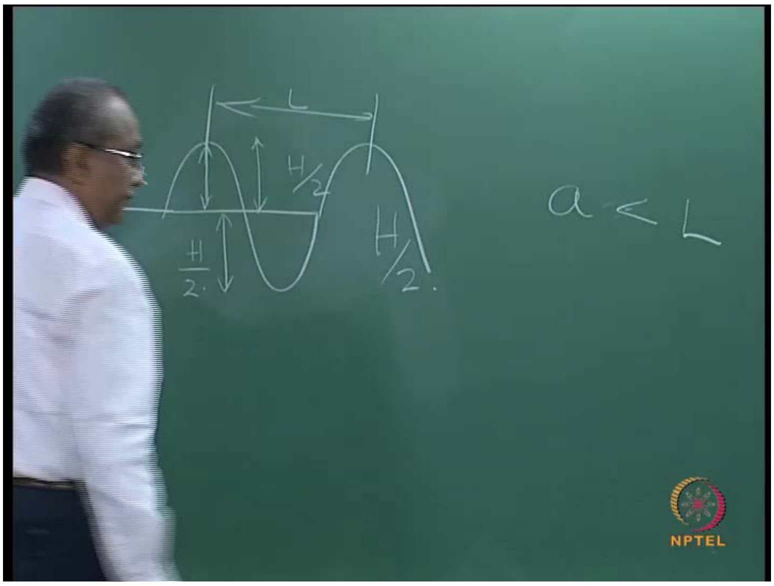

So, in the small amplitude wave theory, we say that H is equal to… So, this is your H by 2. So, this portion about water level which is either high tide line or a mean sea level or a low tide line. This will be the about which the wave surface is oscillating; so you see that this is H by 2 and this is H by 2, such a wave is called as a linear wave. When you want to have the linear wave, the assumption is that, what we saw that amplitude? This is amplitude is much less than wavelength. That is, this is your wavelength, but this may not be the case. As, we have already seen a number of other kinds of waves, or when I gave some of the other examples like Tsunami etcetera.

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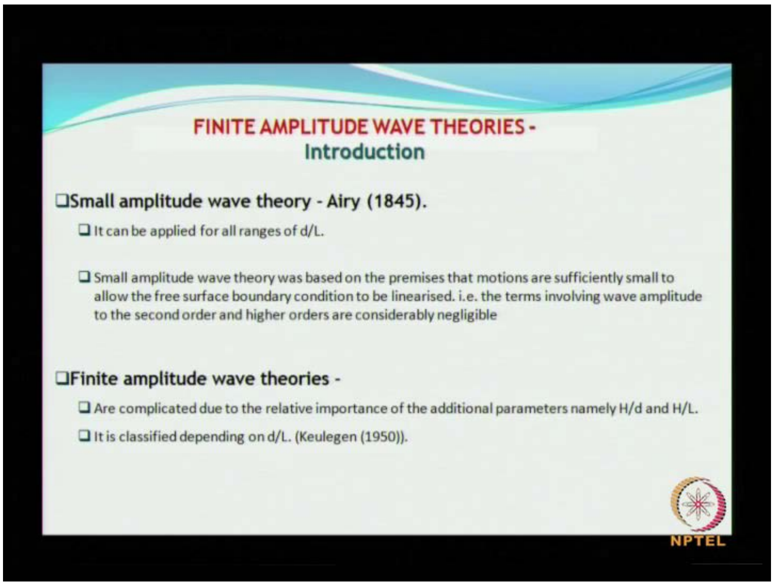

The waves can not be linear and hence we resort to, what is called as the Finite Amplitude Wave Theories. Small amplitude wave theory can be applied for all ranges of d by L, can be applied. There is no hard and fast rule. Small amplitude wave theory was based on the premises that the motions are sufficiently small, to allow the brief free surface boundary condition to be linearised. Please recollect the derivation of velocity potential.

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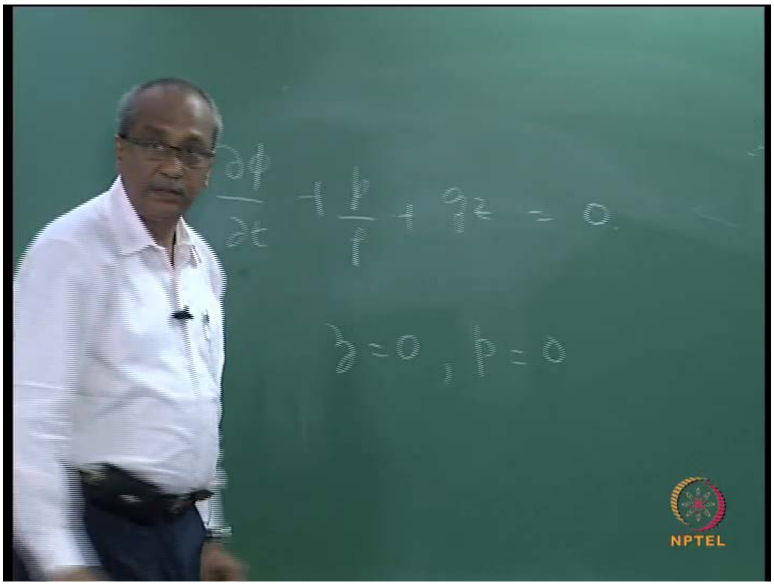

What did we do? We linearised the Bernoulli Equation, dou phi by dou t plus t by gamma and, what we did? We use this Bernoulli Equation at a free surface. When we have the free surface, this is the free surface z equal to 0, which is free surface, free surface is, this is the free surface. But instead of using z equal to theta, we had adopted the simplicity in order to simplify the problem. We adopted this for the dynamic free surface boundary condition. That is at z equal to 0, p equal to 0 and this is called as the dynamic free surface boundary condition.

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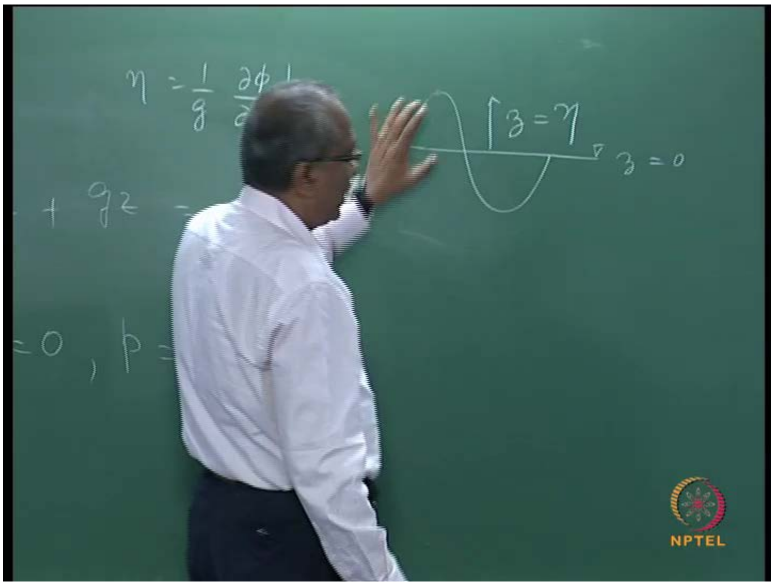

So, from which, remember we get 1 by g into dou phi by dou t at z equal to 0. For the simple reason, we stated that the amplitude is small compare to the (( )) that was the basic assumption, which was made. So, that is, what is illustrated here, as so, what we have done here? Is conveniently, we have avoided the higher order terms. Strictly speaking, those higher order terms have to be considered, and thus we have, what is called as the Finite Amplitude Wave Theories? A quite complicated due to the relative importance of additional parameters namely H by d and d by L.

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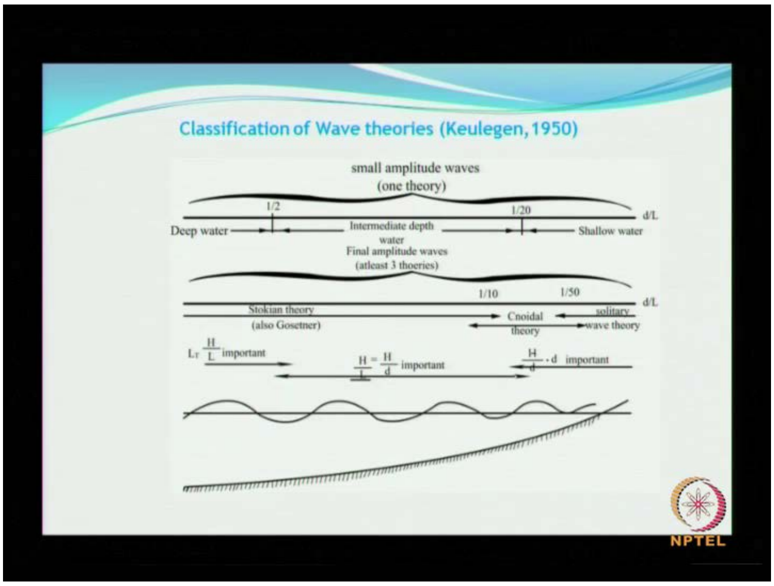

So, based on the variation in d by L Keulegen Carpenter, it is Keulegen in 1950, he classified the waves for the wave theories, as per the d by L.That is, d is the water depth and L is the wavelength so, for the entire water depth situation, that is d by L is equal covering the entire range. You see, there is a single theory, which is the small amplitude Airys Theory. But, for other cases, so as indicated here, you have d by L and beyond this up to this, you have, what is called as your Stokian Theory and within this range, we have, what is called as the Cnoidal Theory. And then d by L up to 1 by 50, that is less than 1 by 50, we have solitary wave. So, here H by L is important, L is also important so, H by d and H by L is important. And all these things are quite important, they plays important role in defining the wave characteristics.

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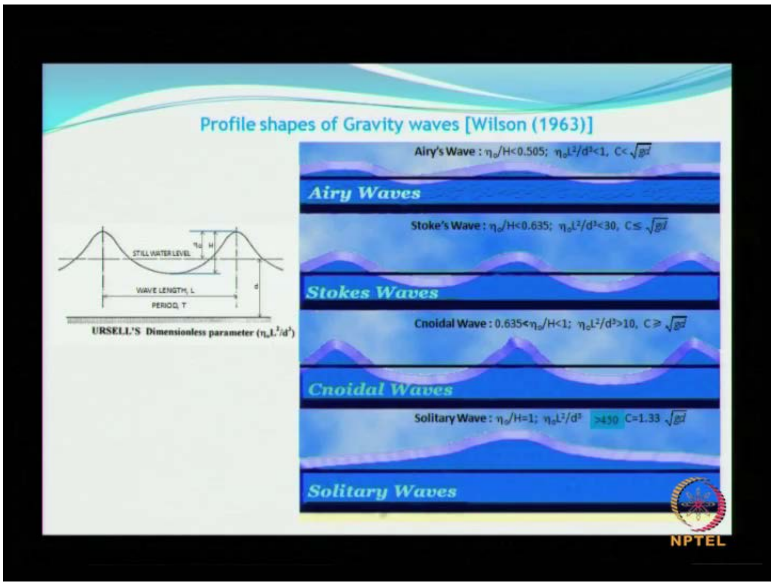

So, Airys Theory eta by H is less than 0.5 and here, you see that what is called as the Ursells parameter. Ursells parameter is this is eta 0 and this is the wave length and d is small d so, this is the Ursells parameter, Ursells parameter is indicated here. What should be the Ursells parameter for the different types of waves that are being considered? For example, Stokes Wave Theory. So, here the celerity is less than root of g d, less than or equal to root of g d. So, here it is greater than root of g d that is in cnoidal waves and, then in the case of solitary waves, the speed of the waves is equal to 1.33 into root of g d. As we have already seen, the Shallow water conditions, in Shallow water the celerity or the speed of the wave is root of g d.

So, the shape of the waves are given here. So, you see this is more or less sinusoidal or it has to be sinusoidal, then we call it as Airys Theory. If the crest is slightly steeper and the trough is slightly flatter, then it enters into the Stokian Wave Theories. Under Stokes Wave Theories, you have several orders that involves. That is, because you have the number of orders, higher order terms you include based on that, you have the order theory. So, you have the 2 order, 3 order, 4^{th} order, 5^{th} order Stokes Theory etcetera. Then, we have the Cnoidal waves, where in the profile is defined by (( )) cosine elliptical function, which we will see later. Finally, we have the Solitary wave, so in the case of cosin, Cnoidal Waves, you see the steepening of the waves.

The trough is quite flat, and long. And these are basically called as long weaves, Cnoidal and Solitary Waves. The importance of Cnoidal and Solitary Wave Theories, as kind rapid importance people are working a lot, in these two waves. Because, the tsunami characteristics can be, to some extent. You cannot call it directly, as a Solitary wave to a certain extent. The characteristics of a tsunami, can be represented to the characteristics of Solitary waves in the case of Solitary waves, you see that, there is no trough. Now, the waves are going to hit like this, at regular intervals. So, it is something like, your wall of water, as we usually call it, in terms of as we usually tsunami is termed as wall of water. So, again I would like to reiterate that Cnoidal and Stokes Solitary Waves are basically, Shallow water waves.

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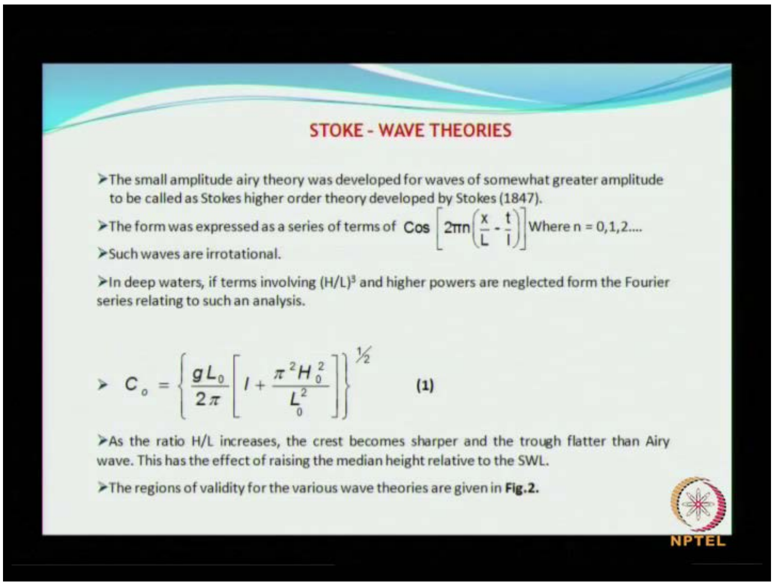

So, let us see Stokes Wave Theories again. I am just giving only the highlight of some of these theories, but you need to go through books, or the references indicated in the lecture. Small amplitude Airys Theory was developed for waves, a somewhat greater amplitude to be called as Stokes higher order theory, developed by Stokes in 1847, quite old theory. The form was expressed as a series of term of cos, cos of, so, you have a n standing, where n equal to 0 1, 0 1 2 etcetera. So, it can go to higher numbers of course, the waves are, such waves are considered to be irrotational.

In deep waters, if the terms involving H by L cube and higher powers are neglected then, you can have a solution, as given here. Under equation 1, as H by L, the rate of H by L increases, the crest become sharper and sharper. Whereas, the trough becomes flatter, and flattered than that of a Airys Theory. So, airys theory, you know, how to calculate the C naught, how do you calculate the C naught in Airys Theory, g L naught by 2 pi. Whereas, here you have that additional term coming into picture, is that clear?

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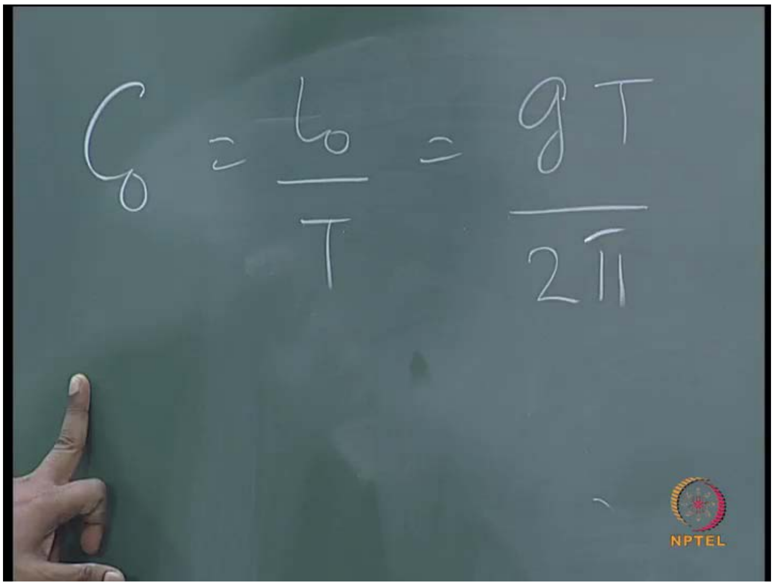

In Airys Theory, C naught equal to L naught by T, this is

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so, the same thing here ,it is represented, you have an higher order terms coming into picture.

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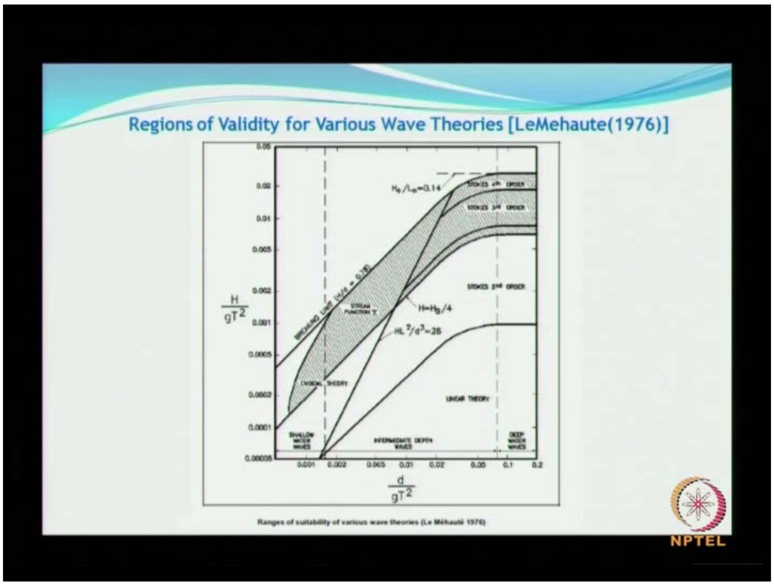

So, this shows, the regions of validity of various theories, and this is after lemehaute in the year 1976. The ranges of the validity of different theories, you see this is the area, where you can adopt linear theory without any problem. This is the region, where you can adopt second order theory. And this is the region, where you can adopt the third order theory, and this is the region, where you can adopt 4^{th} order Stokes Theory. This is the region, where you can adopt your Stream Function Theory and Cnoidal Wave Theory comes in here, and this is the breaking limit.

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So, this is the area, where you have the Shallow water waves. Shallow water waves and this is the area, where you have the Intermediate water waves. And this is the area, where you have the deep water (()). I am sure, you are able to recollect the characteristics of shallow water deep water and intermediate waters, based on the displacement. Under the water particle displacement, under different water depth condition, we have seen enough.

So, on the y axis, you have achieved H by g d square, and on the X axis, you have d by g d square. So, this shows that, the relationship between wave height, wave period, as well as the water depth are the main factors. That dictate, which theory has to be applied for, and why are, we talking about this theory? Where in, you need to consider all these theories.

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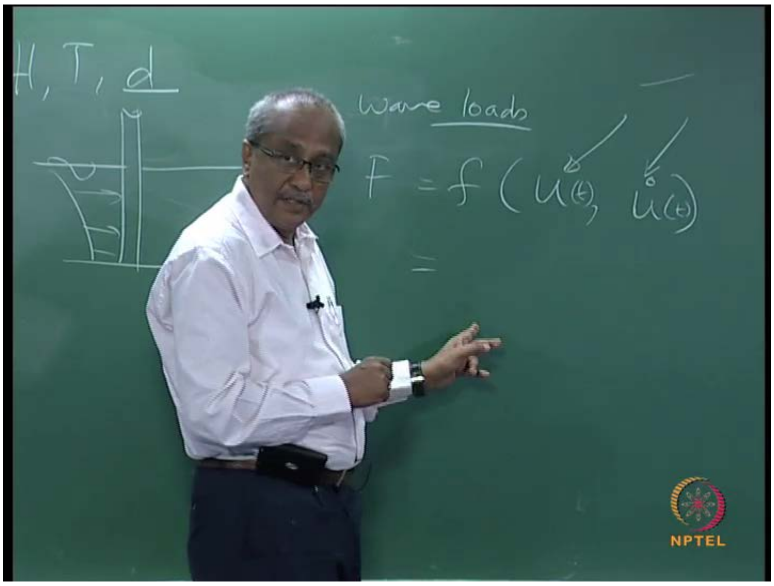

So, you know that, all structures ,or pile structure, this is a small example. All structures are supported on pile, and you need to evaluate the force exerted on this structure. Later on, my lecture on wave forces, or wave loads on structures, you will see that the force is going to be a function of orbital velocity and acceleration. So, this is going to be function of t and also this u and u dot, that is particle velocities and accelerations are going to vary with respect to along the depth. (No Audio from 16:29 to 16:35) So, you need a theory to describe the velocity profile for a given wave height and wave period, for the pile in a particular water depth at t.

So, straight away, if nothing is given, you will be proceeding to calculate these two using linear theory, because it is quite simple and straight forward without, even bothering to check, whether it is applicable. Although, it is the first step, by chance if your parameter that is these two parameters fall in this range, then there is absolutely no problem in using or in proceeding with your linear theory for calculation of the particle kinematics without calculating anything. You have proceeded with Airys Theory, and then finally, if one looks into the variation, and your data falls somewhere in this region, or somewhere, in this region.

Now, you are using a wrong theory for your data, data consisting of wave height period and depth. So, this is a basic step, which you need to do, because you are using a Linear Theory in a zone, where the waves are no longer linear and in fact, it is lying in the shallow water condition. Shallow water situation ,or may be in the deep water situation somewhere here, the particle Kinematics evaluation of the particle kinematics is incorrect.

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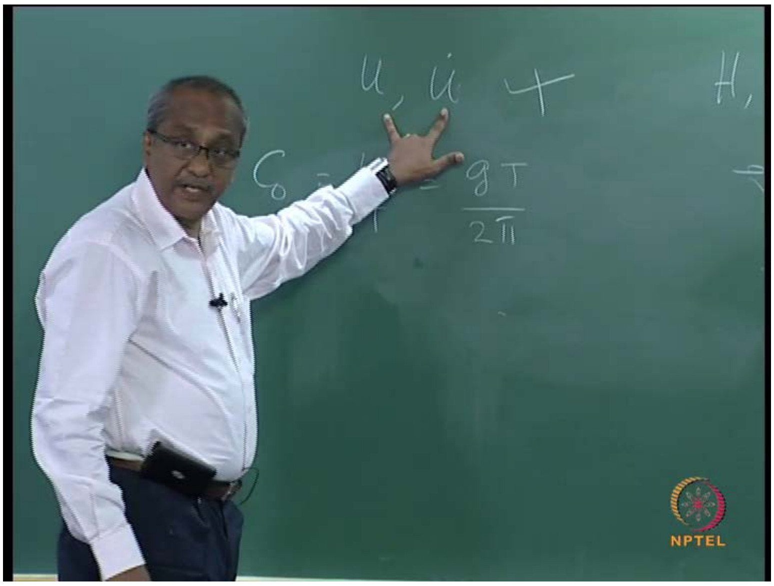

So, you will see that u and u dot evaluation is not wrong. it is, you will still have some values, but it is not 100 percent correct. Now, this is only in the estimation of particle velocity and acceleration. Now, the force is going to be a function of velocity and acceleration so, certain degree of the error involved, now multiplies.

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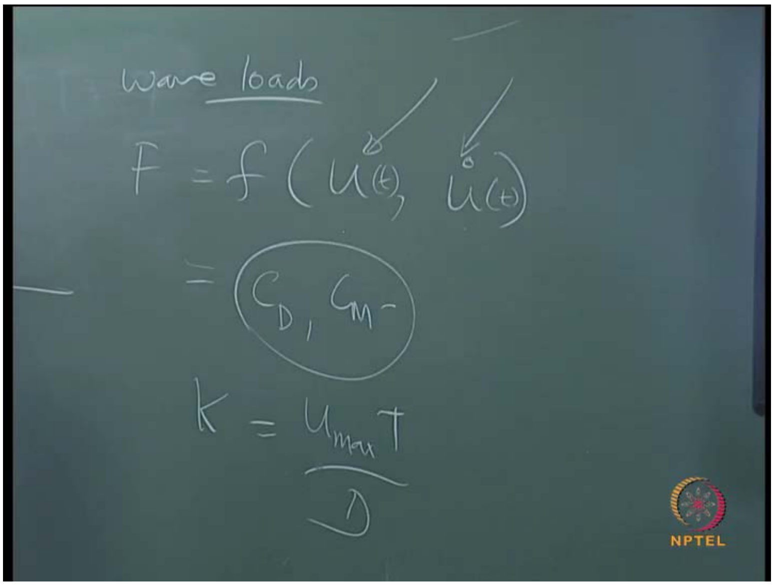

So, for the evaluation of the force, you will also, further see that you need to look at the selection of hydrodynamic coefficients C D and C M. These are again based on a parameter called Keulegen Carpenter number, which is u max T by D. So, here again you see the evaluation is based on this so, your error multiplies. So, the first step is, to make sure, whether your problem falls under Airys Theory. If not under which theory, it follows and only after getting that information, proceed with the calculation of particle kinematics and then with your forces, I hope this is clear.

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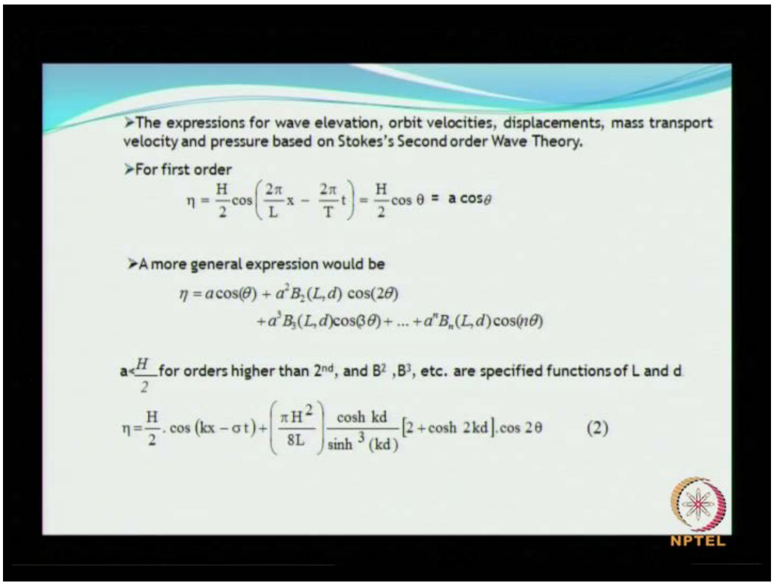

So, expressions for wave elevation, orbital velocities, displacements, mass transport velocity and pressure all these things, based on Stokes second order theory is provided here.

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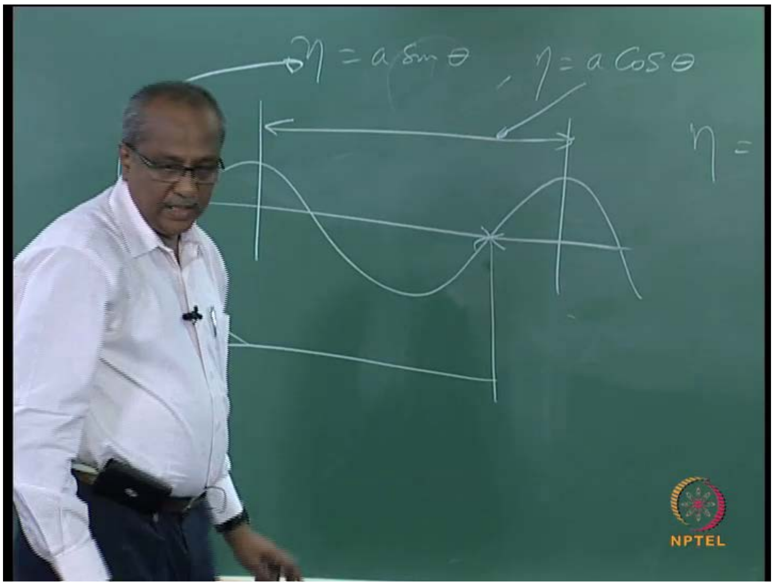

Note that earlier lectures, I have used, a into sin theta for purposefully. I am considering a into cos theta, the main reason is, you should not get confused. Because certain books consider eta as a sin theta, certain books consider does not matter at all, because the properties are going to be same. But only thing is, you should not keep on changing it. When I am discussing about the second order theory, I am not going to change anything so, for the second order theory, what I am trying to do is, I am considering only this portion of the wave.

So, this is going to be represented in this form, this is a sin curve and this is represented by; is that clear? This is very important, because I see some people have, some kind of problem, some of the students have some problem. So, this is very clear, I do not see anykind of confusion here. A more general expression for representing eta is given their, as s squire into beta b b 2 etcetera. It continues, as you can see here square cube to the power n, and also within the argument of cosine. You have the factor n so, that n takes care of the all of the wave theory, that is going to be considered.

So, the second order theory for the wave elevation is shown here, is given by this expression. You can simply derive, I am not going into the derivation part, for the derivation part. There are some books, which do give you the details, but the procedure is as out lined in this small amplitude theory only thing is, you need to considered the higher order terms.

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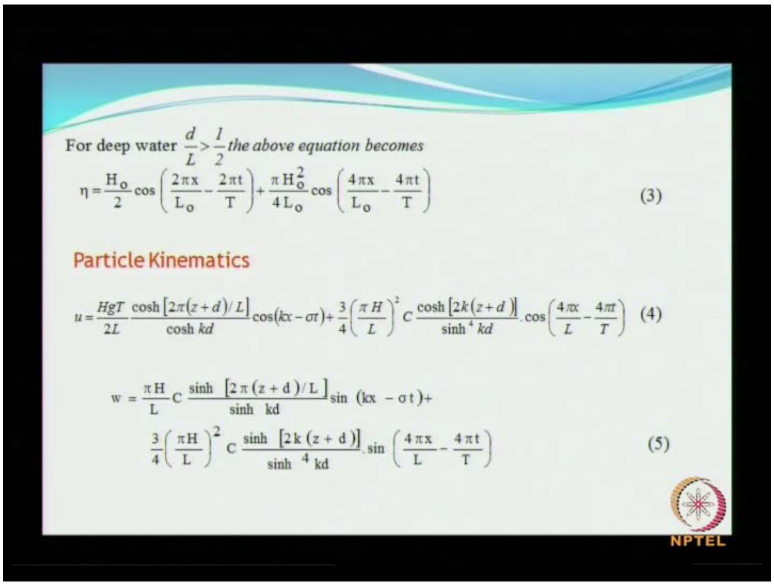

So, for deep waters the above expression will reduce as shown here. Similarly, for the second order theory, the particle velocity is given as shown here. Whereas, I mean for the horizontal direction and this is the vertical direction. So, use the appropriate values for the variable concerned, and then evaluate the particle velocities, which naturally will be different from, what you estimate through the linear wave theory.

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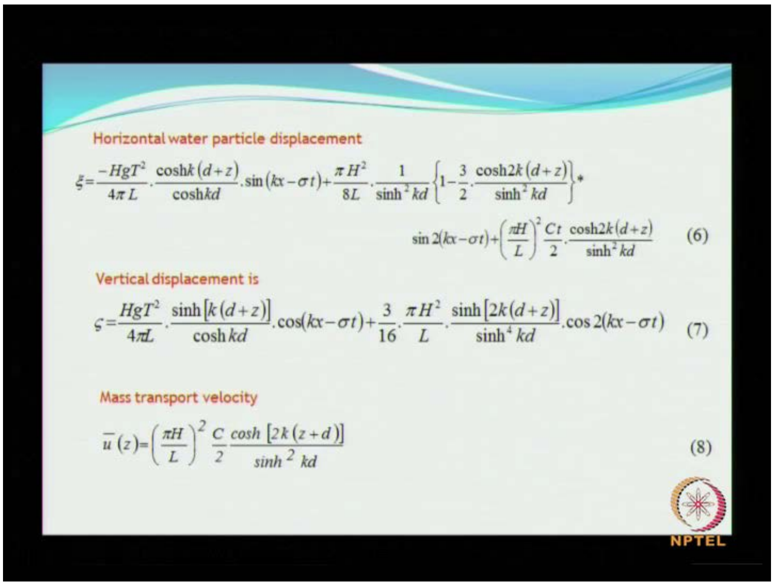

How much is that going to be the deviation, it depends on how much is your crest is steeper than the trough. So, here is the horizontal water particle displacement, vertical particle displacement and the mass transport velocity, all these things, we have seen in the small amplitude wave theory with examples, worked out examples. Is that clear?

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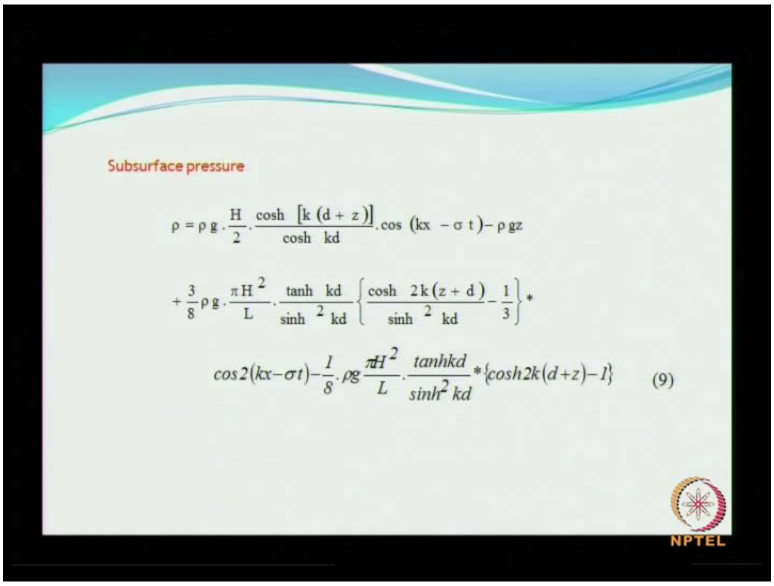

And this gives, the expression for the surf surface pressure all this variables, are known to you, like the k is the wave number, d is the water depth, z is the point of defining the particle velocities, and then sigma is 2 pi by t and load frequency. So, once you have these expressions so, what you need to know is, why you use higher order theories? What happens, if you do not use higher order theories at situations for the conditions. For which, you are suppose to use the higher order theories, that I have already illustrated with the example, that is very important. Now, how to use it is quite straight forward? Because all these variables are already known to you, only thing is the expression is quite long. Is that clear?

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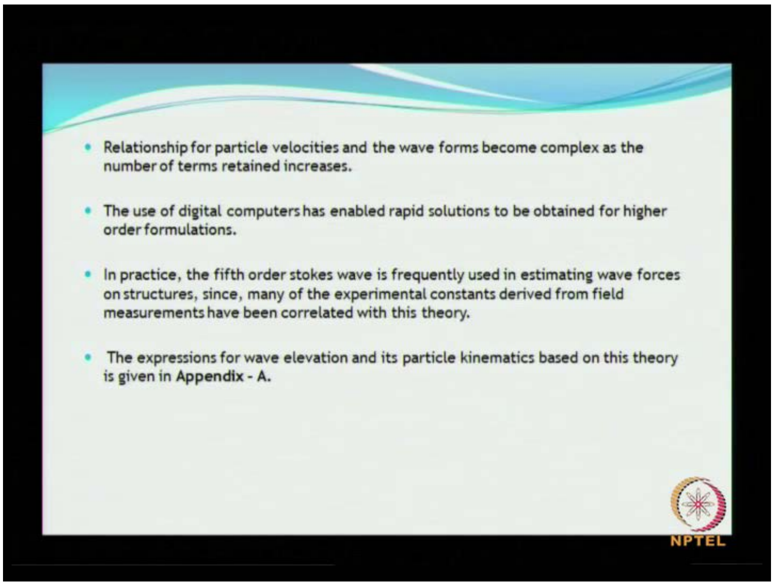

So, relationship for particle velocities and the wave forms, become complex as the number of terms retained in physics. So, usually they go they use 5th order theory, once you have the everything programmed, it becomes very simple, is that clear? So, the use of digital computers, has enabled rapid solutions to obtain for higher order solutions. So, usually 5th order theory is frequently used in estimating wave forces on structures. Since many experimental constants derived from field measurements have been correlated with this theory. What does that mean? See, as I said anyway this protest to the wave forces on the structures.

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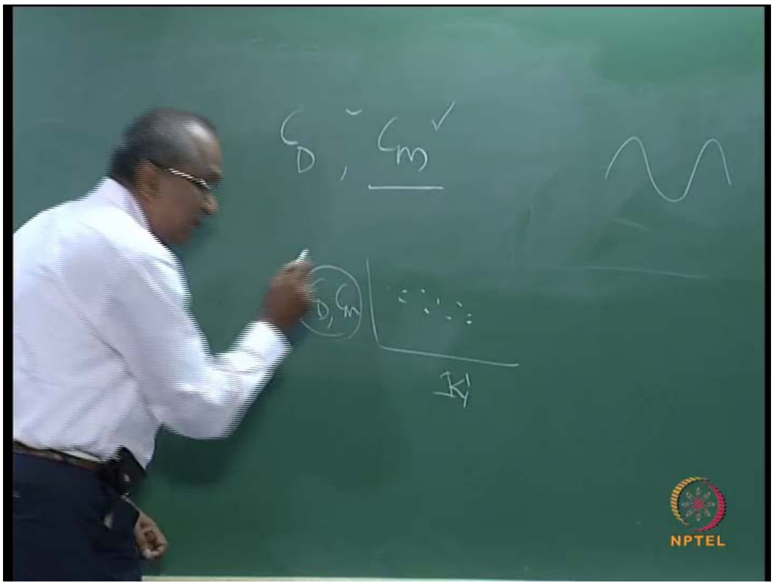

When we talk about the experimental constants, some of the use have two coefficients mainly, which are called as a hydrodynamics coefficients of drag on inertia. These are derived only from experiments, or from field. So, you generate a wave measure, the force and then from the force, and the wave time history, there is a possibility or a method to get the C M and C D. C M and C D are then given as a function of Keulegen Carpenter number, refer to my lecture on wave loads on structures.

Now, you see that these coefficients, where are you getting it from? You are getting it from measurements either in the lab, or in the field. So, it is always, and these waves are not going to be sinusoidal at least the lab. You can try to generate sinusoidal so, that is the reason why they use the 5^{th} order theory the expressions of all these theories is given in appendix a, that will come as a part of the lecture.

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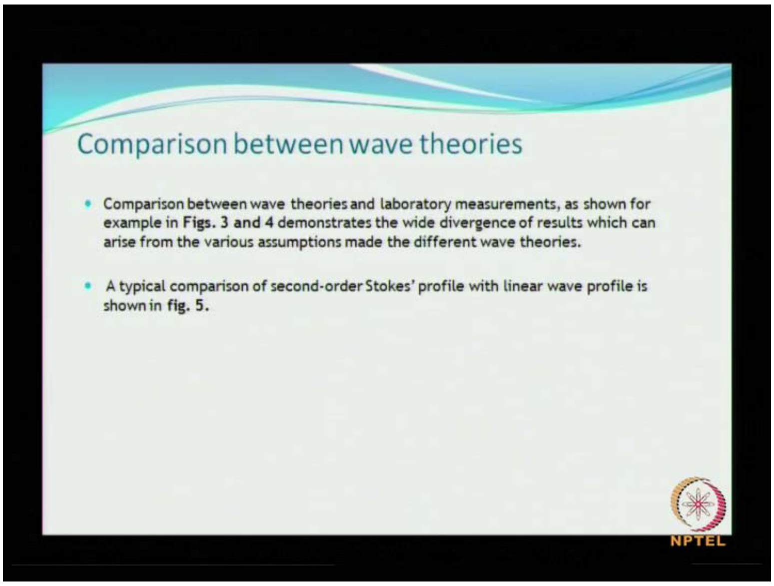

Comparison between the wave theories and lab measurements as shown in example. Demonstrates the wide divergence of the results, which can arise from various assumptions made in the different theories. I just simply highlighted that, there is going to be some difference, but we will see how, what is the kind of difference, we have?

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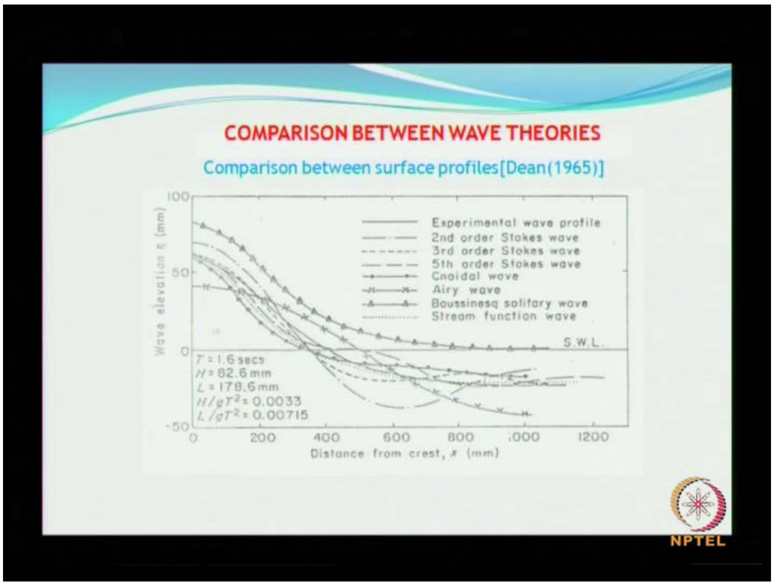

So, this is the comparison between profiles, surface profiles as given by professor Dean in 1965. So, you see the experimental profile, experimental profile is the thick line here. The second order Stokes line is here then stream function theory is here, then you see the Airys Theory here. So, you look at the kind of variation and remember that this is for a given wave height, wave period etcetera. All these data are given here so, this is from the distance from the crest so, the deviation is quite large, somewhere near the crest and this is what really matters.

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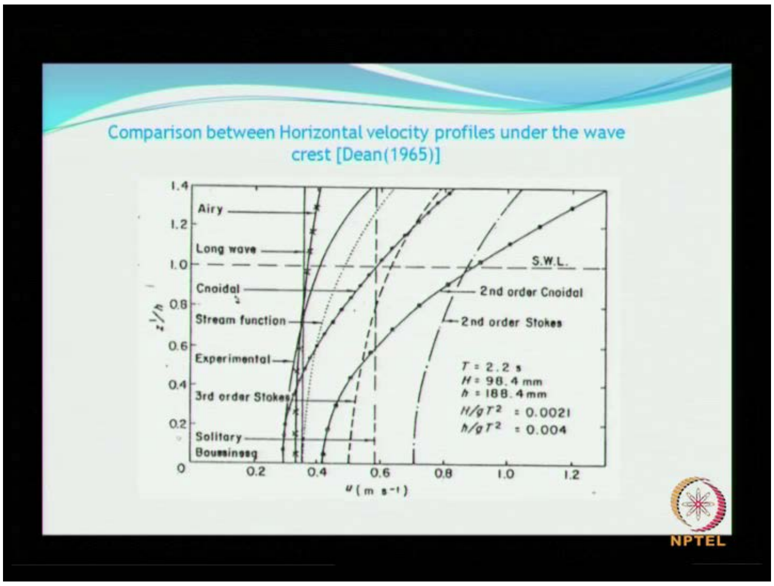

So, this explains, why higher order theories of finite amplitude theories are needed? Yet another comparison here, you compare the comparison of profiles are presented again by Dean, the maximum velocities at each elevation.

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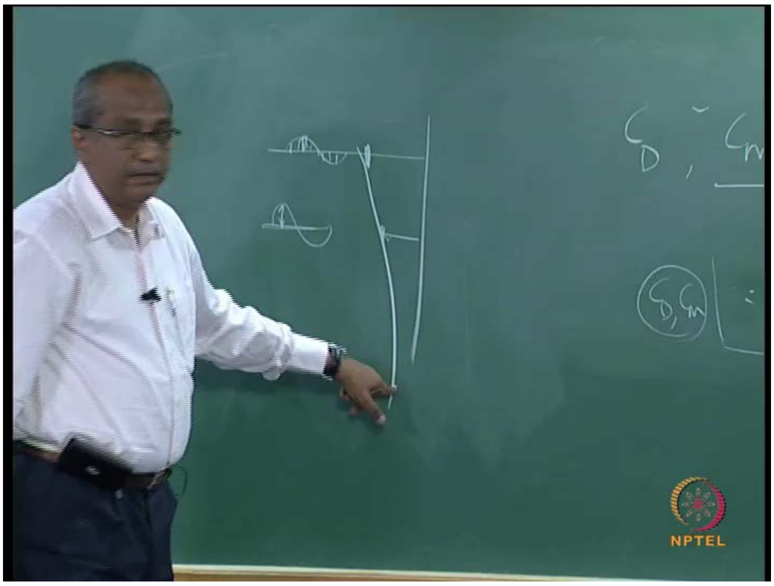

So, at each elevation, in the sense so, you have a time history here. You take only the maximum velocity and that is represented here so, next you have this, and you take only the maximum velocity and that is measured. So, it is plotted along the depth so, the some of the other theories are also included here, like second order Cnoidal Theory, which I am not going to cover here. But you see the Solitary wave theory, Cnoidal wave theory, Airys theory, all these things, how it really differs. The variation is, as from something like around 0.3 and it goes up to about 1 point, and the deviation is significant near the free surface.

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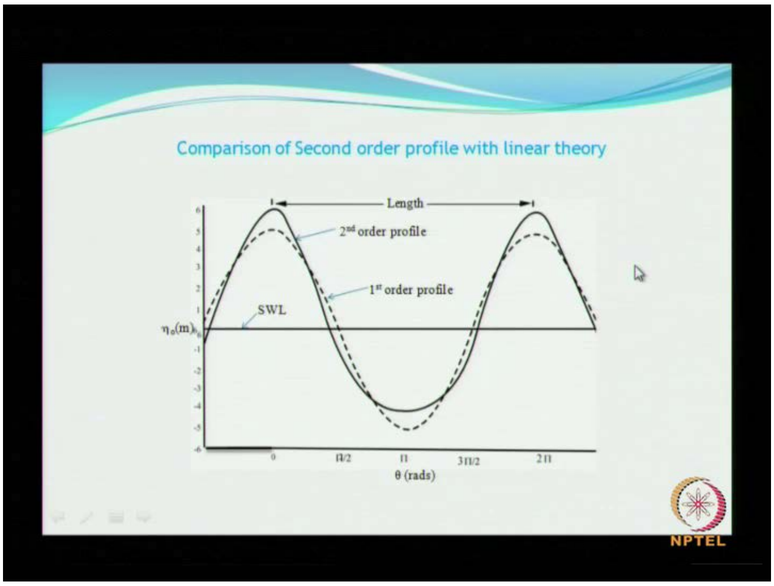

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So, look at this variation, simple variation, the difference between a second order profile and the Airys Theory.

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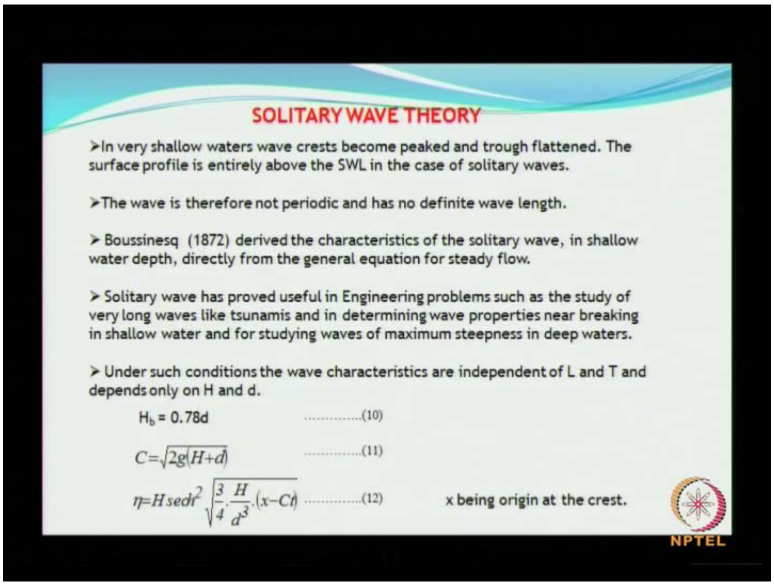

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That explains something about, that has given as brief description, description about the finite amplitude theories. Now, we move on to solitary wave theory, as I said earlier, there are some researches, who are of the opinion that Solitary wave theory. Solitary wave, I mean the tsunami can be the characteristics of the tsunami, can be represented approximately as that of a solitary wave. But there is another set of researches would differ from it. In very shallow waters, wave crest become peaked and trough flattened, the surface profile is entirely above the still water line in the case of a solitary wave.

So, the wave is therefore, not periodic and has no definite wavelength. This is the major difference between Solitary wave and the other waves. Boussinesq derived the characteristics of the Solitary waves in Shallow water directly from general equation of a steady flow, the derivation was quite straight forward. Solitary wave has proved useful in engineering problems, such as study of very long waves like tsunamis. And in determining wave properties near breaking in Shallow water, and for study of waves of maximum steepness in deep waters. Under such conditions the wave characteristics are independent of wavelength t, and will depends only on H.

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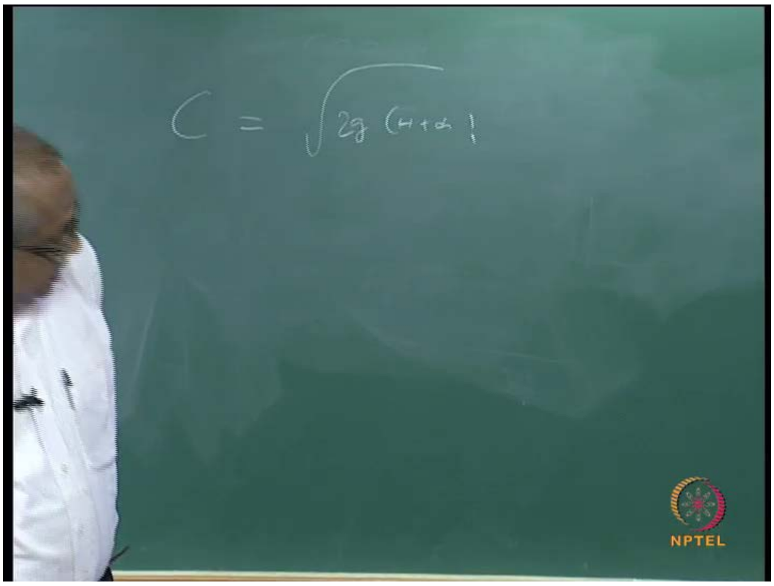

In this case, the celerity the speed of a solitary wave is given as root of, which we will see later. The profile given by, represented by the solitary wave can be represented as shown here, this is sec and H. And here, all other parameters known to you X is being the origin at the crest, based on this only, you can draw the profile, is that clear? A different values for a given C, given characteristics of H, and d use these expressions and traverse the profile from X.(( )) X is starting from the crest of the profile, a crest of the Solitary wave.

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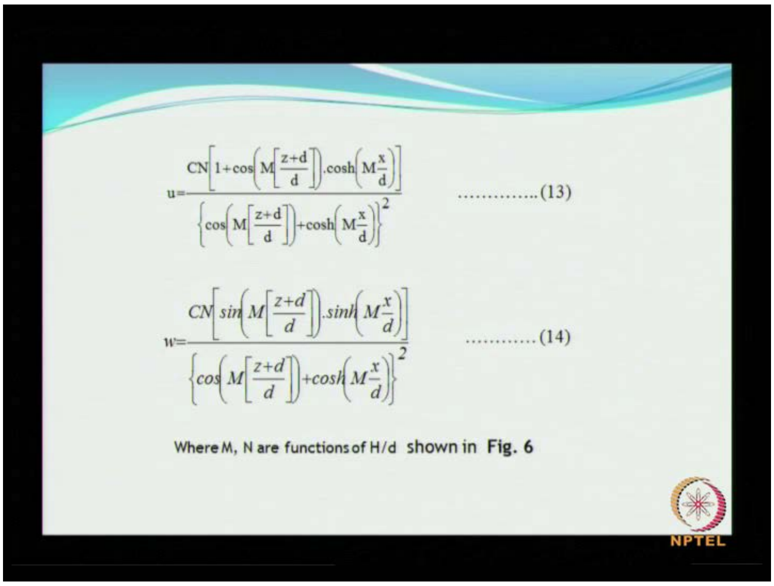

The horizontal and vertical, particle velocities are given by this expression, as C is the celerity as disguise, as discussed earlier. And M, and N are values or functions, which can be obtained from a figure, which represented as a function of H by d, which will be shown soon. All other parameters are known to you, because X is starting from the crest of the wave, as said earlier d is the water depth, s is below the still water line, m and n are obtained from:

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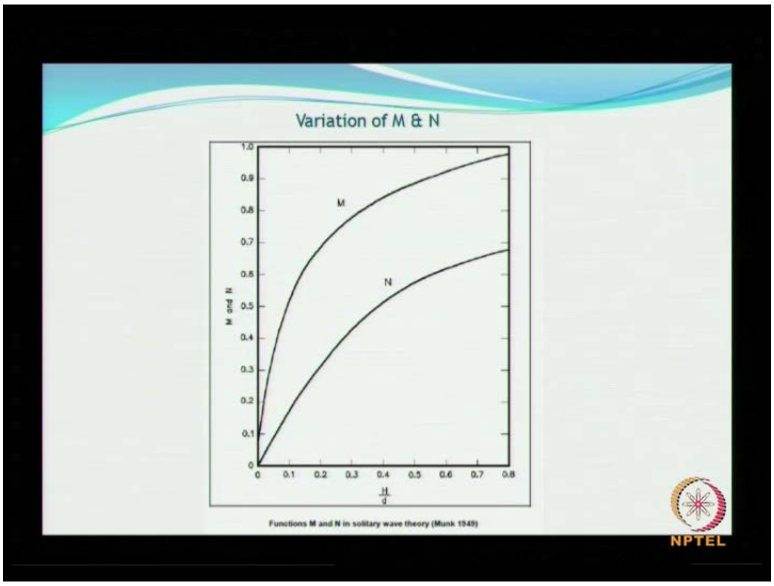

The figure as shown here, once you have H by d for the given wave condition. You can derive the values for M and N, use that expression, earlier expression to get u and w.

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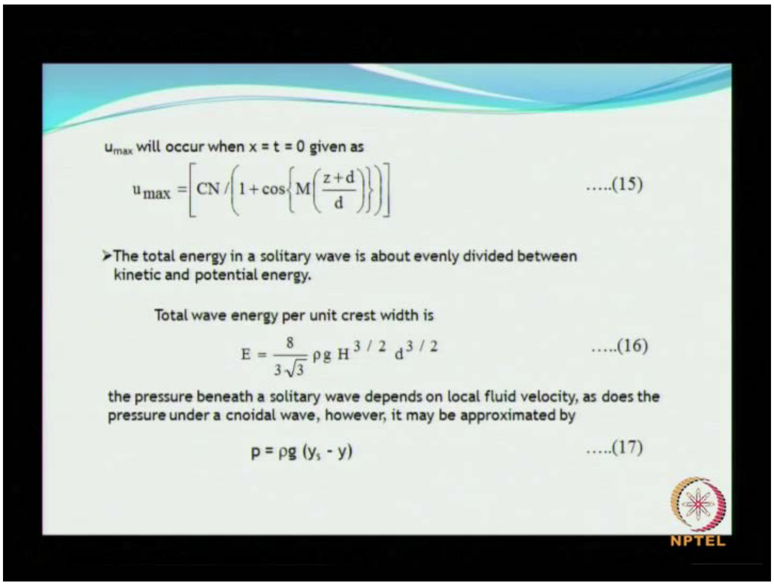

So, u max will occur, when X and t equal to 0 and that is given, as shown here. The total energy in a Solitary wave is about, evenly divided between the kinetic energy and the potential energy. I will not go into the detail of deriving the kinetic energy, although it is quite straight forward. Again use the same principle, as we have done for on a similar procedure. What we have done for linear waves, and you see that the total energy can be obtained per unit crest, is given as this expression whereas, the pressure is given by this expression.

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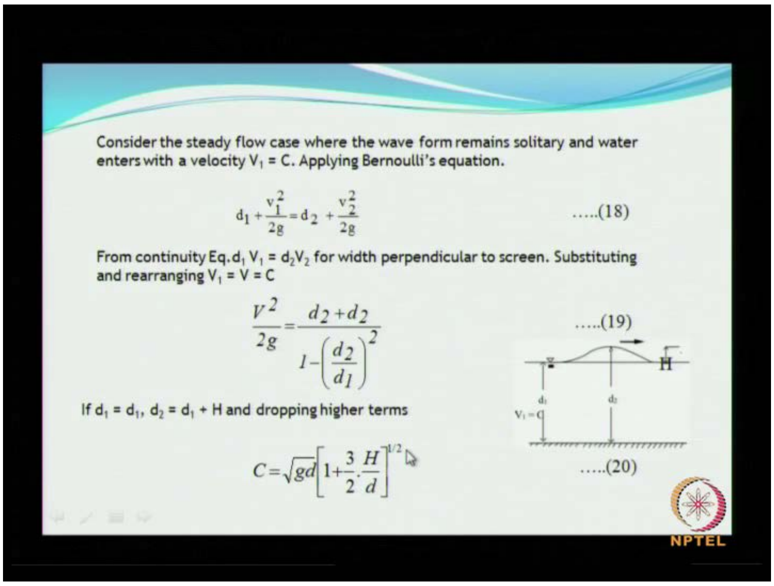

So, considering steady flow case, where the wave form remains solitary and wave enters with a velocity V 1 equal to V C. V 1 equal to C, then you consider these two locations so, this is d 2 and this is d 1, and here V 1 is equal to C. Use the continuity equation as shown here, and then continuity equation is like, applying the Bernoulli Equation as given here, and then use the continuity equation. Form which, I am rearranging V 1 equal to V equal to C, you can obtain an expression as shown here.

But then, d 1 equal to d 1 and whereas d 2 is going to be d 1 plus, this is height of the wave. So, substitute in this equation then you get, an expression for the speed of the Solitary Wave. The speed of the Solitary Wave is nothing, but the Shallow Water Wave Condition, multiplied by this value. You understood? So, if H is large then this component is going to be higher then, you consider just a linear theory propagating in the Shallow water, you understood?

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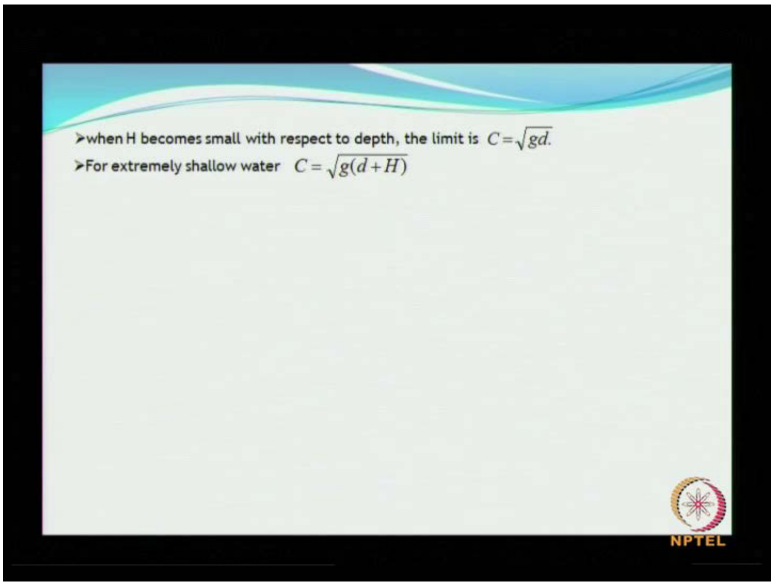

So, when H becomes small with respect to depth the limit, as I have told for extremely Shallow waters, you can also assume it as equal to root of g into d plus H.

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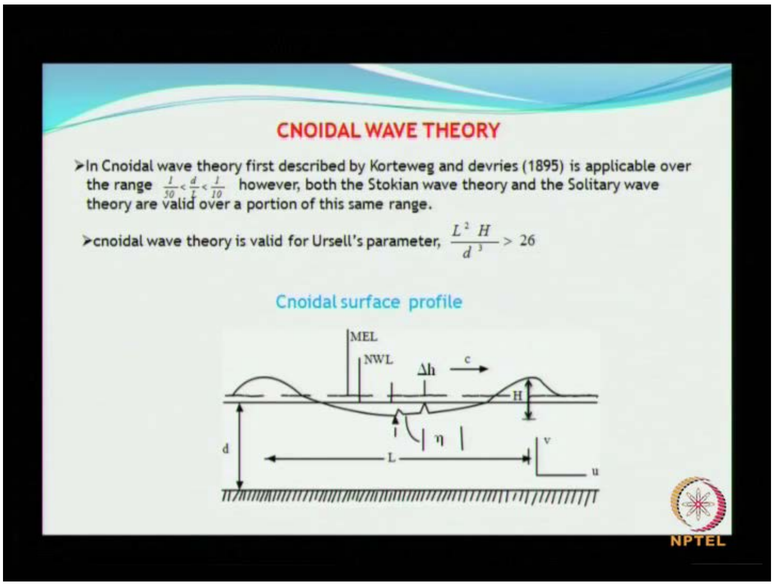

So, that is about Solitary Wave Theory, then we move on to the Cnoidal waves, where in the cnoidal wave? It was first described by Korteweg and Devries in 1895. Applicable over a wide range that is d by L, ranging between 1 by 10 and 150, but however, both Stokian Wave Theory, and the Solitary Wave Theory are valid over. The over a portion of this same range, as we have seen earlier. So, Cnoidal Wave Theory is valid for the

Ursells parameter, greater than 26. In our department, in the department of ocean engineering, our wave maker can generate Solitary Wave Theory.

Solitary wave as well as, Cnoidal waves, leave alone other waves, because other waves any normal wave maker can generate, knowingly or unknowingly or unknowingly you may be generating higher order wave theories. It may be difficult for you to generate a linear wave theory, linear wave whatever, you generate in the lab you assume it, as linear wave which is not the case.

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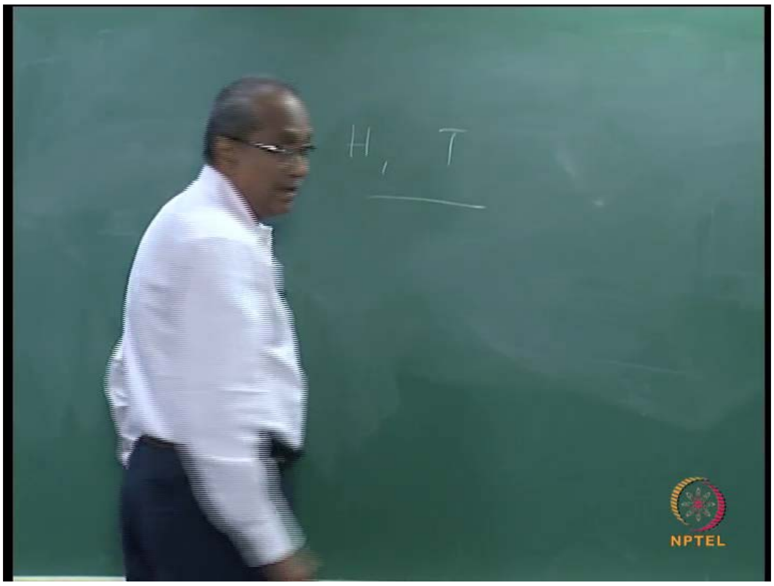

So, unknowingly what I am trying to say is, when you want to use normally fix your wave height and wave period. You do not even bother whether, which theory is going to follow. But in the case of Cnoidal and Solitary, when you want to use your wave maker to generate such waves, all these informations are built in. All these conditions are satisfied only after, which you will see that, such kind of waves are generated in the flume.

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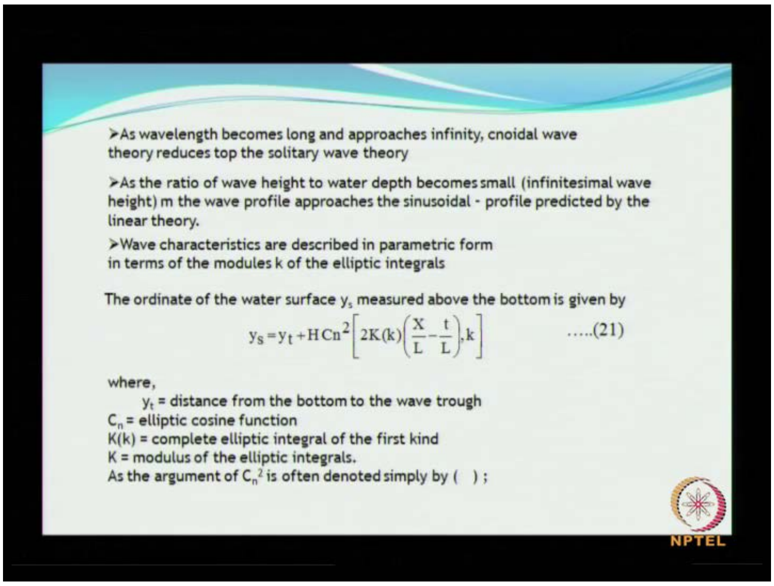

So, the profile looks something like this, it has a very flat, trough and a steep crest, a photograph of, which was already shown to you. As wavelength becomes long and approaches infinity Cnoidal Theory reduces to the Solitary Wave Theory. As the ratio of wave height to water depth become small, infinitised wave. I mean, the wave profile approaches the sinusoidal wave profile, predicted by the linear theory. These are all some of the statements, then wave characteristics are described by what is called as modules k of elliptical interval.

So, the profile is measured from the bottom sea bed and this is the profile, this is the water surface. I mean, the wave profile and this is given by y t, y t is the distance from the bottom to the trough and C n is the elliptical cosin mathematical function and then k is complete elliptical integral of the first kind. k is, k of k is complete elliptical integral of first kind and K is the modules of elliptical integrals as the argument C of n square is often denoted simply by this.

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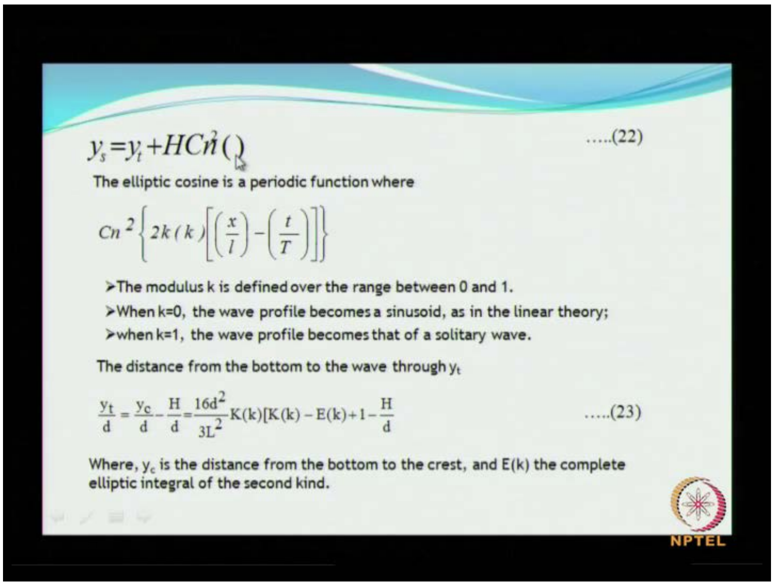

As you can see here, the elliptical cosine function is periodic, where C n is shown as shown here. It is periodic in space and time so, the modules K, small k as we have seen is defined over range between 0 and 1. And when K is equal to small k is equal to 0, the wave profile becomes, linear and in the sinusoidal. And when k is equal to 1, the wave profile becomes that of a Solitary wave. The distance from the bottom to the wave trough y t, this can be obtained by this expression as shown here for details reference refer, I would suggest for this to refer Coastal Engineering Manual.

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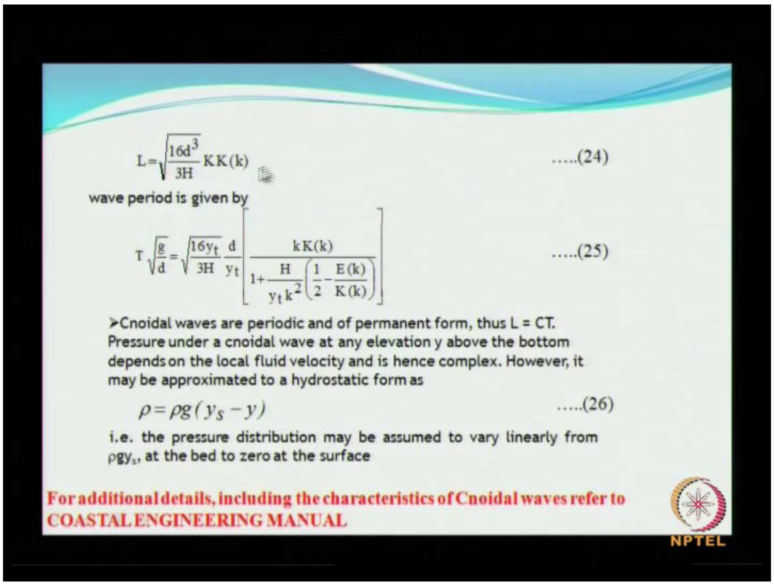

So, the L is calculated as shown here, and these parameters already been explained and the wave period can be given like this. The pressure is provided by this expression, and as I have suggested including the characteristics of the waves, refer Coastal Engineering Manual, or even the book from is also quite good.

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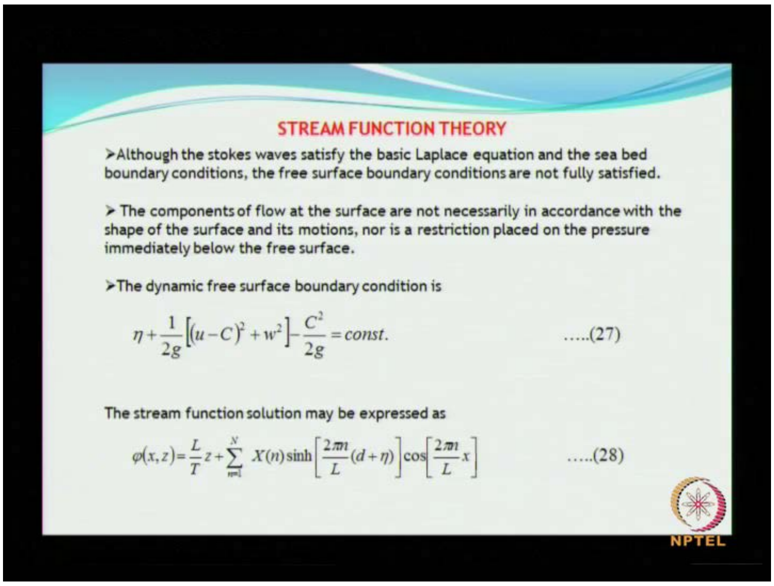

So, that explains the salient features of Cnoidal Wave Theory. Then we move on to Stream Function Theory, although the Stokes waves satisfy the basic laplace equation and the sea bed bottom boundary condition. The free surface boundary condition is not fully satisfied. The free surface boundary condition, I have already explained to you, the components of flow at the surface are not necessarily. In accordance, with the shape of the surface and it is motion, nor is a restriction placed on the pressure immediately below the surface. The dynamic free surface condition, in this case is provided like this, and the stream function solution may be expressed, as given by this expression equation 28.

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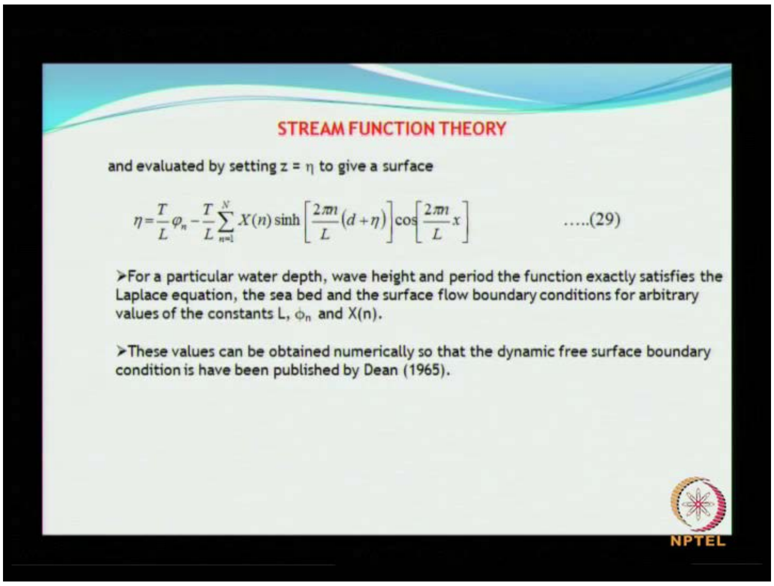

And evaluated at, and evaluating by setting z equal to eta, not z equal to 0, to give a surface as shown here. This is the surface of a stream function that can be described by stream function theory. For a particular water depth, wave height and wave period the function exactly satisfy the laplace equation. The sea bed and the surface flow boundary conditions for arbitrary values of L phi n, that is capital X n. So, these values can be obtained numerically so, that the dynamic free surface boundary condition is can be obtained as published by Dean. So, I would not go into the details of the stream function theory, it is not that widely used (( )) people, there are some cases, where they you need to know the stream function theory.

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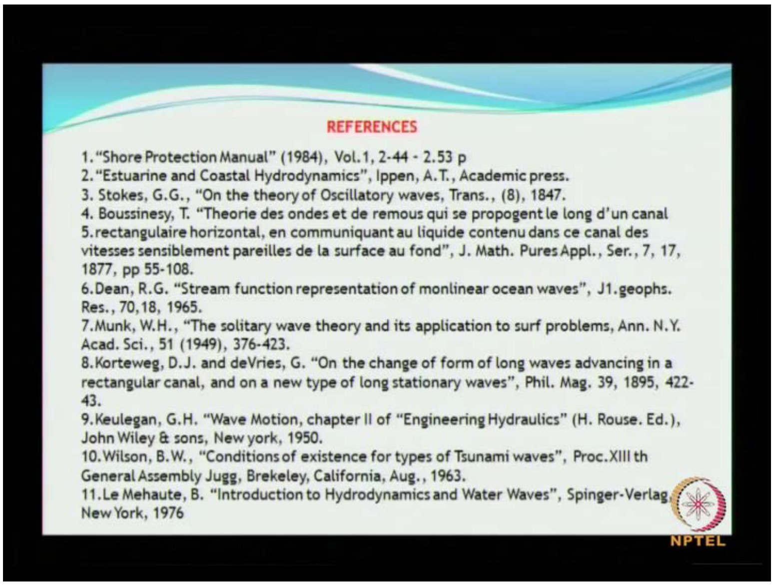

I would, this is a series of references that I have collected and there may be some reason the literature also, but for a class like this. I think it is good, you look at some of these references, in order to brush up and then go for the into your research activity.

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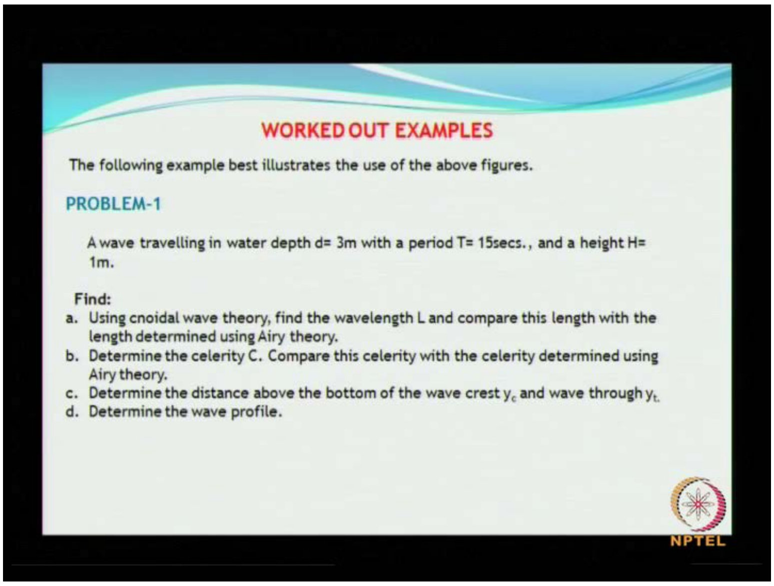

Now, let us continue with a few worked out examples. So, the Cnoidal Wave Theory I will show, how the celerity is calculated the a wave travelling in a water depth of three meters and wave period fifteen seconds and height one meter. Find, using Cnoidal Theory, find wavelength and compare it is, compare this with the wavelength determined by Airys Theory. Determine celerity compare this, with the celerity of the Airys Theory determine the distance between the above, the bottom of the crest y C and trough y t, and determine the wave profile. This is the problem that is, that we need to look into;

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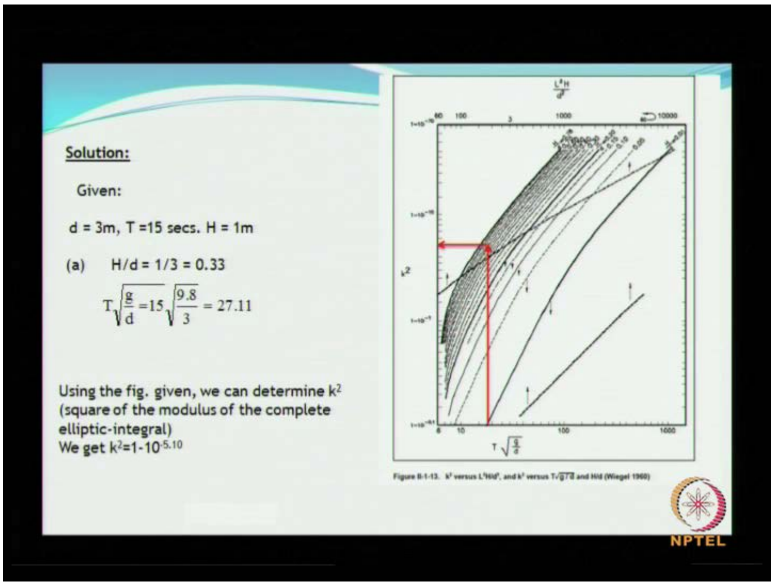

So, these pictures are given available in the coastal engineering manual, or the shore protection manual. So, for the given problem, d is, this muchT so you calculate the value H by d, which is coming to this much, also calculate this parameter T divided by root. I mean, g by d which is working out to 27 point something. So, get into this picture, which shows the determination of square of modules of complete elliptical integral. So, this becomes handy, this nomograms so, get into this picture and get the value of k and we get k as shown here, one minus ten to the power minus 5.1.

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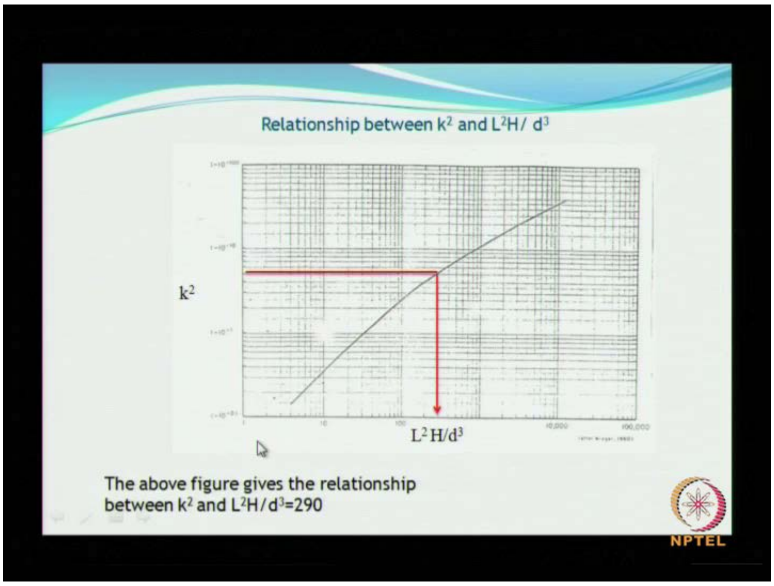

So, this shows the relationship between this k square, which you have determined from the earlier plot to get you the value for L by L square that is nothing, but the Ursells Parameter. And Ursells parameter is working out to 290, naturally it is greater than 26 so, it is following Cnoidal Wave Theory.

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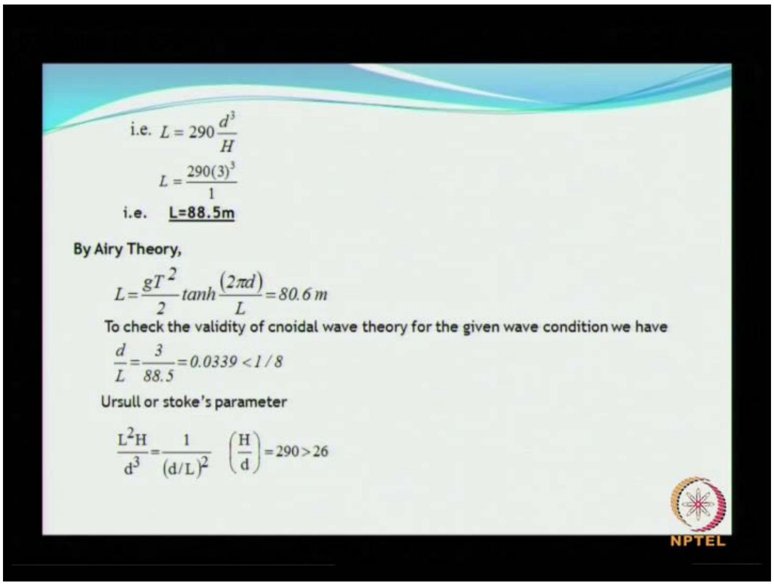

So, once that is known, this L H L square H divided by d cube, you can arrive at the wave length, and once wavelength so, this is the wavelength, and from the linear theory. I do not want to explain to you, how the linear theory is calculated? And from the linear theory, you can calculate this as 80.6. So, that is the kind of difference, you have between the wavelengths to check the validity of linear theory, you have already done. So, d by L and this one, I have already explained about the Ursells Parameter, you need not go into that.

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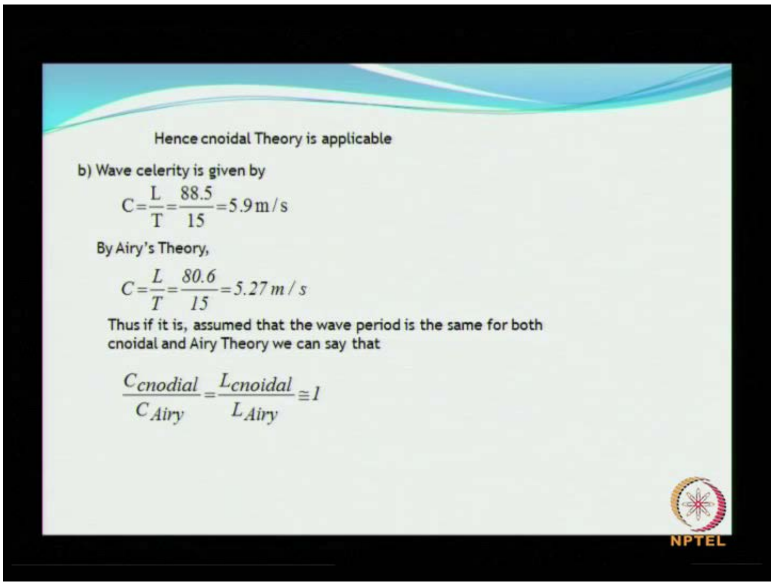

So, once you have calculated your L is quite straight forward, to calculate the corresponding celebrities. So, in this particular case that is not much of difference between the celebrities, and thus if it assumed that, the wave period is the same for both the Cnoidal, and we can say that, it is equal to 1. But that is not correct, there is still a difference between the Cnoiadal and this one.

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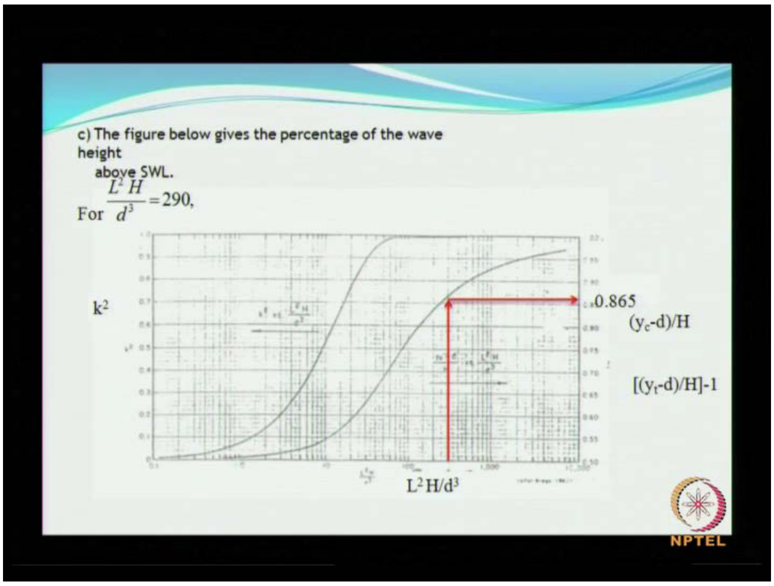

The last one is, about the percentage of crest and trough, so you have already selected this and then you have to go and find out this value, so this is y C the portion of the location of the crest.

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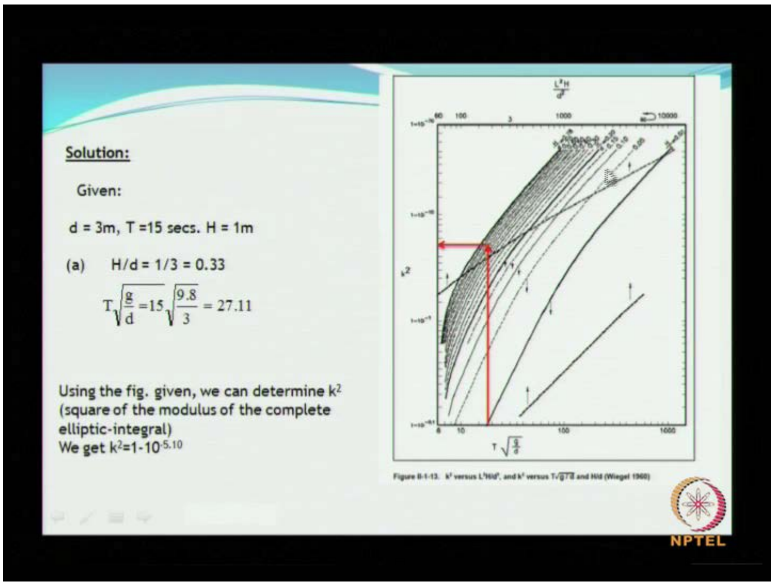

So, in this particular picture, I forgot to tell you, that you need to consider this H by d while selecting this point so H by d here is 0.3, and 0.3 is somewhere here.

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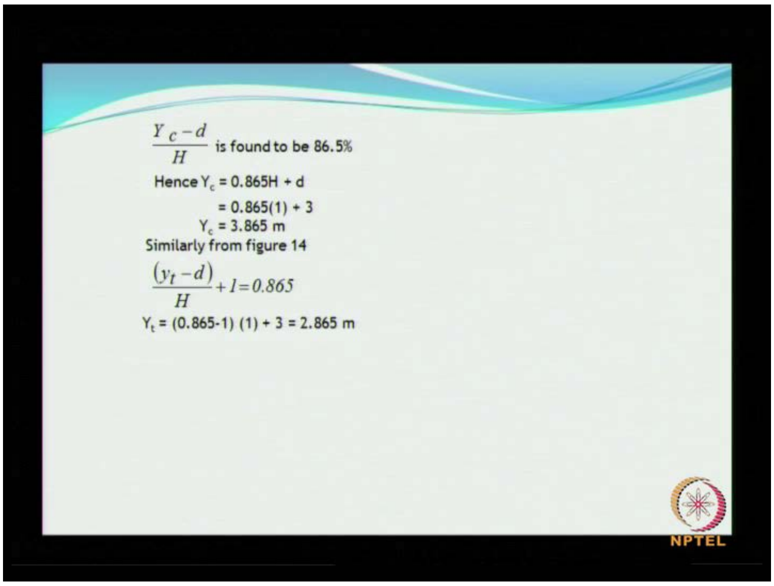

So, from this picture you can arrive at y C and y t.

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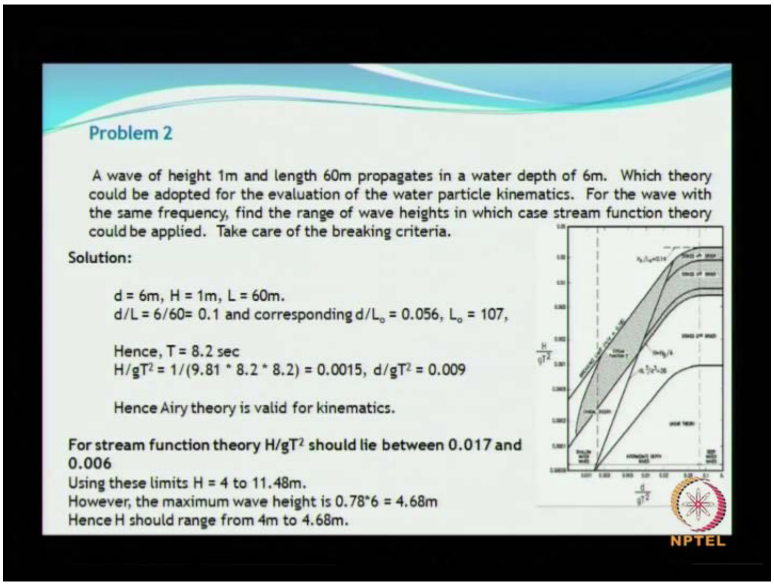

So, we will move on to this second example, a wave height is given as 1 meter, wavelength is given as 60 meters. It is propagating, the wave is propagating in a water depth of 6 meters, which theory could be adopted for the evaluation of water particle kinematics. For the wave, with the same frequency find the range of wave heights in which the Stream Function Theory could be adopted, take care of the breaking criteria. So, the problem is quite simple, only you use this, the problem looks as if it is not difficult, it is a very simple problem. The only thing is, this is a kind of an exercise to make yourself familiar with this graph that is all, which very often students forget.

So, wave height is given wave, water depth is given so you calculate your d by L corresponding d by L naught. All these things are known to you H by dt square also can be calculated. So, for this picture you need H by dt square and d by g T square so since wavelength is given to you normally, which is not the case, normally they give directly the wave period, but in this problem, you are given wavelength. So, you use the dispersion relationship and then, and you get the wave period. So, you calculate the wavelength and then calculate your wave period, and then you get these two pictures.

And then for the stream function theory, this one should lie between 0.17 and 0.017 and 0.0 0 6. Unfortunately this picture is not so clear, so you have to take the corresponding this value, and the corresponding this and using these values for the stream function theory. So, in this case, for these two parameters Airys Theory is valid, using this picture. For stream function H T should be lying in between this value, that is H by; So, the stream function theory is somewhere, here this is the region where the stream function theory to should follow the values.

So, using these limits, H will be ranging between four and eleven meters, but you know that the maximum wave height cannot be greater than 0.78 times water depth. That means, here water depth is 6 meters, so the water depth, the wave height cannot (( )) greater than 4.8 meters. So, the range of wave heights that could be there, is between four meters and 4.68 meters, is that clear? So, this is just an example, to show you, how you can use this unfortunately the picture is not so clear, because it is quite a big picture. So, you need to do this and probably use the hand out, in case if you want to understand better.

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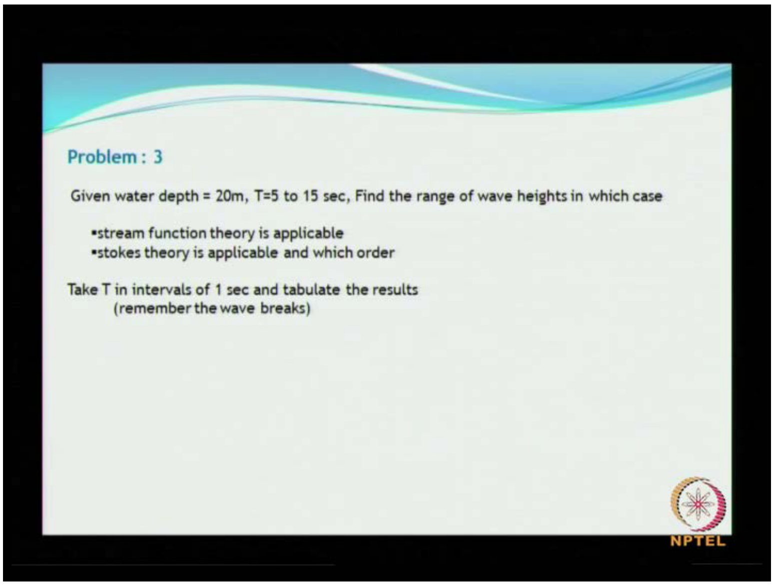

Given water depth to find the ranges of wave heights in which case Stream Function Theory is applicable, Stokes is applicable and which order. So, you take a wave period in intervals of one second and tabulate the results, remember again the wave breaks.

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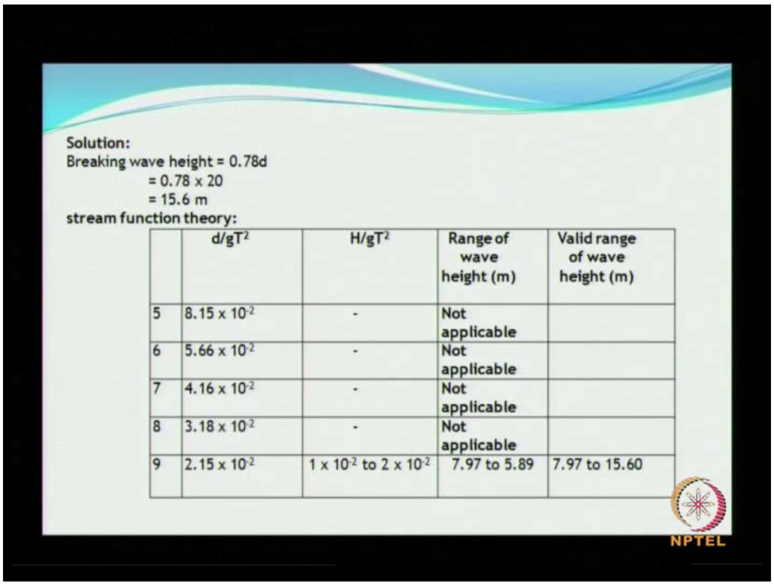

So, what we have done is so here given on the first column, I have given the wave heights, wave periods in intervals of one. Calculated the values of d by g T square, calculated the values of H by g T square and then the range taking into considering the breaking criteria and then range of wave heights, so this is not applicable. Because the wave is going to beak then the range of wave heights, that could be possible for the Stream Function Theory, is that clear? So, use that picture earlier problem, we had that small nomogram, that nomogram need to be used.

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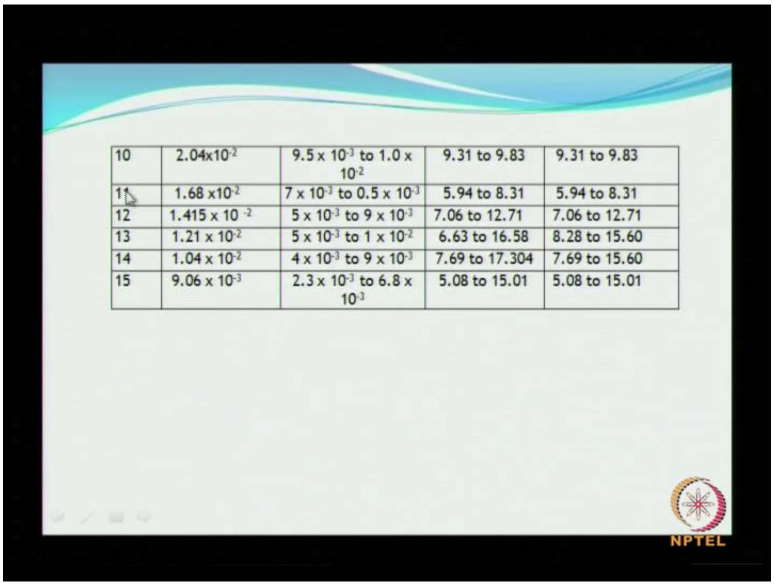

10	2.04×10^{-2}	9.5×10^{-3} to 1.0×10^{-2}	9.31 to 9.83	9.31 to 9.83
11	1.68×10^{-2}	7×10^{-3} to 0.5×10^{-3}	5.94 to 8.31	5.94 to 8.31
12	1.415×10^{-2}	5×10^{-3} to 9×10^{-3}	7.06 to 12.71	7.06 to 12.71
13	1.21×10^{-2}	5×10^{-3} to 1×10^{-2}	6.63 to 16.58	8.28 to 15.60
14	1.04×10^{-2}	4×10^{-3} to 9×10^{-3}	7.69 to 17.304	7.69 to 15.60
15	9.06×10^{-3}	2.3×10^{-3} to 6.8×10^{-3}	5.08 to 15.01	5.08 to 15.01

So, up to 15 seconds we have given the variation of wave heights.

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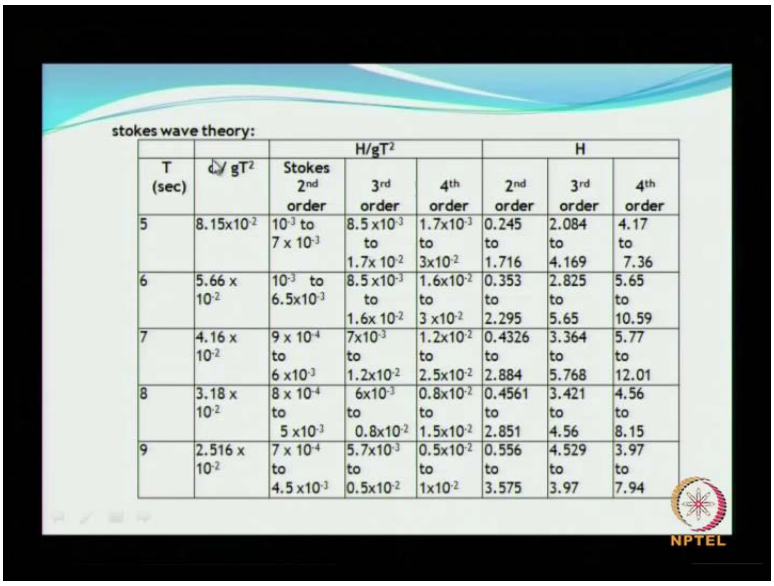

stokes wave theory:

		H/gT^2			H		
T (sec)	d/gT^2	Stokes 2nd order	3rd order	4th order	2nd order	3rd order	4th order
5	8.15×10^{-2}	10^{-3} to 7×10^{-3}	8.5×10^{-3} to 1.7×10^{-2}	1.7×10^{-3} to 3×10^{-2}	0.245 to 1.716	2.084 to 4.169	4.17 to 7.36
6	5.66×10^{-2}	10^{-3} to 6.5×10^{-3}	8.5×10^{-3} to 1.6×10^{-2}	1.6×10^{-2} to 3×10^{-2}	0.353 to 2.295	2.825 to 5.65	5.65 to 10.59
7	4.16×10^{-2}	9×10^{-4} to 6×10^{-3}	7×10^{-3} to 1.2×10^{-2}	1.2×10^{-2} to 2.5×10^{-2}	0.4326 to 2.884	3.364 to 5.768	5.77 to 12.01
8	3.18×10^{-2}	8×10^{-4} to 5×10^{-3}	6×10^{-3} to 0.8×10^{-2}	0.8×10^{-2} to 1.5×10^{-2}	0.4561 to 2.851	3.421 to 4.56	4.56 to 8.15
9	2.516×10^{-2}	7×10^{-4} to 4.5×10^{-3}	5.7×10^{-3} to 0.5×10^{-2}	0.5×10^{-2} to 1×10^{-2}	0.556 to 3.575	4.529 to 3.97	3.97 to 7.94

And similarly, Stokes Wave Theory I have worked out for the different, starting from second, third, fourth Order Theory, calculated the H by g T square. This also is calculated, this calculated then enter into the picture and enter into the nomogram and get the values of the wave height.

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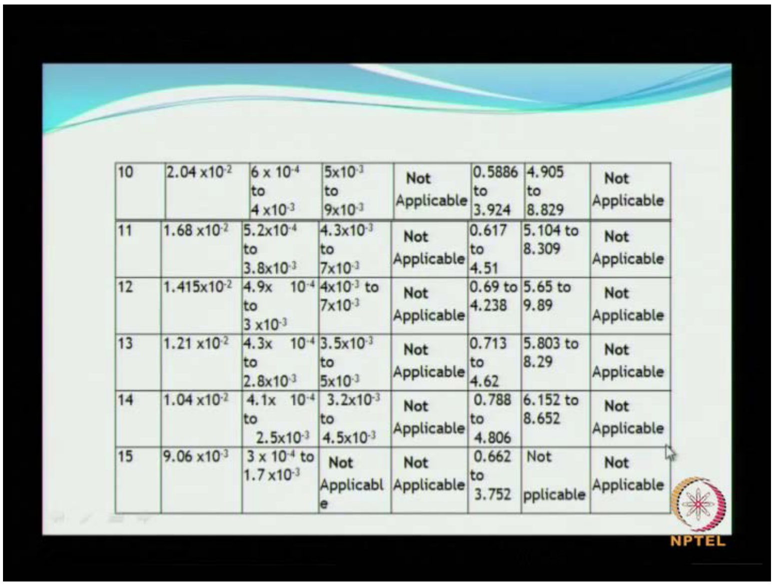

10	2.04×10^{-2}	6×10^{-4} to 4×10^{-3}	5×10^{-3} to 9×10^{-3}	Not Applicable	0.5886 to 3.924	4.905 to 8.829	Not Applicable
11	1.68×10^{-2}	5.2×10^{-4} to 3.8×10^{-3}	4.3×10^{-3} to 7×10^{-3}	Not Applicable	0.617 to 4.51	5.104 to 8.309	Not Applicable
12	1.415×10^{-2}	4.9×10^{-4} to 3×10^{-3}	4×10^{-3} to 7×10^{-3}	Not Applicable	0.69 to 4.238	5.65 to 9.89	Not Applicable
13	1.21×10^{-2}	4.3×10^{-4} to 2.8×10^{-3}	3.5×10^{-3} to 5×10^{-3}	Not Applicable	0.713 to 4.62	5.803 to 8.29	Not Applicable
14	1.04×10^{-2}	4.1×10^{-4} to 2.5×10^{-3}	3.2×10^{-3} to 4.5×10^{-3}	Not Applicable	0.788 to 4.806	6.152 to 8.652	Not Applicable
15	9.06×10^{-3}	3×10^{-4} to 1.7×10^{-3}	Not Applicabl e	Not Applicable	0.662 to 3.752	Not pplicable	Not Applicable

The range of wave height that could possibly cover the different Stokes area waves so with this, I think we have completed the Finite Amplitude Wave Theories. So, I hope this would be useful to you as a beginning so that based on this kind of information, you can just proceed further, by reading additional books.