Wave Hydro Dynamics

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Module No. # 05

Wave Loads on Structures

Lecture No. # 02

Wave Loads on Structures II

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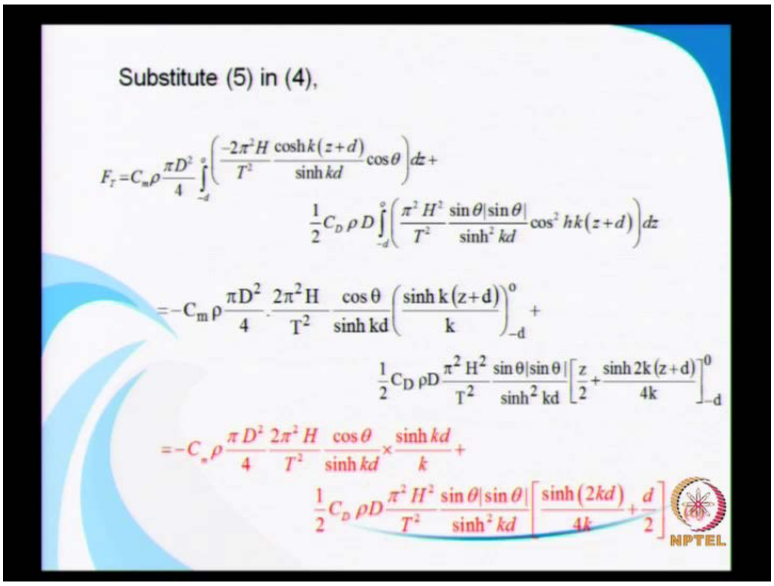

So today, we will again see the expression for total force. So, as per the Morison equation, we have already seen that the total force can be represented as the summation of drag force on an elemental height or area elemental height given by dz and u and u dot are defined by the linear theory.

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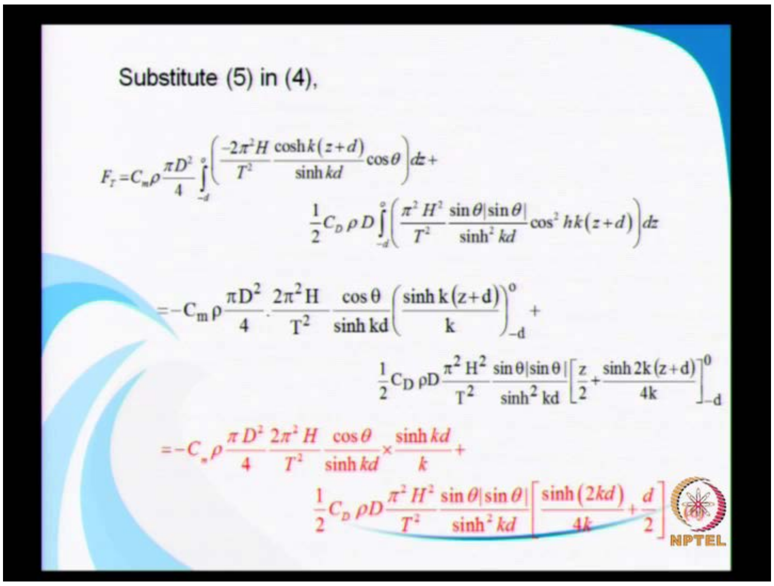

So, once you substitute in the Morison equation, then you will get this kind of expression. As you can see here, this is a portion which is representing the acceleration term. So, this is the initial component; and that you added to a drag component, remember you have u into absolute of u. I told you that absolute of u is one which takes care of the direction. So, we have which is a parameter which is going to take care of the direction that is nothing but your sin theta. So, you see absolute value only for sin theta which will take care of the change in the direction crest of rough, whatever it is, and then I carry out the integration.

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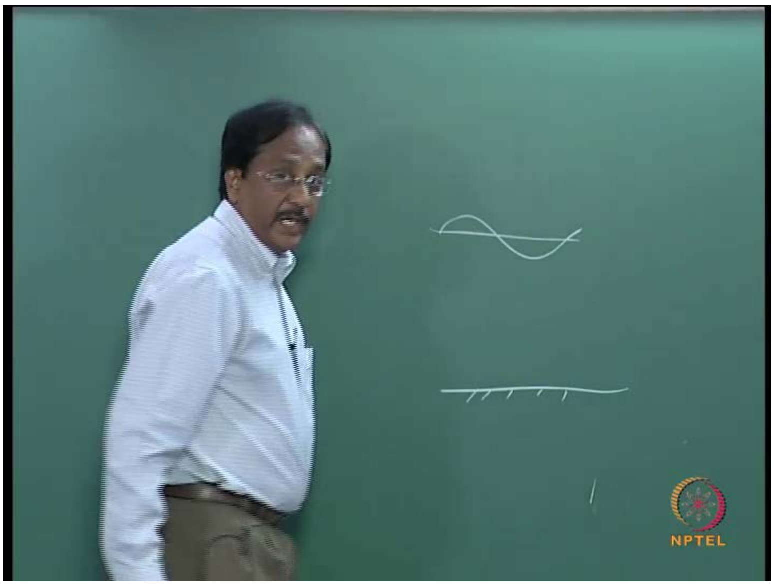

So, this integration is nothing, but from the sea bed up to the, strictly speaking, it should be up to the free surface, but since we are dealing with a linear theory, we integrate from minus d. That is from the sea bed up to z equal to 0 is that clear.

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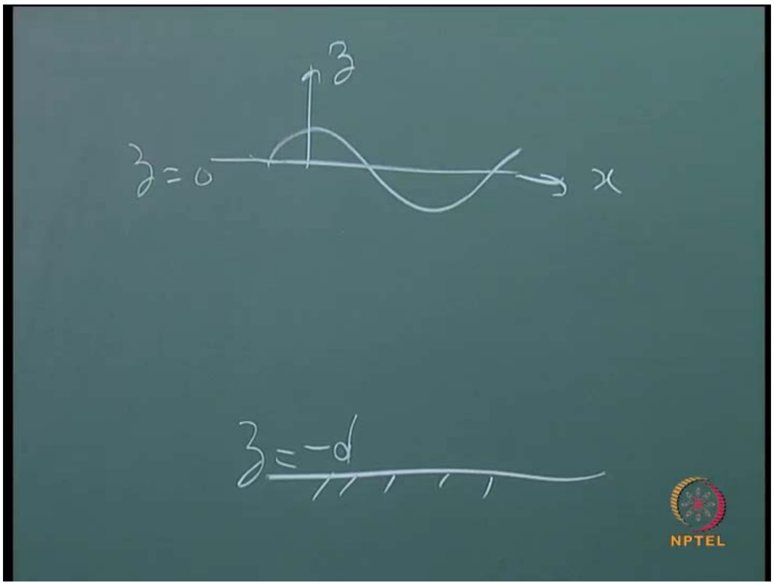

So, this is z and this is here x. So, this is actually a first approximation. The approximation is, we say that the amplitude is small compared to the wave length. So, you look at this, you know that the force is going to vary along the fluid linear along the depth. And we are talking only about the total two dimensional cases. So, the wave force is expected to vary something like this.

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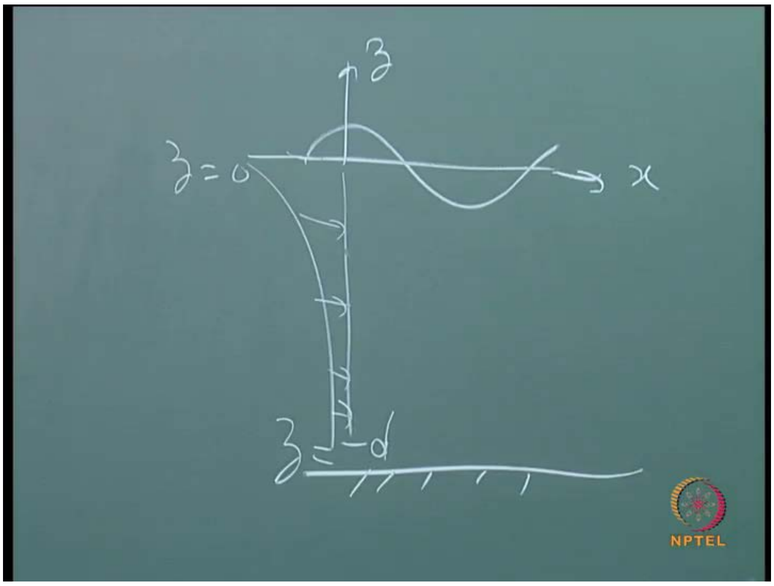

So, that the variation is now taken care by z which is found within the integration. You see z is coming here and in the drag term z is coming here and the integration will be minus d 2 0. Now, you have two kinds of variation; one is with respect to z and since the oscillation is a wave respect to time, the force will be varying with respect to time also.

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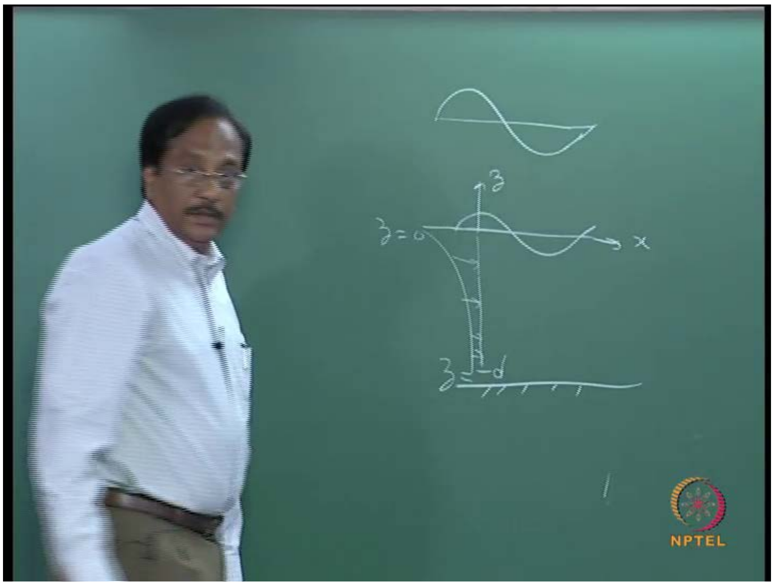

So, what does this indicate? So, finally when you carry out the integration, you get some kind of an expression.

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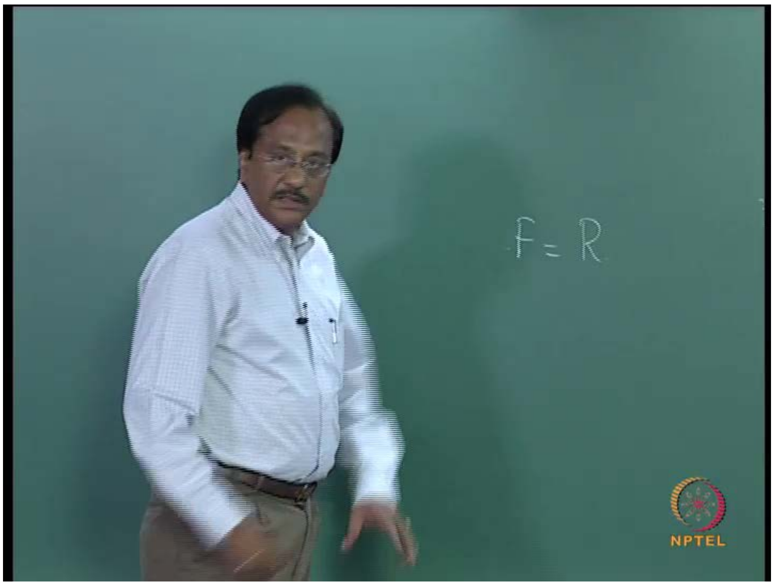

F equals to R some kind of a constant, which you need to evaluate, which will include your include your C D or C M. Then you will have what is this, cos theta for the initial term. I will put this as cos theta plus may be I have a q which needs to be evaluated into sin of theta.

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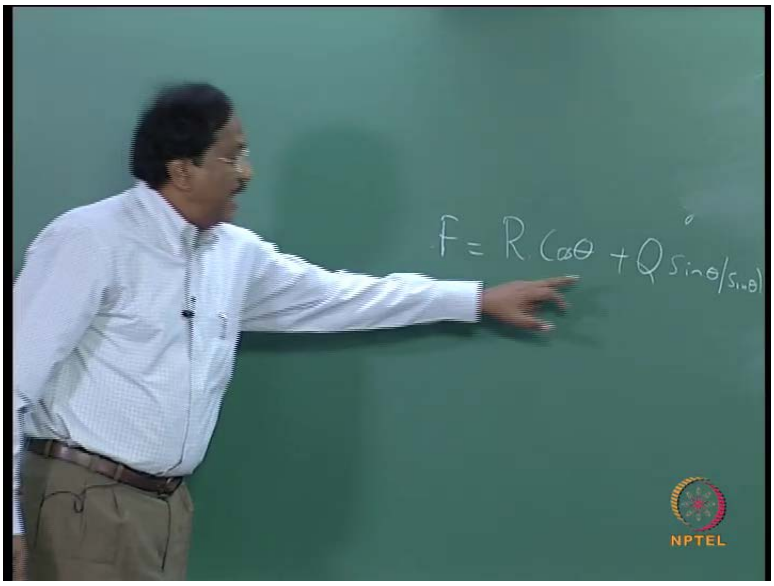

That is how; the wave force is going to vary. So, this R and Q will be a function of z. So, if I consider this as force at any elevation z, then this is how the force will vary, where R and Q are going to be function of certain constrains like hydro dynamic coefficient plus wave height, some variables wave height, wave period etc. As per those two terms given in the two expressions, now, you have carried out the integration. And whatever, I am trying to say R and Q can be obtained as, see all this quantity will be equal to your R and the other one will be leaving your sin theta absolute of sin theta. This whole thing will be become will represent Q. That is how the force will vary.

What is this force? This force is the total force. Suppose, if you want to have the sectional force, if you are interested in finding out how the force is going to vary, how the force will be varying? The force will be varying along the water depth. So, for instance, this is eta of t that is the variation of the wave elevation. And now, I will have at each location z. At each location are variations of the force. We look up the variation later, which is going to change in amplitude. So, you will have again smaller so, if I draw only the magnitude that is what is that at each elevation, the force will be varying as a function of phase.

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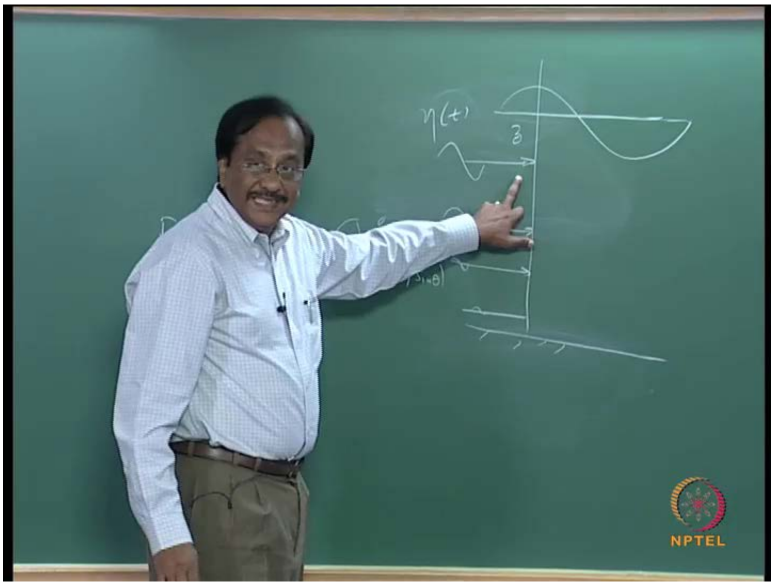

For example, if you consider the condition where you have only initial force, the drag force is completely 0. Then how does a variation look like? It will be a cosine curve as you can see here it will be a cosine curve at elevation one, elevation two, elevation three etc. But the magnitude will be reducing as you go down towards the sea bed. Is that clear? So, if I draw, if I take only the maximum force, so this is the force variation at each elevation. Now, consider only the maximum force that is I am trying to take only this value, then I can represent that as shown here.

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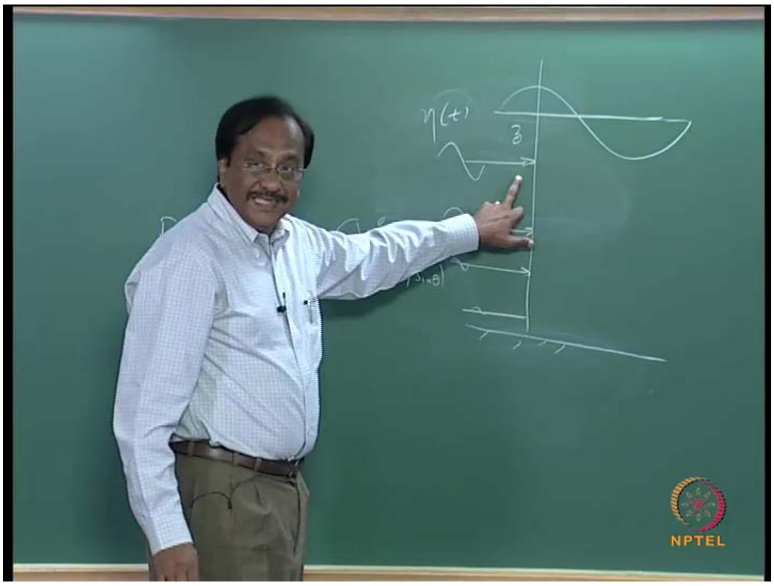

So, and if I join all these things, all these forces, it will be varying as shown here. What is this kind of a variation? This is hyperbolic variation. So, the force will be the force due to waves will be maximum near a free surface and it will reduce as you go down towards the sea bed and the variation is called the variation will be an hyperbolic variation.

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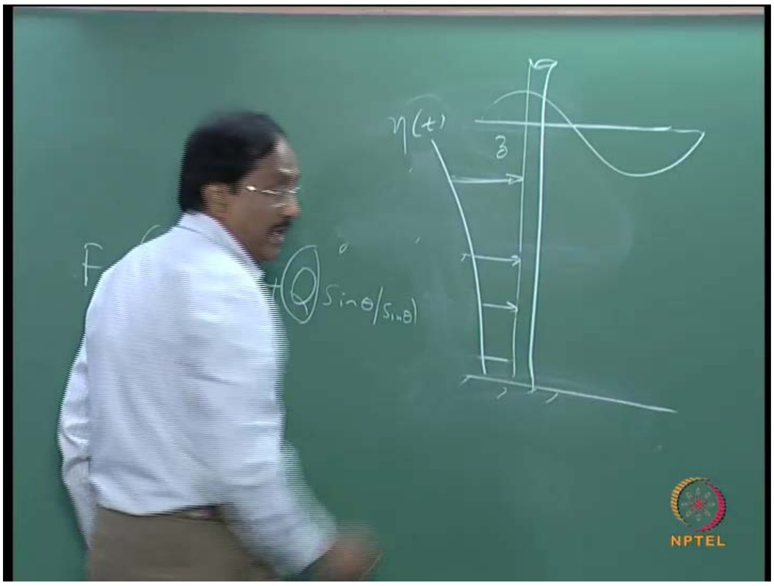

So, either you will be interested in evaluating the section of force, or if you want the total force acting on a pipe. For example so, I have a file here and then I need to evaluate the force total force and moment, then what you do you integrate from this point up to this point? And that is what we have done now, and we have integrated from here to minus d 2 0. And we got a final expression, which I have already explained will be in this form and which when drawn would vary something like this.

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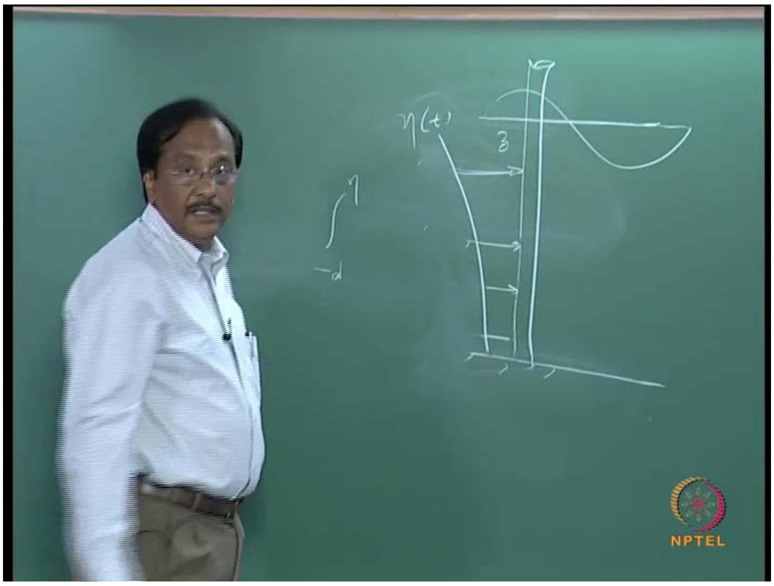

So, now one thing is, if you want to consider the variation of eta, then as I said earlier, the integration will be from minus d to eta, another form of first approximation will be considering instead of d, you may consider d by d plus d eta. That is instantaneous water depths so, it will e a bit complicated, but you are trying to move closer to reality. Closer to what can be I mean the closer evaluation? Close to the real one so when I say accurate wave force system it is only estimation.

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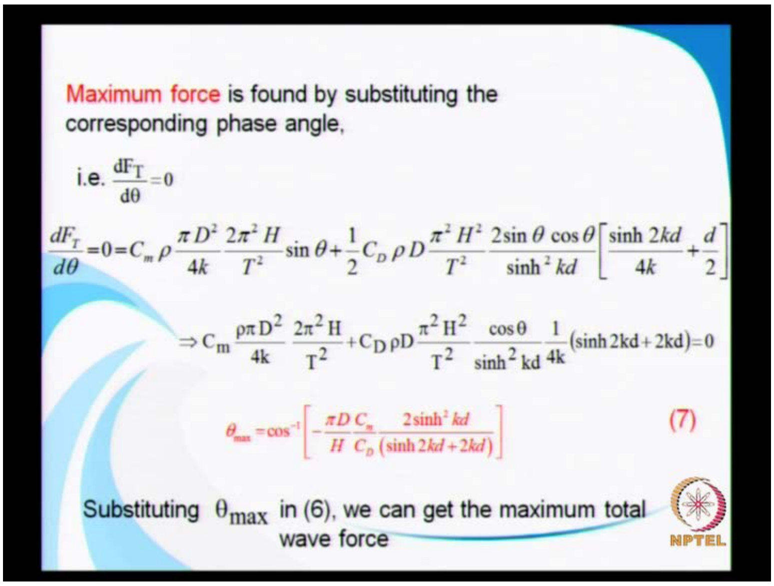

As I have already told you there are some uncertainties so the force what you are getting from the Morison equation is an estimation of wave lengths is that clear? So having seen the total force look at the total force; total force is a combination of sin and cosine and in sin you have also at an absolute value. So, the force variation will be a function of theta total force. Is that clear?

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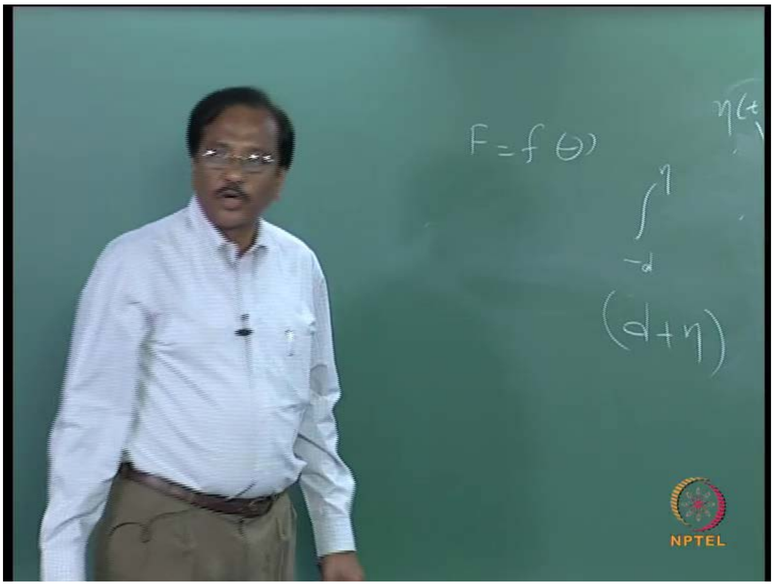

The total force, which we had seen earlier will be a function of eta, but often what are we interested in, we are interested in the total maximum force. So, when we need the total maximum force naturally, you have to differentiate with respect to theta equated to 0.

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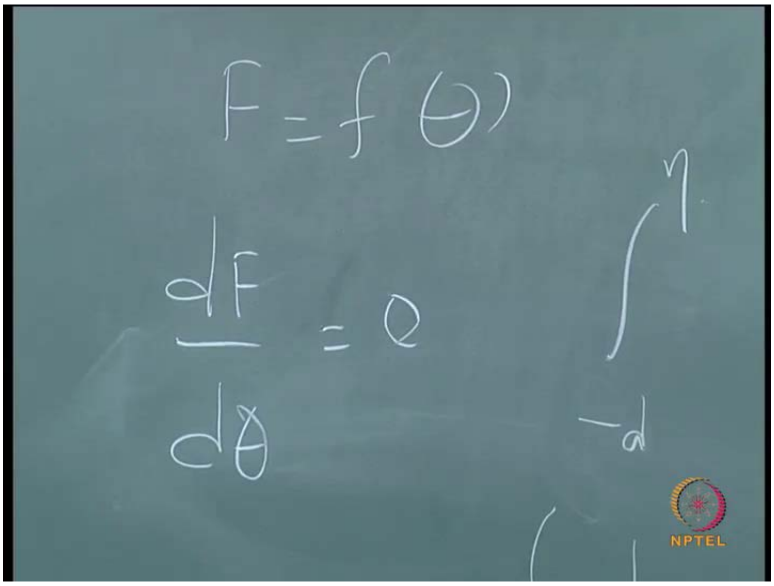

So, take this expression shown in red color differentiates with respect to phase differentiate it with respect to phase and then equate it to 2. Although the expressions look a bit lengthy quite straight forward and easy you can try to get these expressions yourselves as an exercise. So, that you become more familiar with a calculation or how you are getting all this equation for the estimation of phases? Now, that you have determined the theta max and theta max is what does it means that it is a phase at, which the maximum force occurs.

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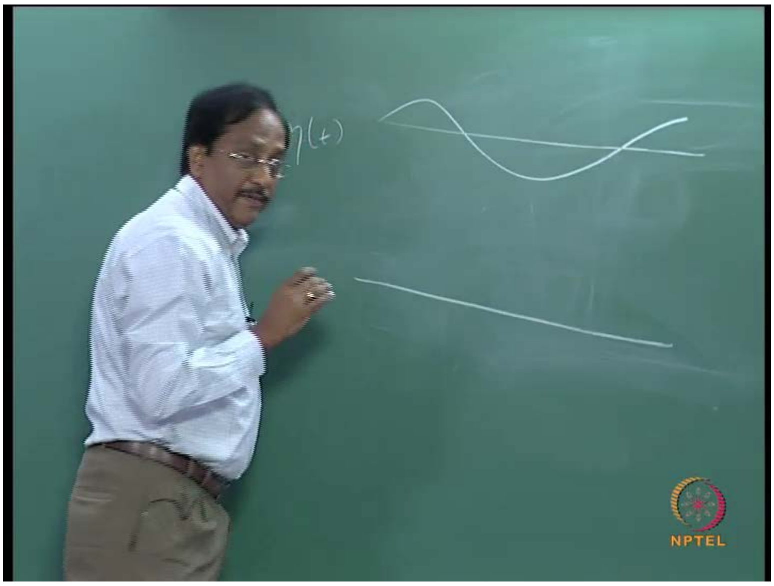

For example, I will explain in this way to be more clear suppose, if this is the wave profile and you have a wave, which is just following only the drag force mostly drag force. For example then naturally, it will be a if wave and force are in the same phase then it will be at this angle at this phase theta e equal to 90 degrees, the force will be maximum. This is going to be theta equal to 0 and I am considering theta equal to 0. Is that clear?

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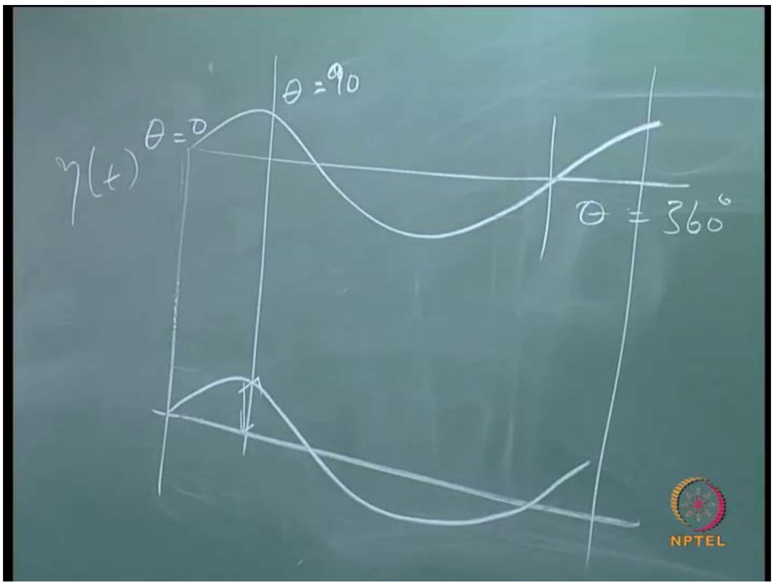

If in case the force is in phase this is how you get? But here, we are not talking about the phase the force in phase we have a force, which may be something like this or it may be perfectly 9 5 8 or it may be so it depends on so many other phenomena.

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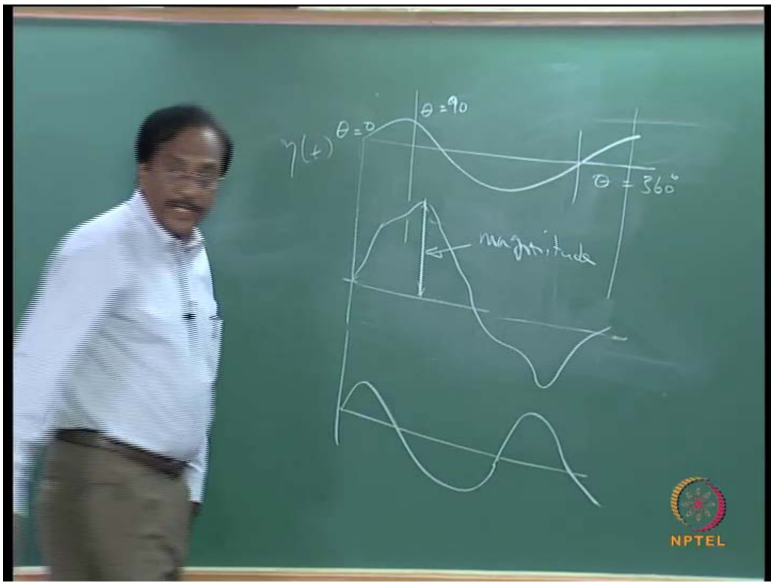

So, if we do not know, we want to estimate by that definition this is the peak force, which is nothing, but the magnitude of the total force. And what phase at what phase is it occurring that is theta max will be this is your theta max with respect to the phase.

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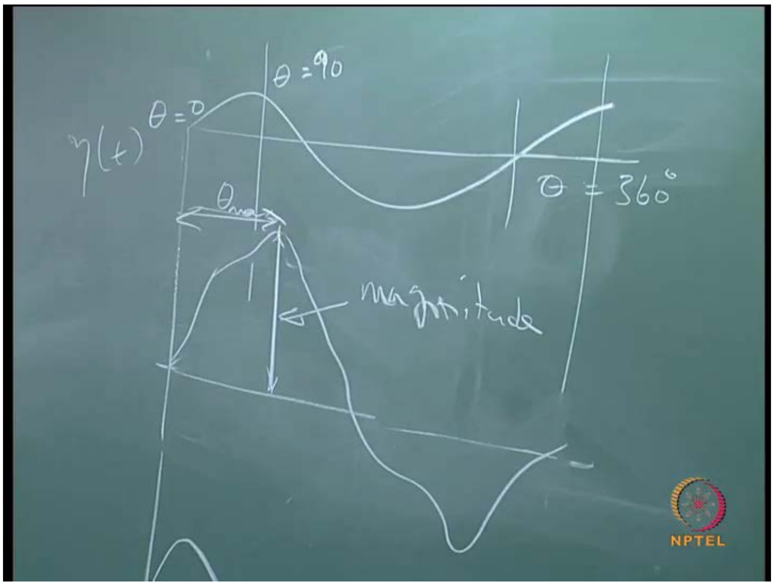

So, the force need not have to occur only when the only at this point it can be slightly away for a simple reason that is the reason why we are estimating the phase at, which the maximum force occurs. Now, looking at this what are the parameters involved as we have seen earlier, it may deal is one important parameter remember H by D greater than one. You have the combination of both drag and inertia, which is going to contribute to the total force. So, that H by D is coming into picture then you also have the cm and cd that itself is going to take care of the inertia and effect of inertia or the drag.

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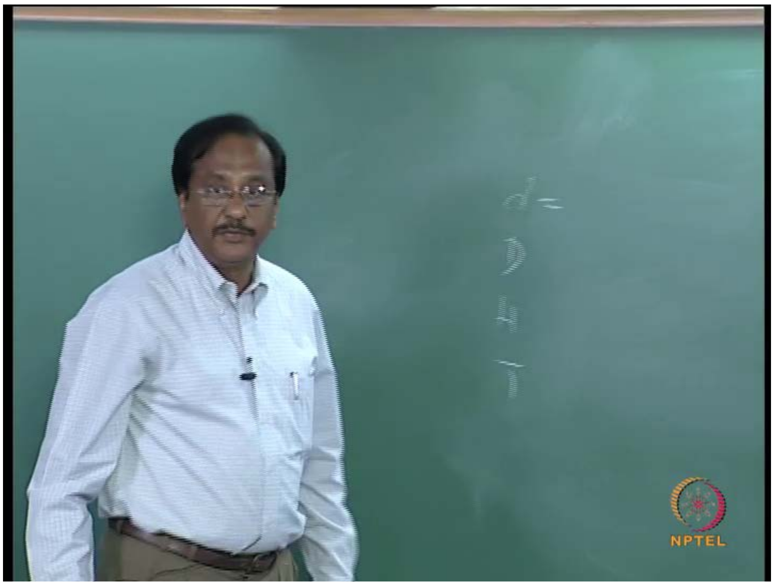

So, as I said earlier, first you have to find out the wave force system and once you are convinced that the data, which is given to you like. For example diameter, water depth, wave height, wave period etc for all these variables you try to compute the H by D and the kc number or D by l which we have seen earlier or the scattering parameter ka or kc.

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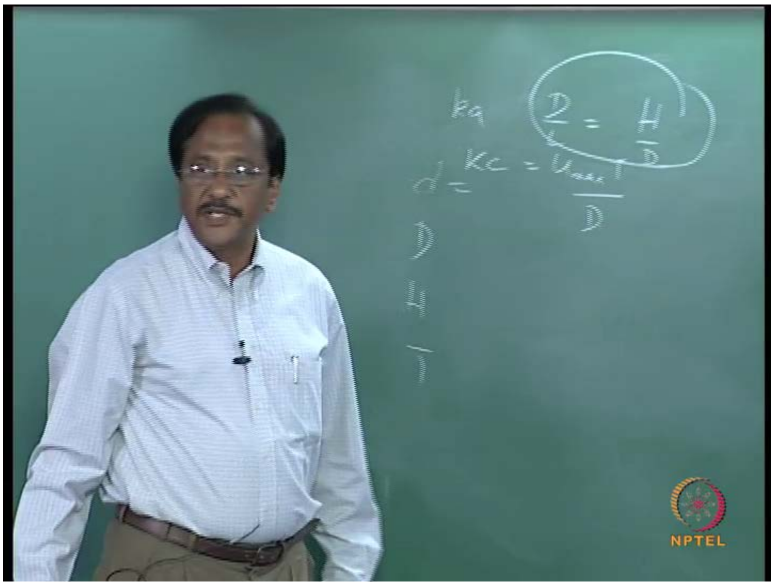

K c is or d by l or H by D so this two combination you know that if H by D is greater than one and D by l is less than one you see that it is going to be predominantly combination of both drag and inertia and hence Morison equation has to be used. And if D by l is less than 0.2 and H by D is less than 1 then we ignore the we can afford t ignore the drag force and consider the total force as just inertia force.

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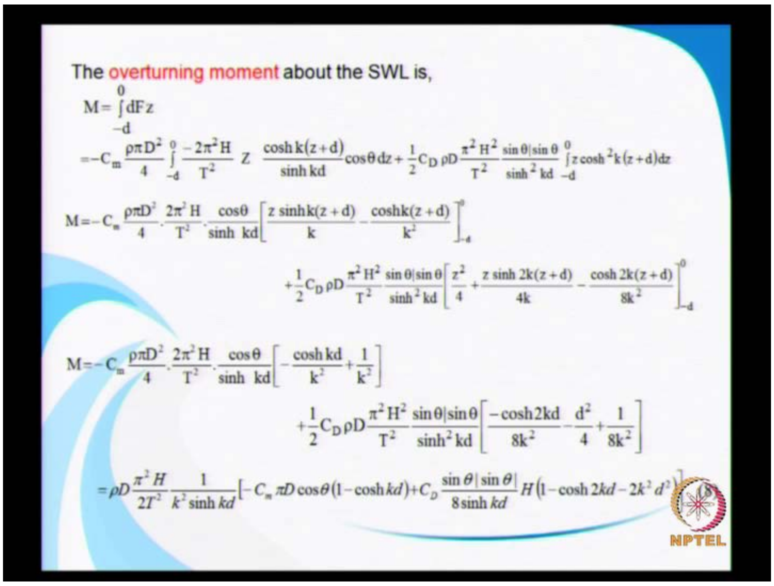

So, now, that you have estimated your theta max you can use this theta max and substitute in this equation. This is the force, which gives this is the expression which the total force substitutes back in this expression in order to obtain the maximum total force. So, once you substitute that total value of theta max you get the total maximum force. We will try to understand this force by taking an example, with some data and then we will try to understand more in much better way. Now, the turning moment about the s w l that is the still water line is now, d F that is force elemental force.

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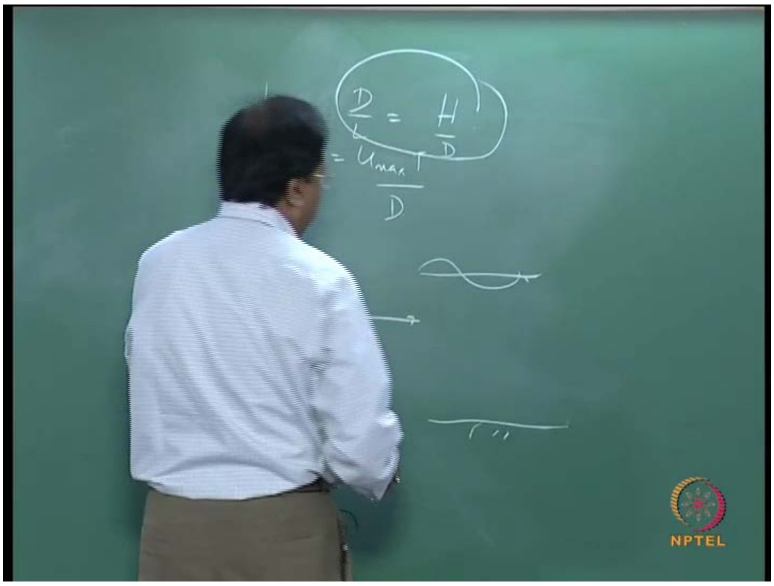

That is you have the so the force is somewhere here now, this force above the still water line is obtained as dF into z. So, dF already we have considered then you carry out the integration all those things and then finally, you will land up with an expression as shown here, which is going to be again a function of theta. Similar to total force you are going to have an expression for moment so again the moment is going to vary as shown here in this sum equation.

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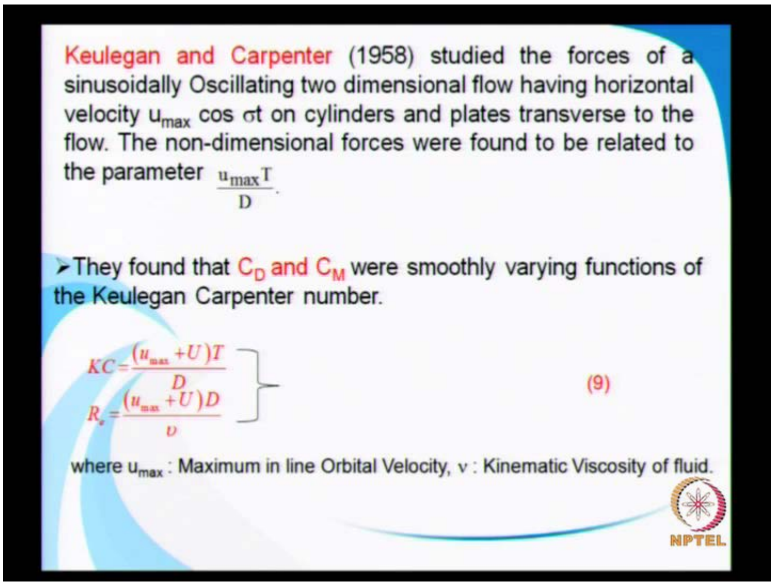

What is this Keulegan and carpenter number Keulegan and carpenter in 58 they studied. The forces on forces on cylinders and plates and they estimated or they represented the dimensional forces, as a function of a parameter which is given as u max T by D. U max is the horizontal water particle velocity.

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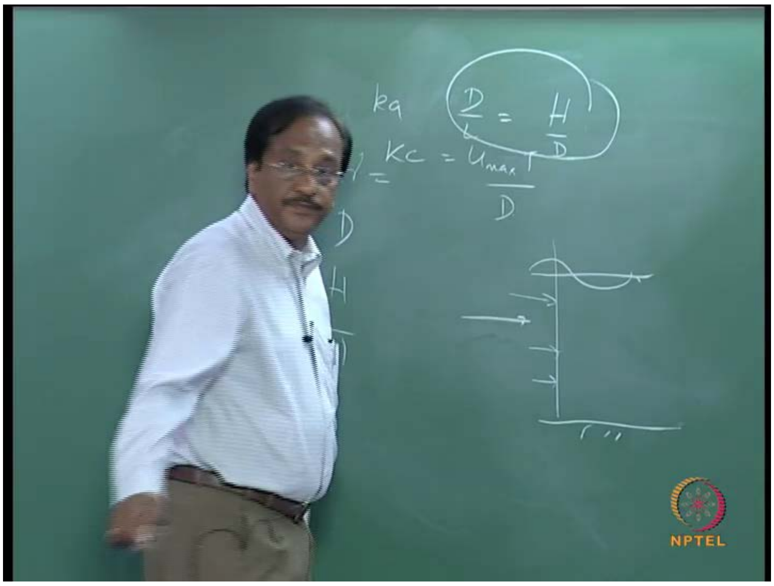

Remember the Keulegan carpenter number itself, will be varying along the water depth am I right? Because at each elevation when you take this at each elevation the Keulegan carpenter number will be different, but here in we are talking Keulegan so you can have a local Keulegan carpenter number. But often when we refer to a Keulegan carpenter number is the one, which is a function of the maximum horizontal vertical velocity, which is going to occur at the still water line, because when you talk about u max you can have local u max that is at this location you have u max here you have u max here etc.

Up to the sea bed you have u max, but what we refer to Keulegan carpenter number is defined by u max, which is at the maximum velocity which is occurring at the still water. And if you want refer the Keulegan carpenter number at any other elevation you will just take local Keulegan carpenter number, which is not that often used. This so they found that C D and C M the smoothly varying functions of the Keulegan carpenter number. So, usually, you see that the variation of coefficient of drag is plotted as a function of Reynolds s number which is Reynolds s number let us recollect Reynolds s.

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Number u max d by mu, which is mu is mu by rho then again we will see what Keulegan carpenter is and what simplifies this. This has something to do only with mainly with the four regions, which is going to control the forces. So, KC number as I said is u max t by D if you have a current associated with the particle velocity. It can be represented as u max capital u plus capital u which is capital u is the current velocity D divided by D and whereas, the Reynolds s number corresponding Reynolds s number in presence of waves.

And currents can be first represented by as shown here, u max plus u into D divided by de kinematic process.

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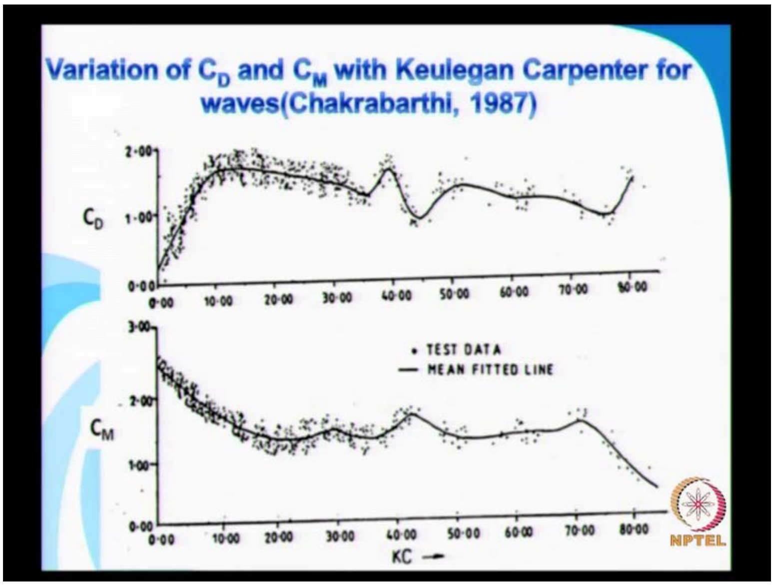

So, now, there have been a number of tests how do you determine the coefficients of drag are coefficient of inertia. Coefficient of drag and inertia has been determined by several researches in the past. Where you know? You have a vertical file and the total force is measured in the laboratory subjected to waves of different magnitudes and frequencies and then, you can obtain the drag coefficient. And how do you experiment the how do you obtain drag and inertia through experiments? I will try to spend about five minutes later, it is quite straight forward, but the problem is if you look at the literature the coefficient and the drag and inertia are the variation, with respect to either Reynolds s number or Keulegan carpenter number its character is quite significant.

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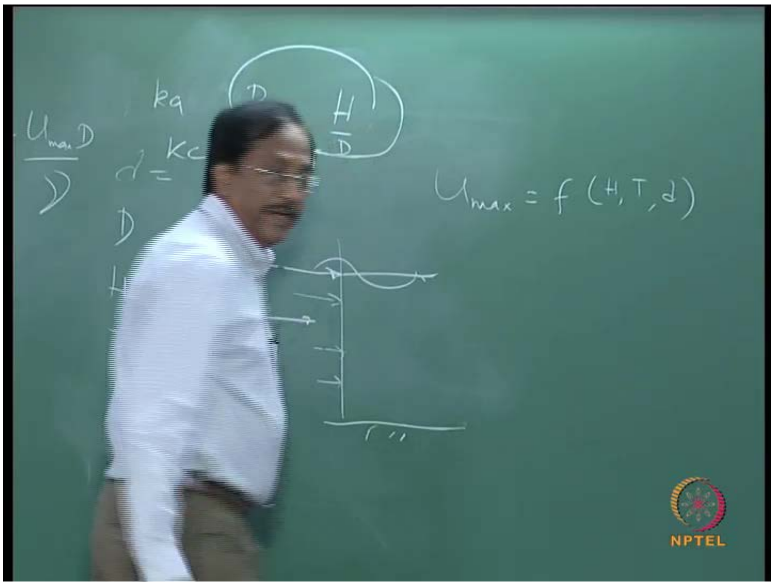

So, it was Chakrabarthi who had published this data, which is widely being used and you should refer to this text book on hydro dynamics of offshore structures. Where in you see he has plotted the coefficient of drag as a function of Keulegan carpenter number and coefficient of inertia as a function of Keulegan carpenter number. So, it is not so easy to obtain a wide range of Keulegan carpenter number in the laboratory as you know, the Keulegan carpenter number is going to be a function of your u max. And the relative diameter u max is going to be a function of wave height, wave period and water depth. So, when you want to do some experimental studies, there are lot of limitations on the wave maker. What kind of waves can be generated in the flume?

And you have to also have you will also have some kind of a limitation on the diameter of the cylinder. It cannot be too slender because installing a force sensor is going to be difficult and the rigidity of the structure also, may be difficult while performing the test. So, most of the literature if you see the Keulegan carpenter the variation of Keulegan carpenter is rather limited up to may be about 20 or 30 within, which within the within that range itself you see a significance character. So, this literature from Charkabarthi these results are found to be quite suitable for evaluating. The wave forces are evaluating the or assigning the values for hydro dynamics coefficients of drag and inertia.

So, how do you select the coefficient of drag or inertia from this plot. So, for your given details, you have to first evaluate the u max we have seen problems under basic wave mechanics. Once you have evaluated u max the wave period is also, known to you and the diameter also, known to you. So, once you have the Keulegan carpenter number so you go into this suppose, if it is ten go into this value and you see that there is a kind of a variation here, but off course this variation is much less. So, you may fix a value something close to this line because, the line is supposed to be the line of best feet.

The amount or the kind of deviation you have in such a case is something like this is our deviation you would. What does this picture represent? This represent picture also, represents one important thing the top one gives that the coefficient of drag is going to increase with an increase in the Keulegan carpenter number. Then c d is increasing that means, the drag component is going to increase which means the viscous component is going to increase so as the KC number increases your viscous affects will become more and more dominant. That is what you should have in mind?

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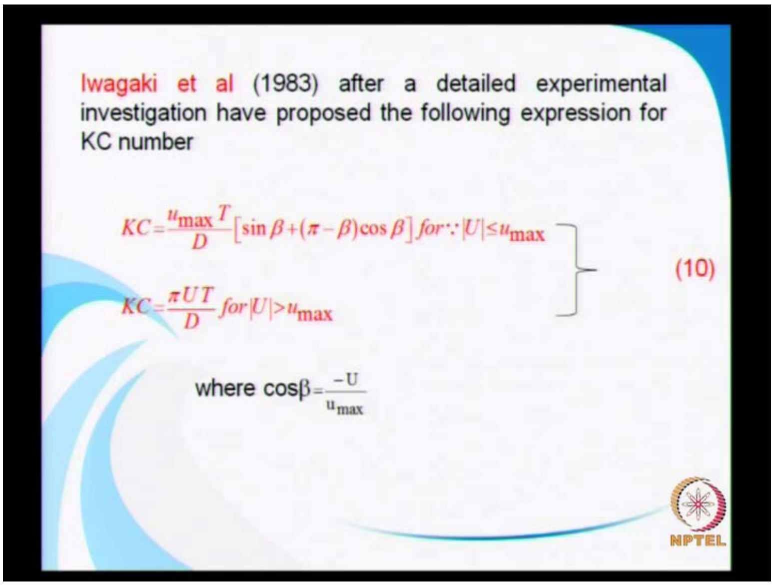

Now, that is about CD and CM for a vertical file, but you have so many other for example, you have a p i code there are so many other codes etc. I suggest some of you if you are interested you should check all these codes. And there are some other published literatures also. Now, we move on to presence of waves and currents how do you get the coefficient of drag and inertia. So, it was Iwagaki at all who defined the coefficient of drag who defined the Keulegan carpenter number. In this form that is u is current and u max is the orbital velocity where beta assigns the direction between the waves and the currents.

So, once you know, once you are given the wave characteristics and you have estimated the u max and if you also know the value of the current, you can estimate all this expression on this equation; from this expression, you can estimate the value of the Keulegan carpenter number.

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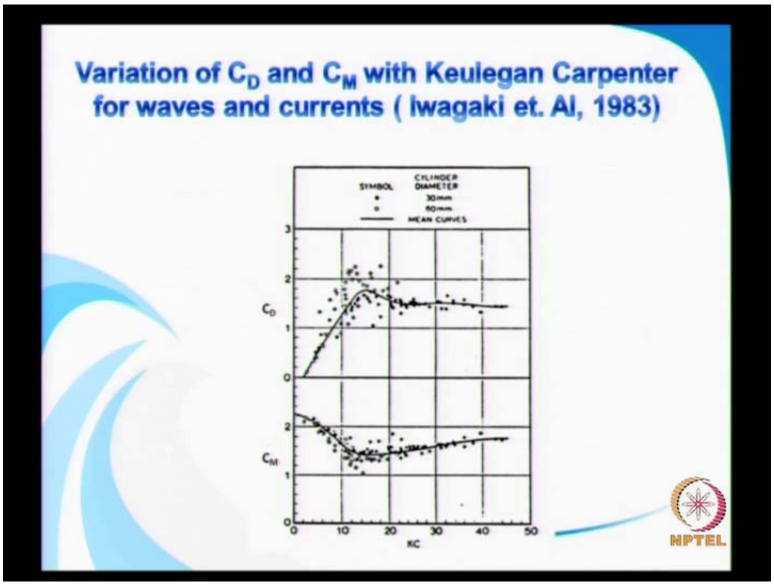

And then move on to this information; these are results provided by Iwagaki at all from comprehensive experimental program, wherein you can arrive at C D and C M for given K C. Is that clear? So, once you have evaluated the K C i mean C D and C M, then things are quite more or less quite. I hope there are not much of questions.

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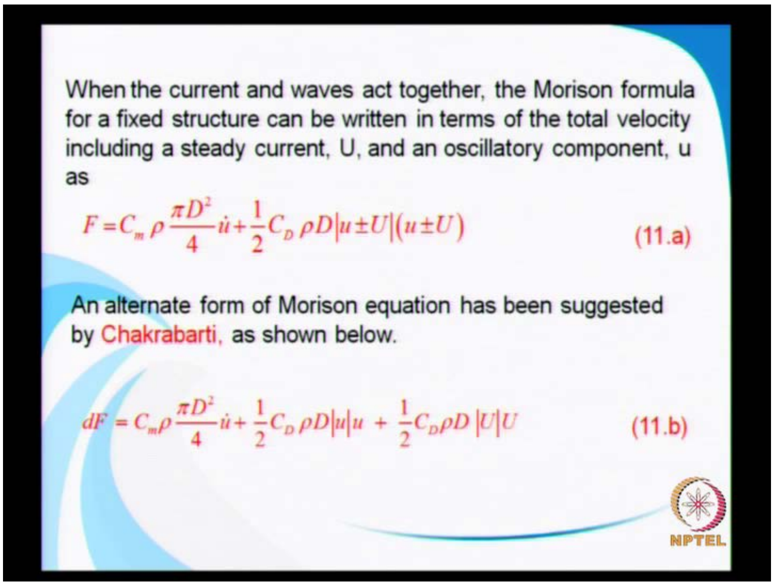

So, now, we will continue to then you have a combination of waves and currents. So, when waves and current act together the Morison formula for a fixed structure can be written in terms of total velocity including the steady current U capital U. And an oscillatory component small U so this will continue to have you will continue to have the initial force due to the propagating wave. But the presence of your current is going to add on to the component that is the horizontal water particle velocity component. So, you can have either the negative current or a positive current either a following current or an opposing current now, depending on this now you can use this expression.

So, these formulas are well written in a text book of a Chakrabarti, which I suggest you have a critical look at those books. An alternate form of Morison equation has been suggested where in the drag component due to wave and their drag component due to the current are separated. The first one is also, referred to as a relative velocity model so this is another form of representing the combination of current and waves acting on exerting the force on a vertical file. So, C D C M and again you can you can still use the once, which I have been given by Iwagaki at all. So, having seen the total force after substituting the expressions for U and U dot then we get the total for here in presence of both waves and currents and then once that is done.

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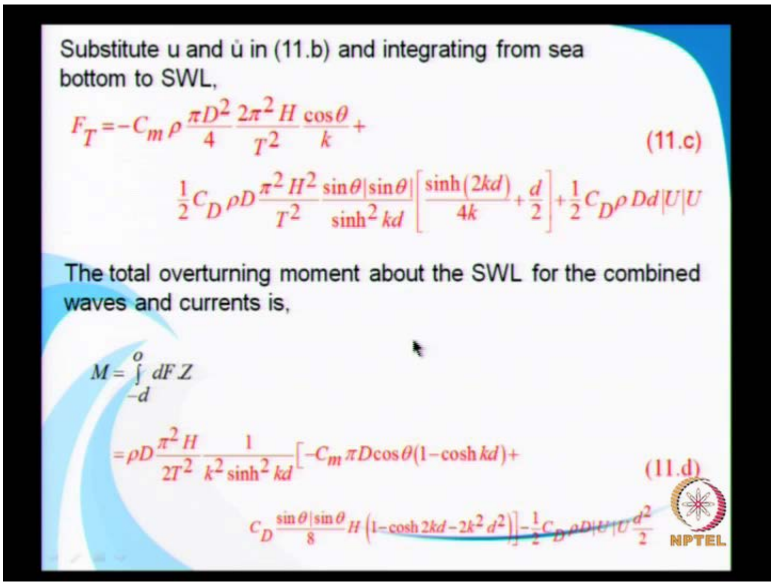

The total overturning moment can be obtained as, similar to what we have done in the case of forces along in absence of current.

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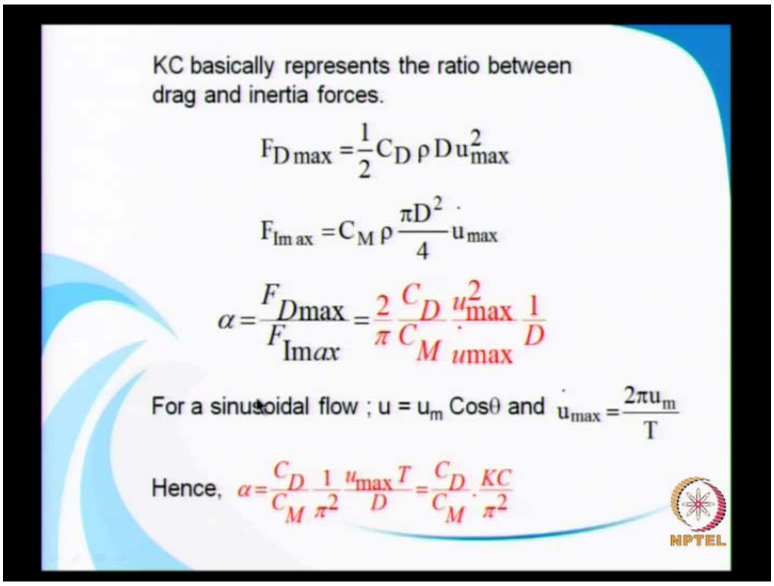

So, this works out to as given in equation eleven point C I hope now, things are quite clear as far as the wave forces are concerned and as well as the forces due to the combination of waves and currents having seen that. So, the problem of selection of C M and C D has been orderly gone through so I am sure that the vision is quite clear. Now, let us try to understand what is meant by K C number. Keulegan carpenter number plays an important role in the case of determination of the wave forces, which is essentially the ratio of wave forces drag to inertia force. This could be easily understood by this small simple derivation. So, F D max we have already seen is given as half C D into rho into D into u max square whereas, F I max is given so, ratio is shown here and then once you substitute for u.

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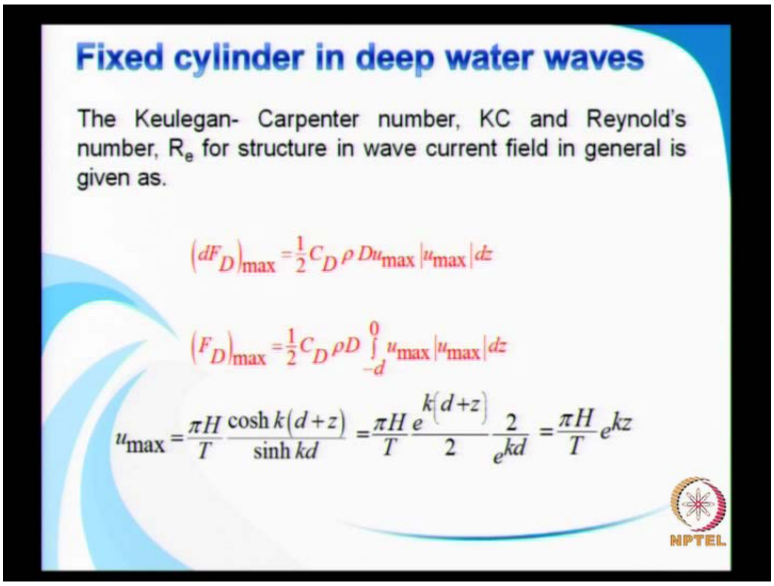

U is can be represented as u max into cos theta or sin theta then you can obtain an expression as shown here. That u max T by D is KC number so this shows the ratio between the drag force and the inertia force. So, again we will come back to the KC number later where in we will try to talk about the ratio H by D later. Now, we will try to move on to we move on to fixed cylinders in deep water waves. So, the drag force in deep waters has can be given as indicated here and once you substitute for the hyperbolic function square in terms of exponential, because it is we are considering deep water conditions.

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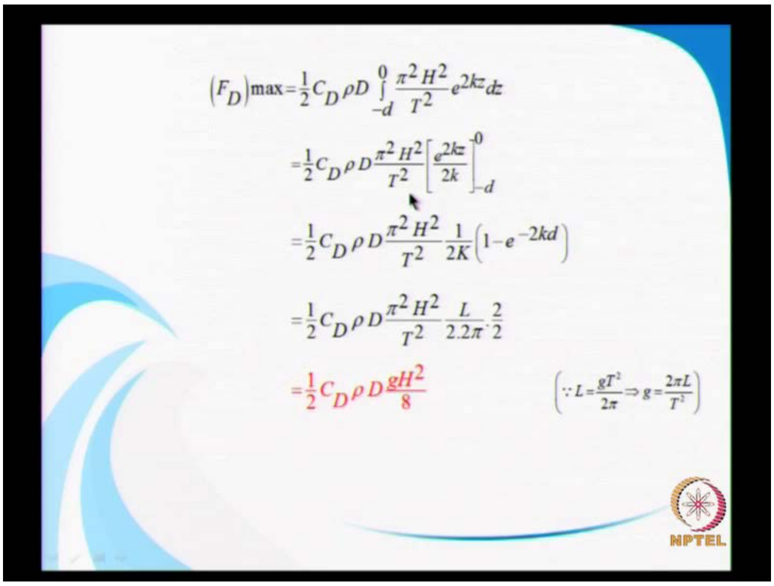

Then you will have u max as equal to this expression. So, these are quite straight forward to derive and then FD max with this approximation can be written and once, you simplify this the derivation is given here you will get this expression for FD max remember, this is only for deep water conditions.

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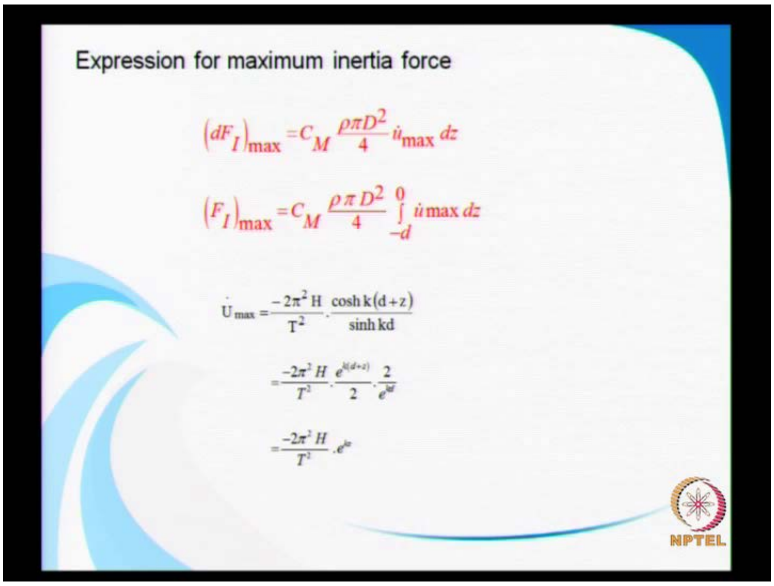

So, now, in a similar way you can derive the maximum inertia force for deep water conditions as shown here once these two are done.

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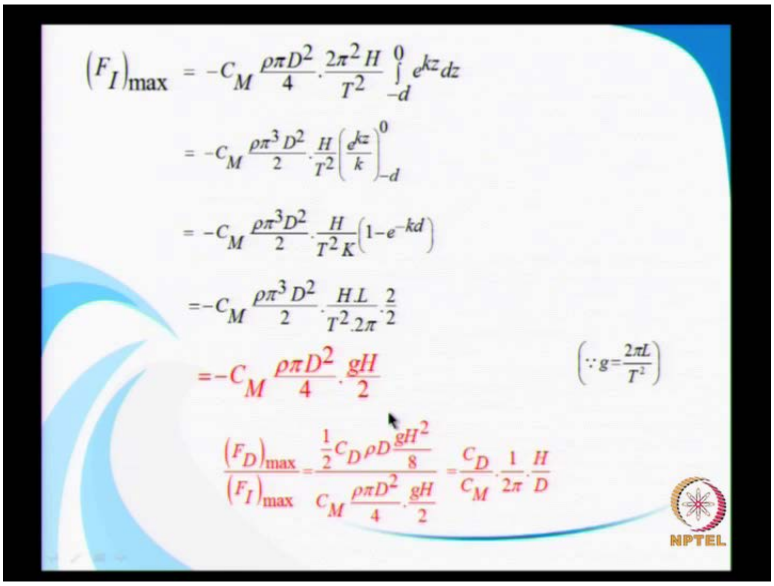

So, this is the final expression for f i max so, I suggest you go through the derivation carefully it is quite straight forward and then I go back again to F D max divided by F I max. When I carry out the simplifications I get this kind of an expression this clearly indicates that, the ratio of f d max to f i max, which is nothing, but the ration of drag two or inertia of course, which is nothing, but the Keulegan carpenter number is going to be also, a function of h by d. So, as h by d into this has H by D increases as H by D increases the viscous effect is going to downwards. So, remember H by D if it is greater than one you have the total force as summation of inertia and drag force. So, what we have indicated or what I have already shown now, we have tried to prove it in what way H by D plays an important role in the categorization of the wave force regions.

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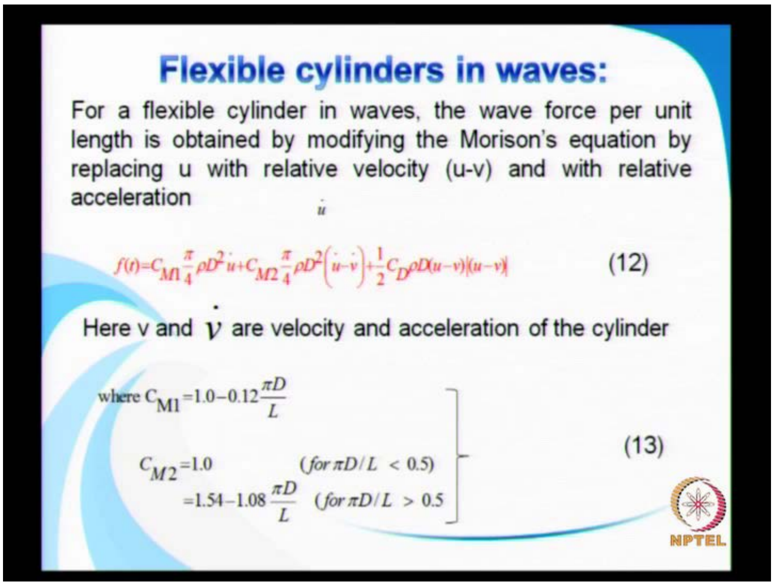

So, there are several cases where you come across a cylinder which is in motion. A classical example, is an articulated tower which is hinged at the bottom and it can be it will be allowed to freely move. So, when you have a flexible cylinder subjected to the action of waves the Morison equation can be represented as indicated here the u will be replaced by relative velocity. And similarly, we have an additional component due to the movement of the cylinder and that is going to be a relative acceleration indicated with.

So, the C M 1 and the C M 2 are the 2 coefficients, which you need to obtain and these are represented as shown here, which will be a function of scattering parameter r pi d by D r. Additional details are available in a number of research papers are by the book written by Chakravarti titled hydro dynamics of offshore structures quite a good book with all this details of all this derivations. So, other kinds of structures are you can have a jacket type of a structure where in the number are inclined also, so you need to evaluate the forces acting on inclined numbers.

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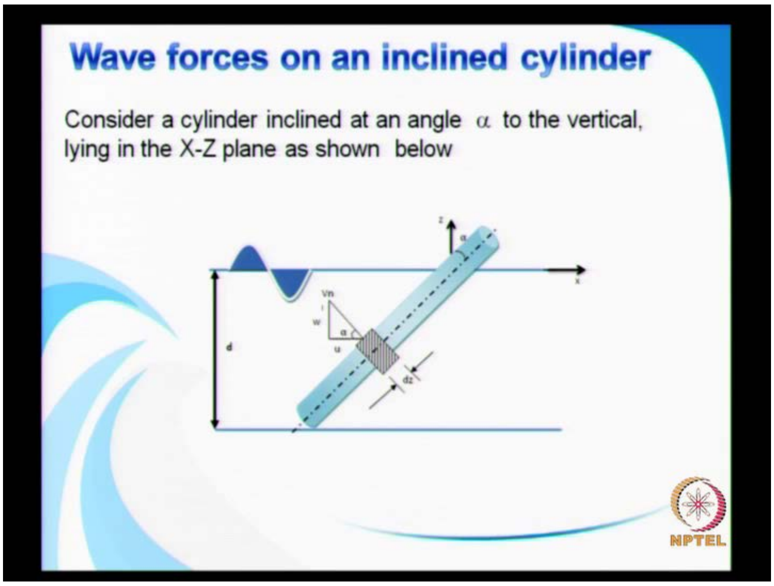

So, for this purpose these figures represent an inclined cylinder and we are taking an elemental element of length dz; now, you have the component normal to the cylinder. The case of a vertical cylinder, you have the component normal to cylinder. The same way now, cylinder is inclined so the component will be normal to the cylinder, which of course, could be resolved in the two components, which will have two components; one in the normal direction, I mean in the inclined direction, and one in the vertical direction.

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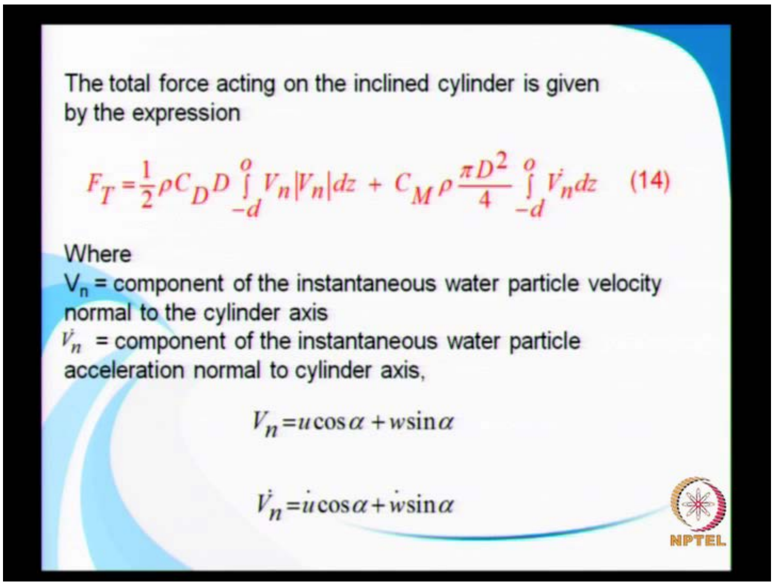

So, now the same Morison equation can be used so the total force in the inclined cylinder on the inclined cylinder is given as equation fourteen. Here in V n, which is the velocity normal to the resultant velocity normal to the cylinder can be obtained as indicated here, which is going to be horizontal water particle velocity and the vertical particle velocity. And the acceleration in a similar way you will get as shown here in terms of u dot and w. So, this is the simplest way of a case where, you have as inclined cylinder subjected to two dimensional flows.

Having been exposed to this you can further go in for additional reading. Where in literature is available, where in you have the cylinder oriented in an arbitrary direction subjected two waves coming from different directions. In which case you will have all the three velocities and you have to consider all the planes in, which the cylinder will be inclined. So, this needs additional reading after having gone through this exercise.

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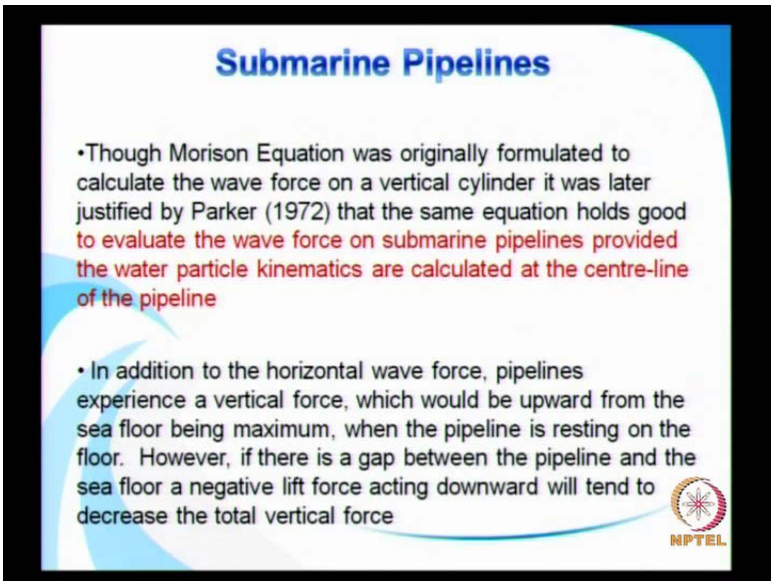

So, again the coefficients of drag and inertia for the inclined cylinders are available in a number of research papers found in the literature. For small angles of inclination the coefficient of drag and inertia may not very much, but far larger inclination it does vary. And it also, depends on how the cylinder is vary whether, it is cylinder against the waves or along the waves. So, all these informations need to be for all these kinds of inclinations the coefficients of drag the hydro dynamic coefficient will is bound to change.

So, now the other kind of structure of tubular number will be horizontal numbers here in I am just calling it as submerged submarine pipelines or it can be also just submerged cylinders. The application of Morison equation for submerged in the pipeline was first recognized by parker in 1972. Where in he convinced that he came out with a suggestion with a large experimental test saying that stating that the particle kinematics, if they are evaluated at the central line of the pipe line. The same Morison equation can be used of course; the coefficient of the hydro dynamics coefficient is bound to change.

So, this submarine pipeline when you have a submarine pipeline you will have both horizontal and vertical force. So, in addition to the horizontal force in the case of a vertical cylinder these pipelines will experience a vertical force, which will be moving upward, which will be binding upwards. And these going to be quite significant when the pipeline is going to rest on the sea bed. When it is resting on the sea bed? For example, for transportation of oil and gas from off shore to on shore facilities there are certain instances location the pipeline is just made to the sandy sea bed.

In that case flow is intercepted floor beneath the pipeline in between the sea bed and the pipeline is intercepted causing a large pressure difference over the pipeline and resulting a huge vertical force. So, the surf weight of the pipeline should be good enough to resist or compensate for this, large vertical force. So, when you design the pipelines all these things need to be carefully taken into account in while you are checking for this stability of a pipeline. And it was quite straight forward to find out the forces the same Morison equation is used.

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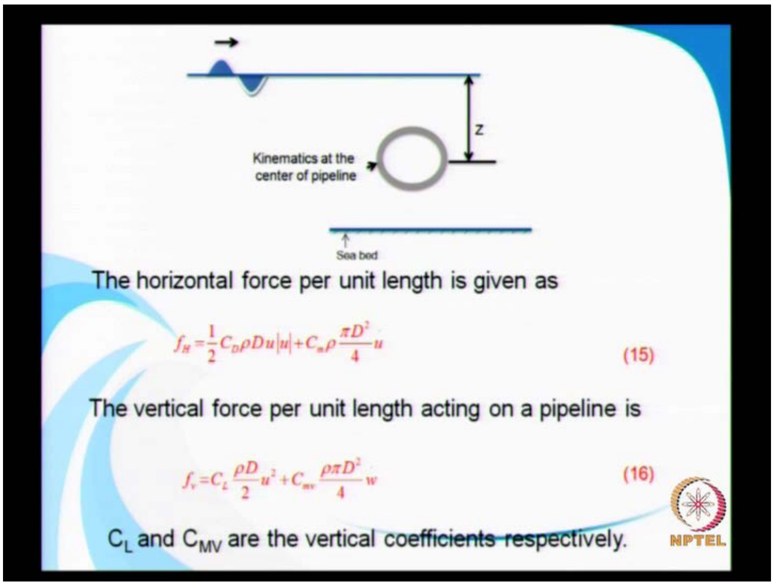

So, you have the horizontal force the same Morison equation in addition to this you have a vertical force per unit line, if you want to have a total force for the entire thing. You know how to this can be easily done? So, the vertical force you would see that this is the lift force mainly due to the horizontal water particle velocity that is going to induce a lift force. This lift force is similar to what you have in steady flow similar to a drag component so, here C L and C M mean are the vertical coefficients respectively. These coefficients again you are need to check available literature for this coefficients are again presented as a function of Keulegan carpenter number as well as, against as well as a function of Reynolds s number.

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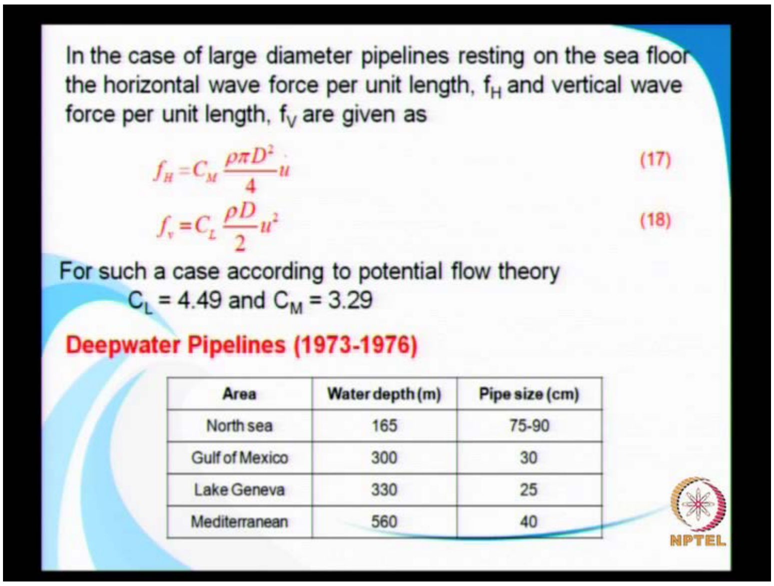

So, there are some you have an idea about the pipe sizes and the water depth, these are of course, a bit old data where in you have. Now, these days you have larger water depth and larger pipe sizes. Just to have an idea the water depths and the pipe sizes are provided here for horizontal numbers. Already we have seen, what is the kind of sizes, which they adopt for different types of offshore structures? They have only for the case of pipelines in case of large diameter pipelines large diameter pipelines and that to when it is resting on the sea bed.

The horizontal it is said in literature the horizontal force can be considered as the constant is large u and you can afford to ignore the drag force. So, the total force is approximately said to be equal to predominantly inertia. And the vertical force is going to be predominantly lift force, and where in this case if it is large diameter pipeline and resting on the sea bed. Then you can get a close some kind of a close form solution, because you already have the C L as 4.49 and c m as 3.29 reported based on the potential force theory by a few authors.

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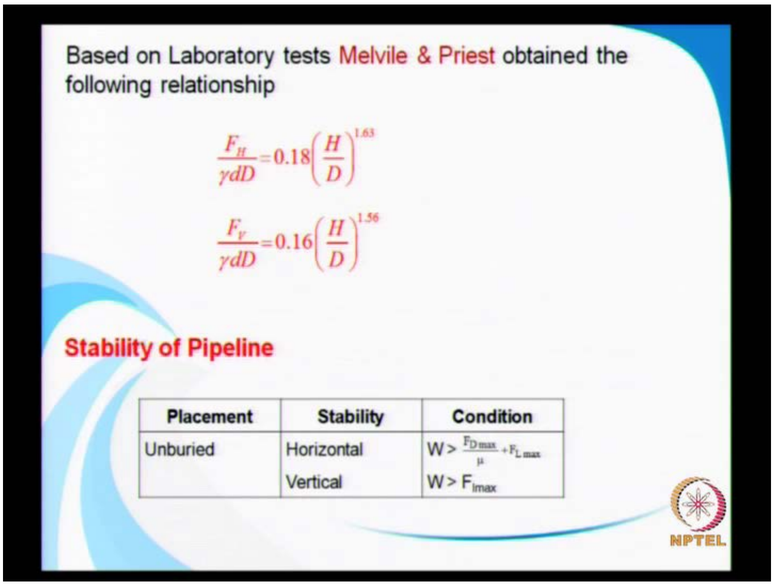

Now, it was this again one of the research that has been published based on laboratory test Melville and Priest had try to obtain the relationship for the horizontal and vertical force for a pipeline as a function of H y D there are several other articles published on the wave forces on pipelines, which needs additional reading. So, why all these things come into picture? Finally, it will boil down to checking for stability of a pipeline. So, you want to check for a horizontal stability of the vertical stability this is the condition where W is the weight and other parameters are already noted.

Thank you.