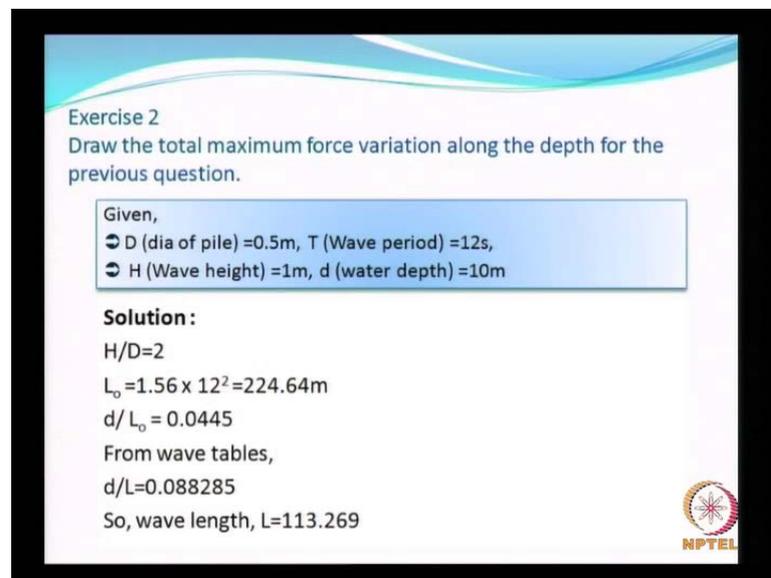


**Wave Hydro Dynamics**  
**Prof. V. Sundar**  
**Department of Ocean Engineering**  
**Indian Institute of Technology, Madras**

**Module No. # 05**  
**Wave Loads on Structures**  
**Lecture No. # 04**  
**Wave Loads on Structures and Problems II**

(Refer Slide Time: 00:11)



Exercise 2  
Draw the total maximum force variation along the depth for the previous question.

Given,

- ↻ D (dia of pile) =0.5m, T (Wave period) =12s,
- ↻ H (Wave height) =1m, d (water depth) =10m

**Solution :**

$H/D=2$

$L_0 = 1.56 \times 12^2 = 224.64\text{m}$

$d/L_0 = 0.0445$

From wave tables,

$d/L = 0.088285$

So, wave length,  $L = 113.269$



We need to draw the total maximum force variation along the depth for the previous question.

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$D = 0.5\text{m}$   
 $T = 12\text{s}$   
 $H = 1\text{m}$   
 $d = 10\text{m}$

$F = (F)$

NPTEL

For the previous question  $D$  is given to us 0.5 meters  $T$  is 12 seconds wave height equal to 1 meter, and  $D$  equal to 10 meters. You need to follow the other all these things already have been explained in the previous problem. So, we need not to go through the same thing again and again.

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$\theta = 90^\circ \text{ and } 180^\circ$

$\eta(t)$

NPTEL

So, what we need, what we saw in the previous problem was a phase variation of the total force and this maximum force was occurring somewhere between theta equal to 90 and 180; whereas the drag force was occurring at a phase angle of theta equal to 90

degrees and inertia force was occurring at maximum inertia force was occurring at a phase angle of 180 degrees, but the total force was occurring in between, the maximum force was occurring in between 90 and 180 degree.

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**To determine the maximum total wave force**

$$\theta = \theta_{\max} = \cos^{-1} \left[ \frac{-\pi D C_m}{H C_D} \left( \frac{2 \sinh^2 kd}{\sinh 2kd + 2kd} \right) \right]$$

$$\theta_{\max} = \cos^{-1}(-0.40846) = 114.108^\circ$$

Substituting this value for  $\theta$  in the expression obtained for the total force at  $z=0$

$$F_T = -C_m \rho \frac{\pi D^3}{4} \frac{2\pi^2}{T^2} \left( \frac{\cos \theta}{k} \right) + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta|}{\sinh^2 kd} \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right] + \frac{1}{2} C_D \rho D d |U| U$$

U, current is zero  
Hence, we get,  $F_{T\max} = 129.1 \text{ N/m}$ .



We have already done the derivation or the calculation of determining the phase angle at which the total force will be an maximum. Again get back to the lecture material on wave forces, and that will clearly show that the equation is as given here. So, for the present problem, you have the values of D H C M C D already evaluated, and then kd also known to us, so I can calculate the phase handle at which the maximum force will be total maximum force will occur.

So, whenever you are refer to maximum force its always good to be very precise. That you are talking about total maximum force, particularly when we are dealing with Morison regime, its always better to indicate that you are talking about maximum total force total or even it is auto automatically understood, it always better to stress that talking about maximum total force. So substituting this value of theta in the expression for the total force, you know the total force expression which we have seen earlier.

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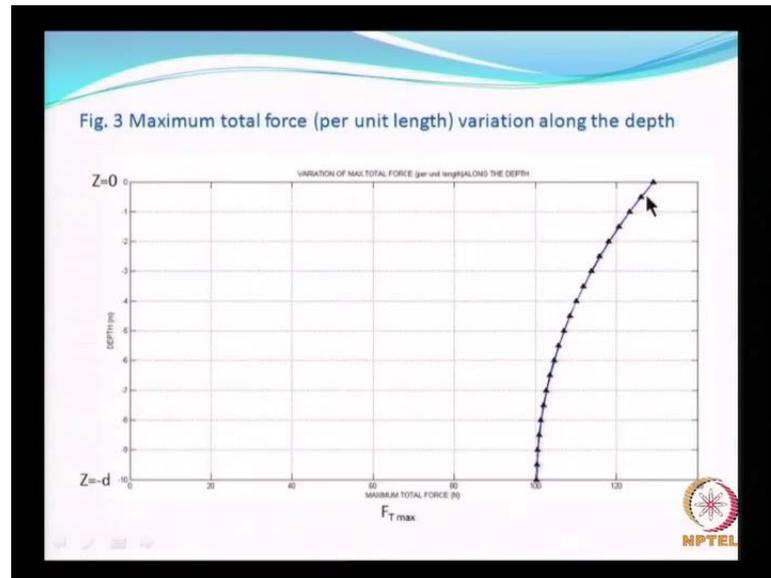
$$u = R \sin \theta$$
$$F_D = (R_1) R \sin \theta \sin \theta$$
$$F_I = 0$$

$Z=00$

So the only thing is using the same thing have we have already determined  $R_1$  and the  $Q_1$  so add these two and uses this theta value of 114 degrees. So, I use the more the other expressions, which also includes the force due to current, because later it becomes handy for me to show how the current effect is included; and what effect does the current has on the total force variation that is the reason. So as of now, this has been derived from the lecture material. You can have a look at the lecture material; and this as of now current is 0, so this term will not appear now, will not be considered so, that will be taken as 0 and hence your total maximum force will be as shown here 5 meters.

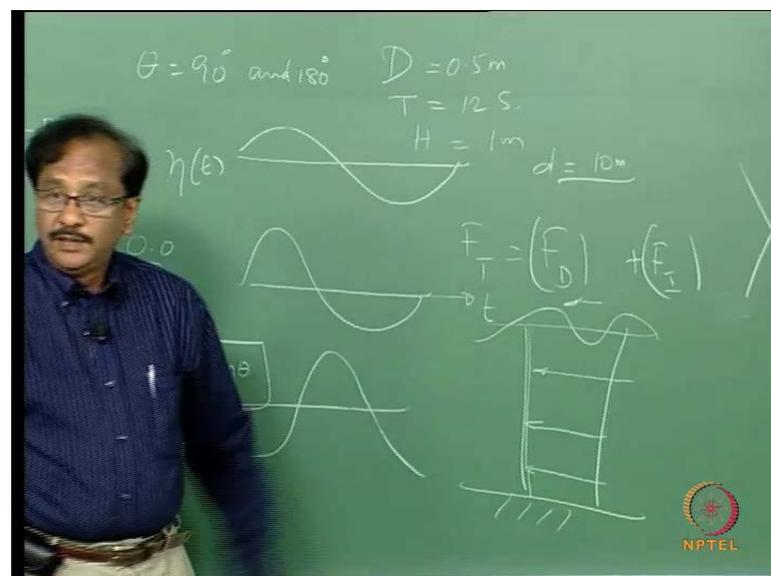
So this total force maximum, total force is computed at a given elevation and that elevation is at still water. So, you need to repeat these calculations at different elevations. There is an expression just prior to this, which gives you the expression for  $Z$ ; this expression does directly for  $Z$  equal to 0. Look at the original expression, the original expression will be nothing but adding this two and having theta replaced by that value that will be the original expression varying effect of  $Z$  will be coming into the picture.

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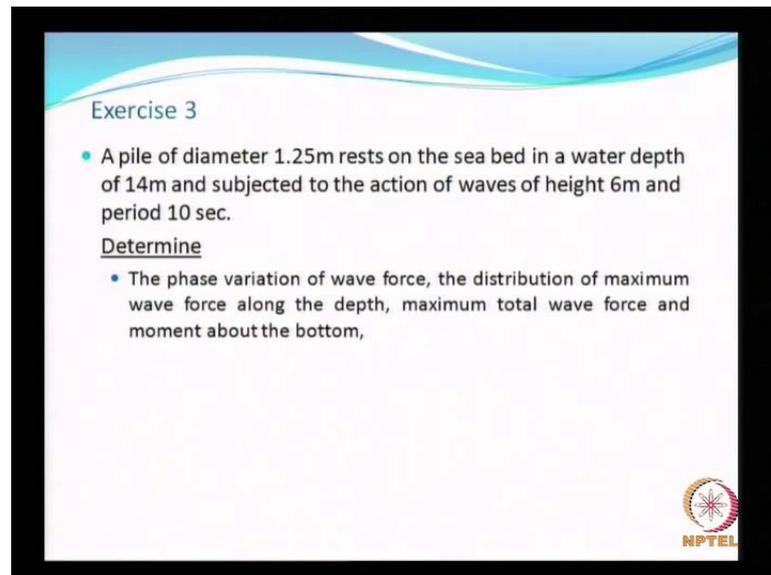
So, at Z equal to 0 you we have computed and similarly, you follow the same procedure for the different elevations, and then you land up with the variation of the total maximum force along the depth. So, how does the force vary? As a hyperbolic function. So, usually you ask this question when you go for interview, they will ask this question how does the wave force vary. So, remember this problem and also the expression, so you will be in a position to clearly say that the force is a hyperbolic variation. How does the velocity varying? Horizontal velocity varying; when do you say, the drag component is maximum? When do you say that inertia component is maximum?

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So, there are certain questions like this. Can we add the drag force maximum and inertia force maximum to get total force maximum? Without thinking, unfortunately the students will say yes? So, this is in order to help you people to know what's going on, when you are dealing with the simple equation which is the Morison equation.

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Exercise 3

- A pile of diameter 1.25m rests on the sea bed in a water depth of 14m and subjected to the action of waves of height 6m and period 10 sec.

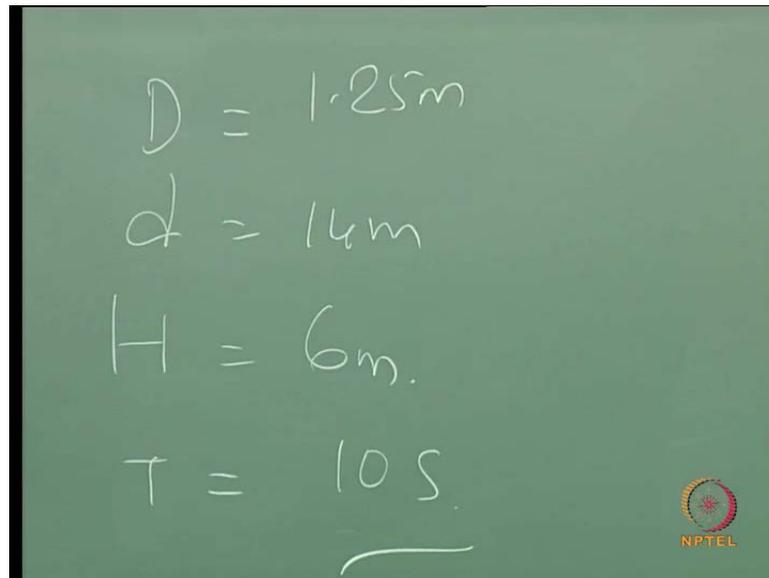
Determine

- The phase variation of wave force, the distribution of maximum wave force along the depth, maximum total wave force and moment about the bottom,



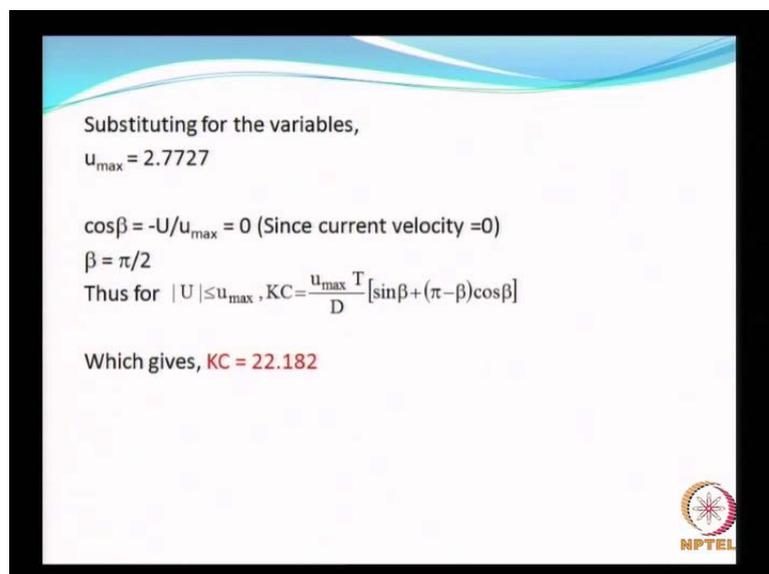
I hope you people are comfortable, so shall we take one more problem, know dia is 1.25 meters, when you are looking into problems also try to have the feelings for the dimensions. What should be the dimensions, so many times students do not have feeling for the physical dimensions of different kinds of structures. Again go back to the lectures wave forces, where in there is a figure, showing different components of cylindrical structures used in different of source structure and what are the approximate sizes, we are talking about. And also the range of wave heights, the range of wave periods, which we deal in the marine environment for design of structures.

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$$D = 1.25\text{m}$$
$$d = 14\text{m}$$
$$H = 6\text{m}$$
$$T = 10\text{s}$$

So here, you see that the pile diameter is 1.25 meters water depth is 14 meters and subjected to the action of wave 6 meters and period 10 seconds. I am sure that you can do this problem having exposed to earlier two problems. What you have added here is, the total wave force and moment about the bottom. When looking at this problem please refer into the lecture material or the for expressions for your moment wave force etcetera; how it has been derived and how it has to be used here. So, you need to calculate all these things, so  $d$  by  $l$  not is known to us so  $d$  by  $l$  all these things are same  $u_{\max}$  is estimated.

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Substituting for the variables,  
 $u_{\max} = 2.7727$

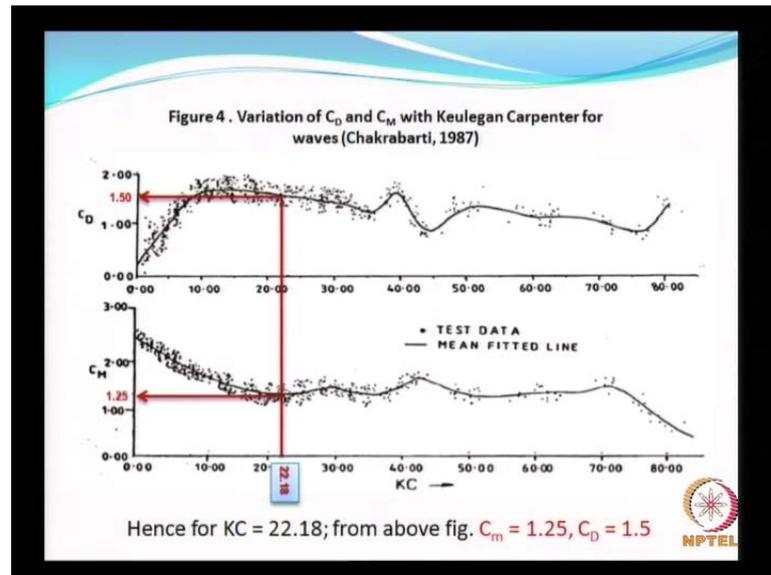
$\cos\beta = -U/u_{\max} = 0$  (Since current velocity = 0)  
 $\beta = \pi/2$

Thus for  $|U| \leq u_{\max}$ ,  $KC = \frac{u_{\max} T}{D} [\sin\beta + (\pi - \beta)\cos\beta]$

Which gives,  $KC = 22.182$

So, then I am using here, the other example refer to the material given to us. So, earlier I have defined the KC as this much that is the expression given by here, and cos beta is shown here. But this will not have any use here, because it will not come here. In this case if you do not consider the current. So using this u max already calculated. So, I will have T is already known and the D is also known, So, you have the KC number as 22.

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And using the same equation, I am in that plot you get here the C M as 1.25 and C D as 1.5

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The sectional wave force is given by

$$dF = \left( C_m \rho \frac{\pi D^2}{4} \dot{u} + \frac{1}{2} C_D \rho D |u| \dot{u} \right) dz$$

Substituting for u and  $\dot{u}$

$$dF = C_m \rho \frac{\pi D^2}{4} \left( \frac{-2\pi^2 H \cosh k(z+d)}{T^2 \sinh kd} \cos \theta \right) dz + \frac{1}{2} C_D \rho D \left( \frac{\pi^2 H^2 \sin \theta |\sin \theta| \cos^2 h k(z+d)}{T^2 \sinh^2 kd} \right) dz$$

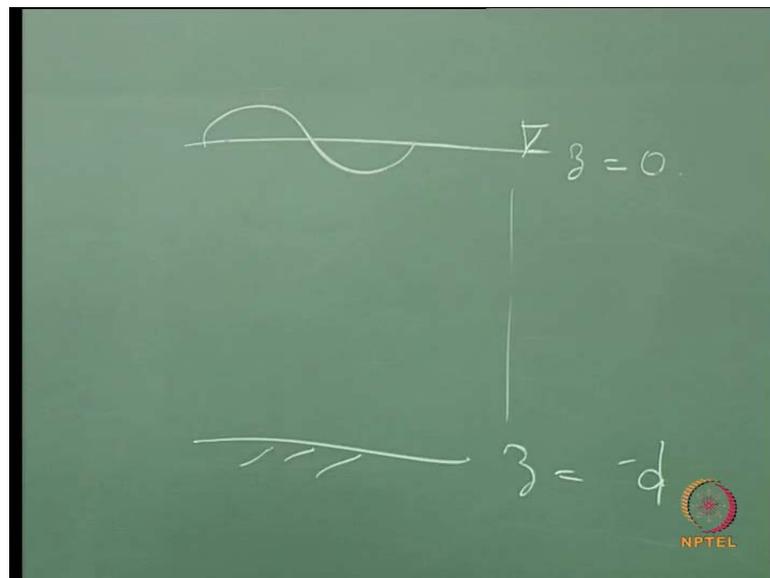
The total wave force is determined by integrating the above expression from seabed to SWL (-d to 0). The resulting expression is given as

$$F_T = -C_m \rho \frac{\pi D^2}{4} \frac{2\pi^2}{T^2} \left( \frac{\cos \theta}{k} \right) + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta|}{\sinh^2 kd} \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right] + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta|}{\sinh^2 kd} \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right]$$

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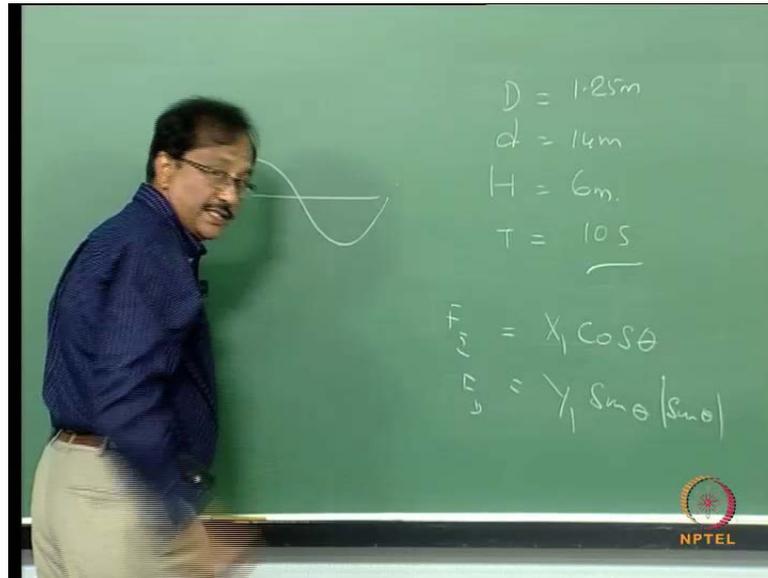
So, once this is known what is the sectional force for any elevation?. This one and then  $dF$ , this will be the sectional force and the total force by integrating from the sea bed up to the free surface after the still waterline is given by this expression, this is what we have appeared earlier when we are calculating. The variation of the total force variation of total force, that is the integration has been done already. Here you have this  $Z$  when I want to have total force, I have integrated the total force from minus  $d$  that is at the still water from the seabed.

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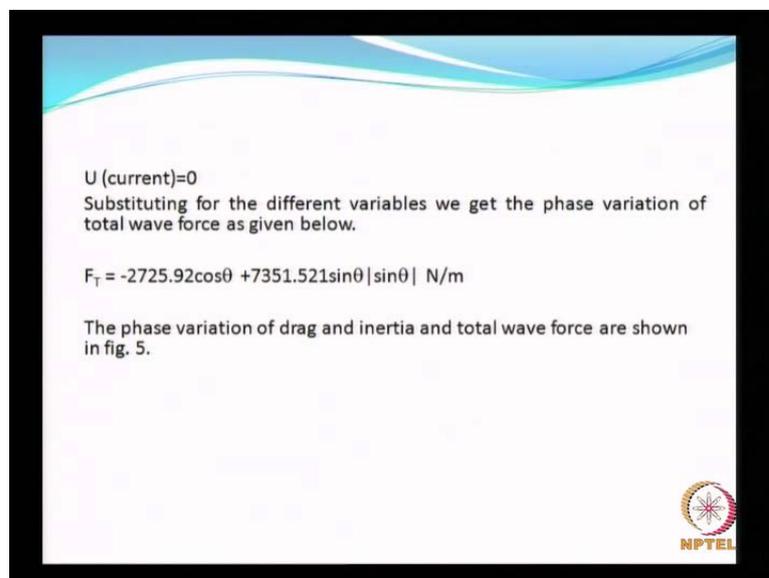
So, I have integrated the wave force variation and that is the expression here.

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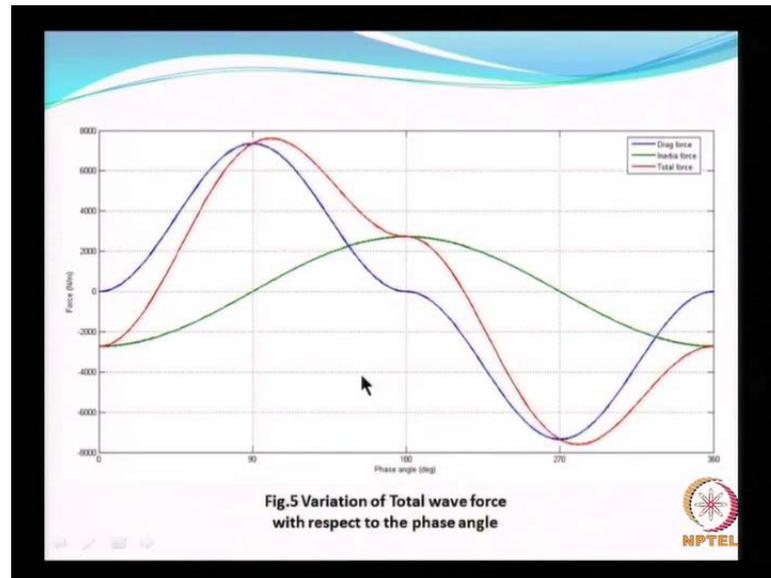
But so this expression will again we will have say may be  $X_1$  into  $\cos \theta$  and may be  $Y_1$  into  $\sin \theta$  this is inertia this is drag that is the total force not total maximum force. So this total force can be something like this.

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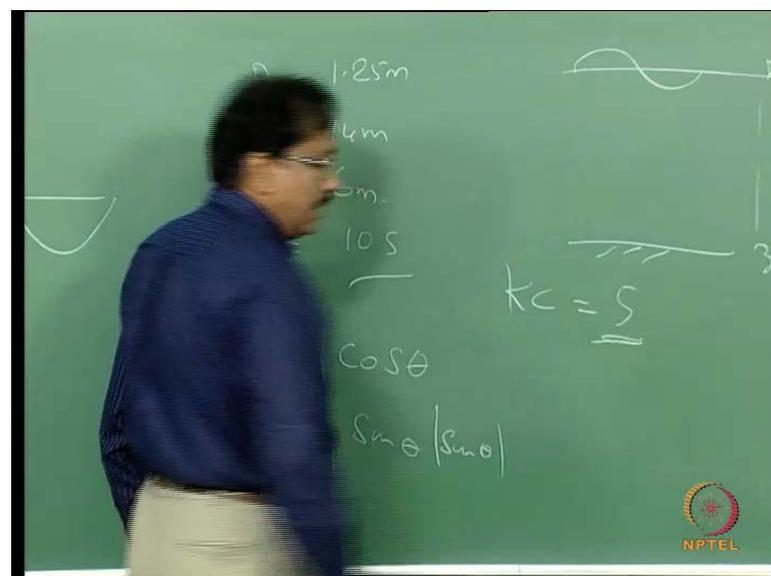
Let see, how this force variations look. So, anyway we are not using the current because current is 0 here. So, these values all can be extracted, all can be substituted, and then you get the variation of the total force of a function of phase as given here

(Refer Slide Time: 13:45)



So the phase variation, of total force are shown here. What does this show, the blue one is the drag force. Note what was the Keulegan Carpenter number it was twenty two. So, which means the drag component will also be present it will quite sizable, comparable to the initial force. So here is the initial force and here is the blue one is the drag force and the red one is the total force. We call these two components, which are dominating here, it is the drag force which is dominating here, it is contributing more towards to the total force. The same variation, if you draw for a lesser value of KC number, say may be value for the different parameters or different variables or given as something else.

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Is that clear and also look at the phase. So all this phase will be somewhere here the total maximum force.

(Refer Slide Time: 16:15)

**To determine the maximum total wave force**

$$\theta = \theta_{\max} = \cos^{-1} \left[ \frac{-\pi D C_m}{H C_D} \left( \frac{2 \sinh^2 kd}{\sinh 2kd + 2kd} \right) \right]$$

$$\theta_{\max} = \cos^{-1}(-0.2239) = 102.94^\circ$$

Substituting this value for  $\theta$  in the expression obtained for the total force

$$F_T = -C_m \rho \frac{\pi D^2}{4} \frac{2\pi^2}{T^2} \left( \frac{\cos \theta}{k} \right) + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta|}{\sinh^2 kd} \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right] + \frac{1}{2} C_D \rho D d |U| U$$

We get,  $F_{T\max} = 7593.295 \text{ N/m}$



So, what we have seen is the total force variation, but now we need the maximum total force. So when you want to have the maximum total force, you have to determine the theta max. So, you calculate the phase at which the maximum force occurs, for the values of the variables given here you get a value of about 1.102 degrees substitute in this expression. This is actually the phase variation expression. This expression gives phase variation of the total force. So, once you have substitute that will be a total force.

(Refer Slide Time: 17:15)

The overturning moment about the SWL is given by,

$$M = \rho D \frac{\pi^2 H}{2T^2} \frac{1}{k^2 \sinh kd} \left[ -C_m \pi D \cos \theta (1 - \cosh kd) + C_D \frac{\sin \theta |\sin \theta|}{8 \sinh kd} H (1 - \cosh 2kd - 2k^2 d^2) \right]$$

Substituting, we get;  $M_{T_{\max}} = -46082.29 \text{ N-m}$



Then move on to the overturning moment, about the still water line is given by this expression look at the derivation in the lecture material and we will use all these values, you will get the total maximum moment as something like 46 Newton meter. So, you have the total moment, you have the total maximum force and total maximum moments.

(Refer Slide Time: 17:56)

Position of maximum total force from the SWL  
 $= M_{T_{\max}} / F_{T_{\max}} = -6.0688 \text{ m}$

Position of maximum total force from the sea bottom  
 $= -14 - (-6.0688) = 7.93 \text{ m.}$



So, you can easily calculate your **(( ))** that is the point of application of the maximum force. So the position of the maximum total force from the still water will be maximum

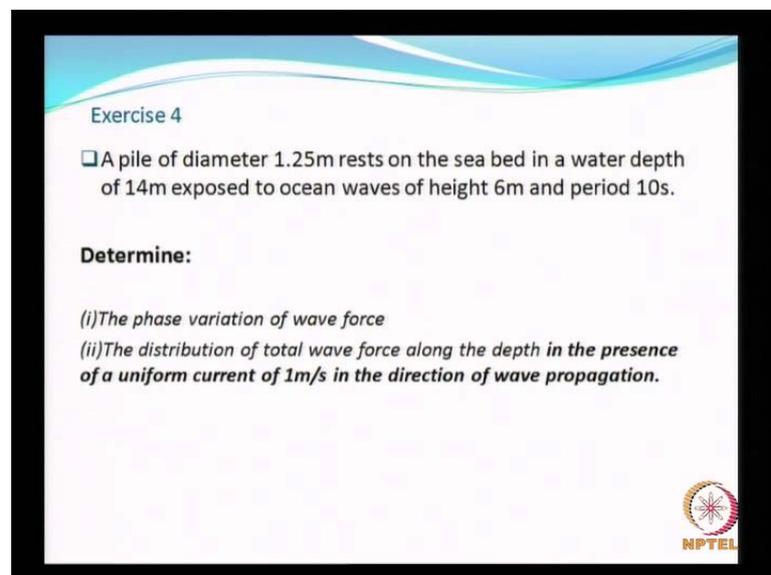
moment divided by the total force and that will be equal to 6 point some meters that is 6 meters below the still water line or 7.9 meters above the sea level.

So, what we have done so far, we looked at the phase variation of the drag force. We look at the phase variation of the inertia force. We look at the phase variation of the total force at each of the elevations and then the next problem was to look at the variation of the maximum force at different elevations by using the seta max. So, you get the hyperbolic variation which you have seen, then we went on to calculate our maximum moment or maximum total force in order to obtain the this what we he done.

With these three exercise I am sure that you should be in a position to solve similar problems. As an exercise you could vary the variables and such a way you use ,you get a smaller KC number a larger KC number and look at the variations of the drag and inertia component and just check whatever we have discussed in the class holds good.

I think that is all you can understand the basic physics lying in the waves and wave forces. So, we have seen three problem on the variation of wave forces, which is quite good enough and I am sure that you have understood, the all the three problems.

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The slide is titled "Exercise 4" and contains a problem statement and two sub-questions. The problem statement is: "A pile of diameter 1.25m rests on the sea bed in a water depth of 14m exposed to ocean waves of height 6m and period 10s." The sub-questions are: "(i) The phase variation of wave force" and "(ii) The distribution of total wave force along the depth in the presence of a uniform current of 1m/s in the direction of wave propagation." The slide also features the NPTEL logo in the bottom right corner.

**Exercise 4**

□ A pile of diameter 1.25m rests on the sea bed in a water depth of 14m exposed to ocean waves of height 6m and period 10s.

**Determine:**

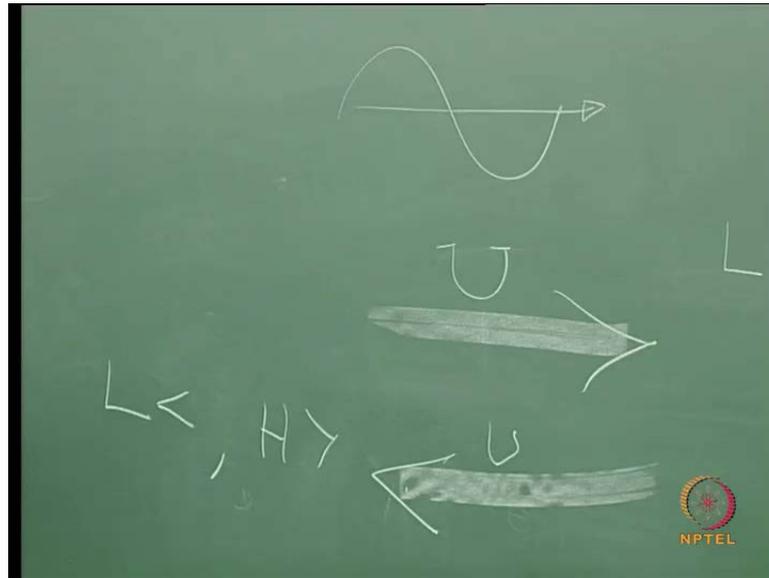
(i) *The phase variation of wave force*

(ii) *The distribution of total wave force along the depth in the presence of a uniform current of 1m/s in the direction of wave propagation.*



So, now I will go into the effect of currents on waves. Please recollect my lecture on the characteristics when of the waves, when propagates over current.

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When wave moves over a current in the same direction, then your wave length will increase and wave height will decrease. On the other hand, when you have current in this direction in the opposite direction, wave length will decrease and wave height will increase.

Please have this in mind, when we are looking at the problem solving, it is better to have a kind of an understanding of the physics behind the wave current or the waves alone. So that it is easy for you to check the values, when you are getting the value, which is similar to what suppose to happen, then you are confident that the problem you have solved is at least that it answer the physics.

So, here again we are calculating the phase variation of the wave force. The distribution of the total wave force, along the depths, in the presence of a uniform current, one meter per second in the direction of wave propagation. See here we have considered a uniform current when you consider a uniform current that means you are slightly over estimating the wave force, I mean the force on the structure.

So, you can think of assuming one formula or you can have the variation of current over the depth, that variation itself can be included or incorporate in the expression in order to arrive at more realistic values. If you do not have any idea about, how the current is varying, then the one solution is, you have to assume this kind of variation.

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Mean surface current = 1.0 m/sec (in direction of waves)

Assume uniform current over the entire depth.  
 $U = 1\text{m/sec}$  and  $u_{\max} = 2.7693$

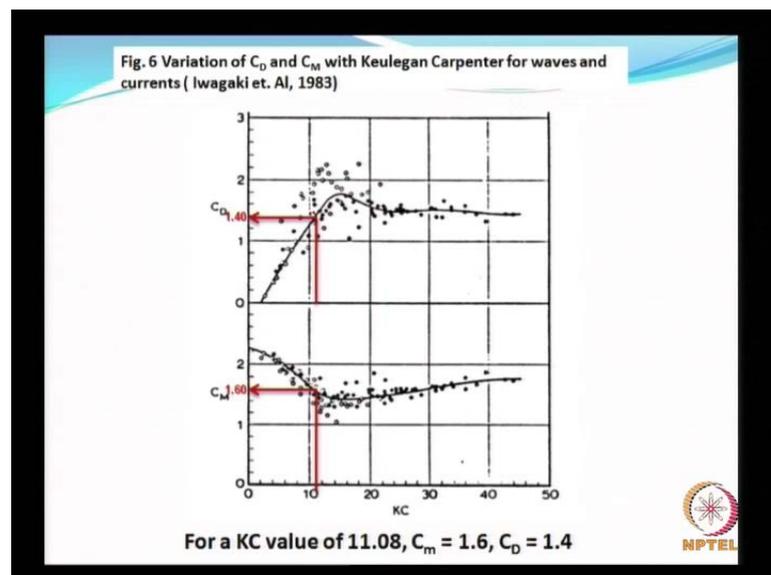
$$\cos\beta = -U/u_{\max} = 0.3611$$
$$KC = \frac{2.7693}{1.25} \times 10 [\sin\beta + (\pi - \beta)\cos\beta]$$
$$\beta = 111.17^\circ$$

Using the above equation,  $KC = 11.08$ .



So, we are considering the current in the direction of wave propagation and one meter per second in the earlier problem, we have already calculated the  $u_{\max}$ . So, we need not have to go to a pure cost to what we have already done. So once this is calculated you can arrive at the  $\cos\beta$ . And once this is obtained calculate your  $KC$  number. So, this  $KC$  number is the  $KC$  number in the presence of waves and currents.

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It was Iwagaki Etal in 1983 who came up with these results, on the variation of the  $C_D$  and  $C_M$  as a function of  $KC$  and the  $KC$  number has been calculated, the way it has

been calculated just shown in the earlier slide. The KC number is well defined, so once you say u is equal to 0 KC number is obtained, whether it is wave or current, but the coefficient of drag and inertial variation this has been estimated only in the presence of current. So, you should not use, the other variation when you are considering a current, in the presence of wave, so this is very important, you will still get the results may not be close to what is going to happen .

So, for waves and current, you need to use only the results of Iwagaki Etal, where in he has used in this experimental set up, he use allowed the current also to propagate over the waves. Not much of a literature is available on variation of hydrodynamic coefficients, and then you have the waves propagating over the current; whereas in the absents of current you have the number of authors, they have published the variation. So now I for the present problem the Keulegan Carpenter number is here. So you using these values on these two graphs you get C M as 1.6 and C D as 1.4.

(Refer Slide Time: 26:11)

Substituting for the variables in the equation for total force given below

$$F_T = -C_m \rho \frac{\pi D^2}{4} \frac{2\pi^2}{T^2} \left( \frac{\cos \theta}{k} \right) + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta| \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right] + \frac{1}{2} C_D \rho D d |U| U$$

We get,

$$F_T = ((-3997.48 \cos \theta + 6492.5 \sin \theta |\sin \theta| + (95.57 \times 14)) \times 9.81 \text{ N}$$

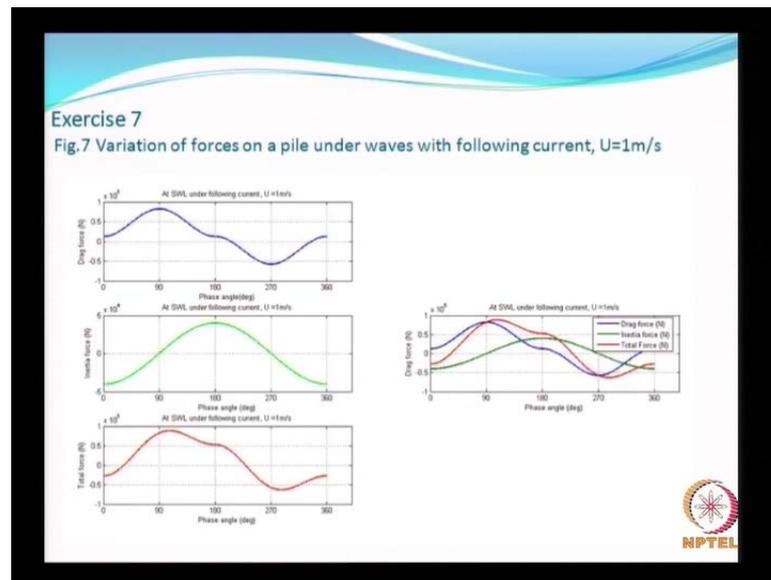
The last term in the above equation is the contribution from the uniform current.

The phase variation of wave elevation, drag and inertia and total wave force are shown in figs. 7 and 8 respectively.



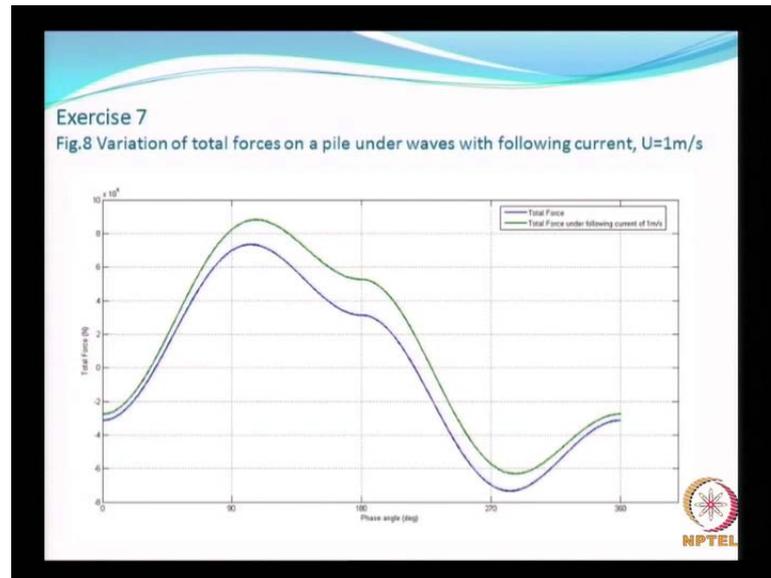
After this, just playing with the expressions, what you have. So, you have the expressions, this is the force and this is the current. So, you substitute in this, and then this is the contribution from the current, and then you will have the total the phase variation of the wave elevation drag inertia and the total force in figures 7 and 8 respectively.

(Refer Slide Time: 26:59)



So, let us see this on the variation of the force on a pile with following currents, that is current is the direction of wave propagation. You have one minute per second. So, you have the top on the left hand side you have the first picture drag force, second is the inertia force, and then the third is the bottom most is the total force. That is the variation is similar to what we have seen earlier; and there on the right hand side, you have the all the three lines are superposed, that is the variation of the drag force, the variation of the inertia force and the total force. So, you see the total force is red in color which is the top, which has the peak slightly more than 90 degrees. So that gives your variation of the three components, which we have considered.

(Refer Slide Time: 28:21)



Now this, the slide shows the variation of wave force in presence and in the presence of a flowing current of one meter per second and in the absence of the current. So, the green one is that with the following current and the blue one is with the total force, just in the absence of the current. So, you see that, here in the total force in the presence of the following current is larger compared to its absence. So, we will come back to it later, when we also consider the opposite current, but have this in mind that what is happening here that you have a wave propagating over a current.

This is the following current that in, the wave length is going to increase. So, when the wave length is going to increase naturally, the force will be more. In order to understand better, we will take an example again to find out the effect of wave period on the wave force, but maintaining the other variable constant. So, you will it will be very clear through the examples, what happens a wave is propagating over the current. So, now let us determine, the total force again the same way you have to consider the total force as shown here, then substituting you have the same expression as we have seen earlier and then we get using all the expressions you will get the total force acting on the pile, in the presence of following current.

So, I give one second, so that you look at the expressions and you see how it has been drawn, because all the variables are known to you just substitute and get the value. But remember that earlier we had the variation of the total force as a function of phase. So,

then we need to determine the phase handle and using that phase handle in the expression for the protocol and get the total force and this is what we did in an earlier exercise where consider only the wave force.

(Refer Slide Time: 31:21)

Position of maximum total force from the SWL  
 $= M_{Tmax} / F_{Tmax} = -6.4019 \text{ m}$

Position of maximum total force from the sea bottom  
 $= -14 - (-6.4019) = 7.5981 \text{ m}.$

The maximum wave force at each elevation is obtained by substituting for  $z$  and  $\theta_{max}$  in the expression

$$dF = C_u \rho \frac{\pi D^2}{4} \left( \frac{-2\pi^2 H \cosh k(z+d)}{T^2 \sinh kd} \cos \theta \right) dz +$$

$$\frac{1}{2} C_D \rho D \left( \frac{\pi^2 H^2 \sin \theta |\sin \theta|}{T^2 \sinh^2 kd} \cos^2 hk(z+d) \right) dz$$

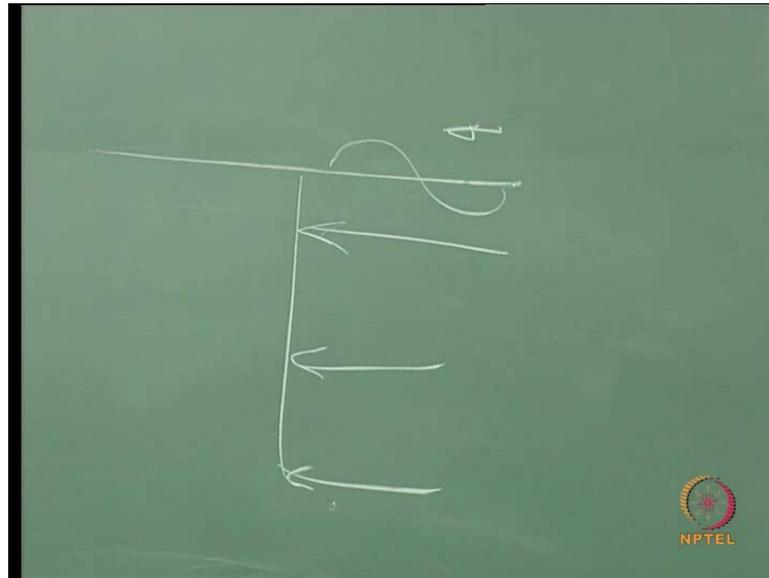
The maximum wave force variation along the depth thus obtained is shown in **Fig. 9**



So, overturning moment is given here and it will be calculated as shown here. After substitution you can also get the point of application of the total force by dividing the moment with the force. See that I came over the force at each elevation is obtained by substituting  $Z$  and  $\theta$ . So what is that you want to have the earlier also we did the same exercise.

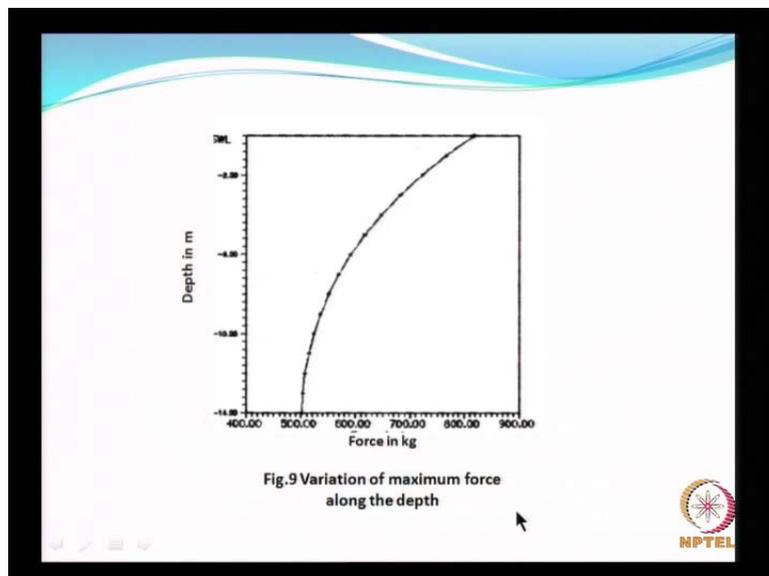
So, what we did we fix  $Z$ . So once you fix  $Z$  here then the total force will be varying with the  $\theta$ . So at each  $Z$  you can get the phase variation or the other ways is you fix here  $Z$  and also you have  $\theta_{max}$ . So at  $Z$  equal to 0 that is at still water line using the  $\theta_{max}$  of whatever you have got here 100 something substitute this expression.

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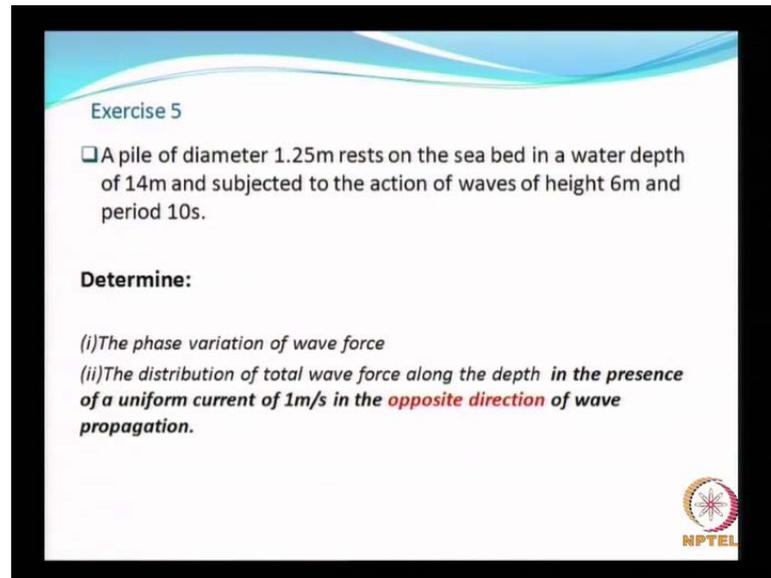
Then, I will get the total maximum force at each of the elevation, as we have seen earlier. The same procedure what we have seen earlier for the force is due to waves in absence of current.

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So, now the maximum force variation along the depth is shown here, how the force varies again it is the hyperbolic variation.

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Exercise 5

□ A pile of diameter 1.25m rests on the sea bed in a water depth of 14m and subjected to the action of waves of height 6m and period 10s.

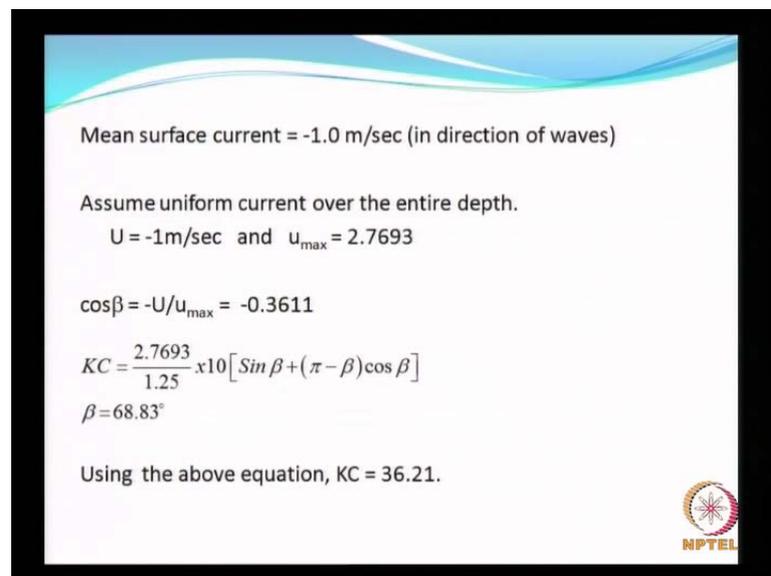
**Determine:**

(i) The phase variation of wave force  
(ii) The distribution of total wave force along the depth in the presence of a uniform current of 1m/s in the **opposite direction of wave propagation**.



Now, we have the force in the opposite direction all other parameters are given here.

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Mean surface current = -1.0 m/sec (in direction of waves)

Assume uniform current over the entire depth.  
 $U = -1\text{m/sec}$  and  $u_{\max} = 2.7693$

$\cos\beta = -U/u_{\max} = -0.3611$

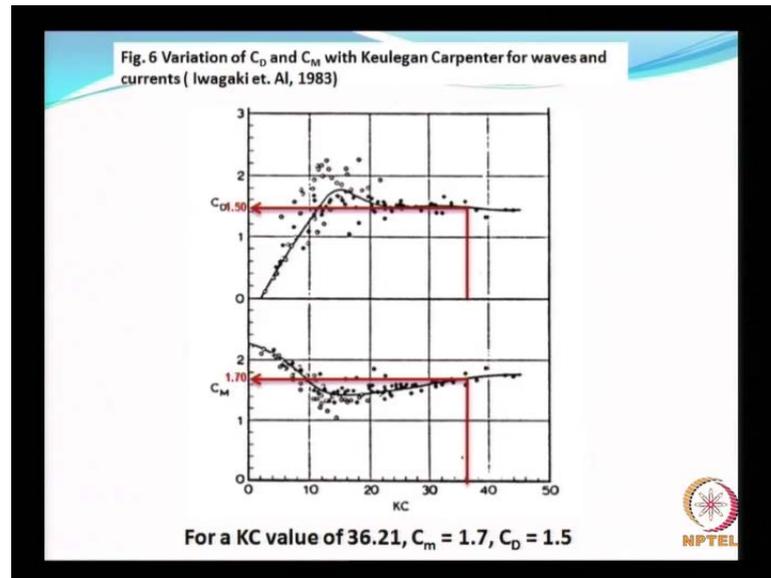
$KC = \frac{2.7693}{1.25} \times 10 [\sin\beta + (\pi - \beta)\cos\beta]$   
 $\beta = 68.83^\circ$

Using the above equation,  $KC = 36.21$ .



The same procedure, we have calculated here beta KC number in this case is going to be 36.

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You get a higher KC number, when you have it flow with the other direction. So, you get a higher value for I mean you have a value for C D is 1.5 and C M as 1.7.

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Substituting for the variables in the equation for total force given below

$$F_T = -C_m \rho \frac{\pi D^3}{4} \frac{2\pi^2}{T^2} \left( \frac{\cos \theta}{k} \right) + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta|}{\sinh^2 kd} \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right] + \frac{1}{2} C_D \rho D d |U| U$$

We get,

$$F_T = (-4247.3225 \cos \theta + 6956.25 \sin \theta |\sin \theta| - (102.39 \times 14)) \times 9.81 \text{ N}$$

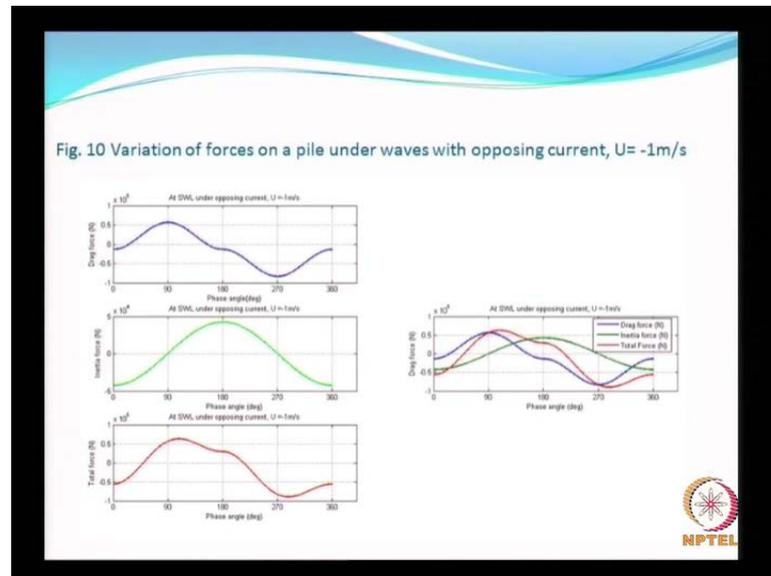
The last term in the above equation is the contribution from the uniform current.

The phase variation of wave elevation, drag and inertia and total wave force are shown in figs. 10 and 11 respectively.

To repeat the calculations has we have seen in the earlier problem. I forgot to mention about the multiplication factor. We multiply this is by 14 which is water depth in order to take care of the current. So, we have we need to use this current right this expression. So that expression you will be this much at a given point, but multiplied by the total because we are assuming it as a uniform current or you can include the way you want in this area

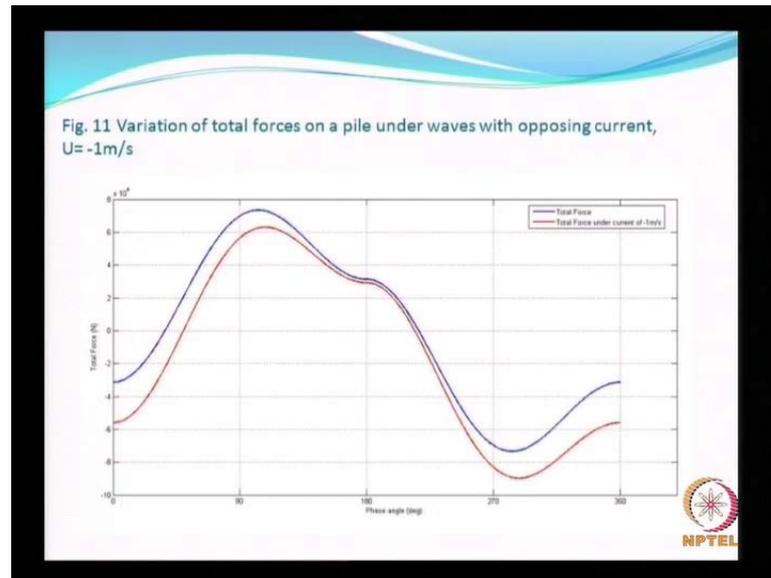
for the current if you have some measurements. The measured value itself can be substituted. So, you calculate the actual current in the expression. So this gives the total force variation as a function of theta for waves with current the opposite direction.

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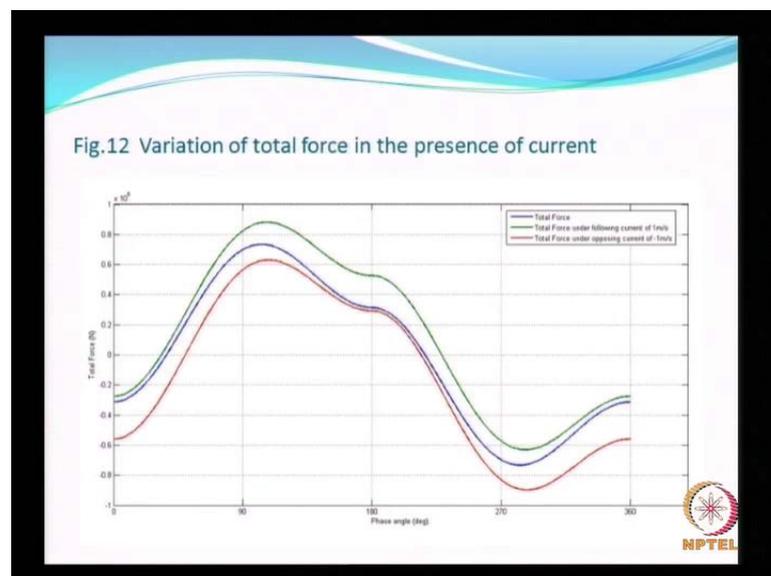
Now for this, let us see what happens on the left hand side we have plotted the variation of drag force inertia, force under total force and variation looks similar to what we have got for the wave force to the pile in four waves in the direction of the wave propagation. It is almost the same on the right hand side, again you see the variation of the different forces that is the drag inertia and total force variation as shown here.

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So, then this shows how the force variation looks like when you have an opposite current, where the current is in the opposite direction, your wave length will decrease have this in mind and wave height is going to increase. This wave length, periods of wave will exert more pressures it will exert more forces on structures. This remember try to recollect we have look at to an example on the variation of pressures. What we have seen, the pressures exerted by long period waves are much higher. Now the same thing would come. So, if you have a lesser wave length the force will be less compared to the without the current and that is what is seen here.

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So, here in I have superpose all the three the blue one, looks shows the forces without current and the green one shows following current of one meter per second and the red one shows the total force with opposing current. So, you see that in this problem all the variables have been maintained constant. Only the direction of current has been changed, on the magnitude of current is also same as 1.1 meter per second. So this gives us clear indication what will happen when you have the current in the opposite direction, what will happen to the forces. So following current is expected to exert more pressure on a more force on a pile on a structure.

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**To determine the maximum total wave force**

$$\theta = \theta_{\max} = \cos^{-1} \left[ \frac{-\pi D C_m}{H C_D} \left( \frac{2 \sinh^2 kd}{\sinh 2kd + 2kd} \right) \right]$$

$$\theta_{\max} = 107.73^\circ$$

Substituting this value for  $\theta$  in the expression obtained for the total force

$$F_T = -C_m \rho \frac{\pi D^3}{4} \frac{2\pi^2}{T^2} \left( \frac{\cos \theta}{k} \right) + \frac{1}{2} C_D \rho D \left( \frac{\pi H}{T} \right)^2 \frac{\sin \theta |\sin \theta|}{\sinh^2 kd} \left[ \frac{\sinh 2kd}{4k} + \frac{d}{2} \right] + \frac{1}{2} C_D \rho D d |U| U$$

We get,  $F_{T\max} = 62988 \text{ N}$



So, again the same story repeats. So, here we are continuing with the opposite currents. So, we have calculated the total force and then we have calculated the moment.

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Position of maximum total force from the SWL  
 $= M_{Tmax} / F_{Tmax} = -1.488 \text{ m}$

Position of maximum total force from the bottom  
 $= -14 - (-1.488) = -12.512 \text{ m}.$

The maximum wave force at each section is obtained by substituting for  $z$  and  $\theta_{max}$  in the expression

$$dF = C_d \rho \frac{\pi D^2}{4} \left( \frac{-2\pi^2 H \cosh k(z+d)}{T^2 \sinh kd} \cos \theta \right) dz +$$
$$\frac{1}{2} C_d \rho D \left( \frac{\pi^2 H^2 \sin \theta}{T^2 \sinh^2 kd} \cos^2 \theta \right) h k(z+d) dz$$

And here in we calculate the total maximum force from the still water, it will be very close to the still water; and in this case, when the waves are approaching in the opposite direction and the position from the sea bottom is 12 meters. So, at the each elevation for the particular case, the same thing is repeated. Here in the calculation of the ratio between the moment and force is done which comes about 1.5 below the still water line. In the earlier case for the following current, we remember we got something like 6.4 meters.

So, how for the opposing current ,we are getting something like 1.5 that is closer to the still water line. Then rest almost are similar to what we have done say for example, trying to get the maximum force at each section, so you calculate your things. So, I do not want to repeat the whole thing, so it is possible to calculate and show the variation of the force along the depth. As we have seen earlier for the following current.

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### Exercise 6

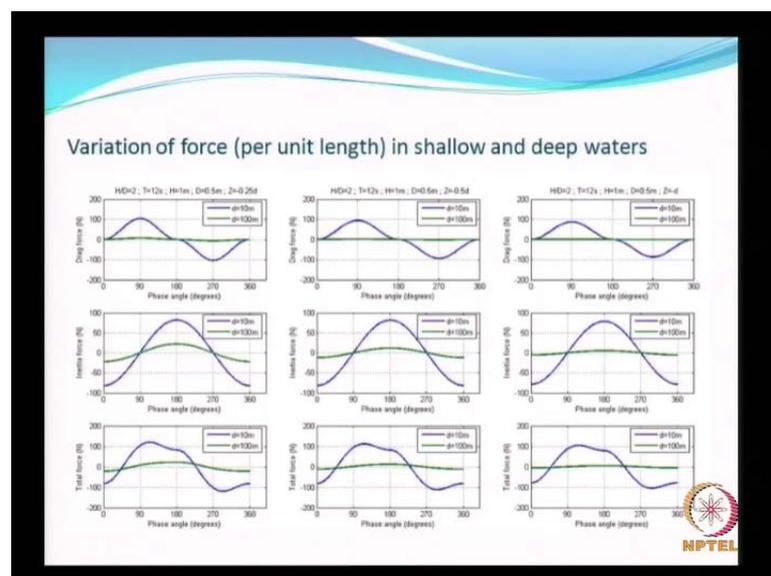
#### Variation of force in shallow and deep waters

- Consider two water depths 10m and 100m
- For each of the water depths, consider different "z", (here,  $z = -0.25d, -0.5d, -0.75d$  and  $-d$ )
- Find out "KC" number at SWL.
- $KC = (U_{max} * T) / D$
- Find  $C_d$  and  $C_m$  using the following chart, for the corresponding KC number  
(Variation of  $C_d$  and  $C_m$  with Keulegan Carpenter for waves (Chakrabarti, 1987))
- Substitute these values in the equation of force (drag, inertia and total force).



Now, this problem demonstrates the variation of force in shallow and deep water. So for this purpose we have considered two different water depth to show the variation one is 10 meters water depth another is 100 meters water depth. So for each water depth consider different Z here in, let us consider Z as minus 0.25d minus 0.5d Minus, 0.75d and minus d. So, we need to calculate the KC number at the still water line and then find  $C_d$  and  $C_m$  using the thought of I mean the results of Chakrabarti, and then substitute these values the procedure is same.

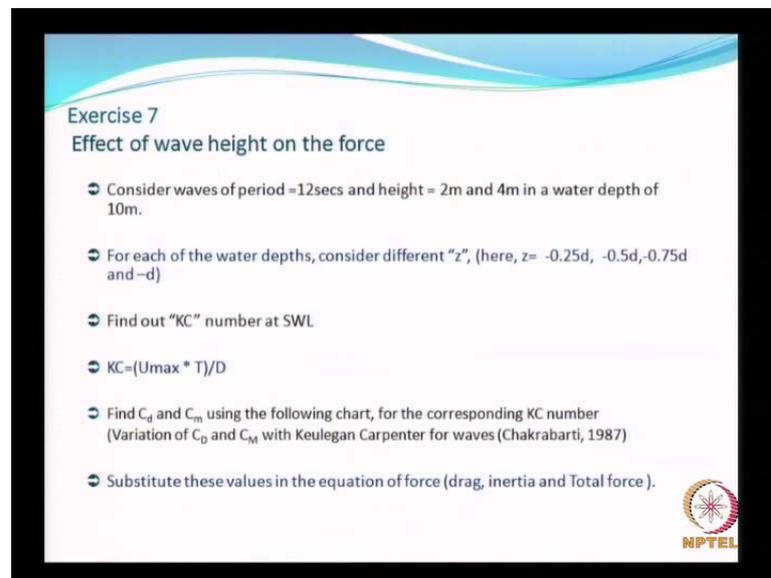
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So, I will not going into the details of the procedure and I simply straight away draw the force variation just to show how the force variation look like. So, you see that the left side column completely gives the values, the variations of the drag force on top the inertia force in the middle and the total force at the bottom. For the two different water depths the blue one indicated for n meter water depth and 100 meter water depth, it is indicated by the green color.

So, you see that the force is very less compare to force in 100 meter water depth is very less compare to 10 meters water depth. The same thing you see for the other 0.5d point I mean Z d of course, the variation is not so big, but still you can see the variation as you go down towards the seabed. The area, the value becomes quite negligible for the variation of the forces. So in this exercise, the problem shows you the variation of the wave force for the case of shallow water depth, I mean lesser water depth compare to greater water depths.

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**Exercise 7**  
**Effect of wave height on the force**

- Consider waves of period =12secs and height = 2m and 4m in a water depth of 10m.
- For each of the water depths, consider different "z", (here, z= -0.25d, -0.5d,-0.75d and -d)
- Find out "KC" number at SWL
- $KC = (U_{max} * T) / D$
- Find  $C_d$  and  $C_m$  using the following chart, for the corresponding KC number (Variation of  $C_d$  and  $C_m$  with Keulegan Carpenter for waves (Chakrabarti, 1987)
- Substitute these values in the equation of force (drag, inertia and Total force).

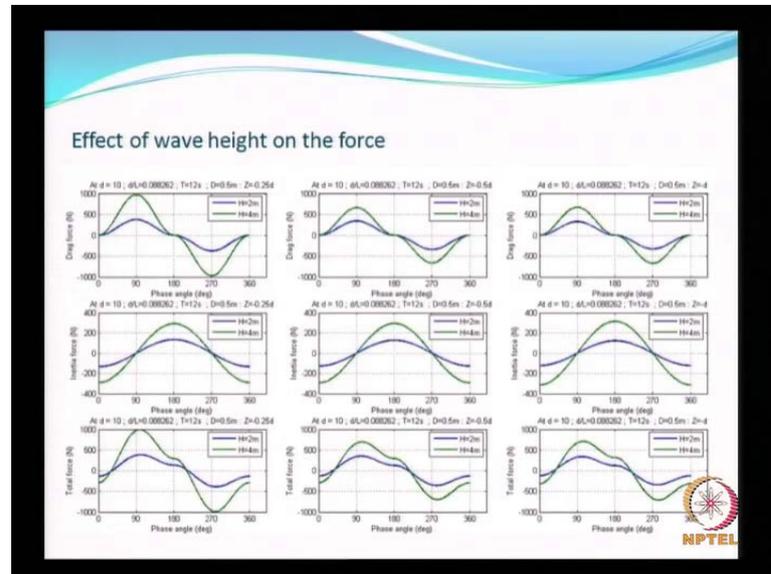
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Now having seen the effect of water depth on the wave force, now we move on to the effect of wave height on the wave force. Now herein is considering wave case period of twelve seconds and we have changed. The so for those when we want to look at the variation of row 8 so naturally we have to maintain all other parameters constant.

And that is what we are trying to do here, we are considering the wave period of twelve seconds with the height is changing from 2 meters and 4 meters we have considered so

water depth is 10 seconds so for each of since we have calculated here KC number and other values.

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And note that we have considered  $d$  as 0.5 meters and water depth as 10 meters and wave period as 12 seconds. So, these variables if you try to do it you can calculate all these values. Now, here in all the left hand side left hand columns we have the variation of the drag inertia and total force for minus  $0.2d$  and then the middle one for minus  $0.5d$  now of the water depth and then the final one is at the seabed. So, the green one is for 4 meters and the blue one is for two meters obviously you see that for a larger wave height the force is expected to be large.

And that is what we get here this information, but the phase variation is obviously same for both. So this when you are estimating the wave force are of pile you would be taking care of the kind of wave height you going to expect in the place where you want to install the file for which all the things need to be considered. So I am sure that this gives you the effect of variation of wave height on the wave force.

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### Exercise 8

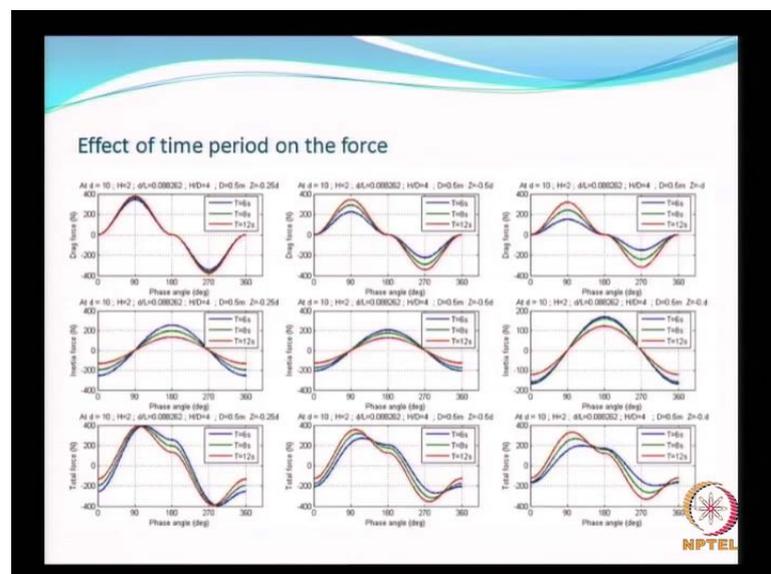
#### Effect of wave period on the force

- Consider waves of height=2m and periods= 6s, 8s and 12s in a water depth of 10m.
- For each of the water depths, consider different "z", (here,  $z = -0.25d, -0.5d, -0.75d$  and  $-d$ )
- Find out "KC" number at SWL
- $KC = (U_{max} * T) / D$
- Find  $C_d$  and  $C_m$  using the following chart, for the corresponding KC number (Variation of  $C_d$  and  $C_m$  with Keulegan Carpenter for waves (Chakrabarti, 1987))
- Substitute these values in the equation of force (drag, inertia and Total force).



Now, comes the effect of wave period on the wave force. So consider wave height following there. We are trying to, the examine the effect of a variable in this case wave period make sure that the maintaining other parameters constant. So, this is the problem when you do some experiments on this kind of problems and when we want to study the effect of one particular variable unfortunately they try to change the other variable also, which to be the case when you want to examine the effect of a particular variable only the particular variable have to be valid by keeping all other variables involved in problem constant. This is extremely important so the same procedure you need to follow.

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And when you do that you have the same thing as shown here. So,  $T$  equal to 6 seconds,  $d$  equal to 12 seconds. So, you see from the picture that, although there is some amount of variation here you see in general, the wave period will be larger for the larger wave period. So, if the wave period is large you see that the wave force is going to be increasing.

Here in the see the variation of the different components, if you examine the row along you see that, this is the variation of the drag force are different elevations, So, you see that the drag force is always higher for a long period wave. And then you examine the variation of the inertia force the inertia force is always less for a long period wave. But what matters to us is for the total force you see that the total force will be always maximum for long period waves.

So, this also proves that you have the wave current interaction, when the value is moving over the following current, then you see that the wave length increases and when the wave length increases, the force increases for the following current; so, this is done. So, next we will try to look into an example for force. So with this, the chapter on wave forces will be completed.