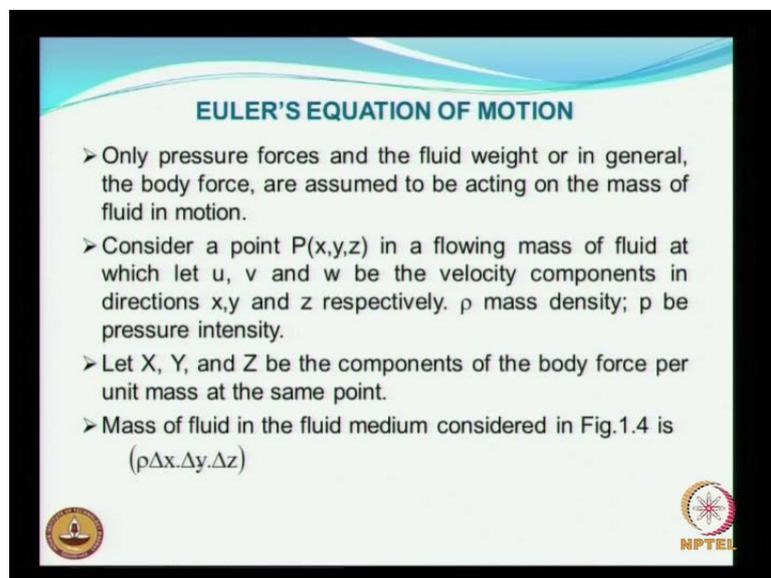


Wave Hydro Dynamics
Prof. V. Sundar
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module No. # 01
Basic Fluid Mechanics
Lecture No. # 02
Basic Fluid Dynamics II

Today, we will see Euler's equation of motion. So, although all these basic fluid mechanics you have been exposed to during your under your study some of this equations **will be** will be mentioned. And it is always good to have brushing up of your fundamentals; this is the purpose of having this as part of the syllabus of wave hydro dynamics.

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EULER'S EQUATION OF MOTION

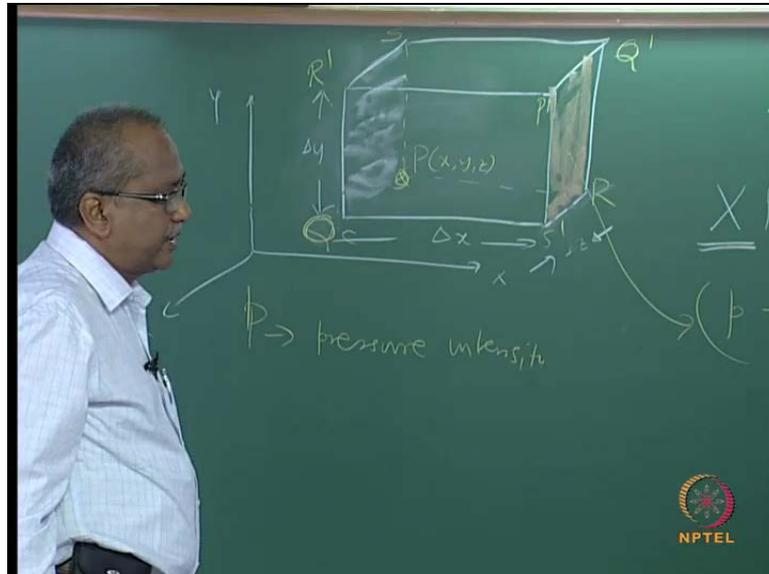
- Only pressure forces and the fluid weight or in general, the body force, are assumed to be acting on the mass of fluid in motion.
- Consider a point $P(x,y,z)$ in a flowing mass of fluid at which let u , v and w be the velocity components in directions x,y and z respectively. ρ mass density; p be pressure intensity.
- Let X , Y , and Z be the components of the body force per unit mass at the same point.
- Mass of fluid in the fluid medium considered in Fig.1.4 is $(\rho\Delta x.\Delta y.\Delta z)$

So, let us proceed with Euler's equation of motion, the earlier class we had seen what are all the different kinds of forces and then the corresponding equations also we have seen finally, it boiled down to Euler's equation of motion. And in this case we consider only

the pressure forces and the fluid weight or in general, the body force they are assumed to be acting on the mass of the fluid in motion.

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I will just just draw this figure, so that it be easy when we are referring to (No audio from 01:42 to 01:53) just make it slightly bigger (No audio from 01:59 to 02:12), so this is x y z, so this will be the length it is a delta x and this is delta z and let me have this point Q this will be R dash and this is S. So, this will be R this will be Q dash, this will be P dash, this should be S dash, no I think let it be, this be point P. So, we will we consider a point at this location and then, we in the a flowing mass of fluid and let us say that u v and w are the velocity components in the x y z directions respectively, rho is the mass density.

And let the P be the pressure intensity P is pressure intensity, so we define x y z are as your body force components in the x y z direction and let let this be the components of the body force per unit mass.

So, now the mass of the fluid, in the fluid medium would be volume into the mass density, so that will be the mass of the fluid in the fluid medium and then now the total component of the body force will be in the x direction it would be x into the mass density I mean the mass; so, x into where you will have multiplied by x, that will be the total component of the body force acting in the x direction.

Similarly, you have with the y direction as well as z direction and p as I said earlier is the pressure intensity **at** at point P, since the length of the edges of the fluid medium are extremely small; let us assume R it may be easy to assume, that on this surface over the entire surface R, R S P and Q that is this face.

Let the pressure intensity be uniform as defined as small p, so what will be the pressure force acting on this surface, this will be p into the area of this which will be delta y into delta z.

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Therefore, total pressure force on PQR'S in x direction = $p \cdot \Delta y \cdot \Delta z$

Since the 'p' vary with x, y and z the **pressure intensity** on the face RS'P'Q' will be = $\left(p + \frac{\partial p}{\partial x} \cdot \Delta x \right)$

Therefore, **total pressure force** acting on the face RS'P'Q' in the x direction = $\left(p + \frac{\partial p}{\partial x} \cdot \Delta x \right) \Delta y \cdot \Delta z$

Net pressure force F_{px} acting on the fluid mass in the x direction is

$$F_{px} = p \Delta y \cdot \Delta z - \left(p + \frac{\partial p}{\partial x} \cdot \Delta x \right) \Delta y \cdot \Delta z$$

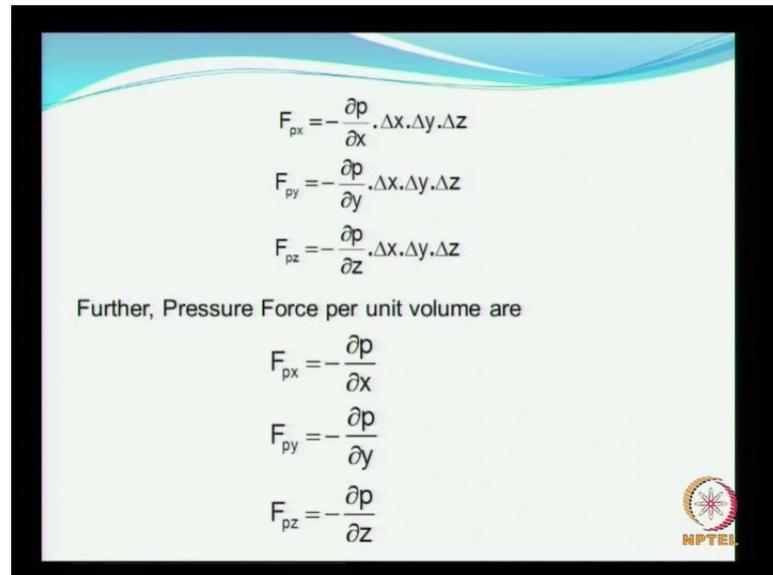
The slide also features a logo in the bottom left corner and the NPTEL logo in the bottom right corner.

Now p varies with respect to in both in all the three direction x y and z, so the rate the pressure intensity on the face are S dash, P dash and Q dash, so that is this face, so this face will be P that is the initial pressure, the pressure changing over x over a distance of delta x.

So, that will be the that is the reason why you have **p plus** p plus dou p by dou x over a distance of delta x but, this is the pressure intensity of this face, so similarly, you have to get the pressure force on that face, which means this pressure intensity multiplied the area that is what is clearly defined here.

So, that total pressure force on this face would be into is that clear, so the net pressure force will be what, on this face you have already evaluated and on this face you have already evaluated (Refer Slide Time: 08:02) the difference has to be your pressure force.

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The slide displays three equations for the pressure force components acting on a volume element of size $\Delta x \cdot \Delta y \cdot \Delta z$:

$$F_{px} = -\frac{\partial p}{\partial x} \cdot \Delta x \cdot \Delta y \cdot \Delta z$$
$$F_{py} = -\frac{\partial p}{\partial y} \cdot \Delta x \cdot \Delta y \cdot \Delta z$$
$$F_{pz} = -\frac{\partial p}{\partial z} \cdot \Delta x \cdot \Delta y \cdot \Delta z$$

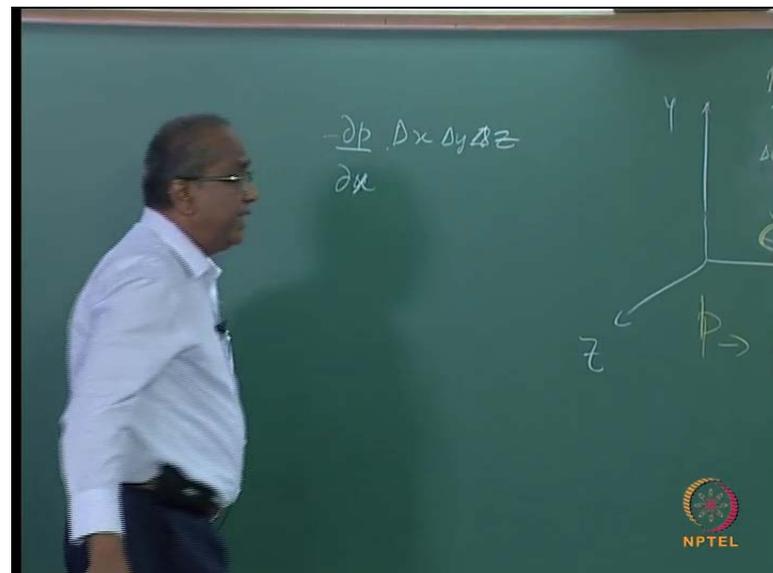
Further, Pressure Force per unit volume are

$$F_{px} = -\frac{\partial p}{\partial x}$$
$$F_{py} = -\frac{\partial p}{\partial y}$$
$$F_{pz} = -\frac{\partial p}{\partial z}$$

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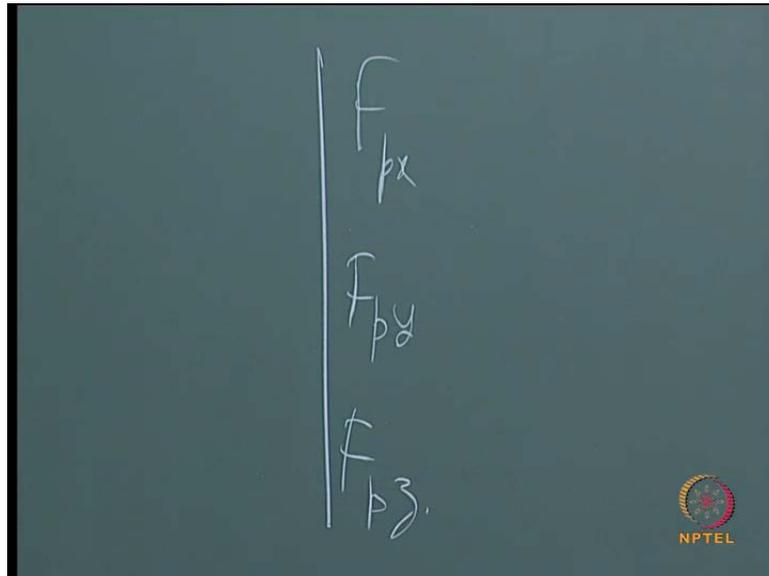
So that is what this given here, the two pressure forces acting on both the surfaces, so when you do that the net force will be **net force will be** take the difference between these two that will result in **do p by minus do p by** minus do p by do x into delta x delta y delta z this will be the net pressure force.

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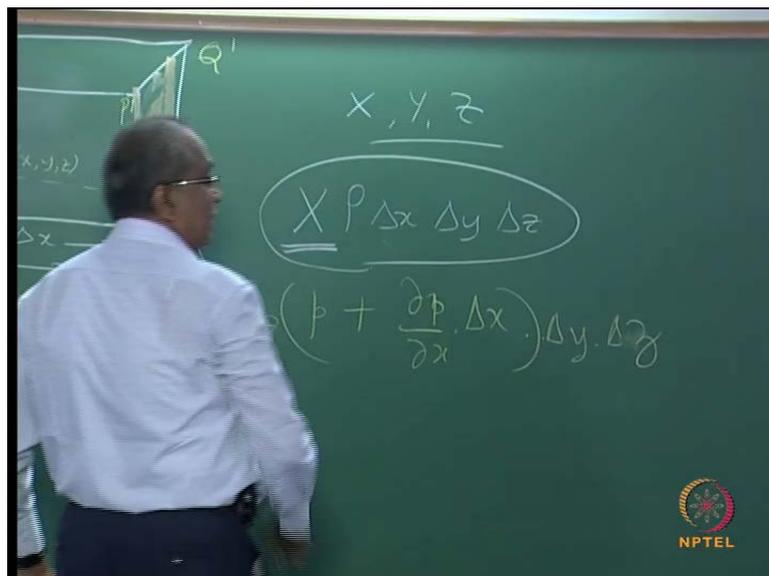


So, per unit volume will be this will disappear and the net pressure force per unit volume for all the in the three directions are given as F_{px} , you have F_{py} and F_{pz} is that clear.

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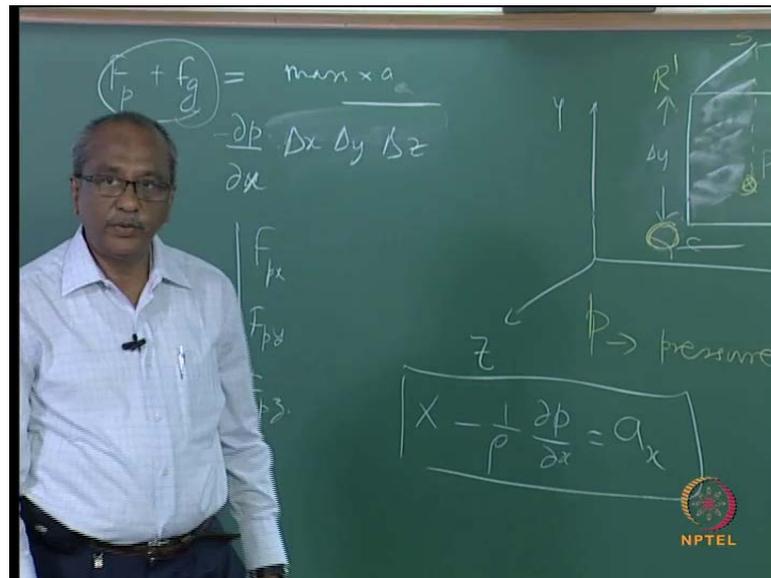


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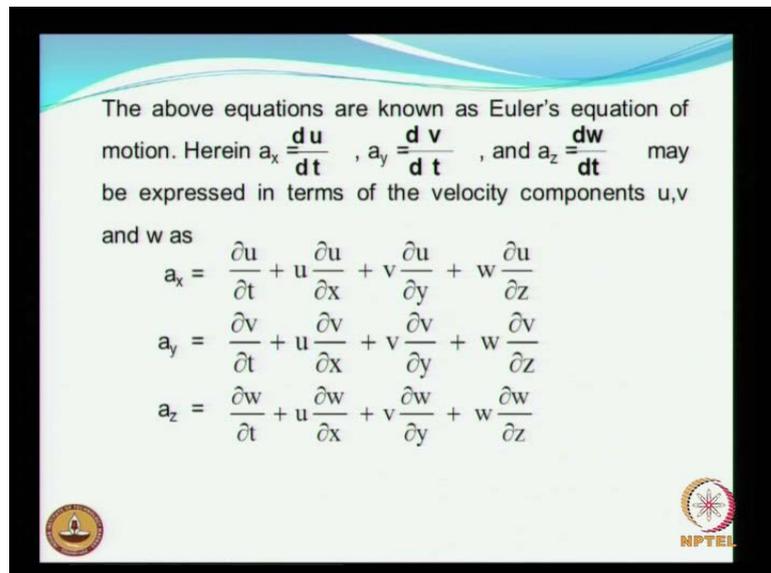
Now adding the pressure force with the body force, so body force we had already I have already explained this is the body force and this is the pressure force, so this two **has to be** have to be added.

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What does that mean, that is F_p plus F_g equal to mass into acceleration **right**, so this is the total force into mass into acceleration, so you would remove the volume. So, you get **x** minus $\frac{1}{\rho} \frac{dp}{dx}$ equal to a_x , so similarly, you have in the other two directions y, z , etcetera.

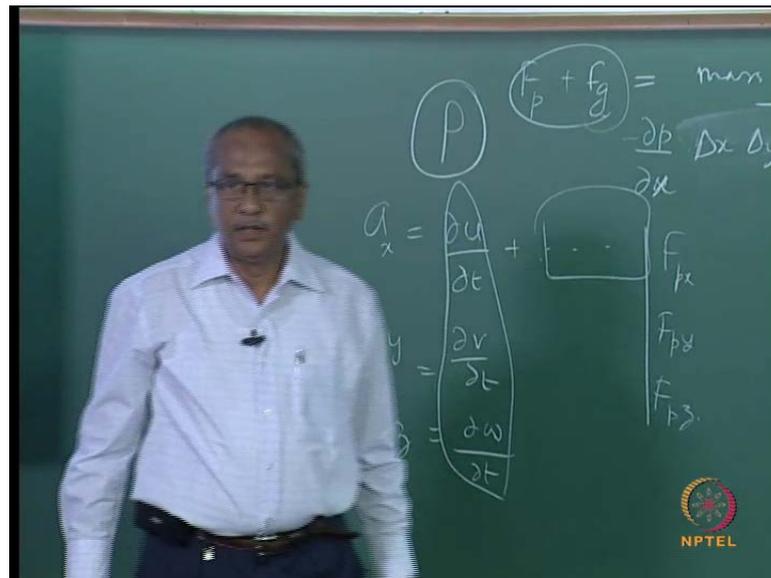
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So, this is actually called as your Euler's equation of motion, so the above equations in the three directions **this is** these are called as the **Euler's** Euler's equation of motion.

Now, you see that there is a term a_x , what is this a_x ? a_x is termed as the total acceleration, total acceleration when you want to define it in terms of the velocity components although we know that it is a definition of velocity; it has two components one is with respect to time and the other one is with respect to space, so wherever you have a...

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So, in the x axis $\frac{du}{dt}$ plus all the terms with respect to space and similarly, a_y and a_z , so here also you'll have $\frac{dv}{dt}$ here you have $\frac{dw}{dt}$, so all these things are called as these terms are called as local acceleration or temporal acceleration, so this are called as local acceleration or temporal acceleration.

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$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ are known as local or temporal accel and

$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial z}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial z}$ are the
Convective accel.

In these derivations, no assumptions have been made that ' ρ ' is a constant. Hence, these equations are applicable to compressible or incompressible, non viscous fluid in steady or unsteady state of flow.



And whereas, all the terms with **with** respect to space are called as or termed as convective acceleration, so in this derivations no assumption has been made concerning rho, rho is retained, since this equations naturally are applicable through compressible, incompressible, non viscous fluid in both **both** for steady or unsteady state of fluid is that clear.

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PATHLINES AND STREAMLINES

- A Path lines is the trace made by a single particle over a period of time. The path line shows the direction of the velocity of the fluid particle at successive instants of time.
- Streamlines show the mean direction of a number of particles at the same instant of time.
- If a camera were to take a short time exposure of a flow in which there were a large number of particles, each particle would trace a short path, which would indicate its velocity during that brief interval.

A series of curves drawn tangent to the means of the velocity vectors are Streamlines.



So, that shows with this we have, now derived the expression for Euler's equation of motion but, where and why Euler's equation and how are used we will just understand

with the help of a simple problem, one or two problems probably we will be able to see later. Now, let us try to understand or recollect it is not understanding most of you should be knowing about all these things it is only kind of recollecting, what we have studied earlier; I will just run through this slide I will not spend much of time, because it is more or less quite fundamental.

Path lines and stream lines, a path line is the trace made by a single particle within the fluid medium over a period of time, so the path, the path line shows naturally the direction of flow or direction of the velocity of the fluid particle at successive instances of time. How the particle is going to move? This can easily be understood when you want to have a some kind of a **a** flow visualization technique you can easily look at the how the different fluid particles can move also.

Stream lines show the mean direction of a number of particles at the same instance of time that is stream line; now in case if you have a camera that has to take that your having a short time of exposure **of a flow** to a flow in which there are a number of particles, each particle will naturally show a short path or short or long path **it is a it is** it is own path, which will indicate its velocity during that period of time.

Since I am saying that it is a short exposure **exposure** naturally you will be the fluid particle will **will** have moved only for a over a short path; a series of curves drawn tangent to the means of velocity vectors are called as stream lines, so these are the definitions of path lines and stream lines.

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➤ Path lines and Streamlines are identical in the Steady flow of a fluid in which there are no fluctuating velocity components, in other words, for truly steady flow. The equation of a Streamline is represented as

$$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$

The slide features a decorative blue wave header, a small circular logo in the bottom left, and the NPTEL logo in the bottom right.

Path lines and stream lines are identical in a steady flow, in which there are no fluctuating velocity components that means, what we are trying to say is, in the case of a truly steady flow a path line and stream line are going to be identical. And the equation for the stream line is given by u is velocity in the direction x divided by the distance dx elemental distance, **distance**, so u divided by dx equal to v divided by dy equal to w divided by dz .

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VELOCITY POTENTIAL

Velocity Potential is defined as a scalar function of space and time such that its derivative with respect to any direction yields velocity in that direction. Hence, for any direction S , in which the velocity is V_s

$$\frac{\partial \phi}{\partial s} = V_s$$
$$u = \frac{\partial \phi}{\partial x}, v = \frac{\partial \phi}{\partial y}$$

when substituted in continuity we get

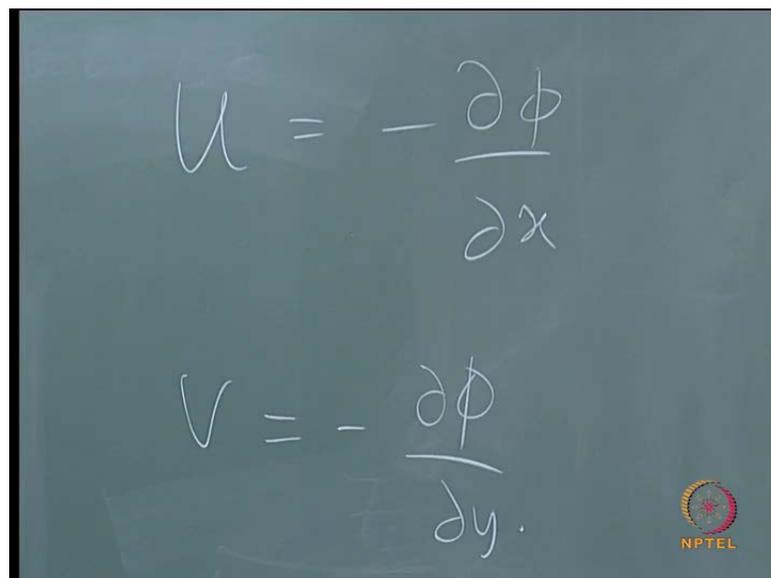
$$\nabla^2 \phi = 0$$

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Now, velocity potential **velocity potential** we will be seeing velocity potential, which will be the basic **basic** parameter or the variable which we will be consider for understanding the motions of a wave. That is when a wave is moving, what happens under the wave that is, it is a kinematics particular a dynamics, etcetera all these things are related to or derived from the velocity potential.

So, before going into all those details first let us understand, what is meant by velocity potential? Velocity potential is defined as a scalar function of space and time, such that it is derivative with respect to a given direction or a particular direction is the velocity in that direction. Hence in general a direction s in which the velocity is v_s , so $\frac{d\phi}{ds}$ equal to v_s that s is the general direction; so if you are considering a two dimensional flow u is equal to $\frac{d\phi}{dx}$ and v equal to $\frac{d\phi}{dy}$.

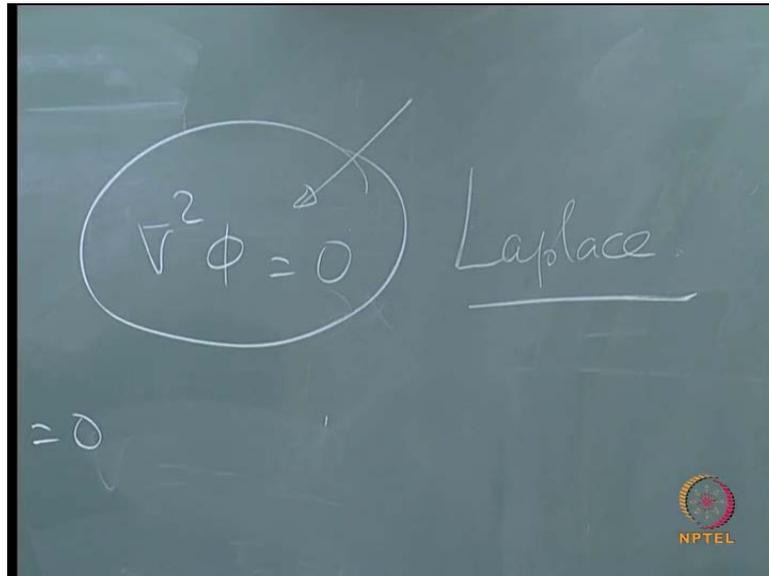
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The image shows a chalkboard with two equations written in white chalk. The first equation is $u = -\frac{\partial \phi}{\partial x}$ and the second equation is $v = -\frac{\partial \phi}{\partial y}$. In the bottom right corner of the chalkboard, there is a small circular logo with the text 'NPTEL' below it.

As I said earlier, there are several books available on the net, there are several e books also available particularly on a fluid mechanics, so you can refer to any of the books and this only just to give you some idea. Now, some of the books may would consider this as minus, so when you there is no harm in either representing this positive or negative but, when you are trying to be involved in solving the problem make sure that, your consistent either you use a negative sign or you use a positive sign for the velocity components, you cannot keep changing it.

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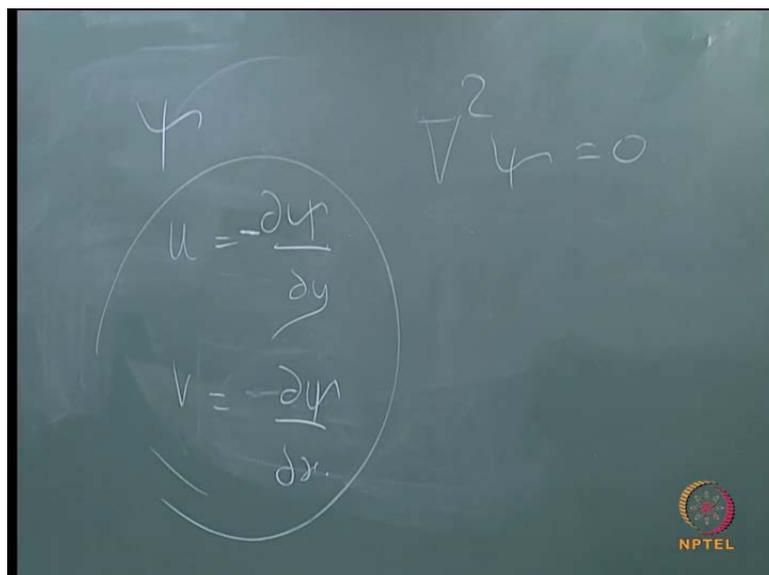
$\nabla^2 \phi = 0$ Laplace

$= 0$

NPTEL

So, we are already seen the continuity equation, so when you substitute for u and v then you will have the del square phi equal to 0 (No audio from 19:15 to 19:27), so this is the equation I mean the equation for the continuity, so once you substitute for u and v . So you are going to get the Laplace equation and this Laplace equation will be the base from which this will be actually a governing equation, when you start deriving the expressions for the kinematics of an ocean wave (No audio from 20:00 to 20:12).

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ψ $\nabla^2 \psi = 0$

$u = -\frac{\partial \psi}{\partial y}$

$v = -\frac{\partial \psi}{\partial x}$

NPTEL

Now, stream function, stream function again is called is a scalar function of both space and time, such that the partial derivative with respect to any direction gives the velocity are right angles to the, right angles I mean the counter clock wise direction to this direction. So, what does this say u equal to y whereas, v equal to since, we are talking about counter clockwise that is going to be a negative sign for u, again if you substitute this you can prove that, this is equal to 0, this is substitute in the continuity equation.

I am sure that all **all** these things have been already you would have seen in your life, while doing your undergraduate course, so now, we will look at the bernoulli equation why bernoulli equation the common example for the application of bernoulli equation is when you have a pipeline and you have a either it is tilted or it is horizontal, then you see that there is a **a** flow taking place pressure at one point is known.

You want to know the pressure at the other end or may be you are interested in finding out what to could be the pressure drop, etcetera so it has a wide application, **in the field of** particularly in the field of fluid mechanics.

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BERNOULLI EQUATION

Let Ω be force potential

i.e.

$$X = \frac{\partial \Omega}{\partial x}, Y = -\frac{\partial \Omega}{\partial y}, Z = -\frac{\partial \Omega}{\partial z}$$

$$u = \frac{\partial \phi}{\partial x}, v = \frac{\partial \phi}{\partial y}, w = \frac{\partial \phi}{\partial z}$$

So, let me start with force potential I define a force potential as shown here, where in x, y, z are represented as a there and you would u are also u, v, w in terms of velocity potential is also given there.

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Substituting these in Euler's equation and using the irrotational flow conditions

$$X- \frac{1}{\rho} \frac{\partial p}{\partial x} = a_x$$

we get the following set of equations.

$$\frac{\partial^2 \phi}{\partial x \partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} + w \frac{\partial w}{\partial x} = \frac{-\partial \Omega}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial^2 \phi}{\partial y \partial t} + u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} + w \frac{\partial w}{\partial y} = \frac{-\partial \Omega}{\partial y} - \frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$\frac{\partial^2 \phi}{\partial z \partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial v}{\partial z} + w \frac{\partial w}{\partial z} = \frac{-\partial \Omega}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial z}$$

Irrotational flow conditions:

$$\omega_x = 0; \quad \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

$$\omega_y = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$

$$\omega_z = 0; \quad \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$


Now, substituting the Euler's equation **substituting the Euler's equation** and using **the substituting the** substituting all these things in the, I mean all these things in the Euler's equation of motion and also adopting the irrotational flow conditions, we can get a set of equations as shown here, in the x direction, y direction and z direction.

(Refer Slide Time: 23:25)

Substituting these in Euler's equation and using the irrotational flow conditions

$$X- \frac{1}{\rho} \frac{\partial p}{\partial x} = a_x$$

we get the following set of equations.

$$\frac{\partial^2 \phi}{\partial x \partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} + w \frac{\partial w}{\partial x} = \frac{-\partial \Omega}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial^2 \phi}{\partial y \partial t} + u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} + w \frac{\partial w}{\partial y} = \frac{-\partial \Omega}{\partial y} - \frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$\frac{\partial^2 \phi}{\partial z \partial t} + u \frac{\partial u}{\partial z} + v \frac{\partial v}{\partial z} + w \frac{\partial w}{\partial z} = \frac{-\partial \Omega}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial z}$$

Irrotational flow conditions:

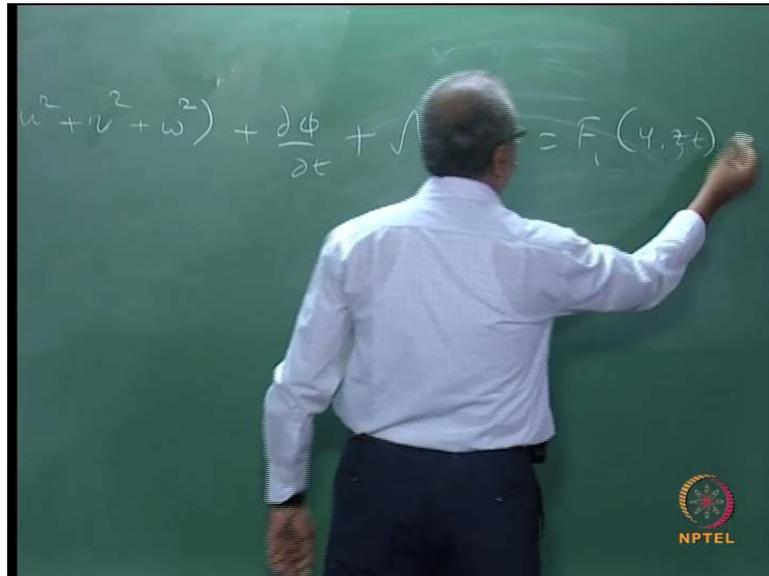
$$\omega_x = 0; \quad \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

$$\omega_y = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$

$$\omega_z = 0; \quad \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$


Here, if the rho is the constant **of rho is the constant** integration with respect to x, y, z look at this expressions, you integrate these expressions you will have half into u, w **sorry** v (No audio from 23:49 to 24:03) of course, you have the time also.

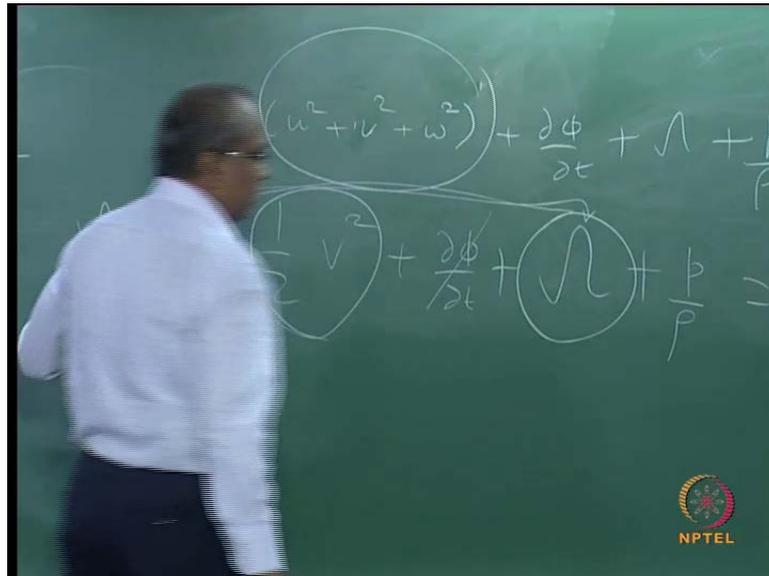
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So for **for** example, in the x direction this is the kind of expression you would have or equation, so what we are trying to do is bernoulli equation we are representing the force velocity potential, substituting this expressions in the Euler's equation. Euler's equation we have derived and at the same time your also taking care of using the irrotational flow conditions.

So once you do this **this** is the kind of expression you will get and once you get this, if rho is the constant of integration **integration** of this expression would result in this expression. Basically what is Bernoulli's equation? You integrate Euler's equation of motion and make sure that you are using irrotational flow conditions, that will yield you the Bernoulli equation.

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So, now if u v and all these things are resolvable of the v, then we can simply say call it as v that is the velocity head or velocity component so half into v square plus dou phi by dou t plus p by rho equal to...

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If u, v, w are resolvable of V

$$\frac{1}{2}v^2 + \frac{\partial \phi}{\partial t} + \Omega + \frac{p}{\rho} = F(t)$$

For steady flow 't' disappears
 We also have $-g = \frac{\partial \Omega}{\partial h}$ if - h is positive upwards

Hence $\Omega = gh$.

Total head = $\frac{v^2}{2} + gh + \frac{p}{\rho} = \text{constant}$

(Kinetic head) + (Potential head) + (Pressure head)

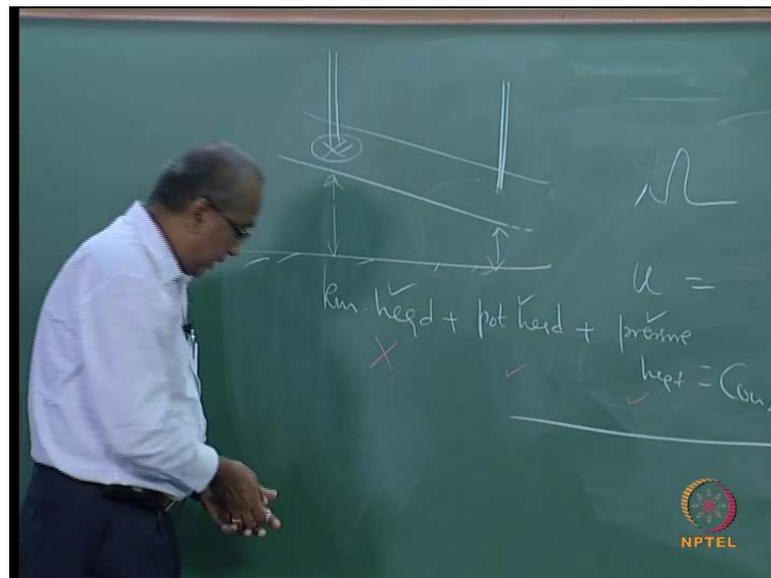


In in a in a general form you can write, represent the bernoulli equation in this form, so when you consider only steady flow t will disappear, so this will go only for steady flow. So, minus g equal to minus if we also have that this acceleration due to gravity and this is with respect to elevation the, then you can represent this as g h. So, now you substitute in

this expression you have the velocity head or the kinetic head, then you have the potential head, which is now this 1 and plus the pressure head is that clear.

So, this is what is called as your Bernoulli equation? Bernoulli equation is kinetic head plus potential head plus the pressure head is a constant, so most of applications, what will happen if you have a pipeline here.

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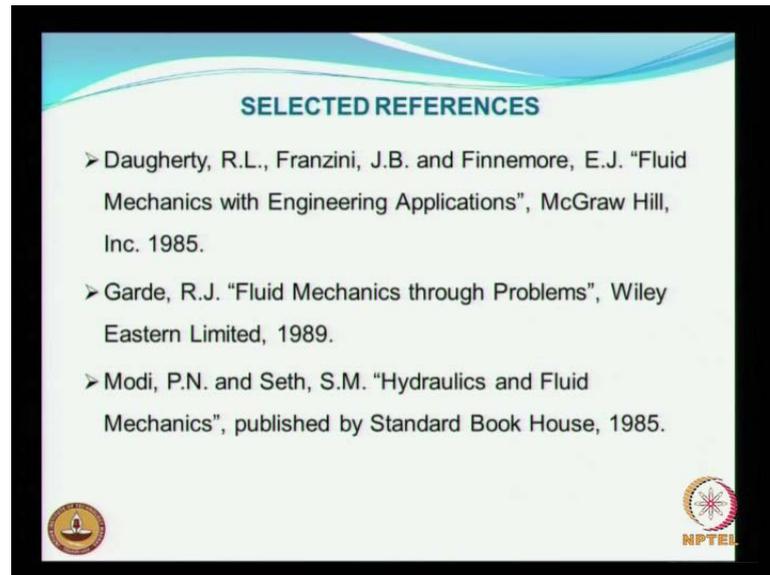


If this is inclined like this, then this will be your head stay it going to take care of the potential head, suppose in case you have the velocity at this location as well as the pressure head at that location and at this location you have the pressure head or one of the either the pressure head or the velocity head.

Using the equation see total energy is a constant at this location as well as at this location the same thing will hold good, so if all these things are available at location one and at location two this is not available but, these are available, then you can determine this by equating the total energy on both the locations.

So, in this way this becomes very handy in applying these two for this kind of a problems, so naturally we will have a one or two problems in order to again recollect whatever, we have studied earlier.

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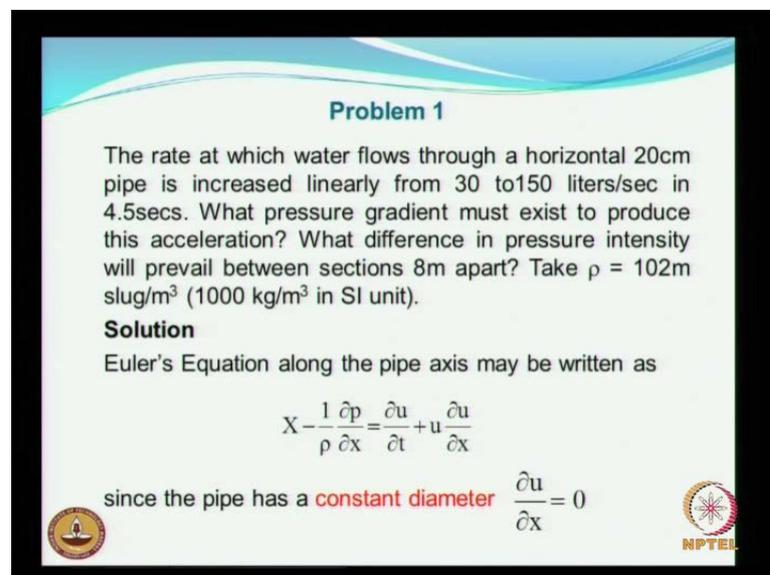
SELECTED REFERENCES

- Daugherty, R.L., Franzini, J.B. and Finnemore, E.J. "Fluid Mechanics with Engineering Applications", McGraw Hill, Inc. 1985.
- Garde, R.J. "Fluid Mechanics through Problems", Wiley Eastern Limited, 1989.
- Modi, P.N. and Seth, S.M. "Hydraulics and Fluid Mechanics", published by Standard Book House, 1985.

The slide features a blue header with the title "SELECTED REFERENCES" in bold. Below the title are three bullet points, each starting with a right-pointing arrow. The slide is framed by a black border and includes two circular logos at the bottom: one on the left and the NPTEL logo on the right.

So, although there are a number of books available for the fluid mechanics, I have referred some of these books, which I found as given very clearly the aspects of whatever the topics, whatever I have covered.

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Problem 1

The rate at which water flows through a horizontal 20cm pipe is increased linearly from 30 to 150 liters/sec in 4.5secs. What pressure gradient must exist to produce this acceleration? What difference in pressure intensity will prevail between sections 8m apart? Take $\rho = 102m \text{ slug/m}^3$ (1000 kg/m³ in SI unit).

Solution

Euler's Equation along the pipe axis may be written as

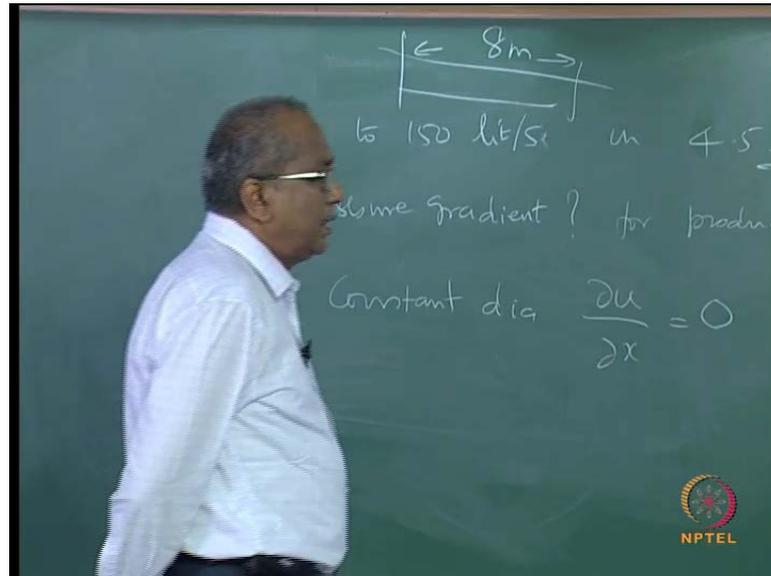
$$X - \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}$$

since the pipe has a **constant diameter** $\frac{\partial u}{\partial x} = 0$

The slide has a blue header with the title "Problem 1" in bold. The text is in black. The solution section includes the Euler's equation in a boxed format. The slide is framed by a black border and includes two circular logos at the bottom: one on the left and the NPTEL logo on the right.

Now let us look into this problem, this is basically to apply the Euler's equation of motion, the problem is, what is the problem? The rate at which the flow is taking place that is the water is flowing **flowing** the rate at which the water flows through a horizontal 20 centimeter pipe is increased linearly from 30 to 150 liters per second.

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And within what time in 4.5 seconds, what you supposed to do? Get pressure gradient, what is a pressure gradient that should exist? To produce this kind of an acceleration you understood.

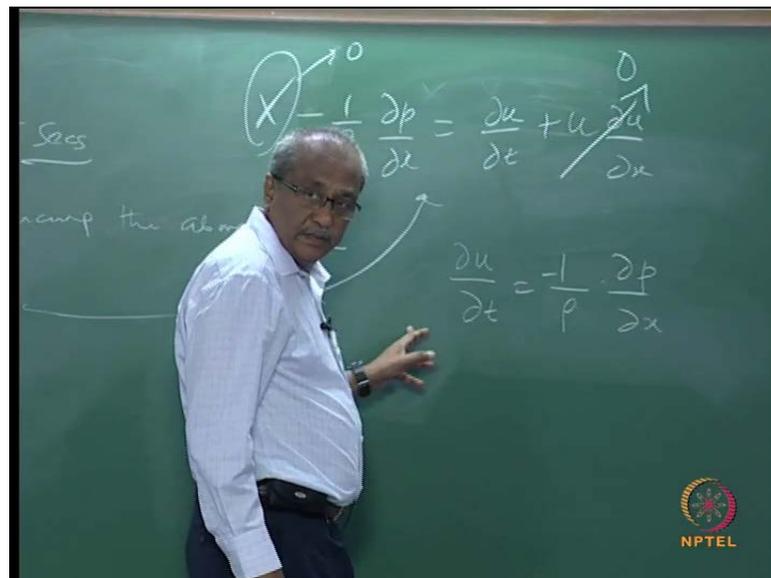
What is the pressure gradient for producing the above acceleration? **What is the difference** what difference in pressure intensity will prevail between, so the pipeline is there, so this is the kind of a difference we are having, what is the now pressure difference and pressure naturally, there will be a difference in the pressure intensity; so the next problem is what is the difference in pressure intensity, that will prevail between any two sections 8 meters apart.

So, all these words are very important, particularly with respect to the position of the pipeline, rate at which it is being pumped all these things, because try to recollect the definitions of the different types of flows. What is a steady flow? What is the difference between steady flow and a uniform flow, etcetera, so when you look at such problems, these sentences really matter a lot.

For example in this problem has anything been said about the pipeline, the pipeline can be horizontal or it can be inclined also, or the pipeline can also be constant diameter or varying diameter. And in fact these were kinds of **a** examples we took, when we were telling about, when we were discussing about the different types of flow.

So, since the pipe is a constant diameter, since a constant diameter what happens constant dia, naturally $\frac{du}{dx}$ will be equal to 0, now since the pipeline is a horizontal, that gives you another indication that the body force per unit volume is also 0 that is **in the direction of wave flow** in the direction of flow.

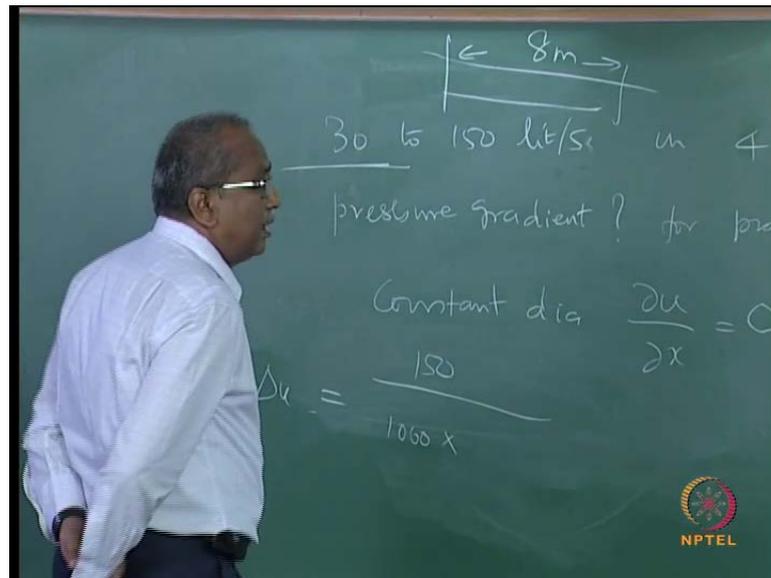
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So, we **we** started this kind of an equation initially for the x direction, which is nothing but, the Euler's equation of motion (No audio from 33:52 to 34:04), so since it is equal to constant diameter, so this will be equal to 0, next is since it is horizontal body force is going to be 0 then you are left only with $\frac{du}{dt}$ equal to, so this takes care of this gives you the relationship of the acceleration and the pressure gradient along the pipeline and that is what you are interested in.

So, the changes in the velocity from 30 to 150 liters what does that mean you can get your Δu , so Δu because the diameter of pipeline is also known to you, so you can calculate the area and velocity will be as shown here 150 that is the discharge, **that is the discharge** divided by your pi by 4 into your diameter square.

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since it is horizontal, the body force per unit volume, X along the flow direction is also zero. The above equation of motion, therefore, reduces to

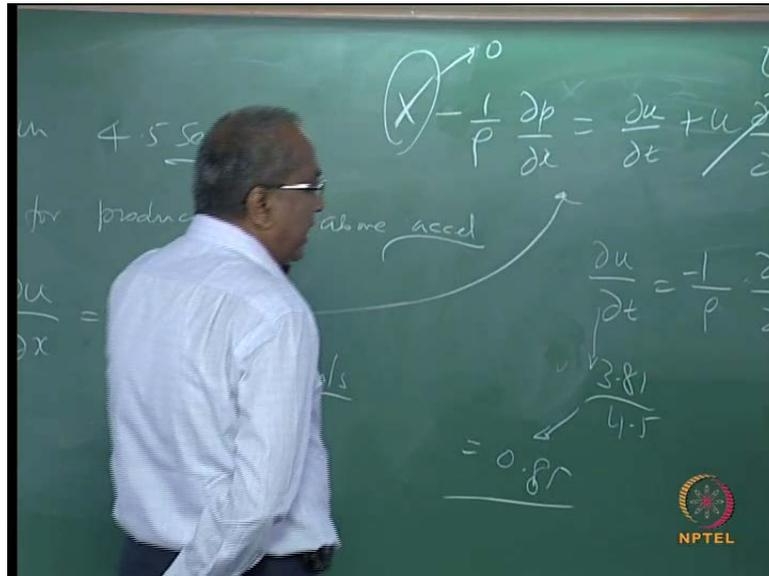
$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

The changes in velocity as the flow changes from 30 to 150 liters/sec in

$$\Delta u = \frac{150}{1000 \cdot \frac{\pi}{4} \cdot (0.20)^2} - \frac{30}{1000 \cdot \frac{\pi}{4} \cdot (0.20)^2}$$
$$= 3.81 \text{ m/s}$$
$$\therefore \frac{\partial u}{\partial t} = \frac{3.81}{4.5} = 0.847 \text{ m/s}^2$$

So, we are basically trying to get the area, so the velocity at the upper point that is at the point where your discharge is more, minus the point at which the discharge is less that is **thirty** 30 liters per second, you have the 1000 to take care of the conversion, so you have delta u will work out to 3.81 meter per...

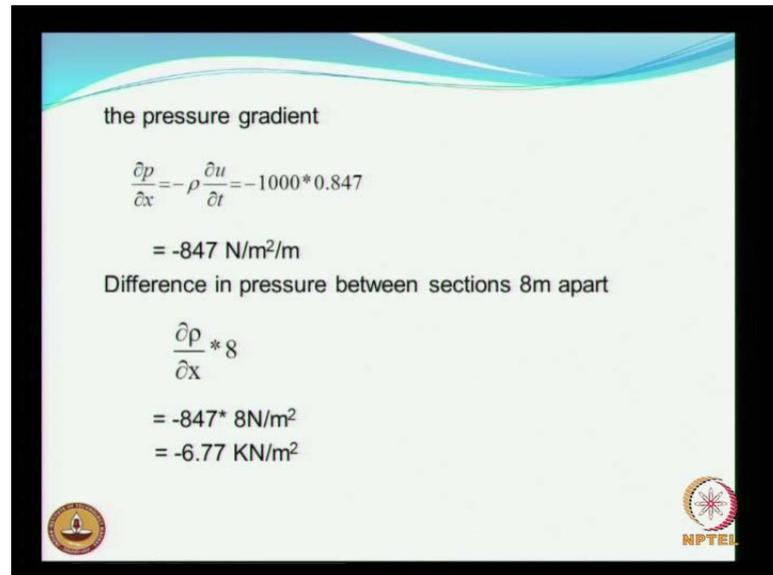
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But, this acceleration is going to take place **within the this** within how much within 4.5 seconds, so from this $\frac{du}{dt}$ is going to be 3.81 divided by 4.5, so $\frac{du}{dt}$ is going to be equal to 0.85.

So, now what we are supposed to calculate is a pressure gradient, so use the same expression, so you get the pressure gradient in terms of Newton as minus 847 Newton's plus meter square per meter, the pressure drop here is per meter; but, what is the problem **asking you** is asking you to calculate the pressure drop over a distance of 8 meters and that is going to be minus 6.77 kilo Newton per meter square, is that clear.

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the pressure gradient

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial u}{\partial t} = -1000 * 0.847$$
$$= -847 \text{ N/m}^2/\text{m}$$

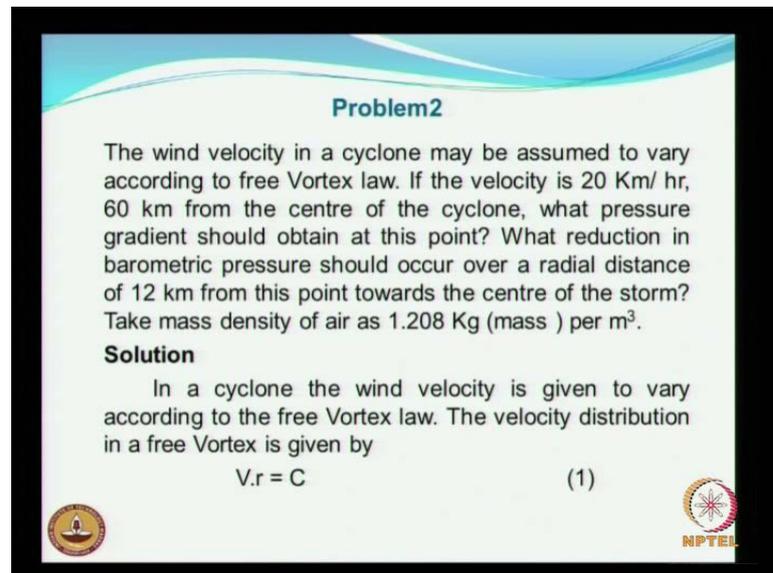
Difference in pressure between sections 8m apart

$$\frac{\partial p}{\partial x} * 8$$
$$= -847 * 8 \text{ N/m}^2$$
$$= -6.77 \text{ KN/m}^2$$

The slide includes two logos: a circular logo on the bottom left and the NPTEL logo on the bottom right.

So, now this **this** is a classical problem which helps us understand complication of all Euler's equation of motion, there are several such problems available in books, you may just go through them and try to familiarize yourself is very interesting any doubts, no doubts.

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Problem2

The wind velocity in a cyclone may be assumed to vary according to free Vortex law. If the velocity is 20 Km/ hr, 60 km from the centre of the cyclone, what pressure gradient should obtain at this point? What reduction in barometric pressure should occur over a radial distance of 12 km from this point towards the centre of the storm? Take mass density of air as 1.208 Kg (mass) per m³.

Solution

In a cyclone the wind velocity is given to vary according to the free Vortex law. The velocity distribution in a free Vortex is given by

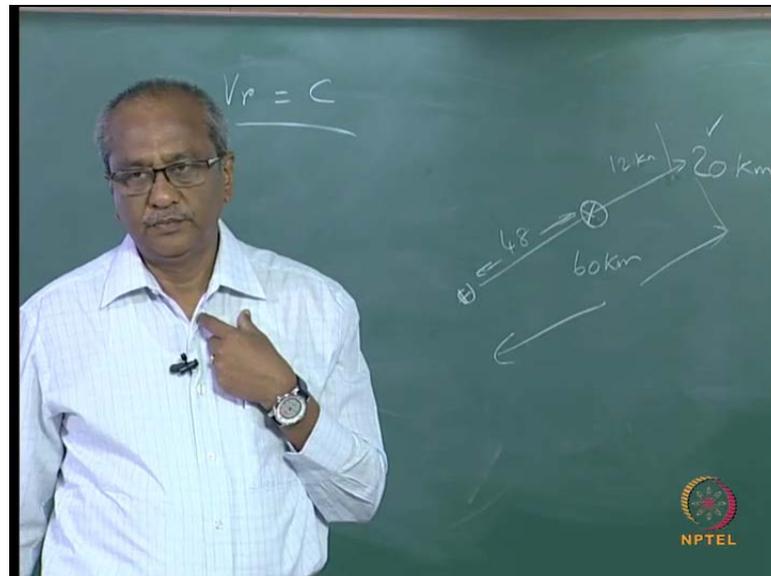
$$V.r = C \quad (1)$$

The slide includes two logos: a circular logo on the bottom left and the NPTEL logo on the bottom right.

We go on to the next problem here, we are talking about the wind velocity in a cyclone, so the problem says the wind velocity in a cyclone may be assumed to follow the free

vortex law, the free vortex law is look at some standard books for getting this details on the vortex law, if the velocity is 20 kmh, 60 60 kilometers from the center of the cyclone.

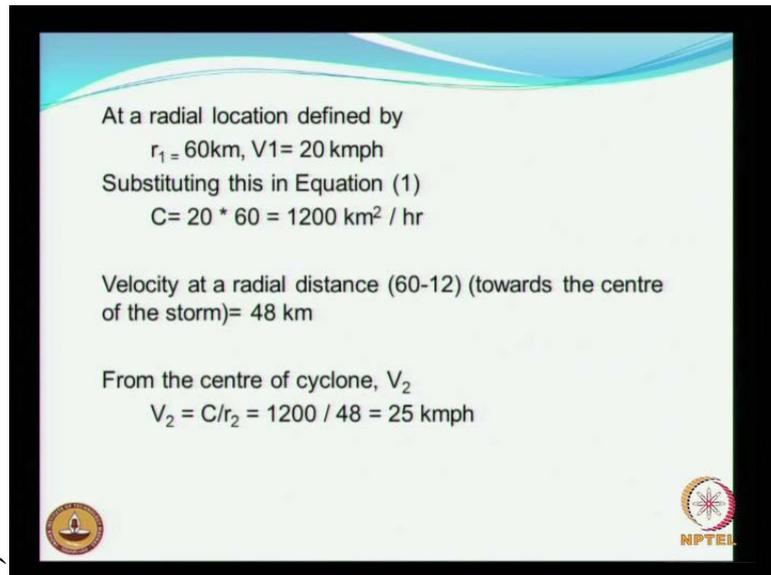
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So, this is the center of the cyclone, we have a distance of 60 kilometers that is the velocity we have, what reduction in the barometric pressure should occur over a radial distance 12 kilometers from this point towards the center; so this is 60 60 kilometers so basically, what we are needing 12 kilometers from this point.

So, this will be your 48 meters from the (()) of the storm, so what reduction in the barometric pressure should occur over a radial distance 12 kilometers from this point towards the center of the storm. So, basically you are trying to find out what is the kind of reduction in the barometric pressure, is that clear.

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At a radial location defined by
 $r_1 = 60\text{km}$, $V_1 = 20\text{ kmph}$
Substituting this in Equation (1)
 $C = 20 * 60 = 1200\text{ km}^2 / \text{hr}$

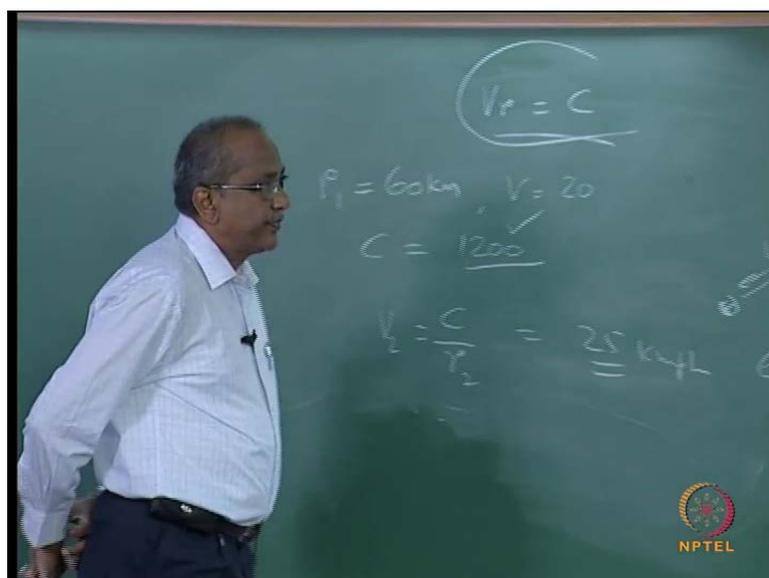
Velocity at a radial distance (60-12) (towards the centre of the storm) = 48 km

From the centre of cyclone, V_2
 $V_2 = C/r_2 = 1200 / 48 = 25\text{ kmph}$

The slide features a blue header with a white wave graphic, a white background for the text, and two circular logos at the bottom: one on the left and the NPTEL logo on the right.

So, in so, we have already been told that it is going to follow this law, so now at a radial distance **let us take** let me take there are two radial distance at 60 kilometers per hour, V_1 is 20, so according to the law you can usually calculate C as 1200, I am not writing the units, because anyway it is there on the slide, so velocity at use the same equation for V_2 , so V_2 will be C/r_2 by C by r_2 , since r is already determined.

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A lecturer in a white shirt and glasses stands to the left of a green chalkboard. The board contains handwritten equations: $V_1 = C$ (circled), $r_1 = 60\text{km}$, $V_1 = 20$, $C = 1200$, and $V_2 = \frac{C}{r_2} = 25\text{ kmph}$. The NPTEL logo is visible in the bottom right corner of the board.

So, you can get V_2 as 25 is that clear, so what is the velocity at this location, so at this location naturally your velocity should be more compared to this location, so it is far

away from the (()) of the cyclone. Now, you use from Bernoulli equation, we are considering the velocity and the pressure head equal to a constant, this is what the Bernoulli equation states, that is $p + \rho v^2 = \text{constant}$.

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$$\frac{dp}{dr} = -\rho v \frac{dv}{dr}$$

$$= -\rho \frac{v^2}{r}$$

When you differentiate with respect to r , because we are interested in finding out the pressure variation **in the** along the radial direction, so when you do that, when you integrate this you will, equal to now, we already know that, this is the vortex formula. So, when you derive this I mean differentiate this from this you can get dv/dr equal to v/r is that clear; so $dp/dr = -\rho v \cdot dv/dr$, so you use this here, so you are going to get $\rho v^2/r$.

So, this will be, so the pressure gradient at a radial distance of 20 kilometers, where a distance of 60 kilometers, where the velocity is **60** 20 kilometers, then dp/dr what is the pressure gradient use this expression. So, **you are** you have the velocity use the velocity here and the radial distance is also known to you use, so you will get 6.09 Newton per meter square per kilometer.

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The pressure gradient at a radial distance of 60km where the velocity is 20 kmph

$$\left(\frac{dp}{dr}\right)_{r=60\text{km}} = 1.208 * \frac{\left[\frac{20*1000}{3600}\right]^2}{60*1000} \quad \frac{dp}{dr} = \rho \frac{v^2}{r}$$
$$= 0.621 * 9.81 \text{ N/m}^2 \text{ per m}$$
$$= 6.09 \text{ N/m}^2 \text{ per km.}$$

Reduction in barometric pressure over a radial distance of 12km from

$$r_1 = 60\text{kms to } r_2 = 48\text{km}$$
$$r_1 = 60\text{km, } V_1 = 20\text{kmph ; } r_2 = 48\text{km, } V_2 = 25\text{kmph}$$


So similarly, reduction in the barometric pressure, where a distance 12 kilometers that is within this distance, so r_1 is already known to you, r_2 is 48, r_1 is 60, r_2 is 48, v_1 is equal to known to you. So, all this values are known to you, then use the bernoulli equation, bernoulli's equation at these two points are applied as shown here, so the pressure gradient can be obtained, this is the pressure gradient use the corresponding velocities and you will get this is the kind of reduction in the barometric pressure, you understood.

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using Bernoulli's equation,

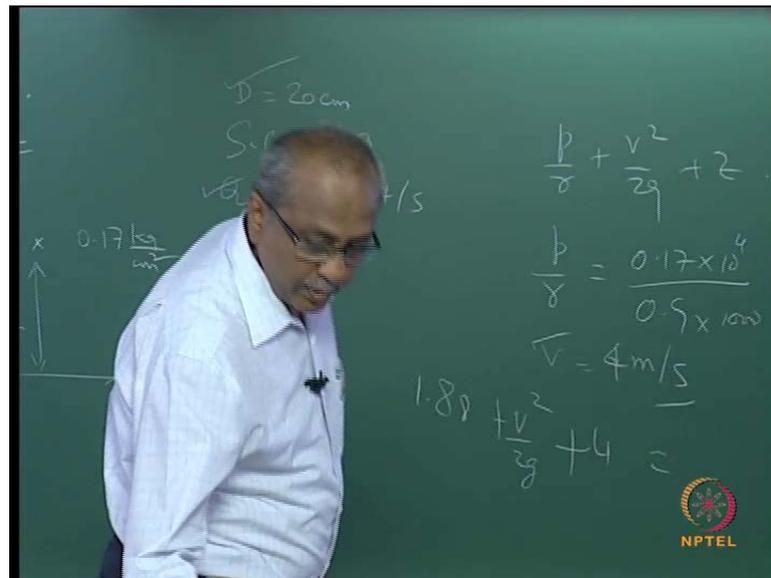
$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

(or)

$$(p_1 - p_2) = (1/2)\rho [V_2^2 - V_1^2]$$
$$= \frac{1.208}{2} \left[\left(\frac{25*10^3}{3600}\right)^2 - \left(\frac{20*10^3}{3600}\right)^2 \right]$$
$$= 10.49 \text{ N/m}^2 \text{ (Reduction in barometric pressure)}$$


So, in this equation you see that we have effectively adopted the Bernoulli equation in order to find out the pressure gradient, we will continue to see some few simple problems. Again now concerning the application of Bernoulli equation, it is a small very simple problem, say diameter is given as 20 centimeters and specific gravity is 0.9, then you have discharge rate equal to 120 liters per second.

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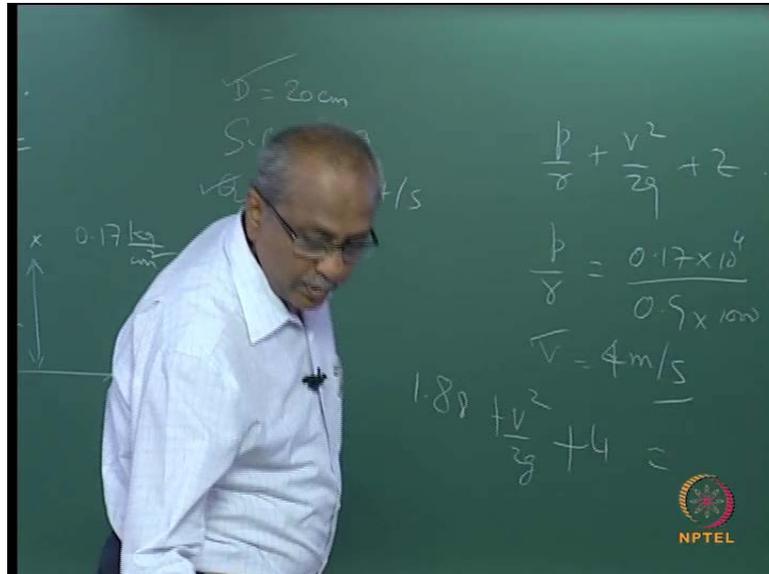
So, at any point, so this is a datum at any point A which is 4 meters above the datum line and here the gage pressure is given as, so calculate the total energy at this point in meters of oil it is a very simple problem. So, you know the total energy is p by γ plus v square by $2g$ plus z , so p by γ is nothing but, you have this, this you need to convert it; so this will be point specific weight, so specific gravity also has to be taken, so this will come to 1.88 meters of oil.

So, that is the potential head, then you have to calculate here because the Q is given, so Q is can be written in terms of a point in terms of meters meter cube per second as it is given the, so that will be and then area of cross section can be obtained, because this is 0.2 meter meters; so you have Q is calculated, Q is converted and a is calculated and so velocity can be calculated as q by v Q by A .

So, the velocity head will be 4 meters per second, so velocity head will be v square $2g$ by v square into divided by $2g$, so that will be the, so the total head will be pressure head that is 1.88, then plus v square by $2g$ v is already known plus you have to make

sure that you include this datum head, that is 4 meters, so you get that is that as 6.69 meters per.

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Problem 4

A 25cm pipe carries water at a velocity of 25.6 m/sec. At a point A, measurement of pressure and elevation were 3.77 kg/cm² and 29.5 m respectively. The pressure and elevation at point B were 3.02 kg/cm² and 32.5m respectively. For steady flow, find the loss of head between A and B.

Solution

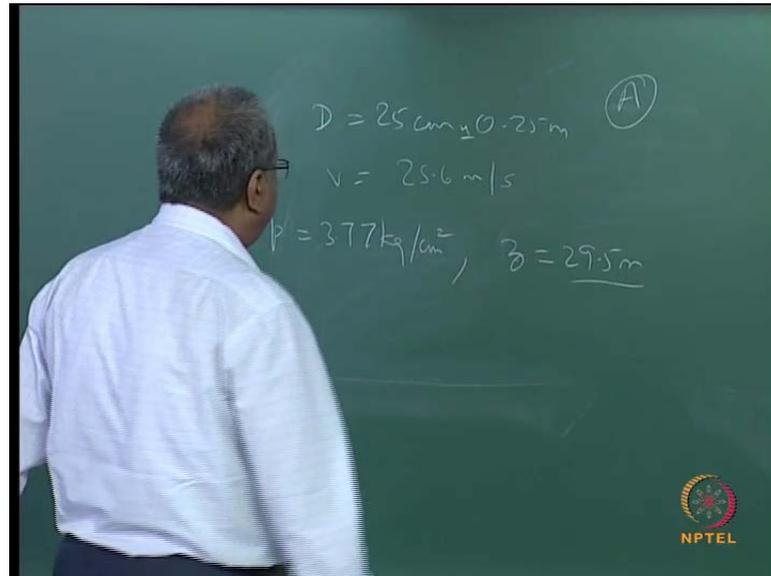
Total energy in terms of meters of water is given by

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z$$

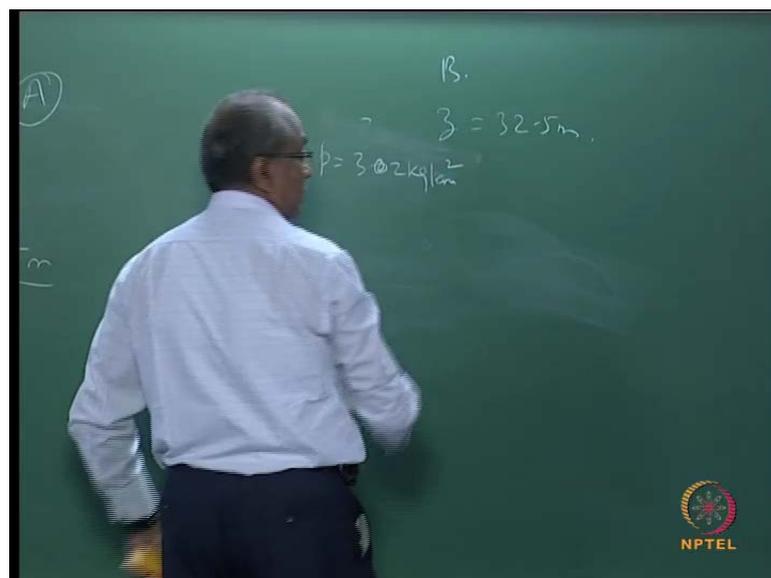



So, this is a very simple problem, so you know next is, this is the next problem, so read the problem keep reading the problem and try to understand, what the problem tries to ask you a 20 centimeters diameter pipe, so diameter is 25 centimeters or 0.25 meters, then what the velocity is 25 meters per second, that is the velocity at any point A at a point A measurement of pressure and elevation.

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So, pressure is 3.7 kg per centimeter square and head, let me call it as is the pressure at some other location that is B, so this is A, so your B is 32.5 meters and your this one is 3.6 kg per, so for a steady flow find the head loss between A and B; so this is A the details are given here and this is B details are given here.

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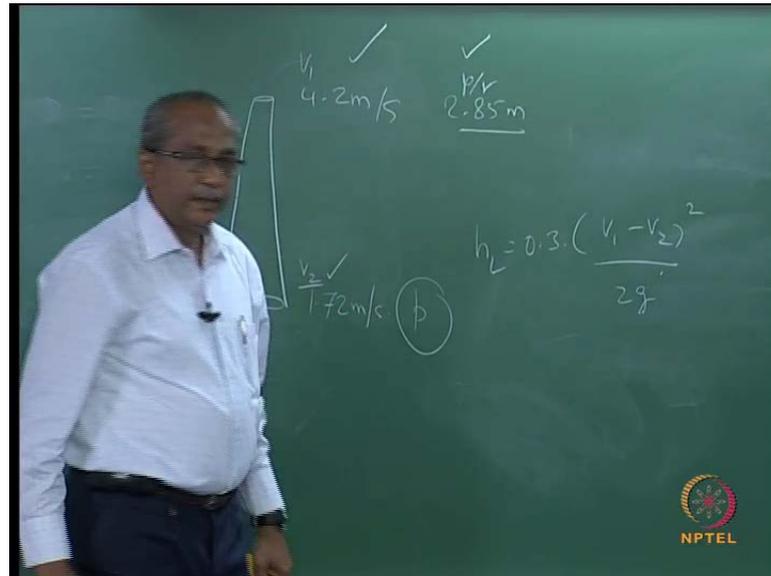
At point A
 $(p / \gamma) = (3.77 * 10^4) / 1000$
 $= 37.70 \text{ m of water}$
 $(V^2 / 2g) = (25.6)^2 / (2 * 9.81)$
 $= 33.40 \text{ m of water}$
 $z = 29.50 \text{ m}$
Total energy at
A = $(37.70 + 33.40 + 29.50) = 100.60 \text{ m}$
At point B
 $(p / \gamma) = (3.02 * 10^4) / 1000$
 $= 30.2 \text{ m of water}$

The slide features a decorative blue and green wave graphic at the top. In the bottom left corner, there is a circular logo with a lamp. In the bottom right corner, there is a circular logo with a gear and the text 'NPTEL' below it.

So, you use the total energy as per the Bernoulli's equation, so at point A you can calculate with this data that will be 37.7 meters of water and with the velocity you can calculate the velocity head and the elevation is given, so the total energy will be total head will be 100.60 meters.

But at point B this is p / γ can easily be calculated, so these can also be calculated, then you have this one. So, the total energy head will be 96.10, so you see that the head loss will be this minus this (Refer Slide Time: 51:00) this I have got which is 4.5 meters, total energy loss.

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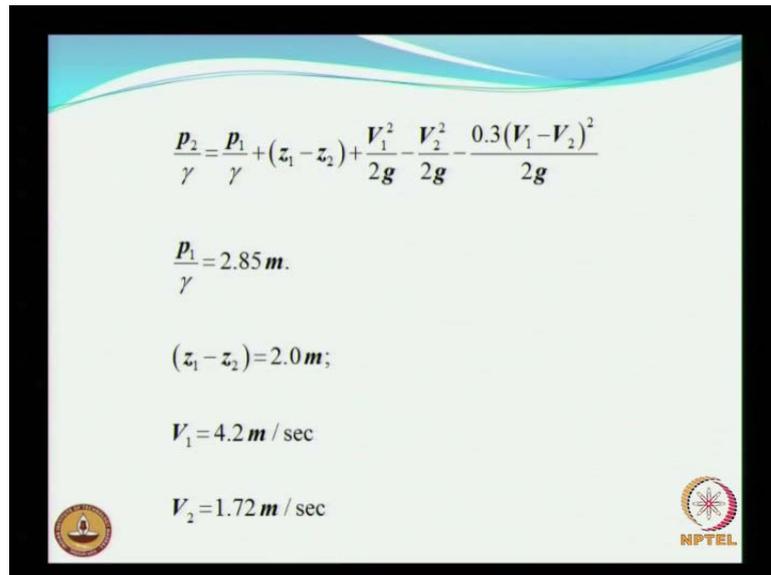
See, all these things are quite important particularly when you are working with **the flow problem** flow related problems, you need to use some of these calculations then, next is a conical tube is fixed vertically **with it is small** with it is small vertically down. So, the smaller **smaller** end is upwards the velocity at the velocity down the tube is 4 meters per second 4.2 meters per second at the upper end and at the lower end it is 1.72 per second.

So, the length of the tube is 2 meters and the pressure head at the upper end is pressure head is 2.85 meters, now the last in the head is expressed; so the head loss is expressed as $0.3 \text{ into } v_1 \text{ minus } v_2 \text{ whole square divided by } 2g$, so where v_1 and velocity, so this will be v_1 and this will be v_2 , is that clear.

So, what is the pressure, so we have all the all the details, so what is the pressure head at the lower end, so the pressure head here is given velocity head is given, velocity head at this end is given but, there are also given the head loss due to friction; so what is the pressure head at this end, so usually the problems will be related like this.

So, according to **the** the moment you apply the bernoulli equation for these two ends, so this is the left hand end and on the right hand side, you have plus the head loss due to whatever head loss it may be due to so many factors is that clear. So, from this what we need is from this equation, what we need is $P_2 \text{ by } \gamma$ this is what we need, so we simplify this and this is the final expression.

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$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} + (z_1 - z_2) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{0.3(V_1 - V_2)^2}{2g}$$

$$\frac{p_1}{\gamma} = 2.85 \text{ m.}$$

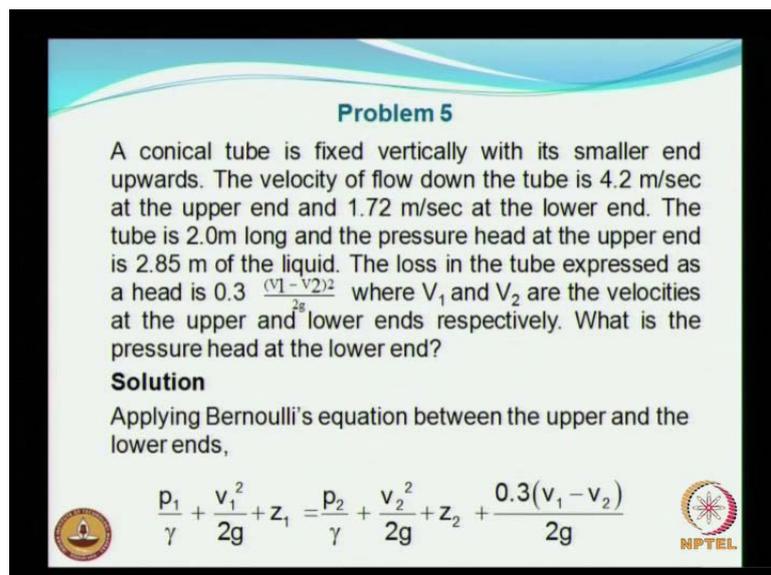
$$(z_1 - z_2) = 2.0 \text{ m;}$$

$$V_1 = 4.2 \text{ m / sec}$$

$$V_2 = 1.72 \text{ m / sec}$$



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Problem 5

A conical tube is fixed vertically with its smaller end upwards. The velocity of flow down the tube is 4.2 m/sec at the upper end and 1.72 m/sec at the lower end. The tube is 2.0m long and the pressure head at the upper end is 2.85 m of the liquid. The loss in the tube expressed as a head is $0.3 \frac{(V_1 - V_2)^2}{2g}$ where V_1 and V_2 are the velocities at the upper and lower ends respectively. What is the pressure head at the lower end?

Solution

Applying Bernoulli's equation between the upper and the lower ends,

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + \frac{0.3(v_1 - v_2)^2}{2g}$$


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by substitution we get

$$\frac{P_2}{\gamma} = 2.85 + 2.0 + \frac{(4.2)^2}{2 \cdot 9.81} - \frac{(1.72)^2}{2 \cdot 9.81} - \frac{0.3(4.2 - 1.72)^2}{2 \cdot 9.81}$$

or $\frac{P_2}{\gamma} = (2.85 + 2.0 + 0.899 - 0.151 - 0.094)$

$= 5.504m$

\therefore Pressure head at lower end

$$\frac{P_2}{\gamma} = 5.504m \text{ of liquid.}$$


So, p_1 by γ is calculated already given then, z_1 minus z_2 can be calculated that is 2 meters the difference then v_1 is already given, v_2 is given so substitute in that expression so you get p_2 by γ that is the pressure head at this location as 5.5.

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Problem.6

In a three dimensional flow, velocity components in any two directions are as given below. Find the velocity component in the third direction such that the continuity equation is satisfied.

i. $u = x^3 = y^2 + 2z^2$; $v = -x^2y - yz - xy$

ii. $u = \frac{-2xyz}{(x^2 + y^2)^2}$; $w = \frac{y}{(x^2 + y^2)}$

Solution:

$$\frac{\partial u}{\partial x} = 3x^2, \quad \frac{\partial v}{\partial y} = -x^2 - z - x$$

Substituting the above in continuity equation we get

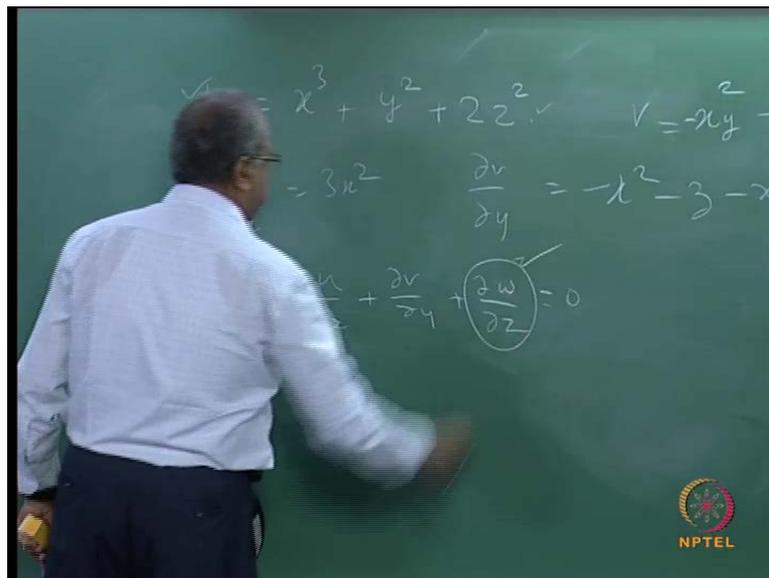


Now, we will just try to familiarize ourselves with this application of irrotational conditions, than **the velocity potential calculations the meaning of** velocity potential the meaning of stream function, etcetera through I think I should have about two or **three**

three or four problems, just three or four problems, so with that we can complete the basics of fluid mechanics any of you have any doubts it is all quite straight forward.

In this problem a three dimensional flow is considered, so the velocity components in any of the two directions are known to us, so we need to find out the **other one** other component and the assumption that satisfies the continuity equation.

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So, u is given and v is also given as, so there is one more problem, so first I will explain the first problem, so do u by do x you can calculate, do v by do y that is calculated, so you know that the continuity equation (No audio from 55:55 to 56:06). So, we need to know this because this is **this is no** there is no substitute in this equation, which is the continuity equation.

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$$3x^2 - x^2 - z - z + \frac{\partial W}{\partial z} = 0$$

$$\frac{\partial W}{\partial z} = x + z - 2x^2$$
 Integrating both sides, we get

$$W = \left(xz + \frac{z^2}{2} - 2x^2z\right) + c$$
 Where C is constant of integration that could be a function of x and y

$$\frac{\partial u}{\partial x} = \frac{\left(x^2 + y^2\right)^2 (-2yz) - (-2xyz) * 2\left(x^2 + y^2\right) * 2x}{\left(x^2 + y^2\right)^4}$$

And then you will get $\frac{du}{dz} = \frac{dw}{dz}$ equal to, so much and integrating both the sides you will get the sum where c is the constant of integration, so in the same way you need to do the next problem $\frac{du}{dx}$.

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$$= \frac{6x^2yz - 2y^3z}{\left(x^2 + y^2\right)^3}$$

$$\frac{\partial w}{\partial z} = 0$$
 Substituting the above in continuity equation we get

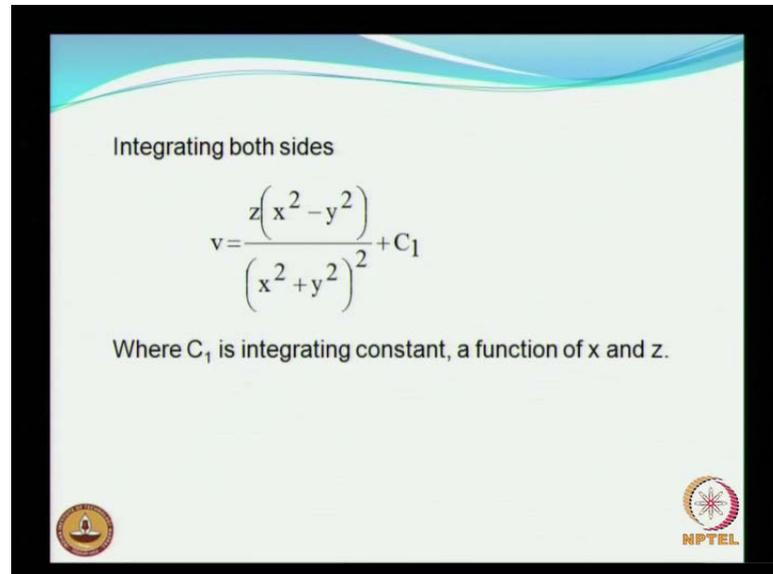
$$\frac{6x^2yz - 2y^3z}{\left(x^2 + y^2\right)^3} + \frac{\partial v}{\partial y} + 0 = 0$$

$$\frac{\partial v}{\partial y} = \frac{2y^3z - 6x^2yz}{\left(x^2 + y^2\right)^3}$$

I am talking about these two problems here (Refer Slide Time: 56:41), in this case you are suppose to find out the velocity in the y direction, so this one is known, then $\frac{dw}{dz}$ by $\frac{dw}{dz}$, so look at this there is no term in z, so this will become 0 $\frac{dw}{dz}$ by $\frac{dw}{dz}$ and

then substituting the above, so the two components you are derivatives, we have found out then substitute in the integration.

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Integrating both sides

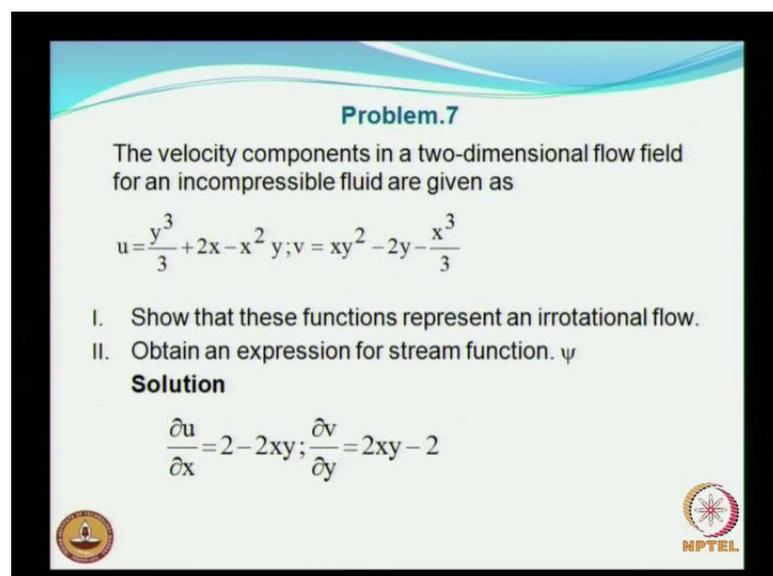
$$v = \frac{z(x^2 - y^2)}{(x^2 + y^2)^2} + C_1$$

Where C_1 is integrating constant, a function of x and z .

The slide features a blue header with a white wave pattern. At the bottom left is a circular logo with a lamp, and at the bottom right is the NPTEL logo.

And then integrate both the sides you will get the velocity in the y direction, where C_1 is the constant of integration, which is going to be function of z and x .

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Problem.7

The velocity components in a two-dimensional flow field for an incompressible fluid are given as

$$u = \frac{y^3}{3} + 2x - x^2 y; v = xy^2 - 2y - \frac{x^3}{3}$$

I. Show that these functions represent an irrotational flow.
II. Obtain an expression for stream function. ψ

Solution

$$\frac{\partial u}{\partial x} = 2 - 2xy; \frac{\partial v}{\partial y} = 2xy - 2$$

The slide features a blue header with a white wave pattern. At the bottom left is a circular logo with a lamp, and at the bottom right is the NPTEL logo.

So, in the same way here there are two components, which are given to you u and v , so that these functions represent condition, I mean irrotational flow and you are supposed to

get an expression for stream function. So, $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ are obtained using those two expressions.

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For a two dimensional flow of incompressible fluid, the continuity eq. (1.5) is expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

By substituting for the two terms

$$2-2xy + 2xy - 2 = 0$$

Thus, continuity equation is satisfied and hence the functions represent a possible case of fluid flow.

Further,

$$\frac{\partial v}{\partial x} = y^2 - x^2 \quad ; \quad \frac{\partial u}{\partial y} = y^2 - x^2$$



Then you add this two, so that can be expressed as shown here, so that satisfies the continuity equation, so it is just checking the satisfying, whether the two flow fields are flow components are satisfying the **the** continuity equation.

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On substituting in conditions for irrotationality we get

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = y^2 - x^2 - y^2 + x^2 = 0$$

Hence the flow is irrotational.

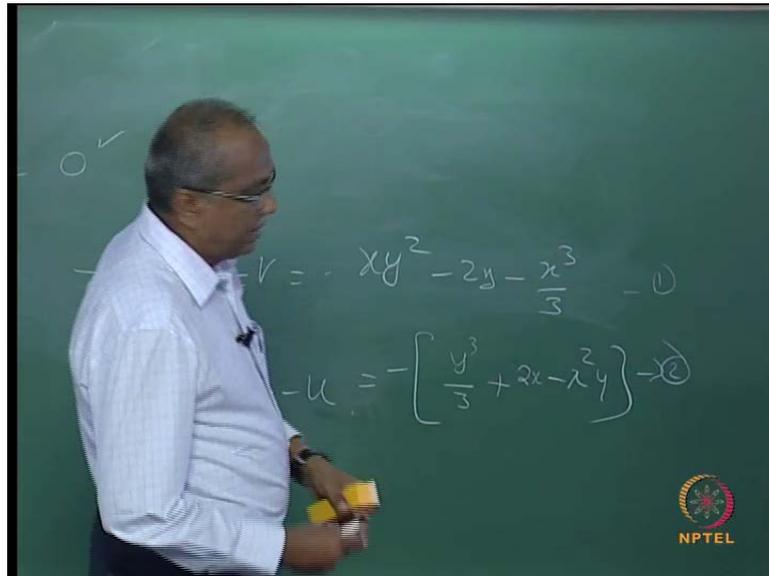
$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - \frac{x^3}{3} \quad (1)$$

$$\frac{\partial \psi}{\partial y} = -u = -\left(\frac{y^3}{3} + 2x - x^2y\right) \quad (2)$$



Then you calculate your further, we can also find out the $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}$.

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So, that is one of the conditions for a irrotational flow, so once you do this you get that this is going to be 0, so that means the flow is irrotational, now $\frac{\partial \psi}{\partial x}$ we have already calculated the we already know the expression for v. Now you can get $\frac{\partial \psi}{\partial x}$ equal to v so you get an expression and similarly, you get an expression for $\frac{\partial \psi}{\partial y}$, which is $\frac{\partial \psi}{\partial y}$ is nothing but, minus u, is that clear.

So, once you have got this, so from the integrating, so **from this equation** from this equation you get the, so let me write this for you whereas, this will be, so let me call this as equation 1 for this problem alone, so easy for you to understand, now integrate this one once you integrate this what are you going to get, your going to get psi.

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Integrating eq. (1) we get

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} + f(y) \quad (3)$$

Differentiating eq. (3) with respect to y we get

$$\frac{\partial \psi}{\partial y} = x^2 y - 2x + f'(y) \quad (4)$$

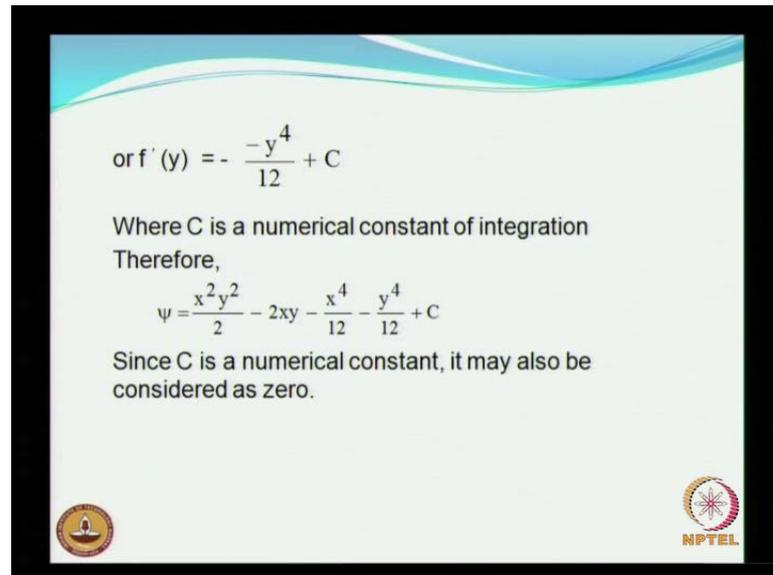
Equating the values of $\left(\frac{\partial \psi}{\partial y}\right)$ from eqs.(2) and (4) we get

$$\left(\frac{y^3}{3} - 2x - x^2 y\right) = x^2 y - 2x + f'(y)$$

So, integrate this and you are going to get the expression for psi, which is this **this** is expression for psi and this is the integrating constant which is going to be a function of y, then differentiating equation three, if you differentiate equation three, with respect to y then you get dou psi by dou y is equal to this much, you just simply differentiate this but, this we have already got here.

So, you equate equation two and this equation four, so that you get an expression for y f y dash, so then you put all these things substitute the numerical, so substitute for f dash y. So, you get the expression for psi with a **constant of integration** numerical constant of integration which may also be considered as 0, so this problem explained to you so.

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or $f'(y) = -\frac{-y^4}{12} + C$

Where C is a numerical constant of integration
Therefore,

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12} + C$$

Since C is a numerical constant, it may also be considered as zero.



So, with this I think we have touched upon, a few sample problems on basic fluid mechanics and then we with this background knowledge, I am sure this is the main information, which you might be needing apart from that there may be some other information also which we will be touching on while dealing with the a hydro dynamics.

So, with the velocity potential stream function all this parameter all these functions will be considered while dealing with the subject on wave mechanics, so any further questions on this close.