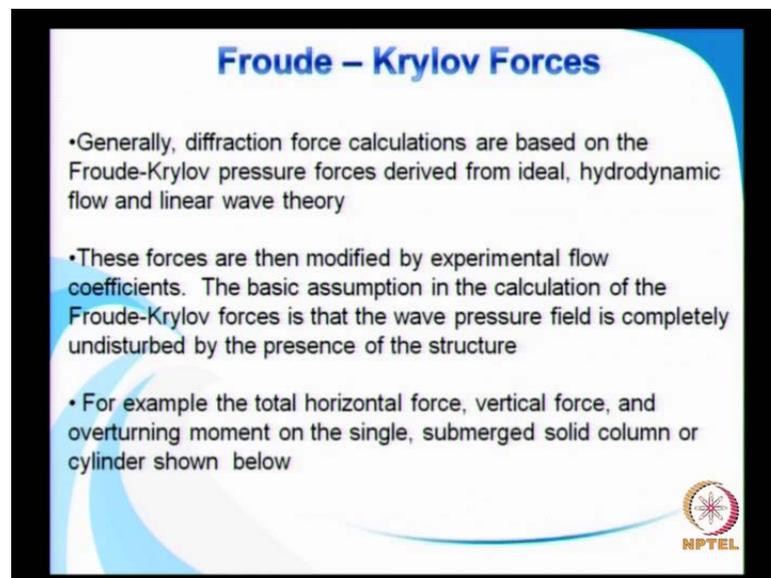


Wave Hydro Dynamics
Prof. V. Sundar
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Module No. #05
Wave Loads on Structures
Lecture No. #03
Wave Loads on Structures and Problems I

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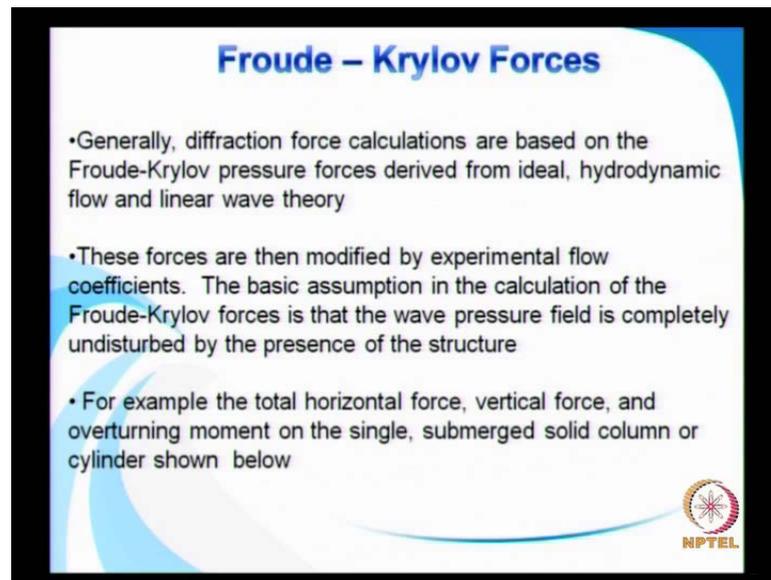
Froude - Krylov Forces

- Generally, diffraction force calculations are based on the Froude-Krylov pressure forces derived from ideal, hydrodynamic flow and linear wave theory
- These forces are then modified by experimental flow coefficients. The basic assumption in the calculation of the Froude-Krylov forces is that the wave pressure field is completely undisturbed by the presence of the structure
- For example the total horizontal force, vertical force, and overturning moment on the single, submerged solid column or cylinder shown below

NPTEL

So, I have mentioned about a classification of wave force regimes. So, one is the Morison, where the regime where the Morison equation can be adopted, and other regime being your diffraction application of diffraction theory.

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Froude – Krylov Forces

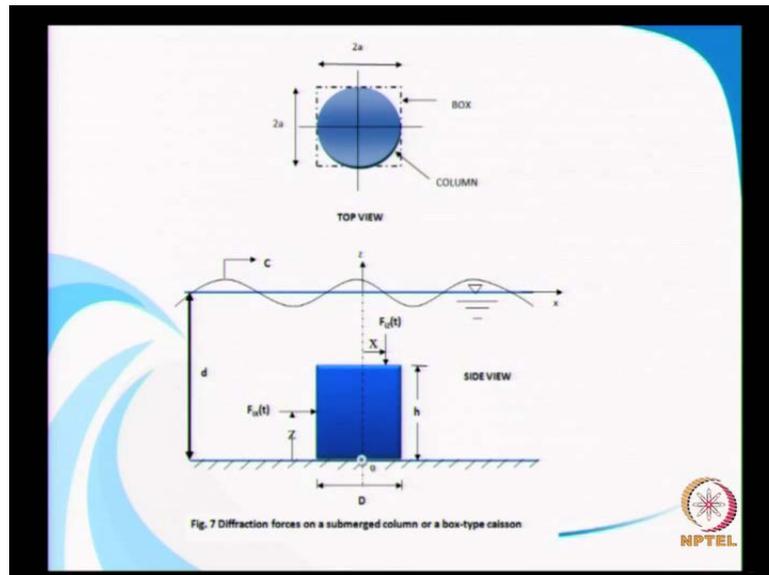
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- For example the total horizontal force, vertical force, and overturning moment on the single, submerged solid column or cylinder shown below

NPTEL

Now, we have a what is called as the Froude-krylov force, and in general or the diffraction force calculations are based on the Froude-Krylov pressure forces, which are derived from ideal flow, and by applying the linear airy's theory. Now, these forces are then modified by experimental flow coefficients, but what exactly is Froude-Krylov force? Froude-Krylov force is nothing but the forces that the pressure field is, we assumed that the pressure field is completely undisturbed by the presence of any structure.

So, a structure is present, wave is moving; and due to the structure, you would have the pressure disturbed, but we assumed that the pressure is undisturbed. And then we try to evaluate the pressures on one side and on the other side and try to calculate, what is a net pressure force acting on the structure. So, this can very well be understood from a simple example, wherein, we consider the variation of horizontal, total horizontal, vertical force, as well as the overturning moment on a single submerged solid column or a cylinder. So, this is an classical example, that is available in published literature. So, and which is a quite handy as an example, for us to understand the Froude-Krylov force.

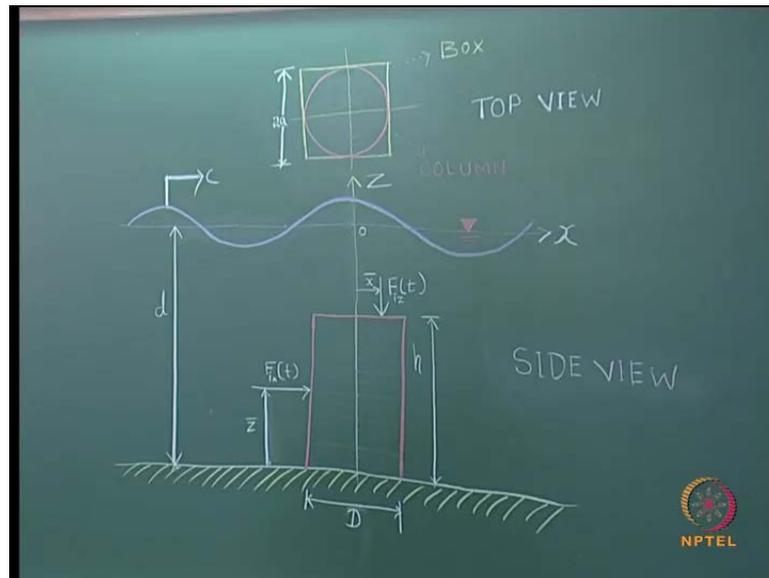
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So, for that purpose, we assume submerged column as you can see here. And the prime view, the top view is I have shown here, the diameter is $2a$, on the height of the column is h , in the water depth of d . So, we consider o - the centre point of the column at the sea bed as the point at which you might be interested knowing the moment. So, the force is acting in the horizontal direction, and the vertical direction are represented can be seen as can be seen here, and here. And this horizontal force is expected to act at a distance of z from the sea bed, and the horizontal force, vertical force is expected to at from a distance x .

So, it is since, it is quite difficult to move again to the move back again to this figure. What I have done is? I have drawn this figure here, for your ready reference.

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So, that while we are, while I am mentioning about the forces, please look at this drawing in order to understand better. So, how are you going to evaluate the forces?

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$$F_{1x}(t) = C_h F_{xk} \quad (19a)$$

$$F_{1z}(t) = C_v F_{zk} \quad (19b)$$

$$M_o(t) = C_o (\bar{z} F_{xk} + \bar{x} F_{zk}) = C_o M_k \quad (19c)$$

where C_h , C_v and C_o are the respective flow coefficients.

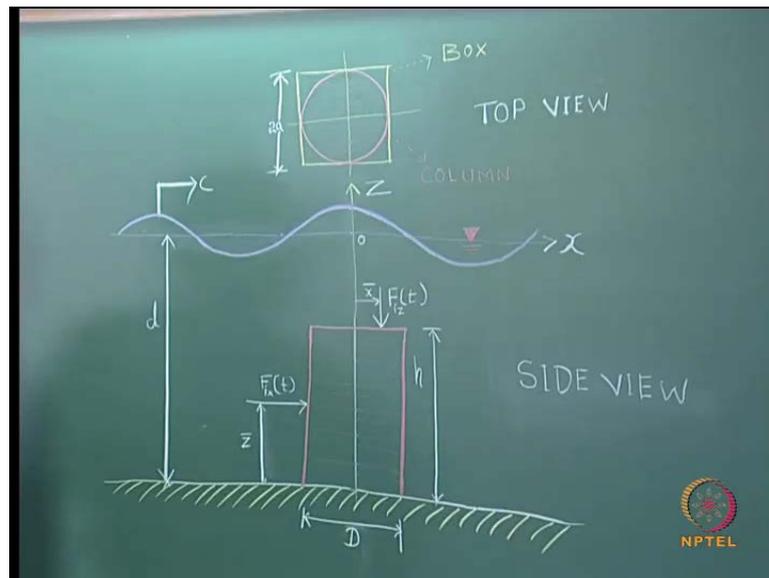
The terms F_{xk} , and F_{zk} which produce the overturning moment, or the Froude-Krylov Forces or the net pressure-induced forces on the vertical sides and on the top horizontal surface, located at the respective centres of pressure



Here, as per the Froude-Krylov force, the horizontal force given by equation 19 a will be a product of some kind of a flow coefficient, which may be determined or given to you by through experimental published literature through experimental investigations. And that will be equal to f into f suffix k x of k . And similarly, the vertical force is given as shown here. The moments or cost by these two forces, at point o which is at this location.

And now, what we are interested is we should know the terms f of x k and f of z k , which produce a overturning moment or the Froude-Krylov forces. And, what is the Froude-Krylov force? It is nothing but the net pressure induced forces on the vertical side, as well as on the top horizontal surface.

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So, we are considering two surfaces, this is surface one. And the horizontal force is going to act in this direction over this surface. And the vertical force is going to act over this surface. And the respective centre of pressure, we are assuming to at a distance of z bar from seabed and from the central axis at a distance of x , the vertical force will be acting.

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Assuming linear theory, the dynamic pressure due to propagating wave is given as

$$p(x, z, t) = \frac{\gamma H}{2} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos(kx - \sigma t) \quad (20)$$

The parameter F_{xk} is found by integrating the pressure difference over the vertical sides of the box normal to a right travelling wave,

$$F_{xk} = 2a \int_{-d}^{h-d} [p(-a, z, t) - p(a, z, t)] dz \quad (21)$$

Substituting Eq.(20) in (21) integrating, and using trigonometric identities we get

$$F_{xk} = -2\gamma a H \frac{\sinh kh}{k \cosh kd} \cdot \sin ka \cdot \sin \sigma t \quad (22)$$


So, now, we know that based on the linear airy's theory, the dynamic pressure due to a propagating width is given by equation 20. So, the dynamic pressure, we have worked out a few problems also. So, the dynamic pressure is going to change with respect to z and it is going to at each elevation; it is going to be a varying as a sin function or a cosine function. So, now the parameter F of F k x can be obtained from this 2 a, that is the area. And the pressure has to be integrated over height as a column, because you are interested in finding out, the pressure the force acting on this column. So, what is the location at this is equal to minus d, z equal to minus d and at this point it will be d minus h but negative. So, you will have minus d to h minus d.

And the pressure, we assume that the pressure has to be undisturbed pressure. So, you have to now remove the column or the structure from your mind. So, when a wave is propagating, you try to find out the pressure at this location and at this location, that is what we are exactly trying to do. We should recollect the problem which we have done under wave hydro dynamics. One or two problems, we looked at the phase at which, the velocity is maximum or the acceleration is maximum etcetera. So, we consider this location and try to find out, what would be the pressure and that in terms of coordinates is given as minus a, that is on the negative side. This is at x equal to minus, but any particular elevation z.

And of course, your since the pressure is going to be a time function so, it has to vary with respect to time. So, all these variables are involved and the pressure is going to be a function of all these three areas and the other side it will be at this location. So, you will have pressure at plus a, because this is 0, this point is 0. So, the difference in these two pressures should give you the and the difference between these two pressures acting over this surface area and integrating sum this height to this height, should give you the force, it is very simple. So, I could find this as one of the best example available in literature to explain the evolution or the meaning of Froude-Krylov force.

So, now, we just use the pressure expression and then substitute in this equation to obtain the F of x into F suffix x k. So, I get the force acting in the horizontal direction, which is now going to be a function of water depth. So, which is going to be a function of water depth of course, it is going to be a function of wave height, water depth and it is going to vary with respect to frequency which is brought into k and sin of k a will be a constant. Now, this going this will give the variation of the force and the phase variation of the force. Is that clear?

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M_k is sum of two integrals. The first is the moment about 0 due to the side pressure forces, or M_{hk} , and the second is the moment about 0 due to the pressure forces on the top, or M_{vk} . These moment components are

$$M_{hk} = 2a \int_{-d}^{(h-d)} [p(-a, z, t) - p(a, z, t)](d - (-z)) dz \quad (23)$$

$$M_{vk} = 2a \int_{-a}^a xp[x, (h-d), t] dx \quad (24)$$

$$M_k = M_{hk} + M_{vk} \quad (25)$$


So, having looked at the horizontal force. Now, if you look at m_k , it has two integrals.

(Refer Slide Time: 12:27)

Approximate formulas for C_h , C_v and C_o is given as

$$\left. \begin{aligned} C_h &= 1 + 0.75 \left(\frac{h}{D}\right)^{1/3} \left[1 - 2.96 \left(\frac{D}{L}\right)^2 \right] \\ C_v &= 1 + 7.3 \left(\frac{D}{L}\right)^2 \left(\frac{h}{D}\right) \text{ for } \frac{\pi h}{L} < 1 \\ C_v &= 1 + 1.57 \left(\frac{D}{L}\right) \text{ for } \frac{\pi h}{L} > 1 \\ C_o &= 1.9 - 1.1 \left(\frac{D}{L}\right) \end{aligned} \right\} \quad (26)$$

The above equations are restricted to the following ranges

- $\frac{h}{d} < 0.6$ for C_h , C_v and C_o
- $0.3 < \frac{h}{D} < 2.3$ for C_h and C_v only
- $0.6 < \frac{h}{D} < 2.3$ for C_o only



(Refer Slide Time: 12:29)

M_k is sum of two integrals. The first is the moment about 0 due to the side pressure forces, or M_{hk} , and the second is the moment about 0 due to the pressure forces on the top, or M_{vk} . These moment components are

$$M_{hk} = 2a \int_{-d}^{(h-d)} [p(-a, z, t) - p(a, z, t)] (d - (-z)) dz \quad (23)$$

$$M_{vk} = 2a \int_{-a}^a xp [x, (h-d), t] dx \quad (24)$$

$$M_k = M_{hk} + M_{vk} \quad (25)$$


So, you have to now, for example, (No audio from: 12:24 to 12:30) in a similar way you need to get the expression for the force in the vertical direction. So, now straightaway I am just using these expressions, for the moment due to the horizontal force and the moment due to the vertical force that is what is given here.

So, M_{hk} which is the horizontal force, will be the $2a$ will be still there. And then this is see horizontal force which we saw by the, from the previous expression. Previous expression we had this; this is what is given here. And that will be acting over a distance

d of minus z(FT). So similarly, you can also derive the forces acting in the vertical direction but instead of repeating that, I am straightaway going into the forces, due to a vertical force a moment due to the vertical force. Now, that is given as now, in this case it is going to vary from minus a so this is your coordinate axis. So, minus a to plus a and that is the area over which your vertical force is going to act in this case and this going to act over a distance of \bar{x} or x as I have indicated here.

So, this will be, this is a elevation up to which, at which the pressure is going to act. So, you need to consider that elevation that pressure exerted at that elevation, you need to consider. And having consider the pressure at that elevation, you carry out the integration multiply it. So, once you carry out the integration that alone will give you the force and multiplying it by the (\bar{x}) is the \bar{x} in this case, you will get the forces due to the or moment due to the vertical forces. So, these two when added, you get the total moment as given by equation 25.

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Approximate formulas for C_h , C_v and C_o is given as

$$\left. \begin{aligned}
 C_h &= 1 + 0.75 \left(\frac{h}{D}\right)^{1/3} \left[1 - 2.96 \left(\frac{D}{L}\right)^2 \right] \\
 C_v &= 1 + 7.3 \left(\frac{D}{L}\right)^2 \left(\frac{h}{D}\right) \text{ for } \frac{\pi h}{L} < 1 \\
 C_v &= 1 + 1.57 \left(\frac{D}{L}\right) \text{ for } \frac{\pi h}{L} > 1 \\
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 \end{aligned} \right\} \quad (26)$$

The above equations are restricted to the following ranges

- $\frac{h}{d} < 0.6$ for C_h, C_v and C_o
- $0.3 < \frac{h}{D} < 2.3$ for C_h and C_v only
- $0.6 < \frac{h}{D} < 2.3$ for C_o only



So, the approximate formulae for the C_h , C_v and C_o is given. Now, try to recollect the first equation, these were the equations which we were discussing.

(Refer Slide Time: 15:16)

$$F_{1x}(t) = C_h F_{xk} \quad (19a)$$
$$F_{1z}(t) = C_v F_{zk} \quad (19b)$$
$$M_o(t) = C_o (\bar{z} F_{xk} + \bar{x} F_{zk}) = C_o M_k \quad (19c)$$

where C_h , C_v and C_o are the respective flow coefficients.

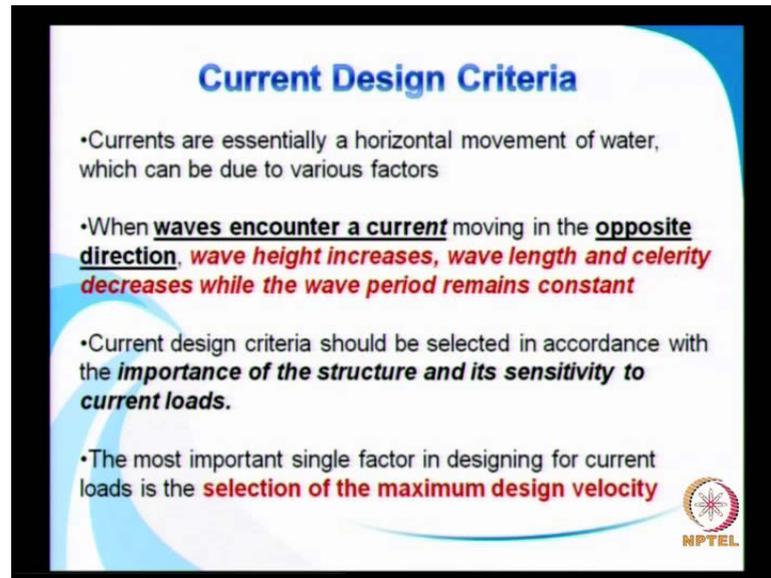
The terms F_{xk} , and F_{zk} which produce the overturning moment, or the Froude-Krylov Forces or the net pressure-induced forces on the vertical sides and on the top horizontal surface, located at the respective centres of pressure



Now, I have explained about the evaluation of forces F_x of k and z of k . And now, once you use these coefficients then you can arrive at the values of the forces moments etcetera. So, all these are equations, these above equations are. So, you have to be careful with a limit, that is being adopted here, for the evaluation of the flow coefficients. And these above coefficients are restricted to the following ranges. And these are all based on experimental investigations.

So, it is quite easy for example, if you have a pipeline on a sloping bed, you can, if you want to evaluate the forces, you can just find out on assumption of the undisturbed pressures, you can evaluate the pressures at this location, at this location. And then you can try to evaluate the force on the pipeline itself. So, this is one simple example, for the application of or the evaluation of the flow I mean Froude-Krylov forces.

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Current Design Criteria

- Currents are essentially a horizontal movement of water, which can be due to various factors
- When **waves encounter a current** moving in the **opposite direction**, **wave height increases, wave length and celerity decreases while the wave period remains constant**
- Current design criteria should be selected in accordance with the **importance of the structure and its sensitivity to current loads**.
- The most important single factor in designing for current loads is the **selection of the maximum design velocity**

NPTEL

Now, having seen the waves and the action of waves as well as the forces on the structures, wave forces on the structures, we also need to have some information about the current forces. We had already evaluated, I mean we had already seen the Morison equation. Application of Morison equation in the case of in an environment, where the waves and currents coexist and that formula we need to and the application of this formula is quite straightforward. But prior to looking at this expression, we will just have, we need to have some kind of an information of the current design criteria or on the effect of. So, currents are essentially a horizontal movement on water and this is due to different factors.

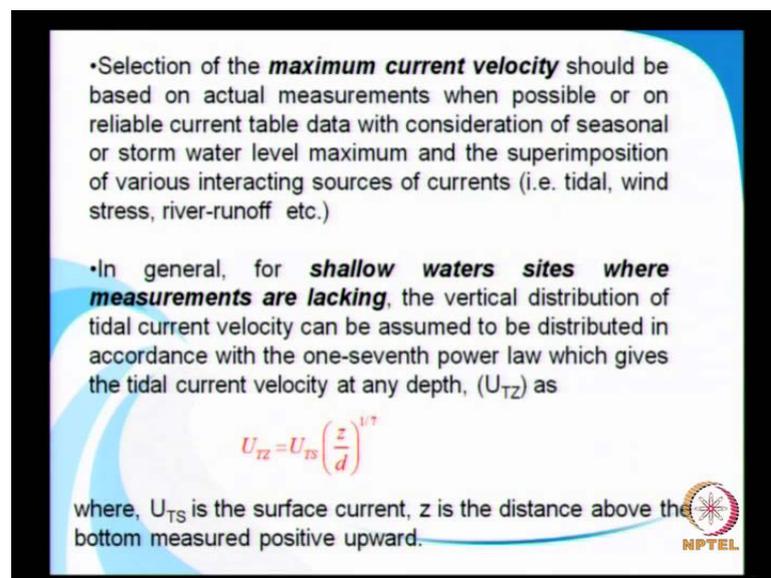
We did have some briefing during the introduction of wave current and tides. So, when so, you can have waves moving along the waves currents moving along the waves or in the opposite direction or the waves and currents can be inclined at an angle. So, if you recollect the results or the coefficients that have been produced by iwagaki. For this particular case the hydrodynamic coefficients will be for the waves and currents inclined at an angle.

So, for just the basic information is that when waves encounter current moving in the opposite direction. So, you need to remember that waves height increases, wave length and celerity decreases, while the wave period remains constant. This can easily be proved; by in fact considering that the wave crust are concerned. This aspect will be

covered in a separate topic and here for this particular course you need to know that the wave height increases and the wave length or celerity decreases whint. And of course, the wave period is going to remain as a constant and vice versa, if you have the current of the same direction.

Now, current design criteria should be selected in accordance to, in accordance with the importance of the structure and it is sensitivity to current loads. So, at certain locations, you cannot afford to miss or you cannot afford to forget about the loading due to current. So, this can be very high for example, some locations you can have as high as even 9 meters or 10 meters per second the currents. And in that case, when you want to install some pipe or pile or whatever it is or any kind of structure you need to know. What is a kind of, how do we treat the currents? The most important single parameter in designing the current loads is a selection of maximum design velocity, that is very clear. So, you need to consider the maximum design velocity.

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• Selection of the **maximum current velocity** should be based on actual measurements when possible or on reliable current table data with consideration of seasonal or storm water level maximum and the superimposition of various interacting sources of currents (i.e. tidal, wind stress, river-runoff etc.)

• In general, for **shallow waters sites where measurements are lacking**, the vertical distribution of tidal current velocity can be assumed to be distributed in accordance with the one-seventh power law which gives the tidal current velocity at any depth, (U_{Tz}) as

$$U_{Tz} = U_{TS} \left(\frac{z}{d} \right)^{1/7}$$

where, U_{TS} is the surface current, z is the distance above the bottom measured positive upward.



And now, how do you select the maximum current velocity? This should be to actual measurements, when possible or on reliable current table data. And we need to consider the seasonal or storm level maximum, and superposition of various interacting sources of current. For example, you have the tide induced current or the due to very often, the most although, we have some of these river-runoff, wind stress etcetera. The tide induced is going to be quite dominant.

So, it depends on the time of project you are dealing with. And if it is a project of major importance, then it is better you carry out a measurement campaign and try to assign or try to fix up your current magnitude for the design of structures. Or in general for shallow water sights, where measurements are lacking, the vertical distribution of the tidal current velocity can be to some extent assumed to be distributed as a function of one-seventh power formula.

So, this one-seventh power formula it comes from oceanography, where this is widely adopted. So, you will see that this gives the tidal current velocity at any depth z , which is equal to $U T S$, that is the surface current and z is the distance above the bottom measured positive upward. So this, in this definition of this formula, the definition of your z axis is just the opposite when compare to what we have done or what we have seen under the wave mechanics.

Why it is represented in terms of $U T S$ that is surface current? Because surface currents are quite easy to measure so you, when you want to design a structure. For example, even you want to design you just design for the whole, first what you do, you design you evaluate the force distribution along your obstruction or the pressure distribution. And then you look at the total force and the moment, in a similar way you need to have the distribution of the currents. So, when you want to have the distribution of currents, one simple method which we have in hand is the one-seventh power formula, which has been discussed here.

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Diffraction regime

- When the relative characteristic dimension, D/L is greater than 0.2 the incident wave gets scattered. Application of Morison equation to this regime is invalid.
- However, it may be used, provided the force is predominantly inertial which means, the drag term be dropped in the Morison equation.
- In the specific case of a vertical cylinder, linear wave theory gives the Keulegan Carpenter number, KC at the SWL as

$$KC = \frac{\pi H / L}{(D / L) \tanh kd} \quad (27)$$



Now, with this, what I will do is, I will, we will come back to the wave forces, but prior to doing the taking up the diffraction regime. We will just look at the some of the problems, through which we can understand the wave forces. I have taken a pile to illustrate, the effect of different parameters on the wave forces. That is the intension of this lecture, one is to demonstrate the calculation process.

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Exercise 1 Problems on wave loads

Draw the phase variation of the wave force on a pile along the water depth

Given,

- D (dia of pile) = 0.5m, T (Wave period) = 12s,
- H (Wave height) = 1m, d (water depth) = 10m

Solution :

$H/D=2$

$L_0 = 1.56 \times 12^2 = 224.64\text{m}$

$d/L_0 = 0.0445$

From wave tables,

$d/L=0.088285$

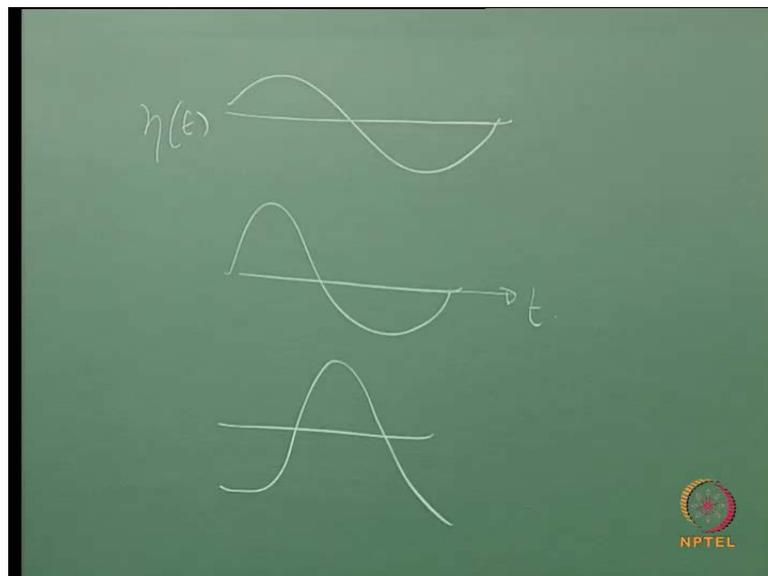
So, wave length, $L=113.269$



The application of the Morison equation, then we will obtain the wave forces and also, we will just try to change the variables involved like your wave height, wave period,

water depth etcetera. Then try to understand the few more process like, what happens to the wave loads, I feel that this is quite important. So, that you have a broader idea on the variation of wave forces with the different parameters involved in the ocean. To start with the exercise number 1 is draw the phase variation of the wave forces, on a pile along the water depth. So, you are interested in, you have supposed to draw the phase variation so this will be the eta. So, we want to know how the drag force varies or how the inertia force varies or how the total force varies with respect to time.

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So, the parameters given are the diameter of pile is 0.5 meters, wave period consider is 12 seconds and wave height is 1 meter and then, the water depth is 10 meters. So, when you are given all these data and ask to evaluate the wave forces, what are the steps you need to follow? As we have seen in the Morison equation. Although, the Morison formulation is quite straight forward and quite simple, the uncertainty lies in the selection of an appropriate wave theory for the description of particle velocities acceleration. And number 2 assigning the values for the hydrodynamic coefficients of drag and inertia. So, what we are considering here in is only the linear wave theory, but the procedures are outlined here. First, we need to find out if Morison equation will be valid or can it be applied for this kind of a problem, for the given set of data. Is that clear?

So, first you have to calculate your, the ratio of wave height to the diameter, in this case it is equal to 2. And you know, the other parameters to be calculate, how to be calculated, your deep water wave length calculate your d by l naught and once that is obtained, then calculate your d by l. And hence the wave length. (No audio from 26:44 to 26:50)

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$d/L=0.088285 \rightarrow$ Intermediate waters
 $D/L=0.004414$
 Here $H/D > 1$ and $D/L < 0.2$
 So, both Inertia + Drag force are dominant

$kd=0.5547$
 $\sinh kd=0.5836$
 $\cosh kd=1.1578$
 $\tanh kd=0.504$

$u_{\max} = \frac{\pi H}{T} \frac{1}{\tanh kd} = 0.51944$

$KC = \frac{u_{\max} T}{D} = 12.466$



What does this imply? We see that the d by L value is indicates that the problem lies in intermediate water conditions. That is T by L I mean equal to 0.0 i, that is in between 0.051 and 0.5. So, this would imply that the wave force, along the water depth will be more or less equal that is, it is the wave force closer to the seabed also, will be, somewhat comparable to that exerted near the free surface. Whereas in shallow waters, you know that the particle displacements are felt over the entire depth. So, in that case the force will be phenomenal and in the deep waters you know that, the force will be acting only near the closer to the free surface up to a distance of l naught by 2. That is please; brush up your basic wave mechanics to understand this.

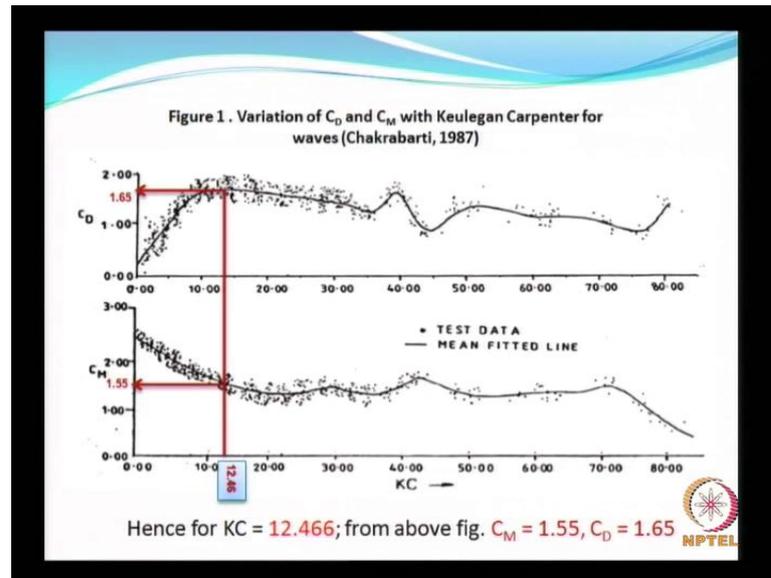
And, when we calculated the D by L , D by L is 0.004. Now, D by L is less than 0.2. So, that is a condition for the application of Morison equation, which is now valid. In addition H by D should be greater than 1, what happens if H by D is less than 1? If H by D is less than 1, then the drag component will become quite negligible. But if it is greater than 1, you need to know that both the inertia as well as the drag component will be contributing to the total wave force acting on the structure.

So, from this problem, you see that both the components have to be, will be dominating and dictating the force acting on the pile. We go further to obtain the values of $k d \sin h k d$, $\cos h k d$ and $\tan h k d$. What I would suggest is such a problem is given and if you would like to look at the wave tables. Once, you calculate your D by L naught immediate at the same time you can just extract the values for all these variables. And keep it ready so that, your time saved in referring to the tables for each and every parameter now and then.

So, you know all the parameters, which are going to be involved in the calculation of velocities and accelerations. Like all these things which are listed here, it is better keep them here and then you can just calculate. Suppose, if that is, if you are adopting the conventional practice of calculations. Once, this is obtained then, you have to calculate your maximum orbital velocity. Maximum orbital velocity will be at the free surface that is, in this case at the still water line that is when z equal to 0. So, you know the expression for u , then just use that value z equal to 0. So, that will boil down to πH by T into 1 by $\tan h k d$.

So, you know the values of all these variables like wave height, wave period and $\tan h k d$ for this particular problem. And then, here, you will see that u_{\max} is 0.5 meter per second. So, T is 12 seconds and D equal to 0.5 meters, when you substitute their here and also the value for u_{\max} , you will get $K C$ number of about 12. What does $K C$ number tell you? Again go back to the lecture on wave forces, if $K C$ number is very less than 2.2, then the drag component is almost going to be very negligible it can, you can conveniently neglect the drag component. And as the, you I mean your $K C$ keeps on increasing, then, you see that the drag component is going to dominant.

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For this problem, you need to get into the, there are so many results or the variation of so many authors have published information on the variation of C_D and C_M . As the function of Renault's number also, there are some literatures which authors, who have reported this variation, with respect to KC number. Well all these things are based on experimental values, experimental investigations. And when you read carefully some of the literature in the past, you will see that it is not so easy to cover. A wide range of keulegan carpenter number, because the laboratory set up will limit as 2 KC numbers of about 12 to 30.

It was Chakrabarti, who has done a lot of experiments. Covering, he has used a set up which is called as you tube also. And then, with this he was able to with this and other facility, he was able to generate keulegan carpenter number up to about 80 or even slightly beyond. So, now, you have a wide range of KC , I mean KC number, the variation of C_D and C_M for a wide range of KC number. You can also refer to a pI code, dean v code, there are other codes, which also gives all these information. So, here in I am using the Chakrabarti this information, this available in the text book of Chakrabarti hydrodynamics of offshore structures.

Then for the present problem, you know that the KC number, we have evaluated as 12.4 or 12.5. So, just get into these 2 curves and then pick up the value of, this is a mean line. So, you see that this scatter in the coefficient of drag is very less. The nature of this very

less scatter in the variation of C_D and C_M is probably achieved by very few investigators like Chakabarti himself. And if you look at the literature, the scatter normally is quite large. So, one need to be very careful, when you are using this results from different authors on the variations of C_D and C_M .

Here in; he has drawn a mean line and this is allowed, because you see this scatter is quite less and it is quite acceptable. And hence from this plot, you see that your C_D is approximately 1.65 and C_M equal to inertia coefficient is 1.55. Having determined the coefficients; you now get into the equations.

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• Equations

Drag Force (per unit length)

$$F_D = \frac{1}{2} C_D \rho D |u| u$$

$$u = \frac{\pi H \cosh k(d+z) \cdot \sin(\theta)}{T \sinh kd}$$

Inertia Force (per unit length)

$$F_I = C_M \rho \frac{\pi D^2}{4} \dot{u}$$

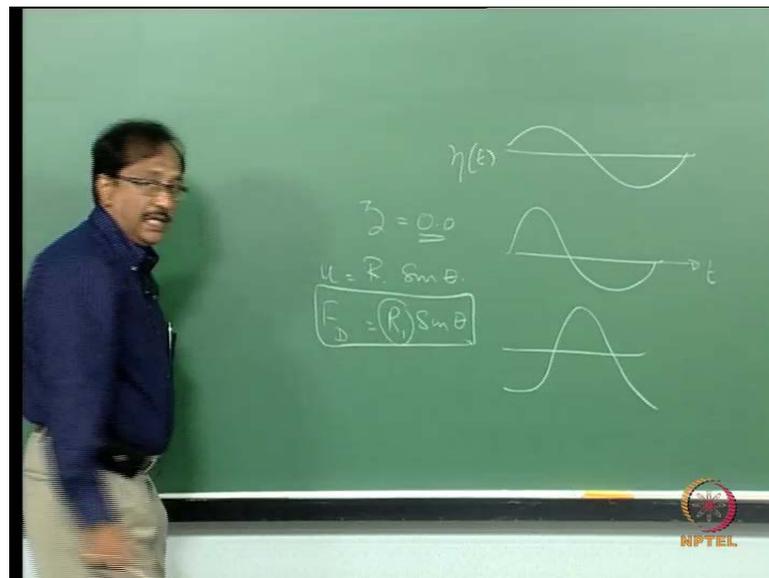
$$\dot{u} = \frac{-2\pi^2 H \cosh k(d+z) \cdot \cos(\theta)}{T^2 \sinh kd}$$

Now, do we have all the information's? Yes, because u is nothing but so this is the drag force component half into C_D into ρ into D into absolute of u into u . Please, recollect that absolute of u is considered herein to take care of the direction of wave propagation. In a steady flow, you do not need that, can just have u square that is why you calculate the force on a structure on a pile, when it is exposed in a river. So, you do not have that absolute value. But in the unsteady case like a wavy flow, you need have that absolute value and then, your u is shown as given here. So, you have wave height, wave period is given, water depth is given all these parameters you have calculated. So, only your z will be varying and your θ will be varying.

And similarly, in the case of \dot{u} , you see that this will be, z will be varying and θ will be varying. But what was a problem? Problem is to find out the phase variation. So,

when wave is acting like this, how the drag force, inertia force and the total force will be varying at different elevations, that is the problem. So, what I do is, I use I take a particular value of z , because all other parameters are known to us now, d h t etcetera everything is known. So, I retain I introduce a value of z .

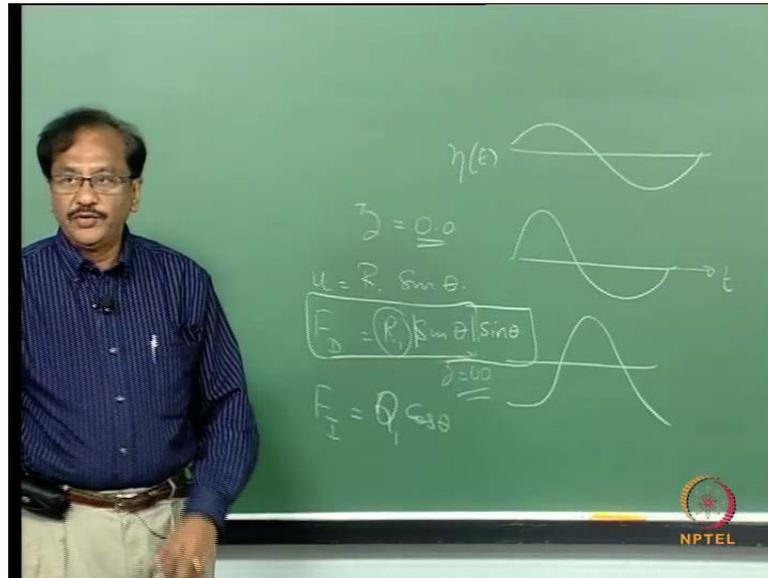
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So, for example, I take z at the still water line $0, 0$. And then, what will I have all parameters are known. So, I will have u equal to $\sum R$ into $\sin \theta$ or since u is also known. So, in the first equation, you see that once u is known u and all these parameters are known. So, F_D is nothing but. So, once I calculate my R_1 , R_1 at particular z in this case it is 0 , you keep changing a different locations. I get the value of R_1 and so I can draw the variation of drag force, that is the phase variation of the drag force.

So, then the same procedure I need to use for inertia force z , I will fix z then vary all the other way, all the substitute the values for other variables. Then I can get F_I equal to q_1 into $\cos \theta$. Of course, this will be (No audio from: 38:41 to 38:47) an absolute value. R_1 into absolute of $\sin \theta$ into $\sin \theta$, that may be the drag force and here it will be Q_1 into $\cos \theta$.

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Once, you get R 1 and Q 1, you can just draw the variation of drag force, draw the variation of inertia force and sum it up to get the total wave force. And this variation will be at different elevations.

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• Sample calculation @ $z = -0.2d$

$$F_i = C_m \rho \frac{\pi D^2}{4} u$$

$$u = \frac{-2\pi^2 H \cosh k(d+z) \cdot \cos(\theta)}{T^2 \sinh kd}$$

Substituting the variables, we get

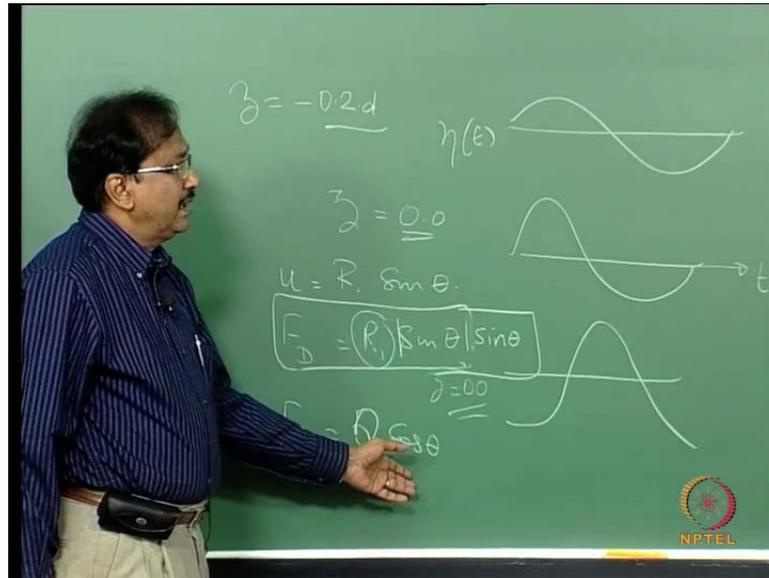
$$F_i = -80.2089 \cos \theta \text{ N/m}$$

At $\theta = 180^\circ$,

$$F_i = 80.2089 \text{ N/m}$$

So, I am just showing you, a sample calculation by taking z equal to minus 0.2 water depth.

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So, the equation is known there and then just follows the same procedure. I will get the value as this Q 1, which I am referring to, will be something like 80 times cos theta, that is going to be in we have adopted Newton per Newton that will be Newton per meter. And note that in the equation, you have a negative sin for the inertia force that is because of the acceleration. So, that will continue to arrive, continue to remain there, that negative signs. So, how does the inertia force vary, looking at this it will be a cosine curve, this is a cosine curve but note that, there is a negative sign. And hence the variation of inertia force will look as shown in that picture, for z equal to minus 0.2 d. And where will be the maximum force occurring, at cos equal to 180. So, you will have 80 point something 0.2 Newton per meter that will be the maximum force. Is that clear? Any of you have any doubts?

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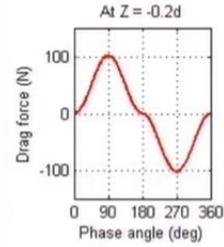
• Sample calculation @ $z = -0.2d$

$$F_D = \frac{1}{2} C_D \rho D |u| u$$

$$u = \frac{\pi H \cosh k(d+z) \sin(\theta)}{T \sinh kd}$$

Substituting the variables, we get
 $F_D = 101.895 \sin \theta | \sin \theta | \text{ N/m}$

At $\theta = 90^\circ$,
 $F_D = 101.895 \text{ N/m}$



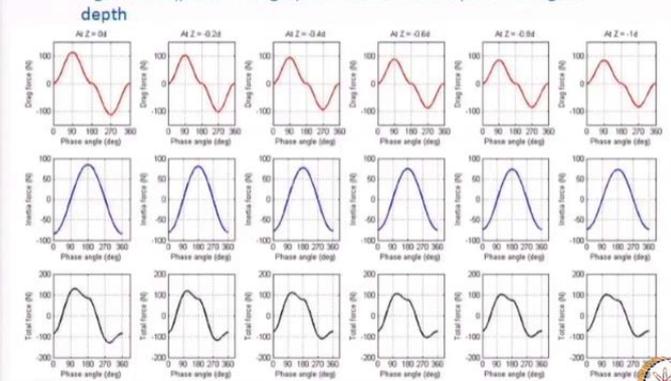
At $Z = -0.2d$



Now, I show you, the sample calculation of for the drag force. Again by considering z equal to minus $0.2 d$. So, you substitute all these things and similarly, as I have explained earlier, R_1 for this case will be 101.9 and that absolute sign of θ will be remaining. And where will this be consider I mean maximum, this is a sin curve and sin has to be maximum, when θ equal to and the phase equal to 90 . So, at θ equal to 90 , you will have the maximum drag force as given here. Is there any doubts? No doubts, if you have any doubts please ask me here.

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Fig.2 Force (per unit length) variation at various points along the depth




So, now in this plot, we have computed the force variation of the drag component, the inertia component and the total force. At what elevations, at different elevation z equal to 0 d, I mean z equal to 0 . Then minus 2 d at a one step below the still water line, then 0.4 d, 0.6 d, 0.8 d and then point I mean 1 d that is at the seabed. Remember that, the variables given to you indicated that the water condition is intermediate water depth conditions. So, you would expect that the force will be quite comparable to that force near the seabed, will be quite comparable near the still water line. And that is what you see here, looking at all the variables. So, this is the variation of the drag force peak.

So, you see there is reduction in the force, as you go towards the seabed. So, but the reduction is not drastic it is quite steady, there is a steady decrease in the drag force as we go towards the seabed. This in the event of a deep water conditions, what will happen? This rate of reduction will be quite drastic particularly after z equal to minus 1 naught by 2 . So, the same thing is observed in the variation of inertial force, again the inertial force will be varying like this. And what about the total force? When you superpose both the total force variation will be as shown here. So, you see that gradual reduction in the force as we go towards the seabed. (No audio from 44:55 to 45:01)

So, please remember, that the maximum force is occurring at this phase that is at 90 . Whereas, the maximum inertia force is occurring at theta, equal to 180 . Many of the students, what they do is when they are asked evaluate the maximum force. It is quite easy to get a get the maximum drag force, maximum drag force it is enough, you calculate up to this and maximum inertia force you calculate only q 1.

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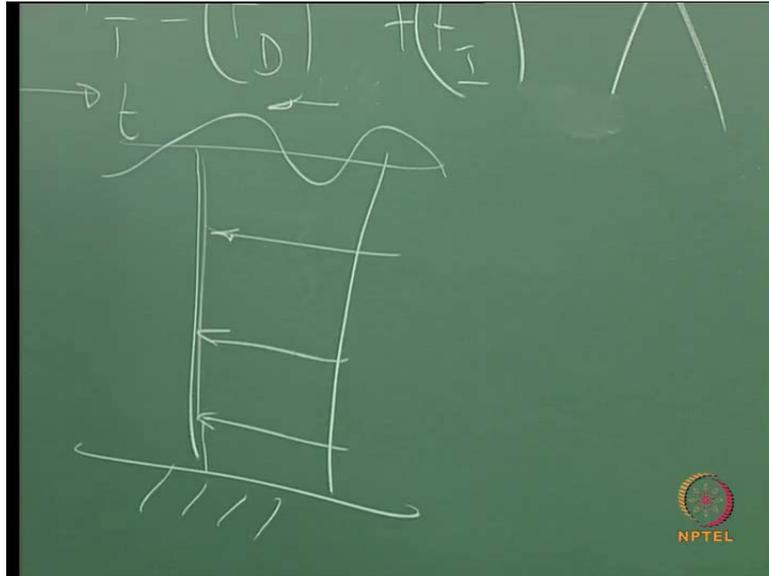
$$F_T = (F_D)_{\max} + (F_I)_{\max}$$

Can we add directly maximum drag force and maximum inertia force, to get the total force? Can I say that total force equal to $F_D \max$ plus $F_I \max$? It is not correct. Because when you look at the values, you are going to be overestimating the forces.

So, this variation, the phase variation clearly depicts that. What will happen if you just add this? Approximately close to 100 and this is approximately close to 75. So, you are getting something like 175. But when you have physically added without considering the maximum drag force and inertia force, you see that the variation looks like this. And a total maximum, your maximum force at any given elevation will be, this will be the maximum force. And the phase at which, the maximum force is occurring maximum total force is occurring is neither 90 degrees nor 180 degrees. Understand, it is somewhere in between and this phase angle we need to determine.

Why you need to determine this? If only if you are interested in calculating the total maximum force, what are we having here? In this plot, we are having only the total force variation, how the total force varies at a given elevation, as a function of phase that is all this gives. So, one way is, you pick up the force at each elevation, the maximum force pick up at this elevation, the maximum force at this elevation maximum force etcetera. So, what will happen?

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You will have maximum force here, maximum force, maximum force like this. So, the force variation will be like this, let me say that the wave is moving in this direction. So, if I have, if I need to get the total force, then I have to integrate this total force. So, I instead of picking it from the figure, we can also evaluate the phase at which the maximum force occurs. And that is what we have done earlier in the lecture material; please go through the lecture material. So, you will understand how this θ_{\max} is determined. So, this problem clearly conveys to us, the phase variation of the wave force, and how what kind of information we can obtain. Thanks.