Wave Hydro Dynamics
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Module No. # 04
Random and Directional Waves
Lecture No. # 06
Directional Waves

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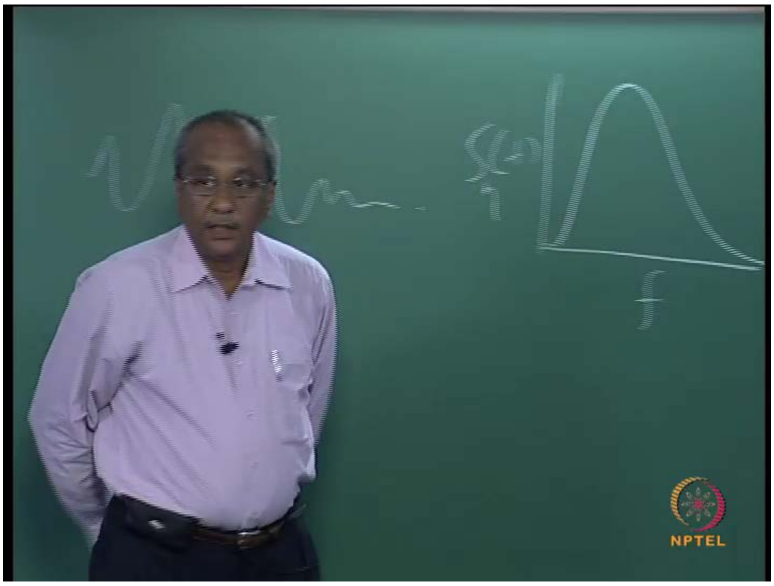

Having seen the description of two-dimensional spectrum, please recollect that a random wave can be described in the frequency domain as a function of and this is how it is represented so on. The y axis is nothing but half into amplitude spectrum. So, the area under the spectrum gives you the significant wave height or the total energy contained to the sea – sea wave.

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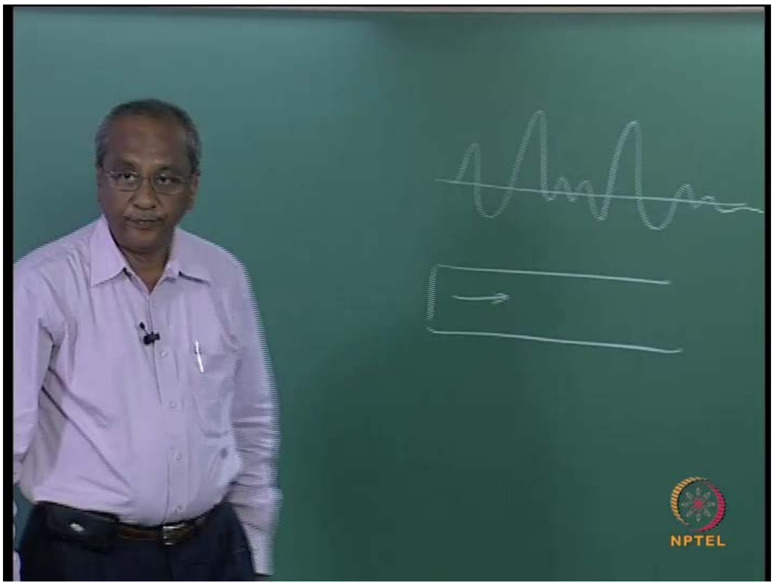

This we have already done and we know what it means this means that waves are travelling in one direction this is true if you are generating a random wave in a wave flume that is in the laboratory, but this is not true in the case of open ocean in the open ocean if you consider the waves propagating from the deep waters.

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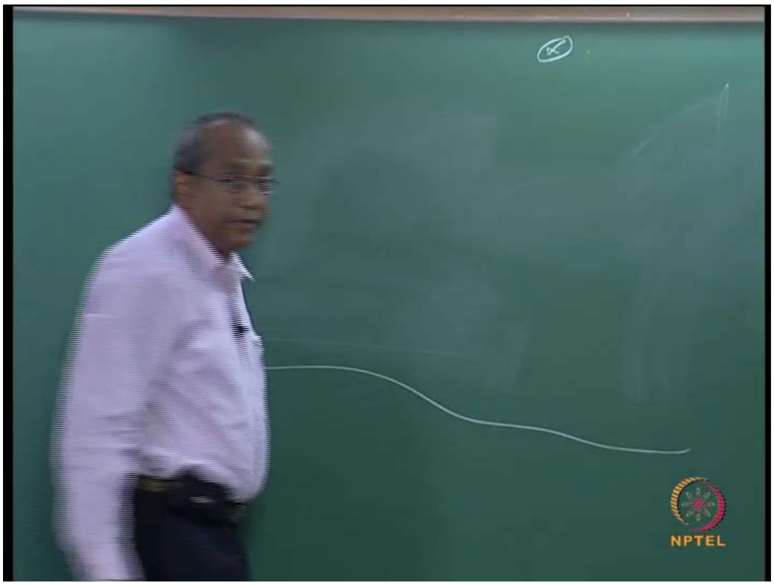

And if you consider a shore line somewhere in the deep waters you have the waves generated because of the blow of the wind blowing over the water surface and the waves are generated somewhere here where in the waves wind will be blowing in different directions so, the waves will also be propagating in different directions.

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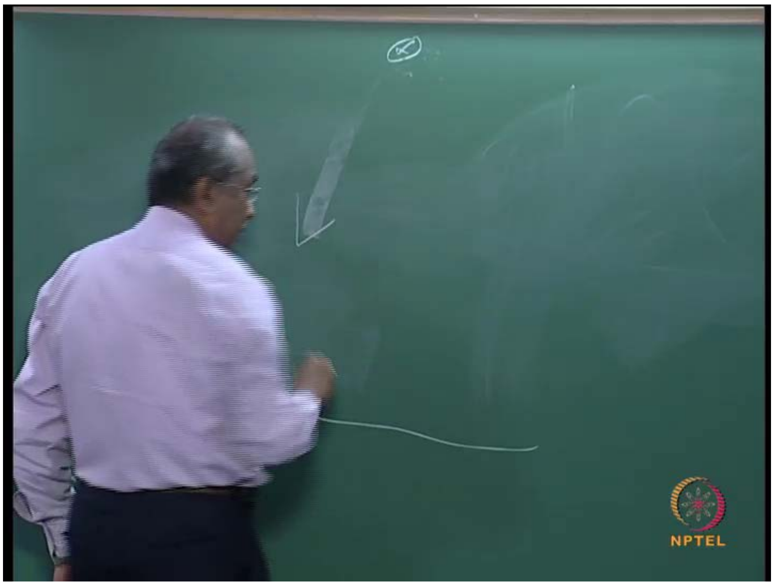

When they propagate towards the shore as you all know that it undergoes the phenomena of refraction which means that the refraction this will orient itself more or less parallel to the bottom depth contours. So, as the waves from the deep water propagates its direction gets aligned to parallel to the break water parallel to the bottom depth contours and somewhere near the coastal waters more or less your direction is well defined. That is you can say that here the waves are more or less coming in almost a particular direction.

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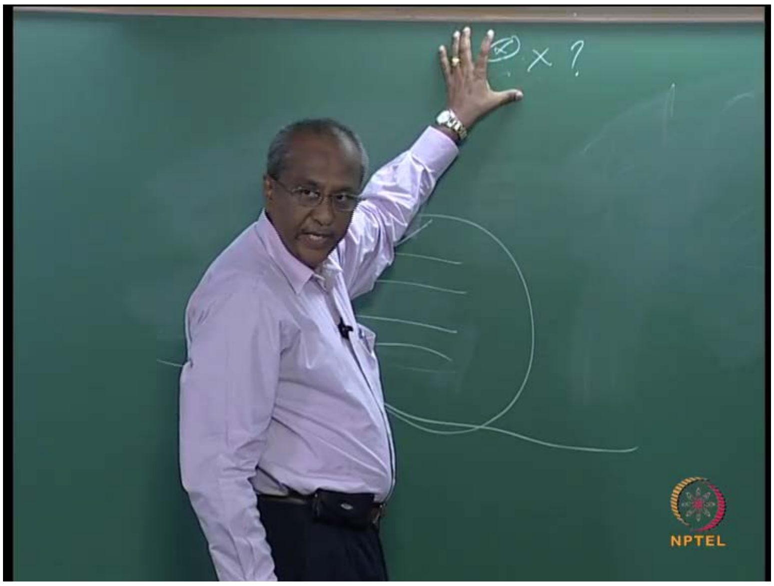

But in deep waters you cannot say that rather questionable so, this essentially means that the waves in deep water will be coming from different directions hence just mentioning about S of eta f or describing the wave profile in terms of only f is rather questionable. So, in deep water you should have another function that is the distribution of the energy or the waves coming from different directions have to be considered. So, this you will be represented as f comma eta you see the difference between S of eta, S of f and S of comma S of S comma, theta. So this is referred to as two dimensional spectrum two dimensional spectrum and this is three dimensional spectrum.

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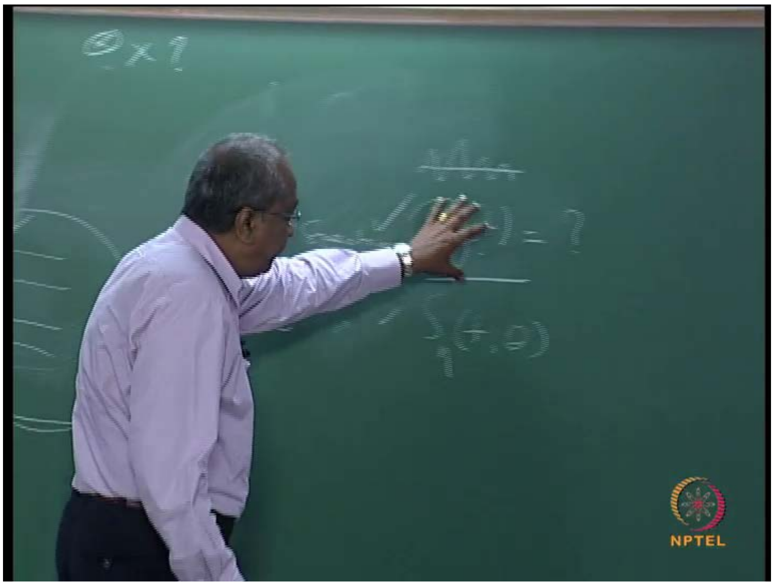

So, we call 2 D waves and 3 D waves 2 D waves their representation as a form in the form of a spectrum it is termed as 2 D spectrum and 3 D waves that is waves from different directions, then they are represented in the form of energy distribution it has to be associated with direction also in addition to frequency as described in the 2 D spectrum. So, that is the basic difference between the 2 D spectrum and 3 D waves or 3 D spectrum.

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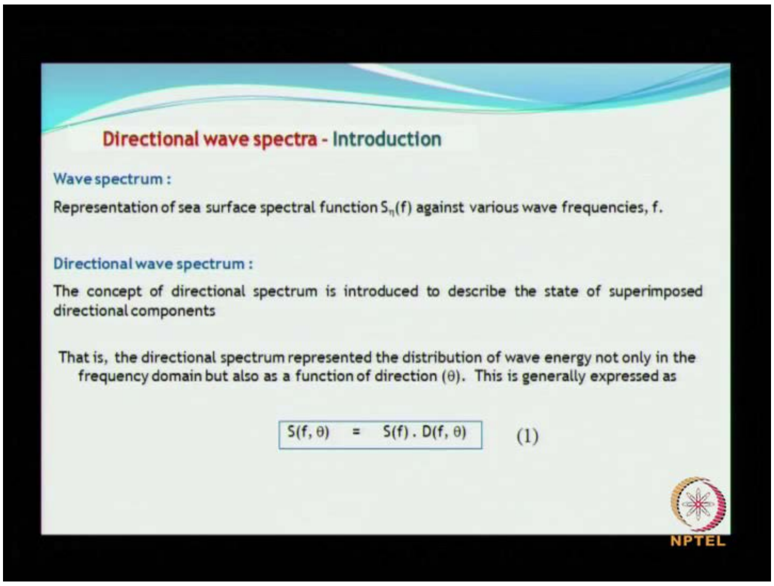

So, representation of sea surface waves spectral functions against wave frequency that is a wave spectrum. Directional wave spectrum the concept of directional spectrum is introduced to describe the state of superimposed directional components that is waves from differential directions so hence there should be a way of obtaining this. How do you obtain that by equation 1?

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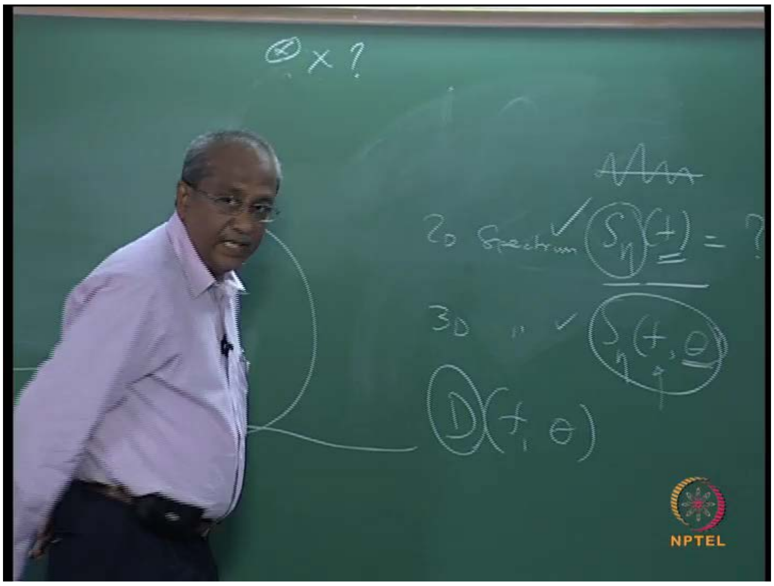

That is you simply multiply the 2 D spectrum which already we have seen under random waves by a directional spread that is how the energy can be distributed. How the distribution of energy distribution of the energy in terms of direction can be described? So, that is obtained by D (f comma theta) which is directional distribution as a function of frequency and eta is that clear. So, now later we will see as we have seen here there are some standard spectra like Bretschneider spectrum, Jonswap spectrum, Pierson Moskowitz spectrum, Scott spectrum so many spectrums are there and we have also seen the usefulness of looking at presentation which can follow some kind of a standard distribution. In a similar way we also have some standard distribution for directional spread.

So, if you want to simulate 2 D waves in the lab you can simulate may be a Pierson Moskowitz spectrum. You want to simulate you know that in the field the waves in the field may be of Chennai. You have waves following say for example, Scotch spectrums then I want to simulate the same kind of waves in the lab and there is a situation that there is not much of a directional spread. Then I will resort to test in the tank in the flume that is capable of generating the wave elevation that will simulate the waves which will follow the Scotch spectrum.

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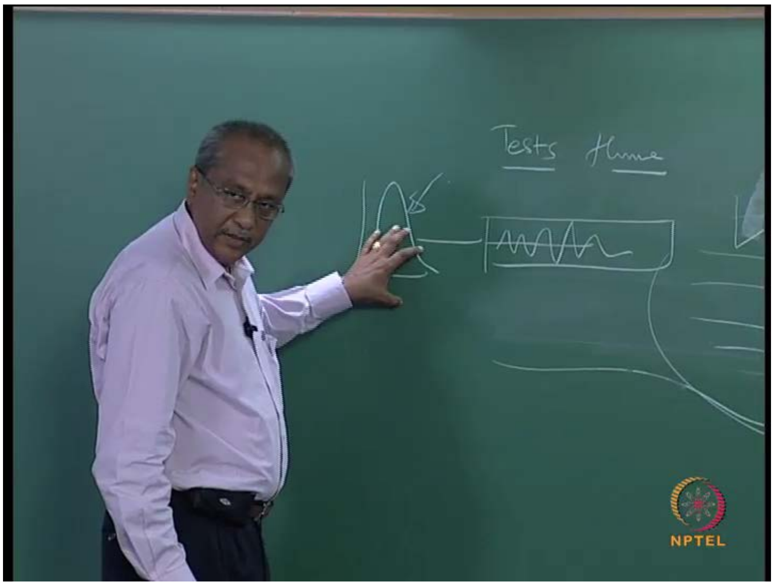

For that matter any spectrum can be this mainly is nothing but sending the signal to your wave maker and dictating how it has to move. So, you will generate a pre defined spectral characters, but the same way you can also generate a directional spectrum. Where in if you have already proved or someone says or some you yourself want to test you can select a particular 2 D spectrum and an associated directional spread.

The associated directional spread it can be a measured spread which can be obtained by doing some measurements and another followed by analysis you can arrive at the directional spread or you can use a theoretical directional spread and test your structures. How it can function or numerically also it can be done? So, basically what we are trying to look at is how you represent the distribution? I mean the simulation of waves how do you represent the wave elevation.

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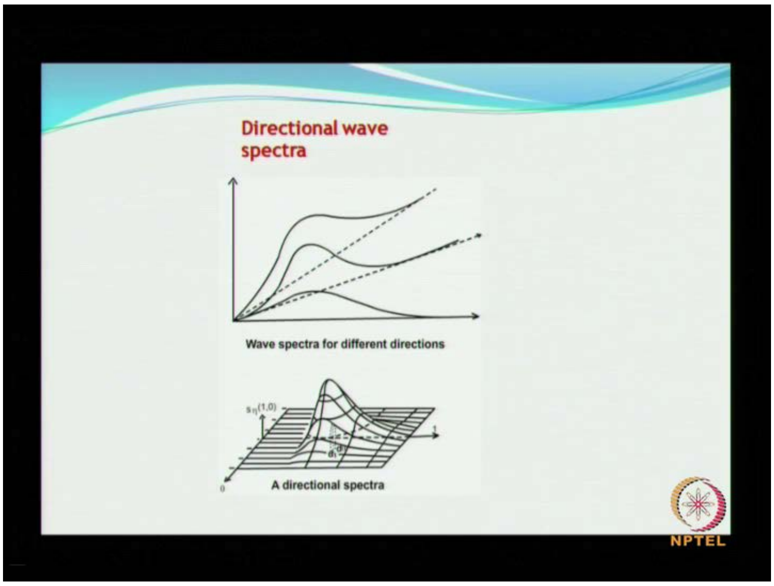

Before going into that this is how the distribution would look like the top figure shows the distribution of energy. If you take one single if you take one single wave I mean spectrum say this can be as a function of frequency for a particular direction or vice versa on the y axis, it will be the energy or of amplitude square which is nothing referred to as spectral density. So, if you superpose all these things then you get a three dimensional view and that three dimensional view is called as the directional spectrum is that clear. So, you will have a long expression, but I will just show some of these expressions and only thing is after having a look at the lecture patiently go through all the equations and try to understand.

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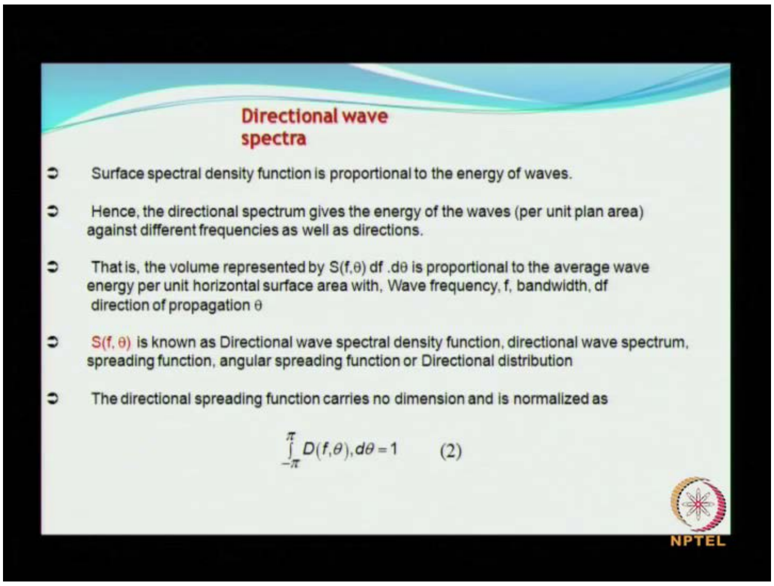

So, although I have discussed something some few points I have listed here directional waves may come surface spectral function is proportional to the energy of waves. Hence directional spectrum gives energy of the waves per unit plan area against different frequencies as well as directions. I explained that that is volume represented by S (f comma theta into D f) elemental frequency and elemental theta directional is now proportional to the average wave energy per unit horizontal area and the variables are mentioned there frequency band with D theta is implemental of directions and S (theta) is known as the directional spectral density function or directional waves spectrum spreading function angular spreading function or directional distribution. So, you have a number of names assigned to S (theta).

Now the directional spreading function carries no dimension and is normalized by as shown in equation 2. S (theta) is equal to S (e f) into D (S comma, theta), so D (f comma, theta) that is as I have written here earlier. So, this directional spreading function carries no dimension and is normalized as shown here is that clear.

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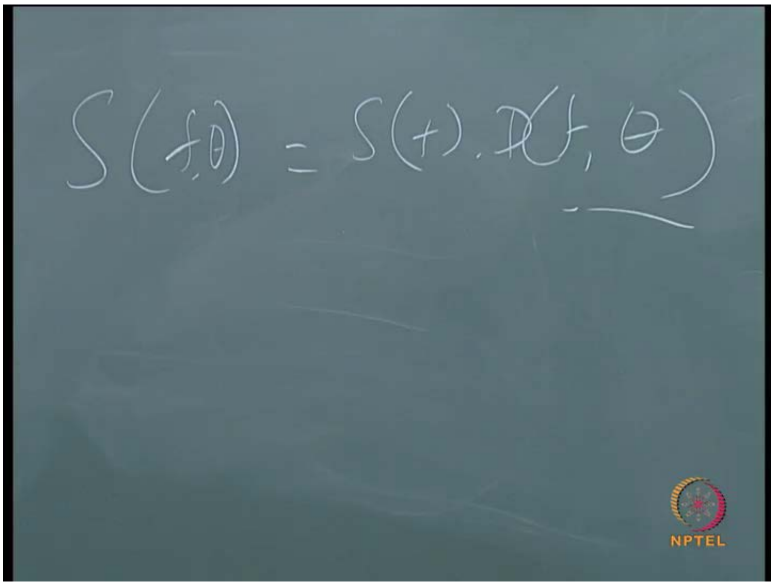

Now as I mentioned earlier why do we have to I have already mentioned earlier why do we have to look at the method of representing the surface elevation or the surface wave profile only then you will be able to describe the waves characteristics. So, there are several there are broadly two methods are more than two methods. But broadly classified it will classify as single summation model and double summation model. What we are looking at is double summation model wherein you have double summation 1 is n equal to 1 to infinity and the next one is m is equal to 1 to infinity.

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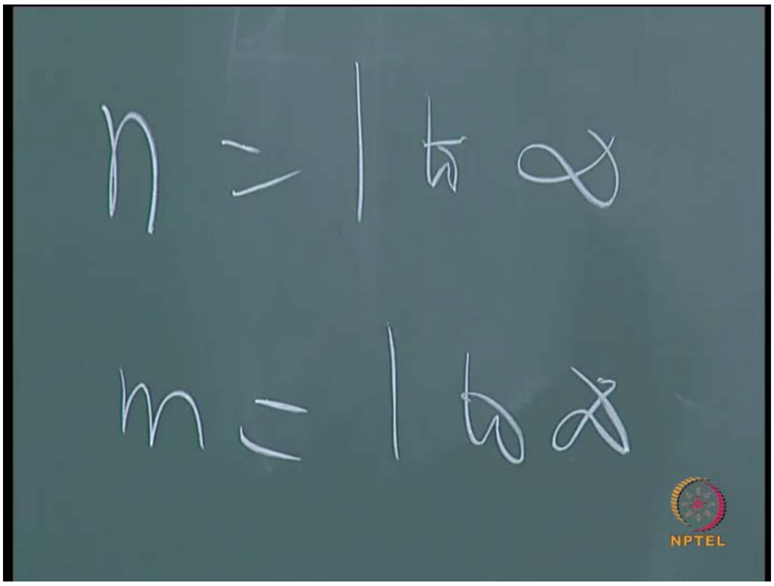

Note that on the hand side.

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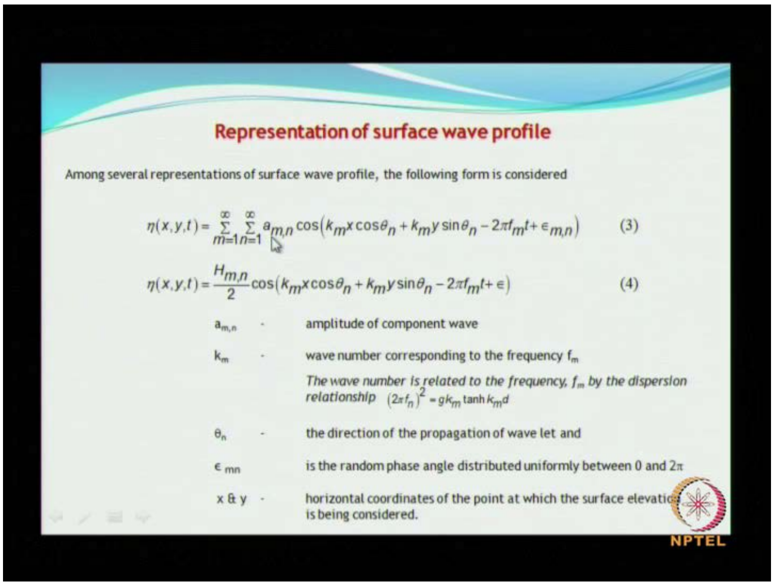

You have the amplitude a of m comma m associated with both direction and frequency n stands for the number of directional components how many directions are you going to include in your simulation and m represents the frequency components.

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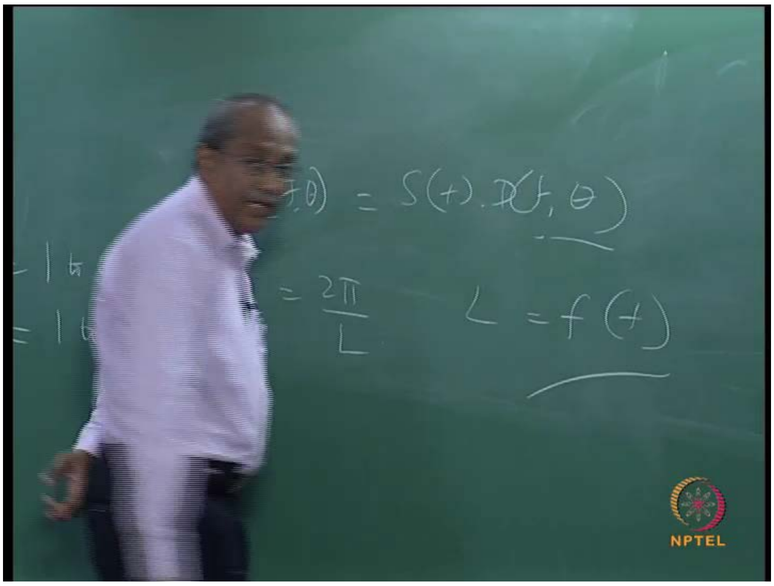

So, you have K m is nothing but wave number for that frequency component you have to K is nothing but you already know K is 2 pi by L and L is going to be a function of frequency. So, naturally you will have to calculate your k and t is the time and then epsilon m is the random phase angle distributed uniformly between 0 to 2 pi which I have already explained under random waves. So, this K m is related to the dispersion relationship as indicated here x and y are coordinates horizontal coordinates at which the surface elevation is being considered at any location x or y, so this general expression rho x u the description of wave elevation very clearly, it has considered the frequency components, it has considered the direction of components and it has also accommodated a particular location in as a function location. So, this is in its generalized form the representation of wave profile.

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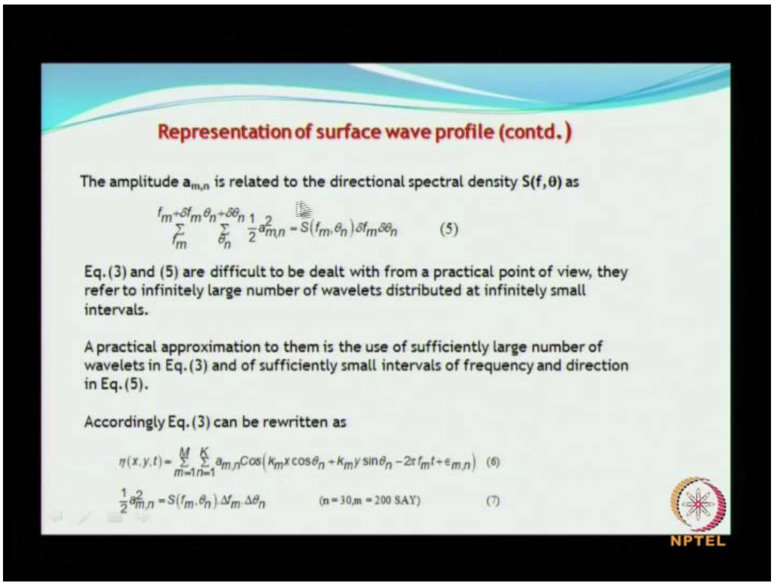

Now the amplitude here is related to the directional spreading function. Now how the directional spreading function how it is related half amplitude square is will be equal to the product of spectral directional spectrum into the delta f and delta three. So, here again you have n ranging there is the theta from n equal to 1 any number of components indicating the theta the directional components and f is the frequency components. Now you look at the equation three equations 3 the upper limit is infinity. So, that means you have a more number of that is the equation 3 as well as 5 it will become quite difficult to deal with from a practical point of view as they refer to infinitely large number of wavelengths.

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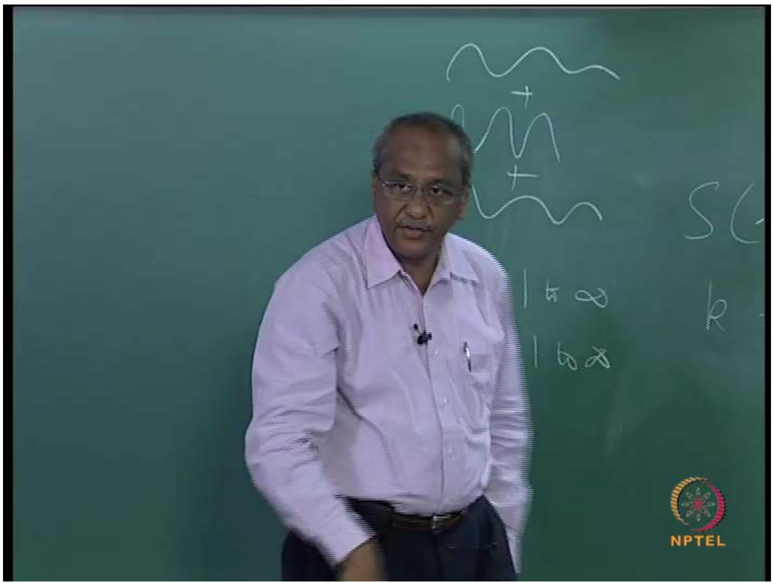

See please recollect that random wave itself a superposition of a number of sinusoidal components. So, based on a number of sinusoidal components you have wave time history. Similarly, here when you want to superpose if you have too many infinity it is not possible practical from practical point of view it is very difficult. A more appropriate or practical approximation is used is to adopt sufficiently large number of wavelengths in equation 3 and sufficiently small intervals of frequency and direction in equation 5. That is in equation 5 smaller intervals of frequency and direction, but in equation 3 affix the number of components.

That is a better way and easier way and from practical point of view it is a quite feasible to simulate the waves close to reality. So, accordingly with this statement equation 3 can be represented as shown here, all the variables are already defined I cannot keep on redefining or keep on telling what all the variables by now you should be familiar and now half amplitude square also provided here. Now I can select the number of directional components as approximately 30 and number of frequency components.

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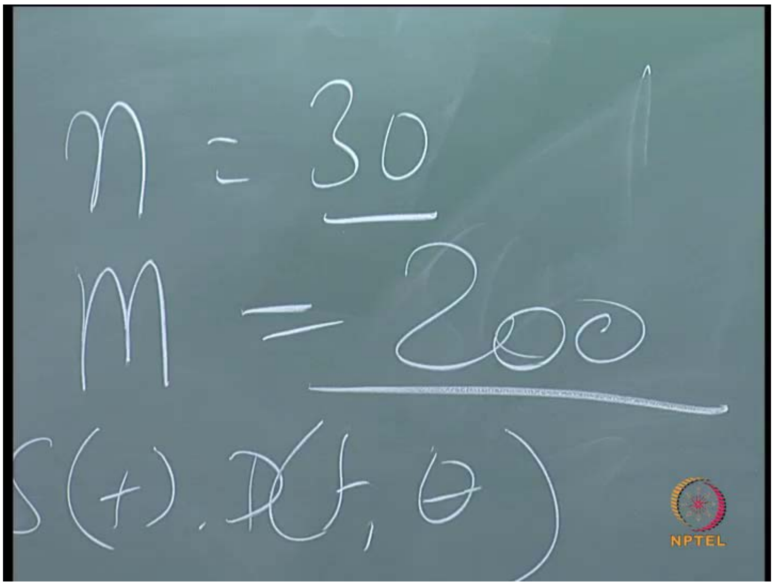

Say about 200 that should be for a representation for the proper representation of the waves.

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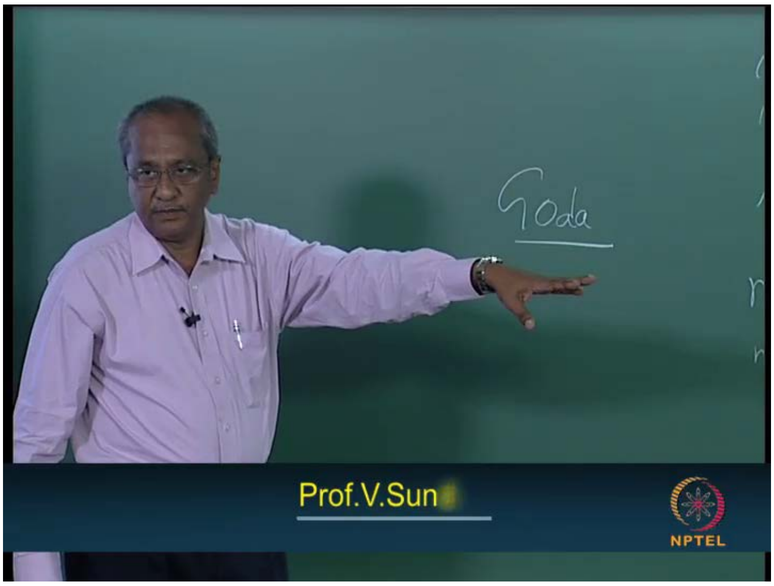

How this thirty, 200 these are all based on a number of calculations that has been done by Goda and you have to refer his book for the complete description of whatever I am giving this lecture. So, spectral density direction of spectrum as I have said is explained mentioned here where this is going to be the directional spreading function and S is the frequency spectrum if the relationship equation 8 is used and then equation 7 can be written as shown here.

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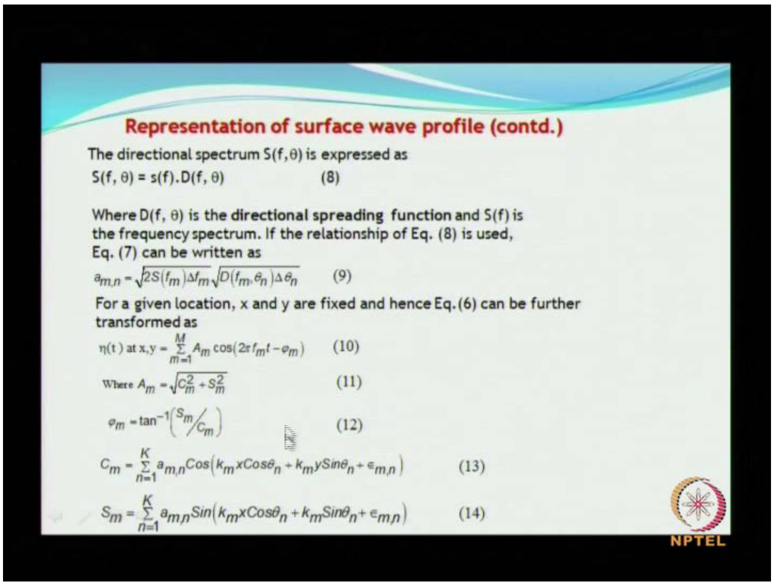

So, for a given location after all you will be interested in particular location x and y are fixed and hence equation 6. Look at equation 6 this is equation 6 and you have x and y, now these are fixed and hence equation 6 can further be transformed as eta at any location x and y as a summation of shown here. Where this is a summation of the frequency components the directional components are coming in picture here.

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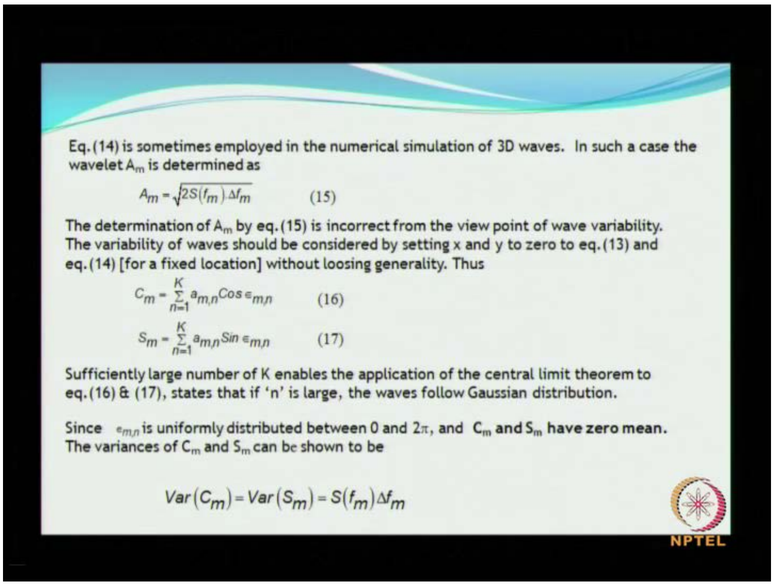

So, this is how you can describe the wave elevation at a given location taking into the directional aspects any doubts. Shall I proceed? Now equation sometimes is employed in numerical simulation naught sometimes it is mostly employed in simulation of 3 D waves. In such a case a m can be represented as shown here. Already I have explained this is how you represent amplitude as a function of spectral density or the spectral density is nothing but half amplitude of amplitude square. So, the determination of a m by equation 15 is incorrect from view point of wave variability. Because that is a general equation that when you set x and y equal to 0 then this equation 13 and 14 for a fixed location without losing its general description can be rewritten as shown here.

These are the equations I am talking about when time permits, you have to sit and derive yourself sufficiently large number of k enables application of central limit theorem to equation 16 and 17 stating that if n is large the waves follow Gaussian distribution are the wave elevation is normally distributed it follows a normal distribution or we say that the wave is a waves are a Gaussian process. Since epsilon suffix are m comma n is uniformly distributed between 0 to 2 pi, that means 0 and 2 pi as stated earlier. We have been seeing this from the beginning from our random 2 D waves and now C m and S m will follow will have a zero mean the variances can automatically be proved to be as a product of frequency interval and vector density in fact we have done this for 2 D waves. Please look at the lecture on two D random waves wherein we have done this.

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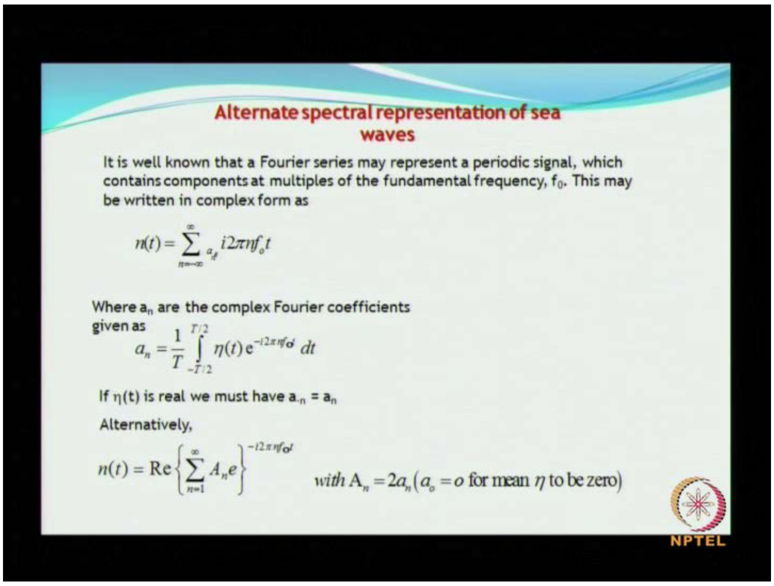

Alternate representation of sea waves alternate representation of sea waves it is well - known that a Fourier series may be it may represent a periodic signal which contains components at multiples of fundamental frequency of peak frequency or fundamental frequency f naught in a complex form it can be represented as shown in this equation. Wherein a n is nothing but these are the complex Fourier components or Fourier coefficients which are represented as shown here. If eta of t is real then we must have a suffix minus n equal to a suffix n brush up your Fourier series.

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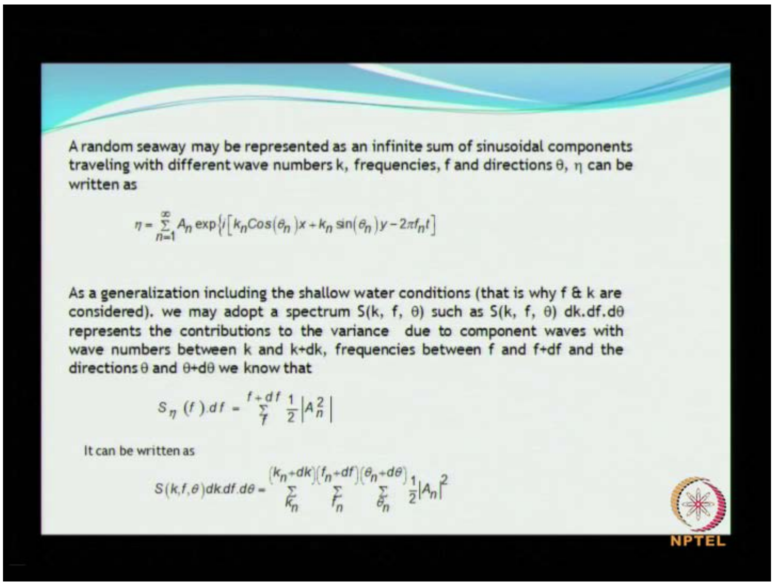

So, alternatively eta (t) can be represented as real part of that summation, which is shown there with a n representing or equal to twice a into the Fourier coefficient, a n in which a suffix 0 equal to 0 for eta to be mean 0 process. So, then with this background a random sea wave may be represented as a infinitely small infinitely sum of a sinusoidal components travelling with different wave numbers and frequencies and directions which can be represented given as the equation represented there and this is referred to as single summation model.

In the earlier case we had double summation, now we are only having single summation let us assess that single summation is much better than double summation you need to go through the literature as a generalization including the shallow water conditions when you want to include the shallow water condition that is the reason. We bring in k because k carries 2 pi by l and 1 takes care of the shallow water condition.

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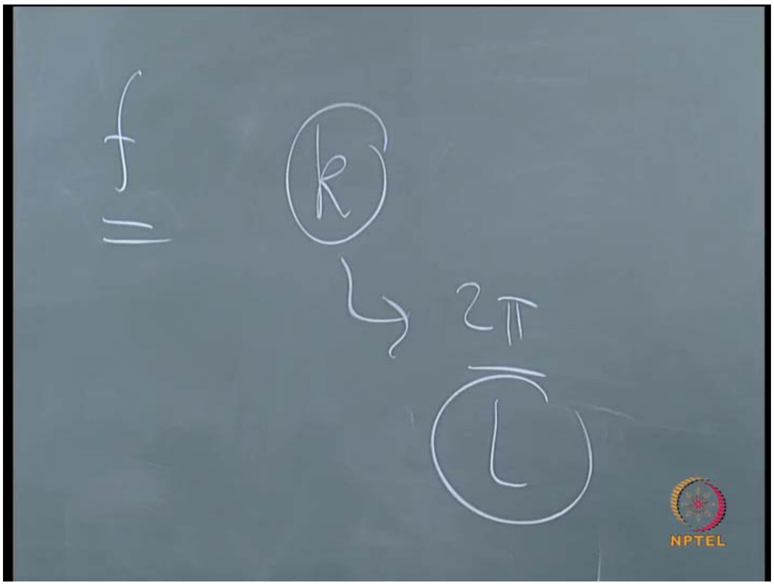

So, we may adopt we can adopt S (comma, f, theta). So, initially we have S (f) then we went in to f (f) comma theta considering the theta also the direction spread. Now we have brought in k also that is to account for the shallow water situation.

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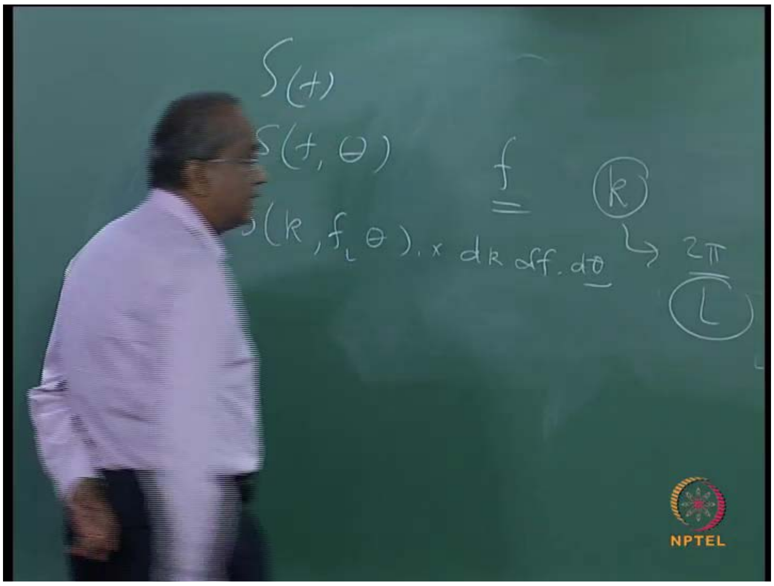

This into d k into d f into d theta is going to give the contribution to the variance due to the component waves with wave numbers k and k plus d k frequencies f plus f d k and theta and theta k and we also know that S eta of f into d f is going to be summation of amplitude square, this is the basic definition which we have already seen. So, it can be re written if you account for all the other components that are being considered in the same form we can bring in the other constituents like your frequency direction as well as the wave number and the whole thing can be represented as shown here.

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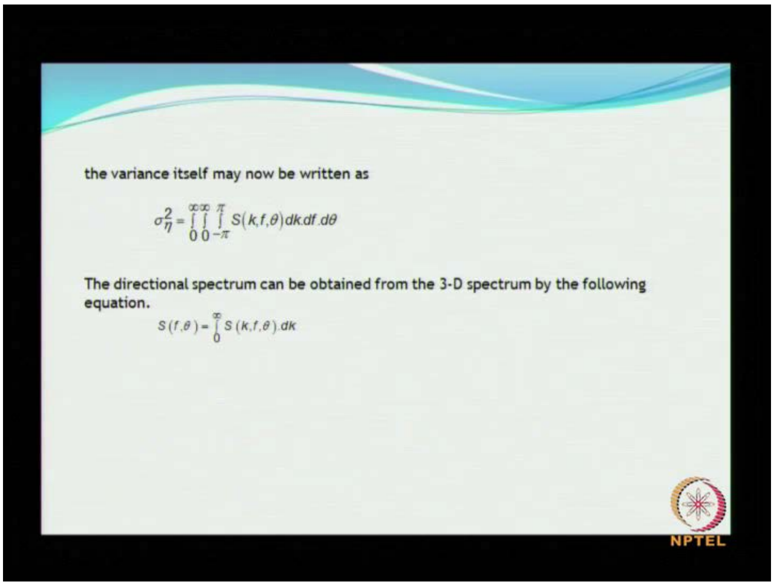

So, hence the variance itself can be may be now written as integration of your wave number spectrum. Now the directional spectrum can be obtained from the three dimensional spectrum by the following equation. Now a directional wave number spectrum may be developed in a similar way similar to what we have seen for the direction of frequency spectrum and the whole kind of relationship which we have been discussing will hold good here and it can be represented the way we have written here.

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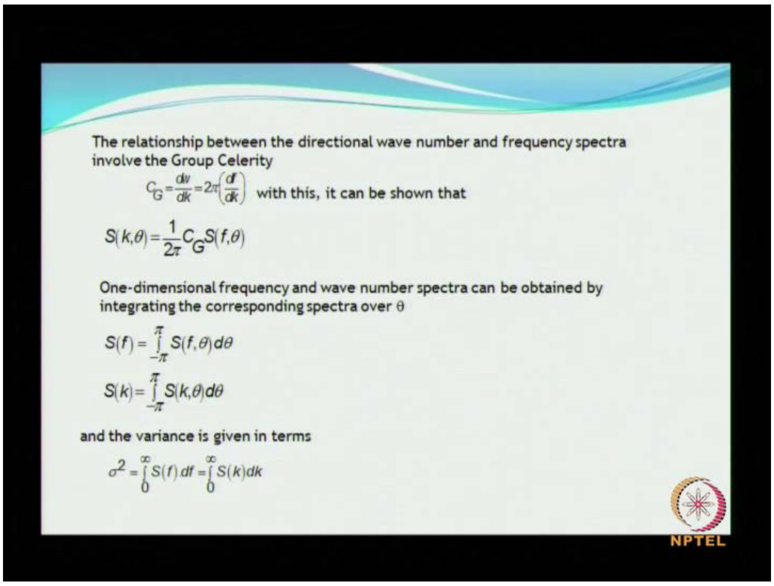

So, which shows S of k into k of theta as a function of has a summation of your wave number spectrum. Now you have brought in the wave number spectrum as well as the frequency spectrum and you know how these two are linked by some kind of as we have already seen the dispersion relationship.

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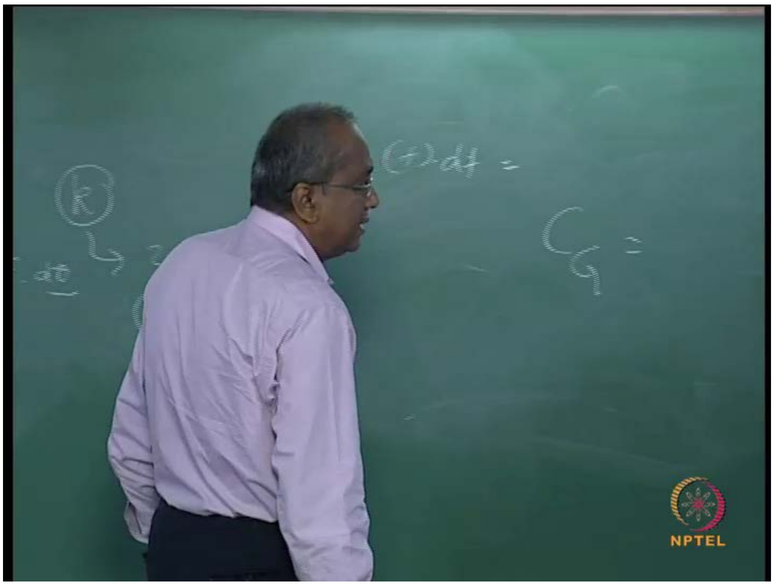

But this inverse let me take group celerity will be nothing but d f by d k into 2 pi that is from fundamentals and hence we can prove that your wave number spectra is here. We are talking only about wave number spectra as a function of frequency spectra both are dimensional spectra you understand what we have seen earlier from this.

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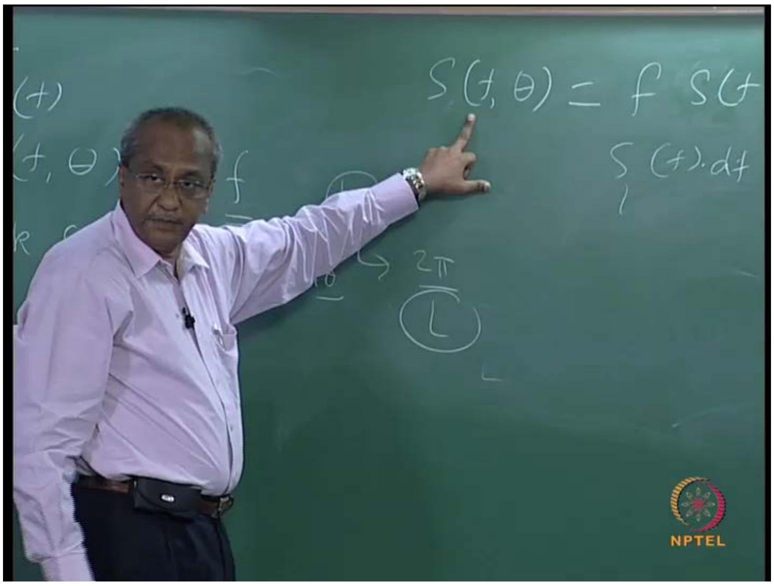

We have this as a function of only the 2 D spectrum from which we got the direction spectrum. Now what we have related your wave number spectrum to your directional frequency spectrum and which is a link the link is provided by the group celerity and the other things are the one dimensional frequency spectrum and the one number. I mean one dimensional wave number spectrum can be obtained as it is the vice versa as I have explained here the other way you can represent as shown here. So, much we have talked the both of the directional spectrum which are two factors are one is your spectral density. Spectral density is one dimensional spectrum and the other thing is other parameter is your d f theta.

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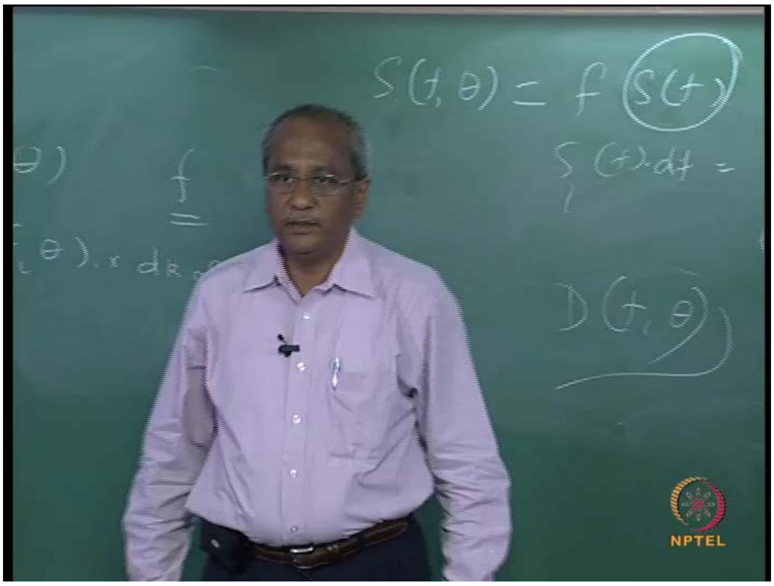

Which is the directional spread as I said earlier similar to number of frequency spectrum standardized the frequency spectrum. We have a number of standard directional spreading functions.

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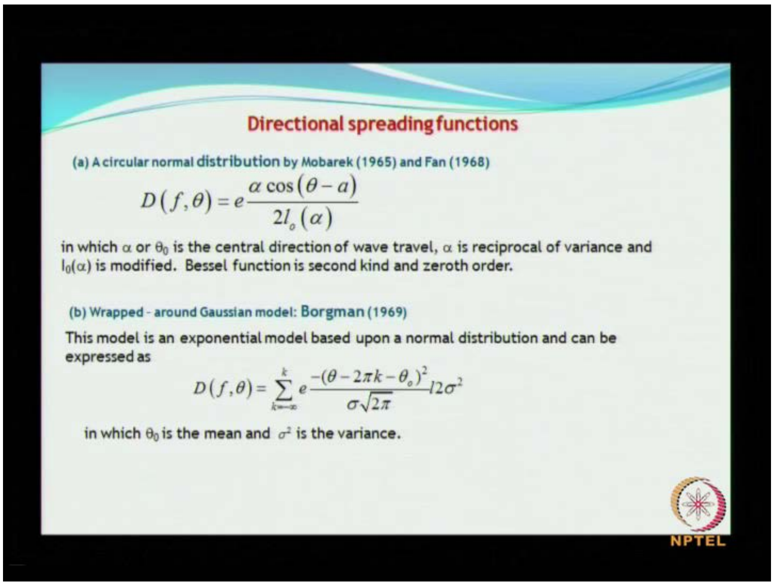

So, there are a number of directional spreading functions which I am not going to detail, but all these things are listed you can have a look, but I will touch upon a few directional spread which is widely adopted first is by Mobarek and Fan. They adopted a circular normal distribution as indicated there all the variables in are denoted there.

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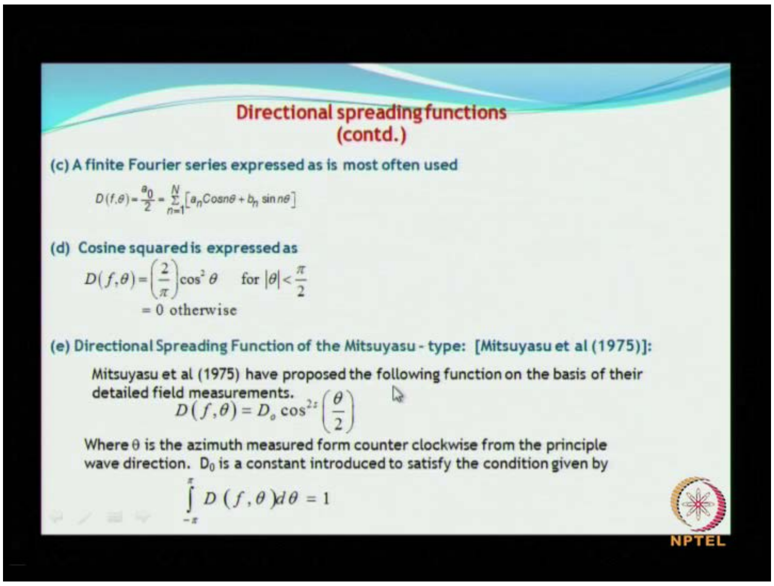

Then you had the we had the wrapped around the Gaussian model by Borgman in 69 and that is given by this expression in the form of finite Fourier's series expression as it is most often used as shown here, but the why the widely spread widely used directional spread are these two, one is the cosine square theta. Where in the cosine square theta the directional spread is given by this expression for absolute value of theta equal to less than 90 degrees or pi by 2 or equal to 0 otherwise. This is one kind of a spectral cosine square spreading function which is widely adopted.

Then the other one is that directional spreading function this is author Mitsuyasu and this has come into existence in 75 they have proposed the same thing except that it is associated with the d naught and your have brought in a parameter S which is called as spreading index. So the spreading index takes care of how the distribution should look like whether it is a broad distribution that is a wave coming from a wide range of directions or it is coming from a slightly narrow range of directions. So, in order to accommodate that they brought in, what is called as the spreading index?

Now here the theta is the azimuth measured from counter clock wise from the principle wave direction and D naught is a constant introduced to satisfy the condition given below. That is the directional spread minus pi by 2 summation of that equal to 1 which already we have seen in the directional spread.

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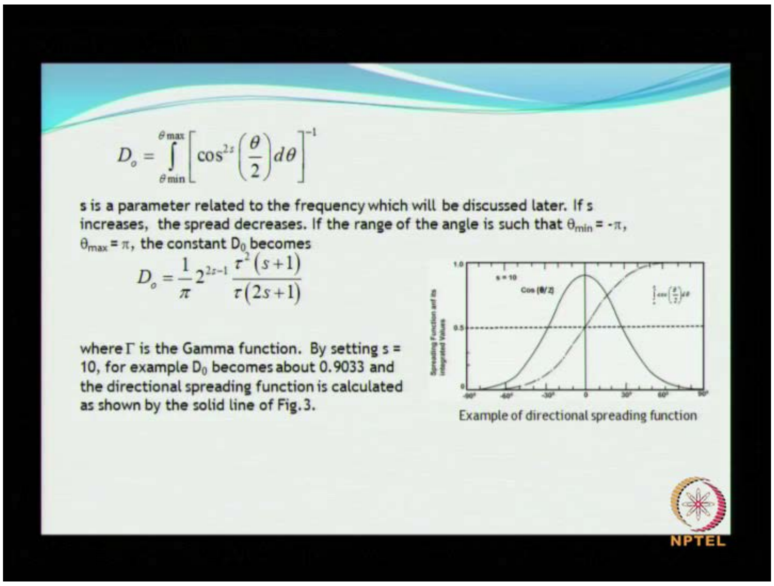

Now this D naught this is given here theta minimum what is the theta minimum to theta max and cos 2 s of theta by 2 to the power minus 1. So, as I said s is a spreading index is a parameter that relates the frequency which will be discussed later, but then one thing is clear, if spreading index that is if s increases the spread decreases from this expression.

So, if the range of angle is eta minimum equal to minus pi and theta maximum plus pi. That is a normal case so the constant becomes as something as shown here where gamma is equal to gamma is the gamma function and s is the spreading index for example, if for just to have a feeling for the spreading by setting s equal to 10 for example, then D naught becomes so much and then you can and the direction of spreading function can easily be calculated. So, direction spreading function is here so that can be calculated and this would be the kind of variation you would have for the directional spread.

This is how you can indicate the spread of the energy as a function of directions the other picture shows the cumulative representation that is so, if you look at the cumulative distribution. Now you can see here that it lies most of the energy lies in this spread is lying between minus 30 and plus 30 degrees, that is where the contribution of the waves directions which are coming from minus 30 and 30 and that is where the majority of the contribution of energy to the total sea wave takes place.

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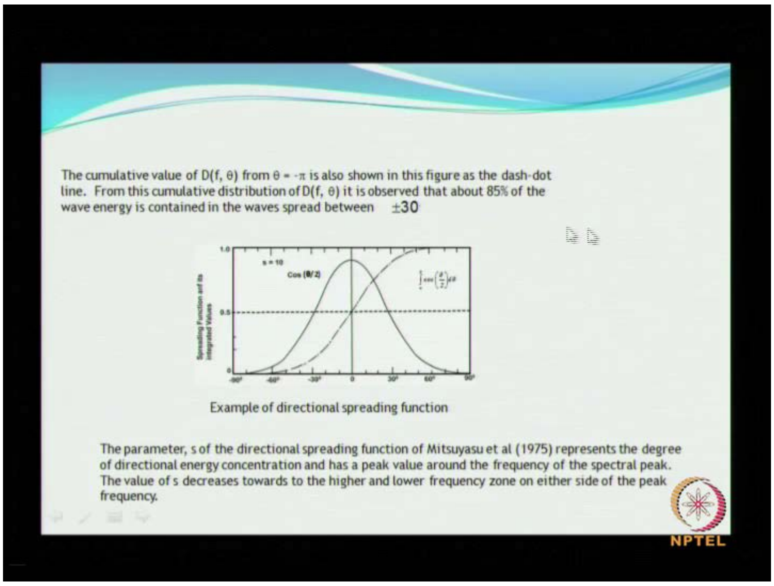

This is what is mentioned here is observed that 80 percent of the wave energy is contained in the wave spread between minus 30 and plus 30. So, this I have already explained here that about the parameter s of the directional spreading function represents the degree of directional energy concentration and as a peak value around the frequency of spectral peak.

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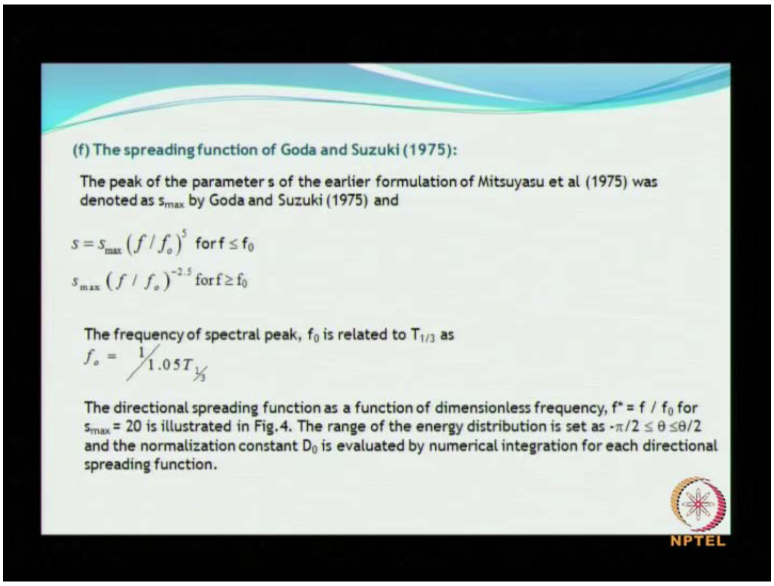

So, value s decreases towards the higher frequency and lower frequency on either side of the peak, the spreading function of Goda and Suzuki this came up in 1975. Where they have represented this s spreading index as a function of S max and relationship are given. Here now this is relating the spreading index as a function of peak frequency, but as the contribution made by Goda and Suzuki in 75 and here the peak the spectral peak is also related to t one third as shown here, now the direction spreading function as a function of dimensionless period frequency. Let me call this as this is f theta is frequency at which the spectral energy is maximum.

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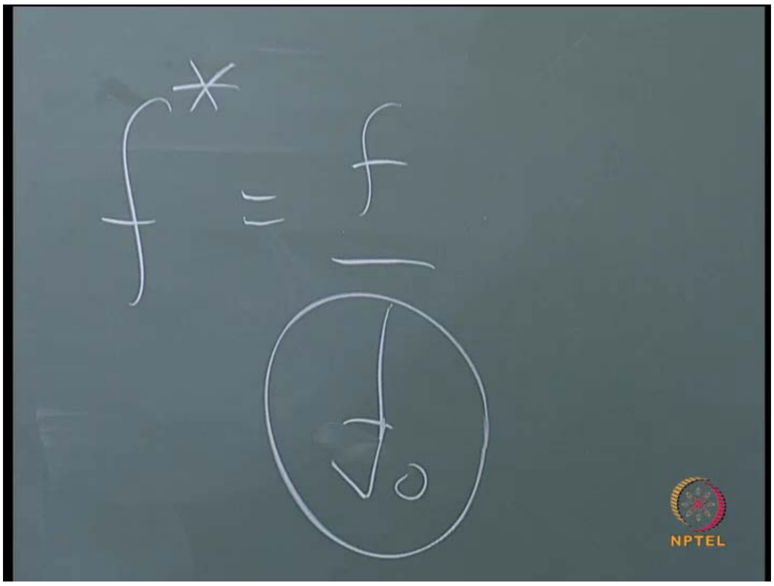

So, the spreading functions for different values of this ratio for a particular S max. S max is provided here S max say for example; for 20 is fixed then this is how the spreading functions.

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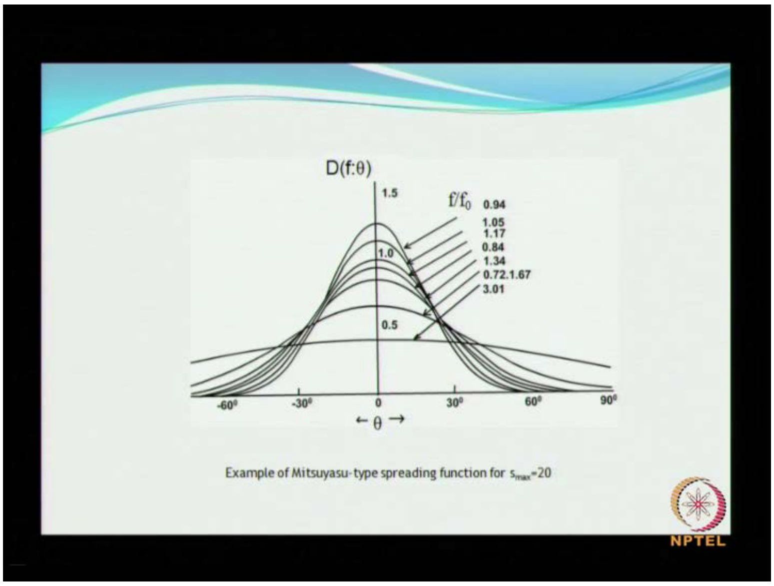

Would look like if you assume S max equal to 20 and this shows how the energy is distributed as a function of a direction. Please remember that it was Goda and Suzuki who brought the relationship between the directional spread and the peak frequency directional spread in terms of your spreading index. As I have explained in the previous slide and this is how it would look like. So, as your f by f naught increases you see that you have a wide range of a distribution. If it is less you have the spreading function narrower. So, if it is narrower most of the waves are coming within a very small range of directions that is what it indicates.

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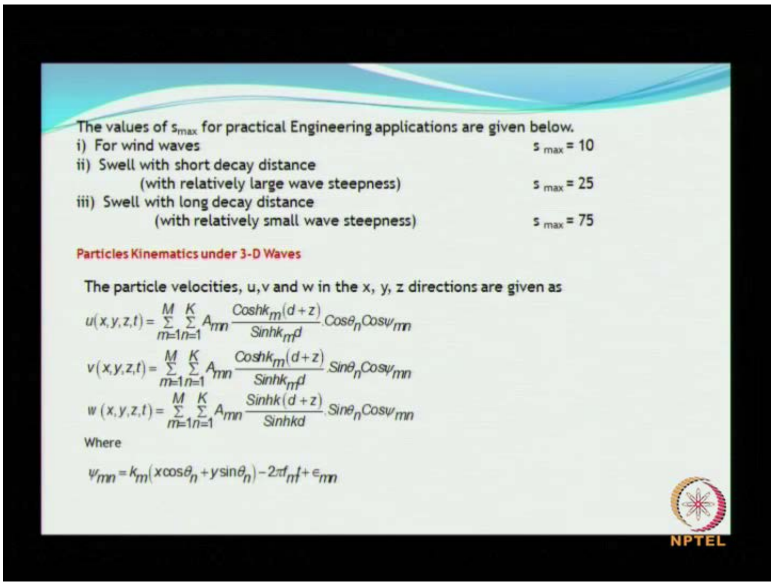

So, the value of S max for practical engineering applications is given here for wind waves. This is the reason why the S max was brought into picture that is for wind waves. It is S max equal to 10 mostly that is what is used around the 10 swell and short decay distance with relatively large wave steepness. We use a value of value of 25 is recommended. Swell with long decay distance S max is recommended as 75 so, having seen in detail the representation of eta. We can use the relationship easily the relationship between eta and the particle kinematics and it is possible for us to obtain the expressions for the velocity particle kinematics.

Earlier we used to see only for u and v that is the one horizontal in the direction of wave propagation, one in the vertical direction. Now you have the directional waves so, you have two horizontal in the horizontal planes on one vertical and that is what is given here u, v and w. So, the expressions are given here. You can use those expressions all the values all the variables are defined, use this formula to simulate your particle kinematics. If you have measured u, v, w and also you know eta at a location and if you want really prove that your measurements are correct use these expressions try to validate your measurements.

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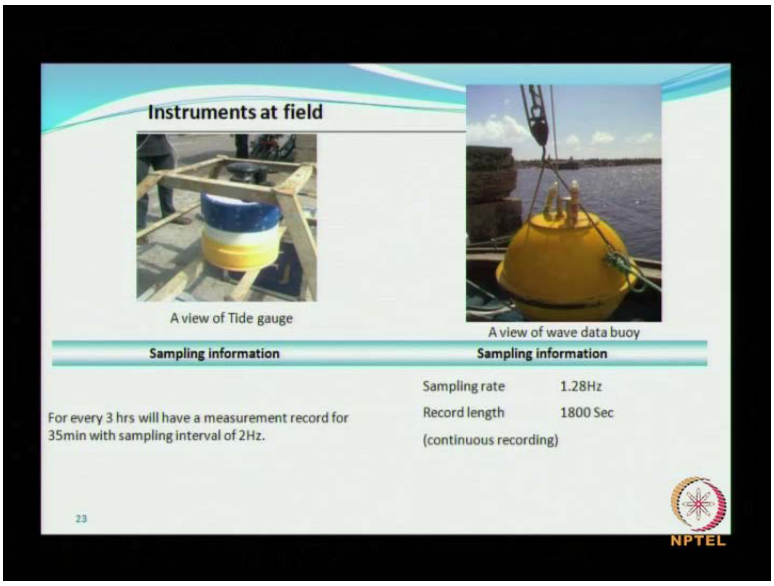

So, instruments used for a just give a small explanation about this usually you have a wave rider buoy. When they say wave rider buoy that means it is not going to measure the directions, if it is installed in a particular in a location at that particular location. It will do the wave elevation. So, it will give only you can get only the two dimensional spectrum, but if you have a directional wave rider buoy here it is a wave rider buoy, but in this case the direction also is measured. If we have a directional wave rider buoy so, the left hand side shows the tide gauge and we have used all these equipments to monitor for the purpose of obtaining some detailed investigation for carrying out some detailed investigation on the wave and the flow field conditions within a groin field.

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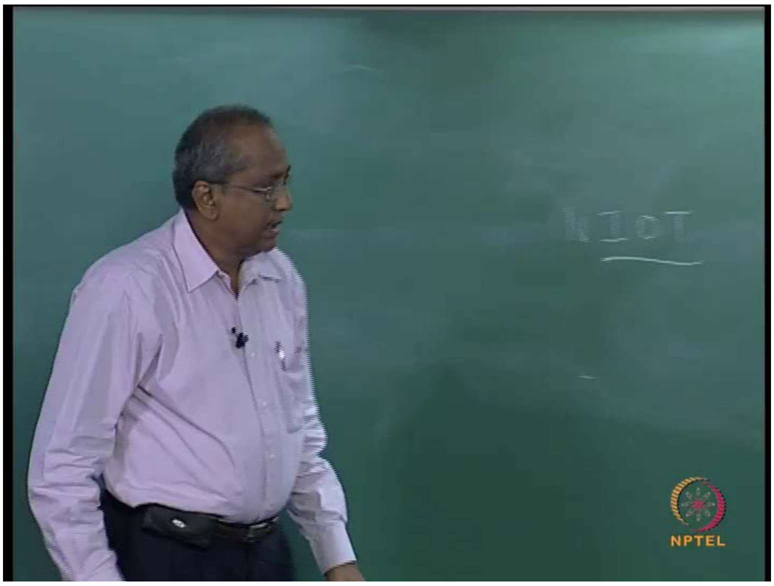

We have used some of these instruments with in collaboration with NIOT is a part of a major project. So, I will just show you some recording what we have done. So, this tide gauge will record for every 3 hours it will have a measurement or record for 35 minutes with the sampling interval of 2 hertz and in the other case it will be a sampling rate adopted as the 1.28 and the record length was about 1800 seconds.

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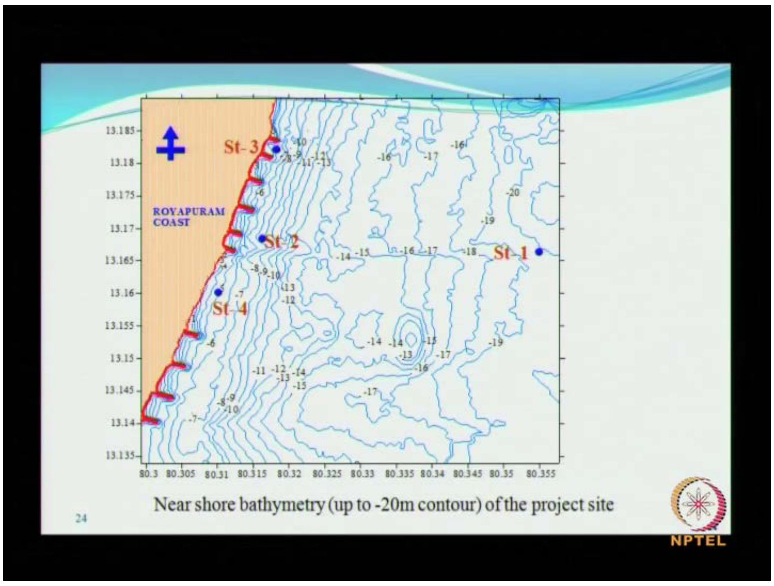

Here in the recording was on a continuous basis the data was transferred online and then we used to then take the data and for further analysis. So, this is a groin field along north of Chennai harbor. So, Chennai harbor again some of those.

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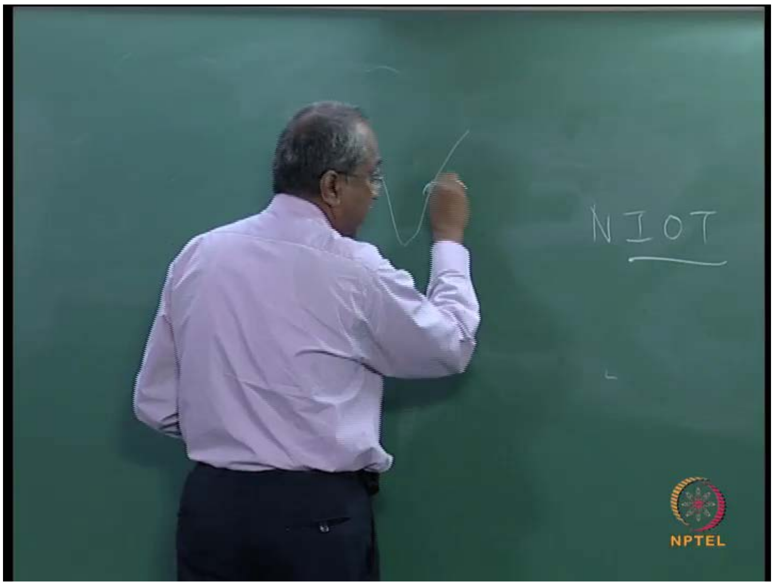

So, this is India map something like, so where here is your Chennai? So we have a groin field where in we wanted to work on the do some measurements some of the some of the stations are located here. We had the measurement of waves and other parameters here.

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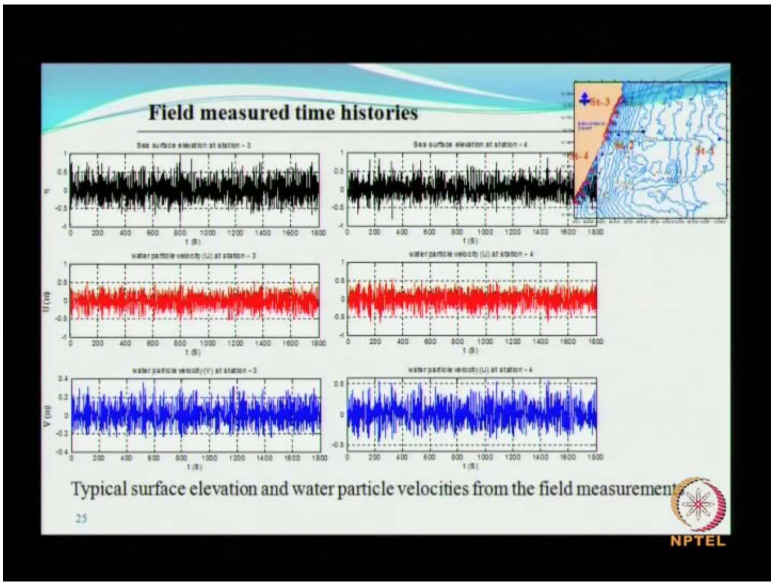

This station 2, station 3 or station 4 and station 3 are also there. So, then this is within a groin field all this red color indicate the groin field drawings. So, this is a station 4 so, the typical time histories recorded at station 3, station 4. So, station 3 and station 4 are indicated here. So, the black one gives you the wave elevation the red indicates the inline velocity or the horizontal both are horizontal one is into horizontal direction and other one blue is the transverse direction.

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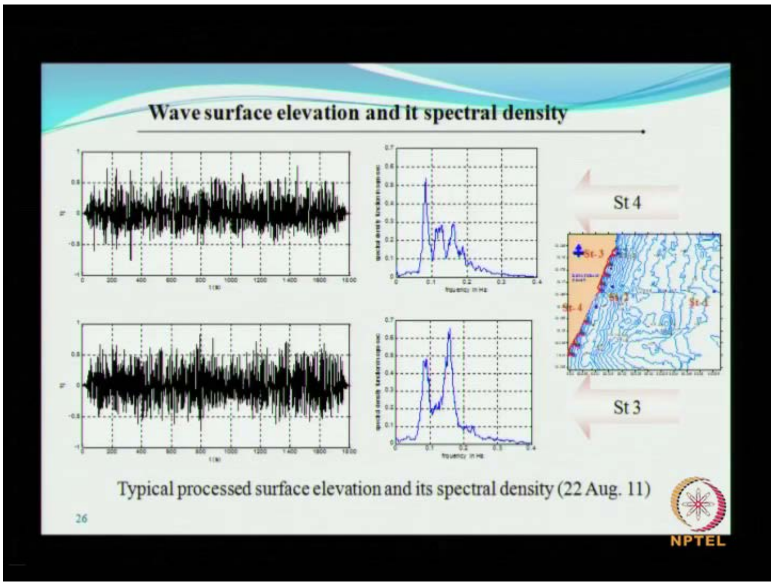

So, with which you can also find out what is the directional spread at that location?

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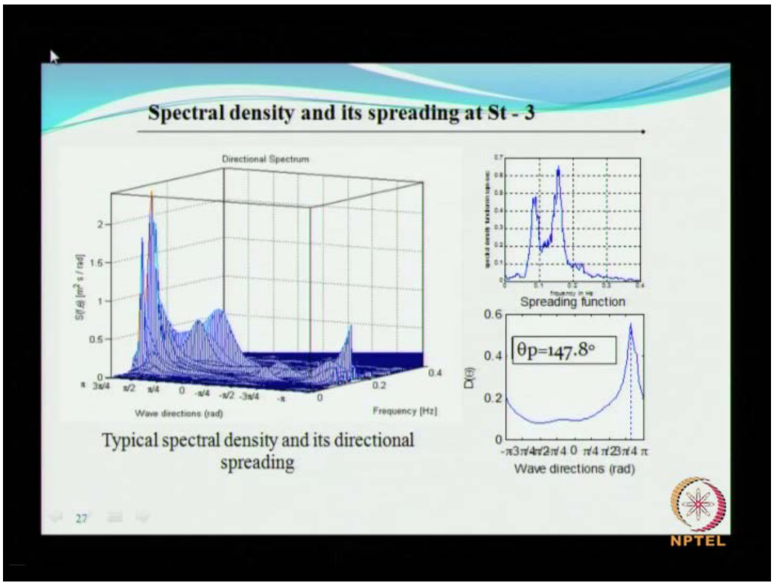

So, this is a typical surface elevation and it is spectral density at this is in station 3 and 4. So, this is station 3 one of the stations so, this is how it provides the information. What is a kind of a frequency range for which the energy is really dominant etc all this information can be obtained from such a result? So, this is the directional spectral density as you can see here direction is there the frequency is also there and from an analysis. You can we have obtained the wave direction and the predominant or the principle direction is as indicated here.

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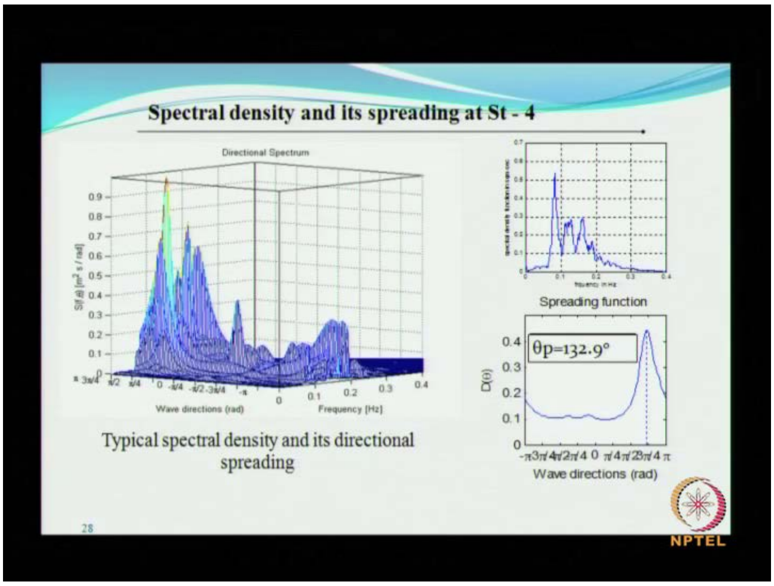

So, this spectral density shows exhibits two peaks and this is another location wherein in station 4, how the directional spectrum looks like and its distribution of its directional spread and the spectral density the two dimensional spectral density.

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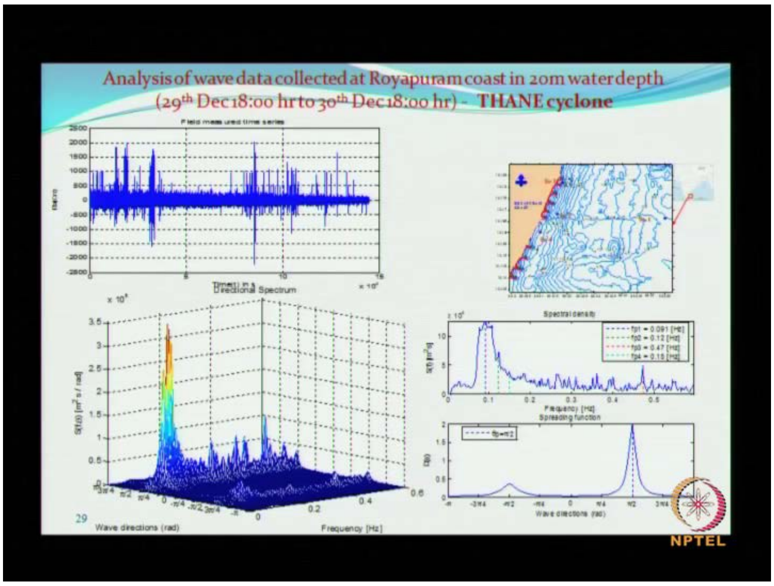

Also the analysis collected during the THANE cyclone reported here. So, the wave rider buoy measured all this information so, you have the raw data here you have the three dimensional spectrum. How the theta is distributed, how the spectral density looking all the information can be obtained.

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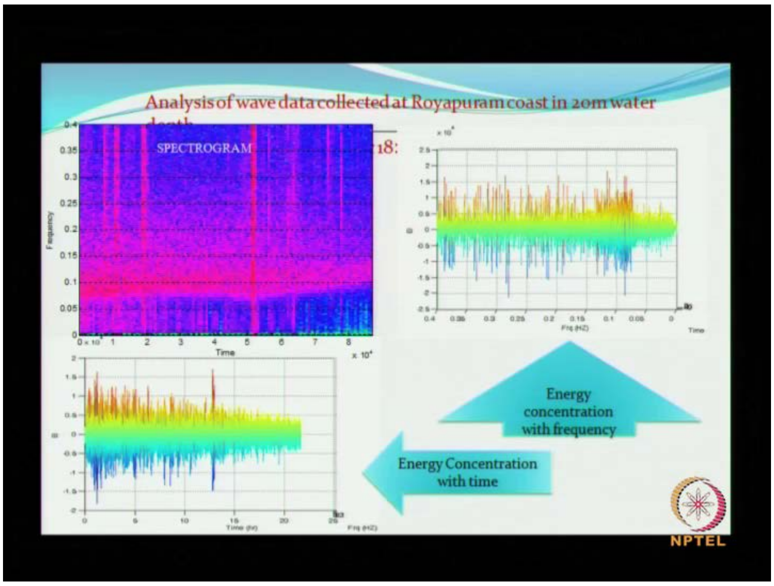

This is the spectrogram where the raw which shows the relationship between time and frequency on the side. You see what you see on the right side is the energy concentration as the function of frequency and on the bottom figure. You see the energy concentration as a function of time.

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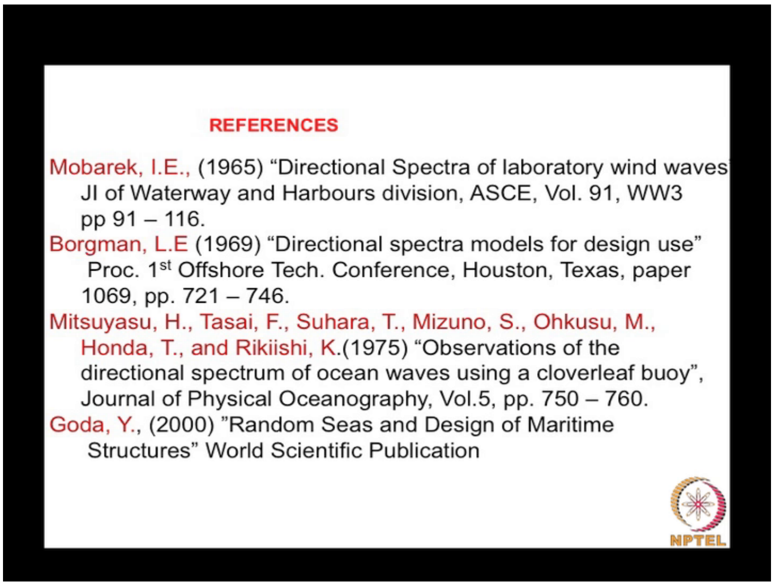

This demonstrates that what you can do with the information on the directional waves I think with this I will close.