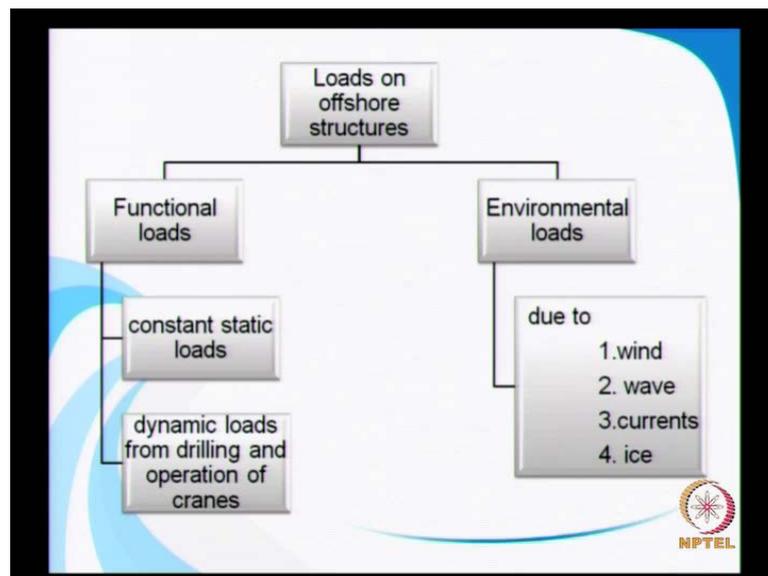


**Wave Hydro Dynamics**  
**Prof. V. Sundar**  
**Department of Ocean Engineering**  
**Indian Institute of Technology, Madras**

**Module No. # 05**  
**Wave Loads on Structures**  
**Lecture No. # 01**  
**Wave Loads on Structures I**

So, today we will just see the details about wave loads on structures. This part of the lecture is extremely important for the design of offshore structures. The design of structures, I mean the onshore structures and the offshore structures differ mainly, because of the action of waves on structures in offshore. You should recollect the basics of wave motion, wherein we saw that waves are unsteady motion. And the particle velocities and acceleration are very important and we looked at a theory which is called as linear array's theory which is used to describe the particle motion as well as the dynamic pressures, all other information what is going on below the ocean waves.

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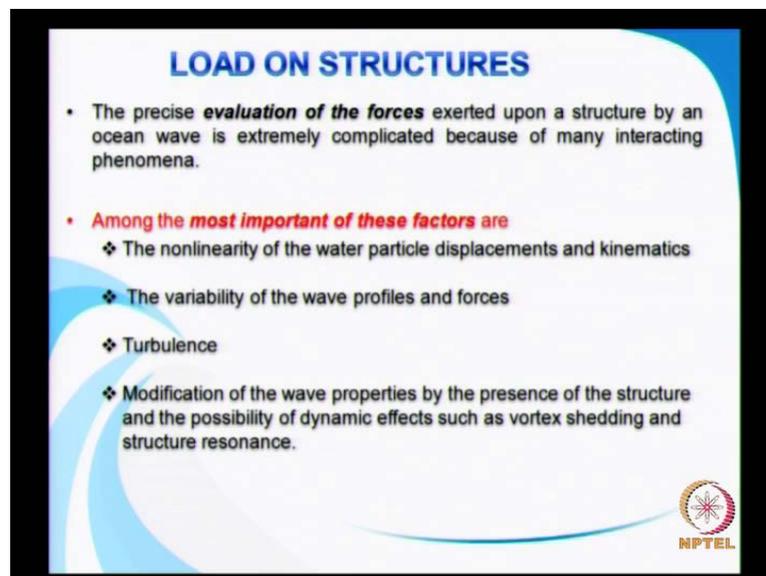


Having in that, today we will see this topic loads on offshore structures can broadly be classified as environmental loads and functional loads. Functional loads are more or less

well understood. And these kinds of loads are almost similar to that what we have on land structures. So, this can be either constant static loads or dynamic loads or varying load. Due to the operation of drilling cranes or some of the offshore platforms will have helipad where in the, that kind of loads also have to be taken care etcetera. So, these are all more or less like well understood kind of loads, and it is not so new for at least for the civil engineers.

But, what is important for us apart from the functional loads for which this lecture is devoted is a prediction or the evaluation of the environmental loads. So, the environmental loads are due to wind, due to ocean waves, due to ocean currents and then finally ice which is not directly relevant to tropical countries. But of course, in certain part of the world parts of the world you need to consider the action of ice also. However, in this lecture this part will not be considered, we will be looking at the wind wave and currents of all these environmental loads. The loads due to waves very often dominate and these loads only dictate the design of any offshore structures. So, you see that evaluation of the wave loads have to be done as close as possible to reality.

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**LOAD ON STRUCTURES**

- The precise **evaluation of the forces** exerted upon a structure by an ocean wave is extremely complicated because of many interacting phenomena.
- Among the **most important of these factors** are
  - ❖ The nonlinearity of the water particle displacements and kinematics
  - ❖ The variability of the wave profiles and forces
  - ❖ Turbulence
  - ❖ Modification of the wave properties by the presence of the structure and the possibility of dynamic effects such as vortex shedding and structure resonance.

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The precise evaluation of the loads exerted on any structure in the offshore by ocean waves quite complicated and that is, this is because of several phenomena interacting with each other. But we do have some simple empirical relationships etcetera to determine the wave loads. So, among the factors which cause some problems for us in

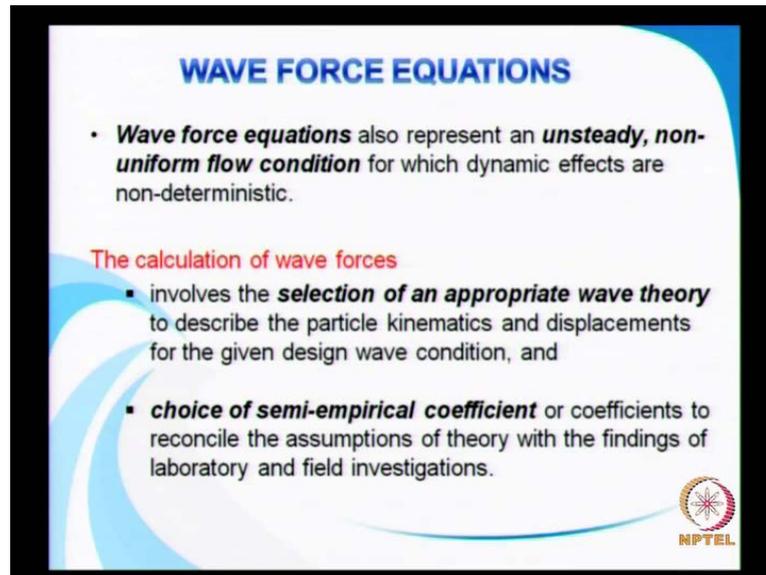
the evaluation of the wave forces are the non-linearity of the particle displacements and kinematics. What we have seen so far is? The particle kinematics and displacements due to linear waves.

When I say linear waves is a just regular sinusoidal wave. But waves cannot need not be linear very often, it not, will not, need not be linear. What do you mean by non-linear simple definition? the crest portion would be higher than the trough portion. The crest may be steeper and the trough may be flatter. So, in that case you cannot really define the wave as an in form of a sine curve sine definition. So, there are so many theories to describe the, such kind of non-linear waves which we will take up later.

But this non-linearity of the particle displacements and kinematics is one important factor which is contributing to errors are may be leaning to complications in the evaluation of the wave force. The next is variability of wave profiles that is what I was just telling? You may have a steep crest and a flat trough, you will not be in a position to define. You cannot define that kind of a profile by a simple sinusoidal wave which we have seen earlier.

Then turbulence, this is not another which is not well understood that posses some difficulties. Then the presence of the structure modifies the wave properties, the waves get scattered depending on the size of the structure which we will be seeing later. And that may be the possibility of dynamic effects, such as vortex shedding and structure resonance. So, this again depends on the type of fixity you are having for the structure how slender. The structure is etcetera structure responds as well as vortex shedding. I am sure that you would have studied in your basic courses in civil engineering or any other fields of engineering. But anyway we will be looking at this late, I hope things are clear now.

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**WAVE FORCE EQUATIONS**

- **Wave force equations** also represent an **unsteady, non-uniform flow condition** for which dynamic effects are non-deterministic.

**The calculation of wave forces**

- involves the **selection of an appropriate wave theory** to describe the particle kinematics and displacements for the given design wave condition, and
- **choice of semi-empirical coefficient** or coefficients to reconcile the assumptions of theory with the findings of laboratory and field investigations.

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Now, we will proceed on the wave forces the equations for defining the wave force, wave force equation represent an unsteady non-linear flow conditions for which the dynamics affects are non deterministic. There are a few steps, immediate steps to be adopted for the calculation of wave loads or in other words these few steps in one way or other may be responsible for also leading into certain degree of uncertainty in the evaluation of wave loads. For instance you have to describe a particle motions, a wave is moving and you have the particles moving under the waves and its motion only is going to induced loading on the structure.

This movement of the particles is very clearly understood as far as the linear theory is concerned. Depending on the water depth condition for instance, it is if the water depth condition is the deep then, you will have circular orbits size reducing at the distance of  $\lambda$  naught by two below the means sea level. It is almost negligible whereas in shallow water you will see that the particles will be moving over the entire depth of almost same magnitude. So, you will see that the force exerted is almost same this, I have also illustrated when I have, when I spoke about the wave theories. I mean the linear wave theories.

So, if you are using a linear wave theory in the location where the waves are not following a linear theory. Than a certain degree of error may would result not if you adopt a linear theory for describing the particle velocities, accelerations etcetera. So,

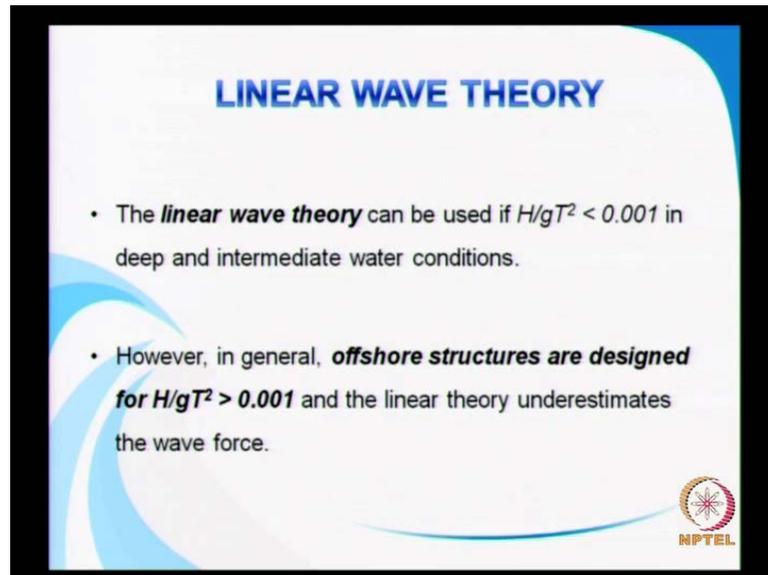
before you get into solving a problem for wave forces, first you have to check whether, we can apply a linear theory for evaluating the particle velocities. And later, I will also be covering a topic on finite amplitude wave theories, where in you will be exposed the reasons of applicability for linear theory and other kinds of wave theories. Now, we will try to adopt only the linear theory.

But with the caution start, you would have uncertainties or the uncertainties in the valuation of forces may be due to inappropriate selection of the wave theory for the description of particle displacements are velocities or the second one is as indicated here. Choice of empirical coefficients, there are certain coefficients, please recollect what we have understood in basic fluid mechanics. I am sure all of you would come across coefficient of drag, it is an empirical coefficient. And how do you get this coefficient of drag coefficient of drag in basic fluid mechanics is obtained from published literature.

Where in several authors would have done carried out experiments determined with the speed, with the velocity flow takes place and then determining the force on a single pile or structure. And then coming out with that coefficient, so, usually the drag coefficient is presented as a function of Reynolds number. So, the in a similar way so this coefficient of drag is nothing. But an empirical coefficient and we use only the experimentally available values. So this mere result in some kind of a variation later, if you see there so much of literature available on the variation of coefficient of drag as a function of Reynolds number or any other form of flow parameter, dimensionless flow parameter which we will be seeing later.

So, remember that there are two types of uncertainties which may result. One is in the selection of the empirical coefficients of drag or inertia which I will introduce the inertia term later, and the first one is the selection of appropriate wave theory.

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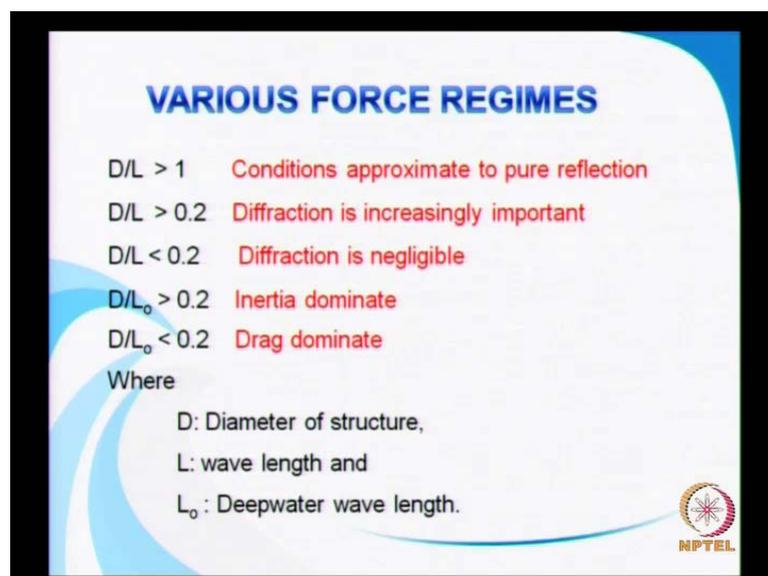
### LINEAR WAVE THEORY

- The **linear wave theory** can be used if  $H/gT^2 < 0.001$  in deep and intermediate water conditions.
- However, in general, **offshore structures are designed for  $H/gT^2 > 0.001$**  and the linear theory underestimates the wave force.

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Since, we will be applying linear wave theory to begin with, let us see where the linear wave theory is applied. Linear theory can be used if  $H$  by  $g T$  square is less than 0.001 in deep and intermediate water depth conditions deep is  $D$  by  $L$  greater than 0.5 and intermediate  $D$  by  $L$  in between 0.5 1.5. However in general offshore structures are designed for  $H$  by  $g T$  square greater than 0.00 and linear theory is expected to underestimate the force. So, this should be at the back of your mind when you are working with linear theory.

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### VARIOUS FORCE REGIMES

- $D/L > 1$  Conditions approximate to pure reflection
- $D/L > 0.2$  Diffraction is increasingly important
- $D/L < 0.2$  Diffraction is negligible
- $D/L_0 > 0.2$  Inertia dominate
- $D/L_0 < 0.2$  Drag dominate

Where

- D: Diameter of structure,
- L: wave length and
- $L_0$ : Deepwater wave length.

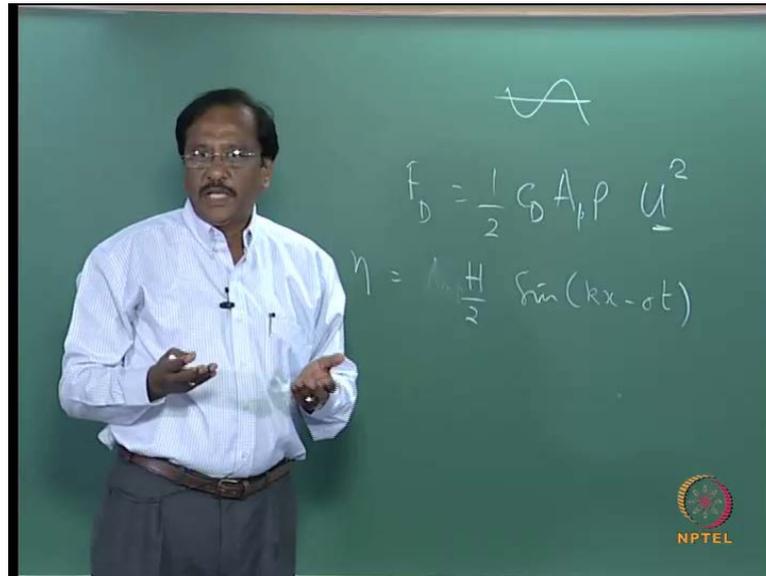
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Now, let us get into various force regimes. I will be introducing certain words like diffraction, drag inertia etcetera. In a broader sense the wave forces can wave force regimes can be classified as in the form of as a function of the relative characteristic dimension, which is diameter of the structure divided by the wave length. If  $D$  by  $L$  is greater than one condition of pure reflection takes place we will look at refraction etcetera later about the phenomena. If  $D$  by  $L$  greater than 0.2 diffraction increasingly important, what does that mean?

That means the diameter of the structure is quite large compared to the wave length. That is, it is greater than 0.2. So, if you have a larger structure and the waves go on hit the structure waves get scattered. Due to the presence of this, large structure and scattering of the waves because of the presence of large structure is called as diffraction. Or sometimes it is referred to as scattering. The next case is  $D$  by  $L$  less than 0.2 in which case diffraction is said to be negligible not that we have used wave length. But then you have  $D$  by  $L$  naught greater than 0.2  $L$  naught is the deep water wave length.

Deep water wave length is usually greater than the wave length decreases as it go towards the shore from deep water. The wave length keeps on decreasing which we are seen earlier. So,  $D$  by  $L$  naught if that is greater than 0.2 we say that the force is predominantly inertia and if  $D$  by  $L$  naught is less than 0.2 we say that drag is dominate. All of you would have heard about drag force, you have a structure, you have the force wave, I mean flow taking place.

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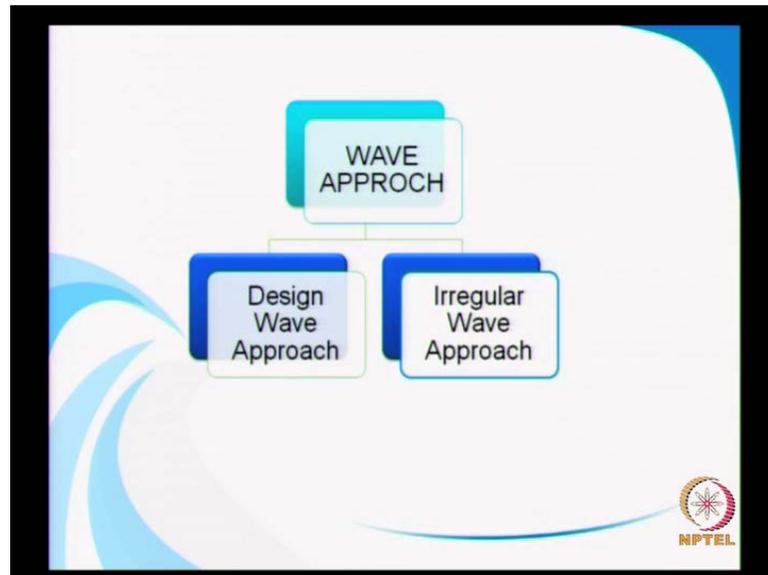


How do you write the drag force is? I am sure all of you would have seen this half into C D into projected area into rho into u square. See this, the usual drag force we have all learnt during our graduate course. But here we are considering an unsteady flow, what is, what does that means? They are considering a wave interacting with a structure. How you define a wave? A wave is going to vary with respect to time remember eta is some amplitude or I would just simply say h by two into sin of k x minus sigma t. And I have also explained earlier, what is the, what does x and t convey at a given location.

The variation of the profile with respect to time that is your wave profile in a particular location. And then the other thing is at a given instant of time, if you have a tank at a given instant of time. What is amplitude here? What is amplitude there? That is with respect to space both have to be are included in this, so now you see that eta is varying with respect to time. So, we have to consider that, so that is going to induce acceleration, because it is going to vary with respect to time. It is going to, I mean insert an acceleration term which is going to be responsible for the inertial force acting on a structure.

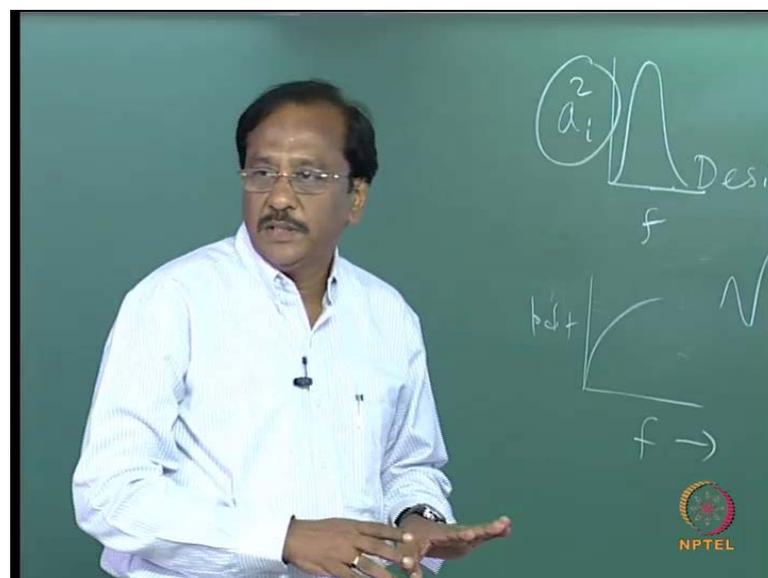
So, here if you see inertia dominant means that it is acceleration induced force that is higher drag. Dominant means the viscous effect is more so that is, so we say that is drag dominated. So, these are the broader classification of the wave force regions.

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When you look at a wave is wave forces in the broader sense, you can have either design wave approach or irregular wave approach. What is the design wave approach? you want to design a pile and it has to sustain the extreme conditions in the ocean.

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So, you have to evaluate what is called as the design wave? Civil engineers you remember we when we want to design a damp, we look at the design flood. The design flood is hundred year design flood. So, here again you have to arrive at the design wave for a given particular site. I will introduce later the concept of random waves, but as of

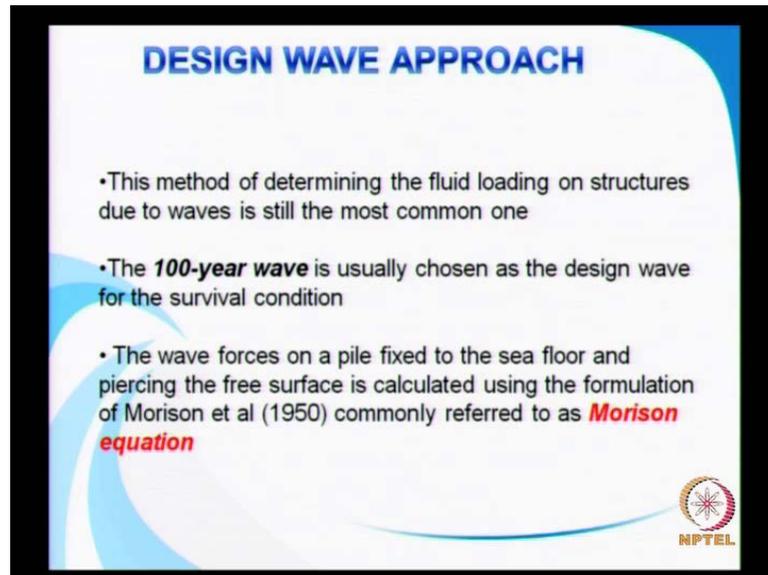
now, you should have in mind that the ocean waves. If you measure the water surface elevation in the ocean, the time is  $t$  will be something like this, so this is a random wave and you need to analyze. This random wave and you arrive at certain statistical parameters from which you finally, you will come up with a design wave, which will have a design wave height and a corresponding wave period.

For which, you will be evaluating the wave force is that clear, that is, what is called as design wave approach? The other approach is that in the design wave approach, the irregular wave is represented as a statistical parameter which will be a single parameter. But in the irregular wave approach, see first there is a wave coming and hitting a smaller wave, then you have a larger wave, then a medium wave again. So, each time it is hitting the structure with particular amplitude and a particular frequency. So, you consider all these things, or all these combination of a number of waves, with different wave heights and frequencies and evaluate the wave force and then present, how your force will look like you understood?

So, that is what is called as irregular wave approach? So, usually irregular wave approach you will be presenting the result in what is called as a frequency? Which will be looking like this, you will have frequency here and you will have some kind of amplitude on the  $y$  axis. Which is referring to as spectral density? So, this Figure will give you, how the forces vary for different frequency components in the random in the design wave approach? You are going to deal only with one frequency, that is the entire frequencies are represented by a single wave period, you understood.

The other way of presenting this will be probability approach, you can have the force and you can get the probability. I mean cumulative probability density function. So, it will be something like this, which I am not going to take up now Only to introduce you to the two approaches. But when we look at the shorten statistics etcetera, that time we will have a more better understanding of the formulations and other aspects.

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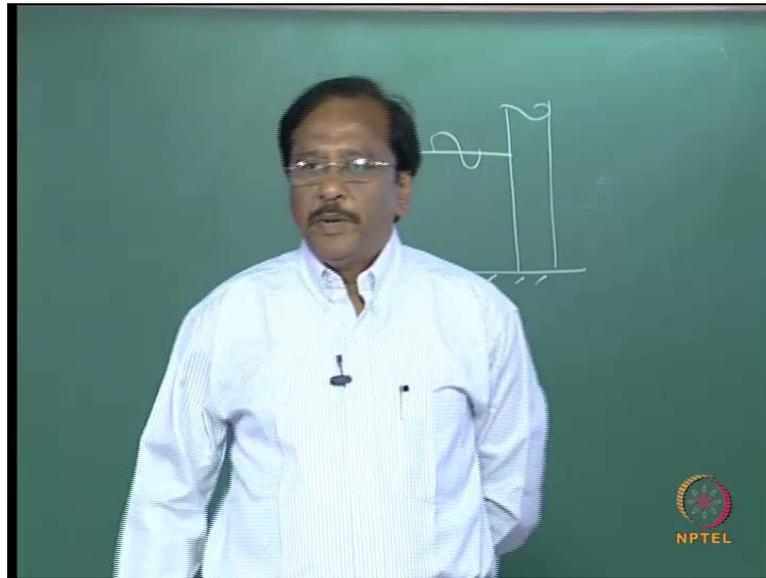
**DESIGN WAVE APPROACH**

- This method of determining the fluid loading on structures due to waves is still the most common one
- The **100-year wave** is usually chosen as the design wave for the survival condition
- The wave forces on a pile fixed to the sea floor and piercing the free surface is calculated using the formulation of Morison et al (1950) commonly referred to as **Morison equation**

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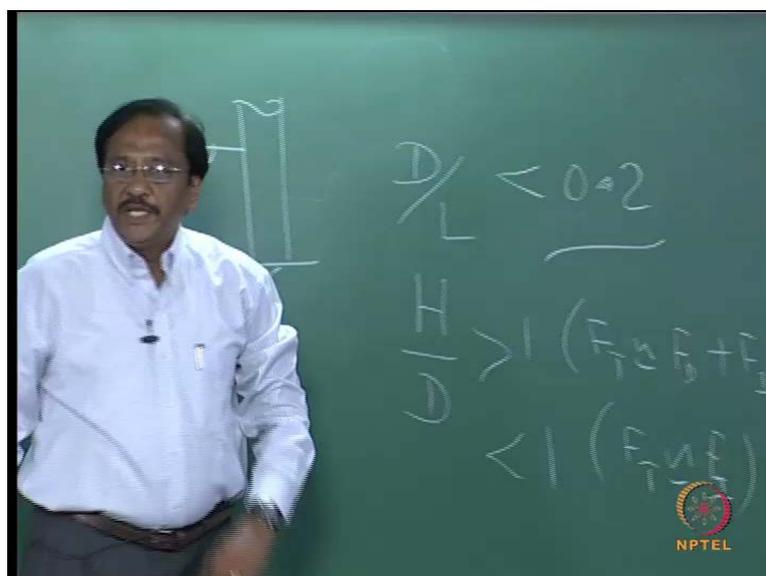
So, this is, what, I had already explained in the design wave approach. This method of determining, the fluid loading on structures due to wave is still, the most common one it is being still used. The hundred year wave is usually chosen as the design wave for the survival condition. So, when you design any structure there are two aspects one is the operating wave conditions and another is the survival condition. So, naturally the wave height or wave climate for the survival condition, will be much higher compared to the operational conditions having had some amount of exposure to the basics. Now, we will go again, we still there in a fundamental only. So, the wave force on a pile looks at the boundary condition pile.

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Resting on the bed and piercing, the free surface for this condition only the formulation was proposed by Morison. At all as early as 1950 for the evaluation of wave force, even today, this equation is being widely adopted remembered that Morison is equation should be applied. Only when you have the viscous effects and also when you have the combination during, which you have the combination of both drag and inertia component for certain conditions. That inertia may be dominant or certain conditions wave conditions the drag may be dominant. So, let us see, what is meant by Morison equation? The most important thing is that Morison equation is valid.

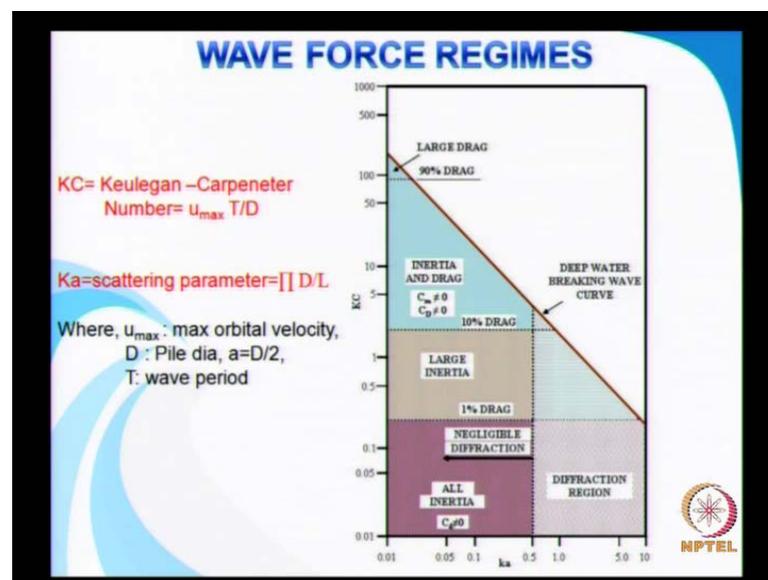
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Only for  $D$  by  $L$  less than point two, we also have later, we will see, there is another parameter which is also important that is relative wave height divided by the diameter. If it is greater than 1, then we can say that a total force will be drag force plus inertia force and if it is less than 1. We can conveniently say that, the total force is approximately equal to  $F I$ , that is under this situation and this is one condition this is one single condition  $D$  by  $L$  less than 0.2. Under this  $D$  by  $L$  less than point two, you have two conditions  $H$  by  $d$  greater than one or less than one. This drag force and inertia force we will be seeing immediately after this slide. So, total force acting on the structure will be orbital particle velocity dependent drag force.

Which is referring to as  $f d$  and particle acceleration dependent inertia force? For large size numbers more complex theories are necessary in order to take into account. The scattering radiations of the incident waves from the number, because here in this case for  $D$  by  $L$  less than point two, the size of the structure is less. So, you do not have the problem of scattering, so, it is quite easy to work with but you have uncertainties.

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So, this is, we have seen this condition for application of Morison equation is used. When you have the drag force dominating, what is drag force? Drag force is particle velocity dependent now, look at the wave force regime now, I introduce a parameter similar to Reynolds number. We have, what is called as ceruleans carpenter number? So, which is  $u_{\text{max}} T$  by  $D$   $u_{\text{max}}$  maximum horizontal particle velocity over a time period. That

is something like displacement to divided by the diameter of the structure, will try to understand more about cerulean carpenter number later.

But right now, what it is  $u_{max} T$  by  $D$   $u_{max}$  is our orbital velocity? So, displacement divided by your time, the other parameter on the x axis is what is called as scattering parameter?

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$$k_a = \frac{2\pi}{L} \cdot \frac{D}{2} = \frac{\pi D}{L}$$

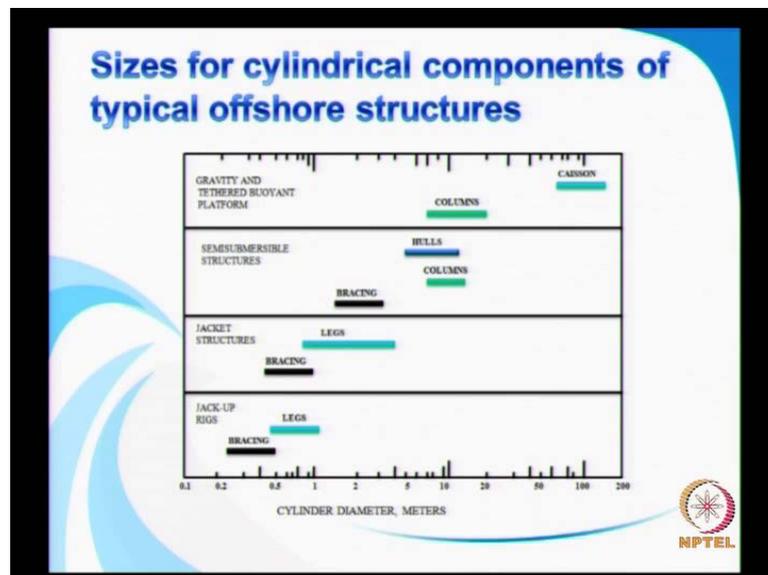
Scattering parameter  $K$  is small  $k$ , it is two pi by  $L$  into  $a$ ,  $a$  is radius of the structure which is  $d$  by  $l$ . So, this can be nothing but  $\pi D$  by  $L$  as indicated in the so this shows that different force regimes. So, for example, if you are given structure and the wave characteristic, I mean if  $k a$ ,  $k c$  if you are given a structure and the wave climate what are you supposed to do? You are supposed to evaluate, the parameters which we have seen right now, either this parameter as I indicated or those parameters. Which are indicated there this gives a very clear picture. For example, if your regime falls under this, then this indicates, that the total force is inertial the viscous force being negligible.

And above, that this is the zone, where you have large inertia a percentage of drag being just one percentage. And this is the area, where you have  $a c m$  and  $c d$  both have values, and we have at least ten percent of the drag force in the total force. Because the total force is a combination of drag force and inertia force. The top one is large drag nearly 90 percent of the total force will be and this is a line providing you that deep water wave

curve. Now,  $k a$  as you see for all these things for the Morrison equation to be adopted, this is the regime is that clear.

So, you are not supposed to strictly adopt the Morrison equation for this regime, which is nothing but the diffraction regime before understanding, the Morrison equation be clear about, where you can adopt this equation? Under what conditions, what are the parameters you need to evaluate? Before, you can assign the Morrison equation for the evaluation of the wave of course the lecture will be followed up with one or two examples worked out examples wherein things will be much more clearer.

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You earlier saw that, the wave force region one is the flow parameter comes into picture. Apart from, the flow parameters the structural dimension also is very important in deciding the regime, the flow regime or the force regime. So, in order to have a some kind of feeling, for the size of the structure. Which, we are talking about in ocean in the ocean structures in the ocean. This slide is quite useful, which gives the variety of structures, that could be found in the offshore or that needs to be designed, gravity and tethered buoyant platforms, semi submersible structures, jacket structures, jack up rings.

And each of these structures have different components like for example, in this case you will have columns and caissons. Here you can have, you will have hull columns against dressings note that the structures diameters is given in meters here. So, you see that, the diameter can be anywhere between point three two as high as about hundred meters.

Now, this gives us a feeling for the size of structures, which we are going to deal with in the field of ocean engineering for the design of offshore structures. I hope this is clear.

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### MORISON EQUATION:

**Definition sketch  
(Vertical Cylinder)**

#### Fixed Cylinder in Waves

- The principle involved in the concept of the inertia force is that a water particle moving in a wave carries a momentum with it.
- As the water particle passes around the cylinder it accelerates and then decelerates
- This requires that work has to be done through the application of a force on the cylinder to increase this momentum

Now, let us get into the Morison equation, I am taking up a pile as shown here resting on the sea bed fixed to the sea bed and piercing. The free surface and the diameter is  $d$  and I am taking an element of height  $dz$  at an elevation  $s$  from the sea bed. And a wave is propagating from left to right  $\phi$ . What is the force total force acting on that element? The total force acting at that element, which is refer to as sectional force is summation of the drag force and the inertia force acting over this element. If we can determine this force then, you know that, this force is going to vary from the sea bed up to the crest of the wave and you can integrate to get the total force moment etcetera.

So, the principle involved in the concept of inertia force is that, a water particle moving in a wave carries a momentum along with it that is clear. A wave is moving as the water particle passes around the cylinder it accelerates and then de accelerate, because it is a wavy flow. So, that means, this requires, that work has to be done through application of a force on the cylinder to increase its momentum.

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The incremental force on a small segment of the cylinder,  $dz$ , needed to accomplish this is proportional to the water particle acceleration at the centre of the cylinder (in the absence of the cylinder)

$$dF_I = C_m \cdot \rho \left( \frac{\pi D^2}{4} \right) \dot{u} dz \quad (1)$$

Where

- $dF_I$  = inertia force on the segment  $dz$  of the vertical cylinder
- $D$  = Cylinder Diameter;
- $\dot{u}$  = water particle acceleration;
- $C_m$  = inertia coefficient (for uniformly accelerated flow,  $C_m \approx 2$ )
- $\rho$  = mass density of fluid



So, the incremental force on a small segment, as we have seen earlier that is the  $dz$  needed to accomplish. This, that is, the above aspect is now, will be proportionate to the water particle acceleration at the center of the cylinder is that clear. So, I can simply say that the force acting on that particular element  $dz$  is equal to  $C_m$ , that is the empirical coefficient of inertia. Where a certain degree of uncertainty prevails as stated earlier into mass density  $\pi d^2$  by four and you have the acceleration coming into picture. So, this is your inertia force and inertia force coefficient would also vary other terms are given here.

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➤ The **principal cause of the drag force component** is the presence of a wake region on the "downstream" side of the cylinder.

*The wake is a region of low pressure compared to the pressure on the "upstream" side* and thus a pressure differential is created by the wake between the up and downstream of the cylinder at a given instant of time.

➤ The pressure differential causes a force to be exerted in the direction of the instantaneous water particle velocity.

$$dF_D = \frac{1}{2} C_D \rho D |u| u dz \quad (2)$$

where

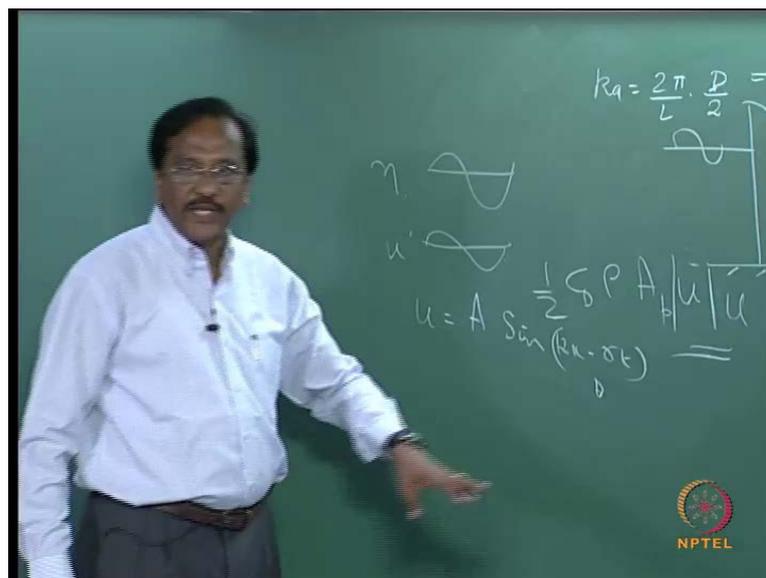
- $dF_D$  = drag force on an incremental segment,  $dz$ , of the cylinder
- $u$  = instantaneous water particle velocity;  $C_D$  = drag coefficient



Now, if you look at the drag force, the principle cause of the drag force component is due to the presence of a wake region on the down downstream of the cylinder. The cylinder is here, the wave is moving into this direction so, you will have a wake on the downstream side. What is a wake? Is nothing but a region of low pressure on the downstream side compared to the pressure on the upstream side. So, there is a kind of a pressure difference created by the wake between the upstream and the downstream of the Cylinder. At any given point of time, the pressure difference causes a force that will be exerted on the cylinder and it will be in the direction of flow taking place.

So, you have half into  $C_d$  into  $\rho$  into  $d$  into absolute of  $u$  into absolute of  $u$ . So, this is one important thing, which you need to remember you look at the absolute value as I have said earlier.

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The drag force is half into  $C_d$  into  $\rho$  into projected area into  $u$  square. Am I right, this is the usual way of evaluating the drag force now, you have an absolute here absolute value. This absolute value is to take care of the direction it is nothing but to take care of the direction, because you see that when, the wave is moving like this the velocity is going to change, because velocity is sum into sine of  $kx - \sigma t$ . That is, if you call this as equal to  $\eta$  or a phase, then you see that, the direction is going to change depending on this one cycle, full cycle you have plus and negative, if you do not have it have the absolute sign then what will happen?

You are not really representing, the direction of the flow, it will take positive all the way through and the negative component will not come, at all so in order to take care of the direction of the flow. We have an absolute value and again you see that there is a coefficient of drag.

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Combining the inertia and drag components of force, the Morison Equation is written as,

$$dF = \left( C_m \cdot \rho \frac{\pi D^2}{4} \dot{u} + \frac{1}{2} C_D \rho D |u| u \right) dz \quad (3)$$

If the cylinder extends from the ocean floor to the SWL, the total force on the cylinder is given by the integral,

$$F_T = \int_{-d}^0 \left[ C_m \cdot \rho \frac{\pi D^2}{4} \dot{u} + \frac{1}{2} C_D \rho D |u| u \right] dz \quad (4)$$

where  $u$  and  $\dot{u}$  are given as

$$\left. \begin{aligned} u &= \frac{\pi H}{T} \frac{\cosh k(z+d)}{\sinh kd} s \\ \dot{u} &= \frac{-2\pi^2 H}{T^2} \frac{\cosh k(z+d)}{\sinh kd} \cos \theta \end{aligned} \right\} \quad (5)$$


So, combining the inertia and the drag component, we have the force on that elemental height of  $d z$  given as  $d f$  etcetera that is equation d. Now, if the cylinder extends from the ocean floor to the sea water line. Then, the total force on the cylinder is given by an integral minus  $D$  by minus  $d$  remember that for us  $z$  is zero at this point. So,  $z$  will be minus  $d$  at this point so from the sea bed up to the still water line in fact strictly speaking. You have to carry out this integration up to  $\eta$ ; because you can have, you will have the wave coming up to this level. So, you need to include that  $\eta$  also for simplicity, I have just stopped at 0.

So, here in you have to substitute the expressions for  $u$  and  $\dot{u}$ , which we have seen earlier so, this should be so this is the Sin theta and then Cos theta. So, this two components will be out of phase.

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Substitute (5) in (4),

$$F_z = C_m \rho \frac{\pi D^3}{4} \int_{-d}^0 \left( \frac{-2\pi^2 H \cosh k(z+d)}{T^2 \sinh kd} \cos \theta \right) dz +$$

$$\frac{1}{2} C_D \rho D \int_{-d}^0 \left( \frac{\pi^2 H^2 \sin \theta |\sin \theta|}{T^2 \sinh^2 kd} \cos^2 k(z+d) \right) dz$$

$$= -C_m \rho \frac{\pi D^3}{4} \cdot \frac{2\pi^2 H}{T^2} \frac{\cos \theta}{\sinh kd} \left( \frac{\sinh k(z+d)}{k} \right) \Big|_{-d}^0 +$$

$$\frac{1}{2} C_D \rho D \frac{\pi^2 H^2 \sin \theta |\sin \theta|}{T^2 \sinh^2 kd} \left[ \frac{z}{2} + \frac{\sinh 2k(z+d)}{4k} \right] \Big|_{-d}^0$$

$$= -C_m \rho \frac{\pi D^3}{4} \frac{2\pi^2 H}{T^2} \frac{\cos \theta}{\sinh kd} \times \frac{\sinh kd}{k} +$$

$$\frac{1}{2} C_D \rho D \frac{\pi^2 H^2 \sin \theta |\sin \theta|}{T^2 \sinh^2 kd} \left[ \frac{\sinh(2kd)}{4k} + \frac{d}{2} \right]$$


So, substitute in this equation carrying out all the integration, you come across this expression and we will continue this later.