Wave Hydro Dynamics

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Module No. #04

Random and Directional Waves

Lecture No. #05

Simulation of Random Waves

I hope things are clear with spectrum, standard spectrum why you need it? How you do it? And where you apply it? Etcetera. If you have any doubts please ask? If you do not have doubts, I just proceed.

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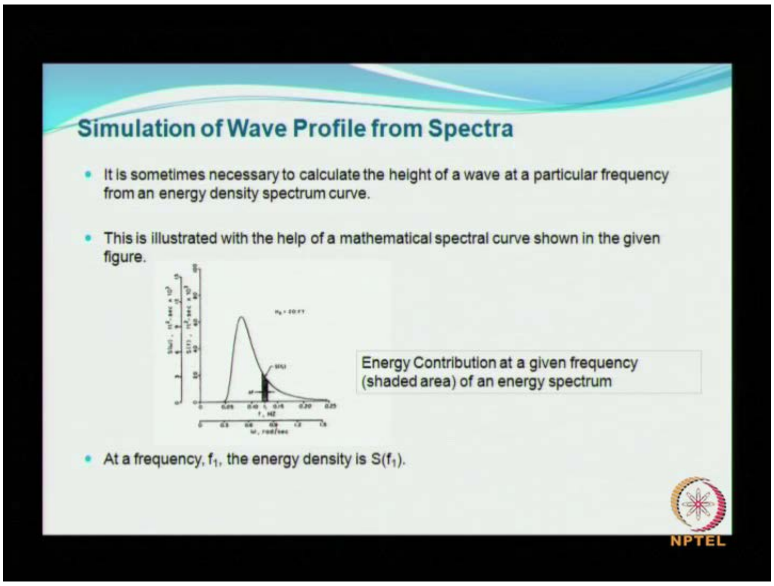

And, I have given most of the salient information, if you want to adopt or apply all these information. I suggest you got through some books as well as, and lecture material available along this lecture. So, that things will be more clear. And also, you try you can try use this vapoo, you get lot of experience how to handle all these spectrum, etcetera.

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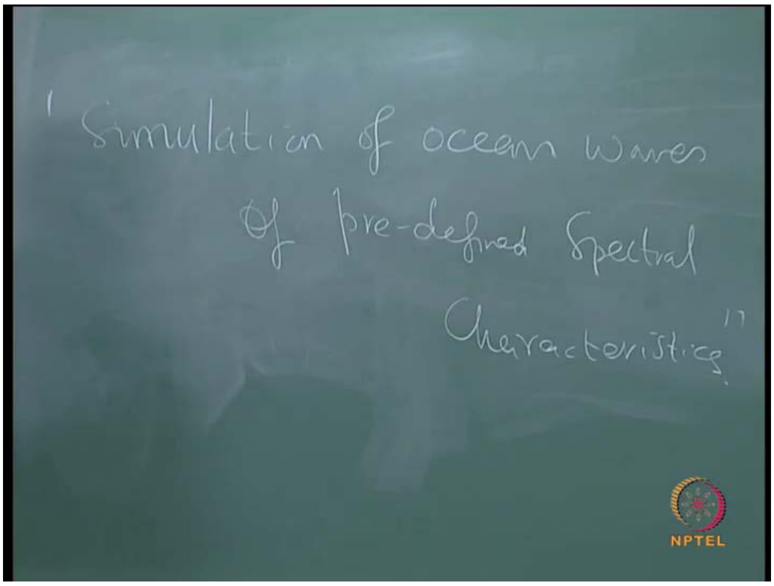

Earlier, I have just highlighted about the necessity of simulating away profile from a spectra. This is often referred to as, simulation of ocean waves of predefined spectral characteristics. So, if you are having the kind of facility, where in you can simulate in the lab ocean waves of pre defined spectral characteristics, that means you have an excellent control over the wave maker. And these days several labs worldwide have this kind of facilities, and so there is a lot of work that has been done by using different kinds of spectrum.

Now here, we just look at some of the mathematical aspects of this energy contribution or this simulation of waves form a spectrum. So, if you look at this picture forget about the units, but you just look at for instance this frequency, over this frequency this is the amount of energy that is contributing to the total sea wave, at frequency that particular frequency. This is often referred to as peak frequency. Apart from that if there are any other frequencies where you are interested in knowing, the energy content that is possible.

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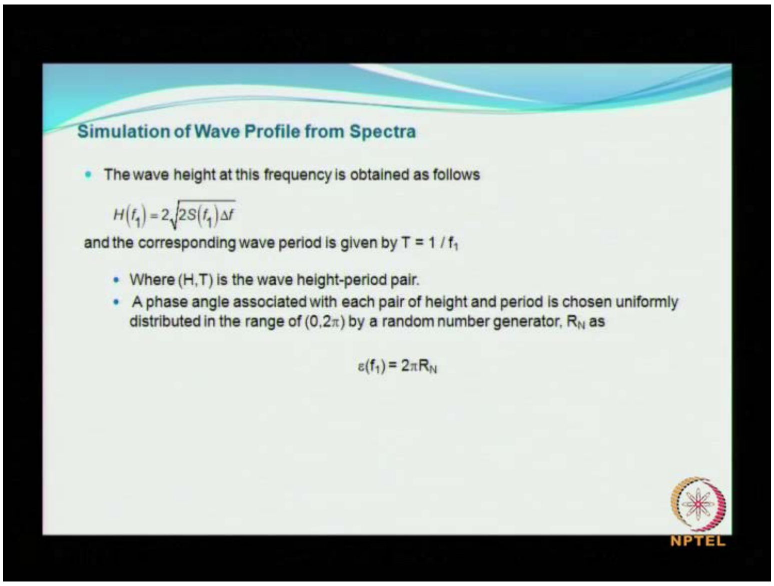

That is, how do you simulate from a spectra here, the wave height at that particular frequency which I was mentioning that is that we call that as f1.

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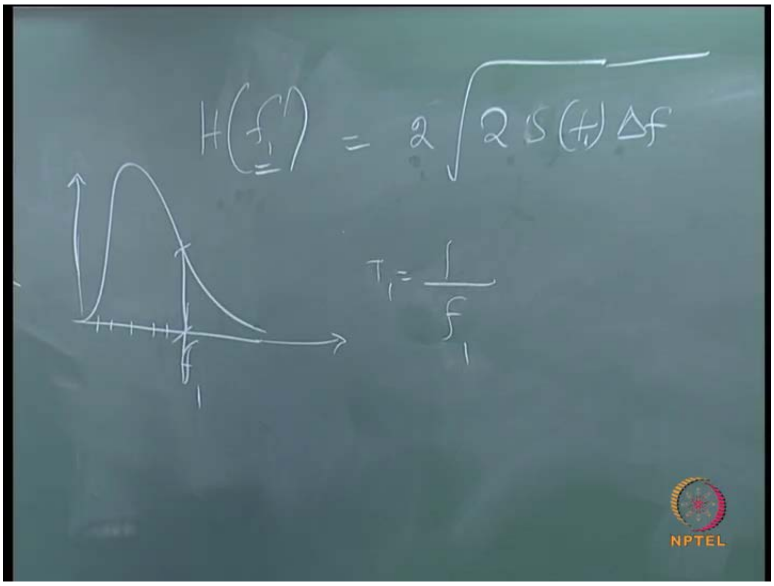

H of f1, that is the frequency at sorry the wave height at frequency f1 so the wave height are at frequency 1 will be 2 times of the spectral density at f1 into delta f. So, delta f is frequency interval, the spectrum will have a constant frequency interval. So, that delta f you have to multiply it, with 2 times square root of twice t spectral density at that particular point. If it is amplitude this will not be there since, we talking about wave height we do this two times. And you also try to associate what we are discussing now, with the problem which we worked out, where we try to obtain the spectral density for different frequencies.

We try to obtain so, once the row heights given H1, H2 etcetera were given for frequency one frequency two. I think we took over 5 frequencies then we found out what energy content for each frequency. And then some uneven we followed the total we obtain the total energy and such a combination of waves. The same exercise only we are doing it here. And, the corresponding wave period that is when we say here, what is wave period at this, one by wave period is 1 by f1 this can be referred to as T1 or Tx. So, H n T will constitute a wave height period. So, that is the simulation of wave profile form spectrum. But where does the random number comes in, a random number comes in when it is associated with the phase.

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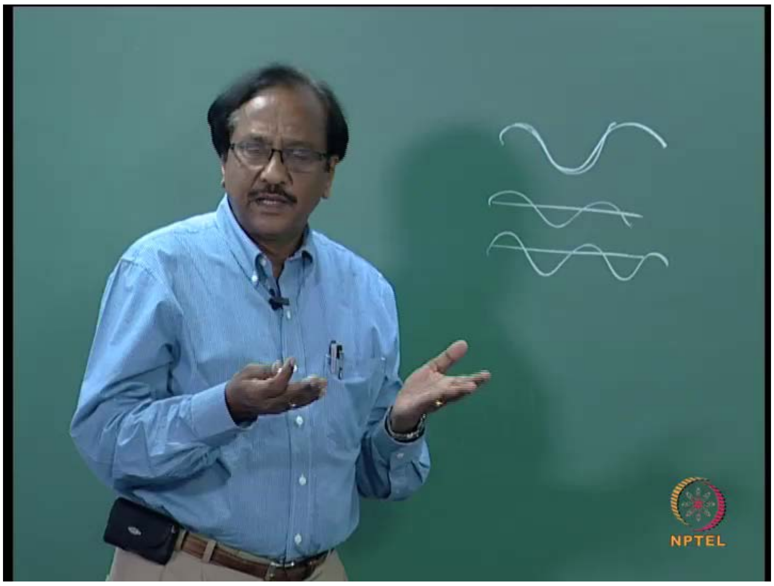

See, you can have a wave so if all this waves the number of waves if it hires the same phase and you superpose, you are not going to get a random wave. For want to have a random wave then they have to be superposed different frequencies etcetera. And a random number the phase has to be set as a random. I am sure, all of you know about what is meant by phase.

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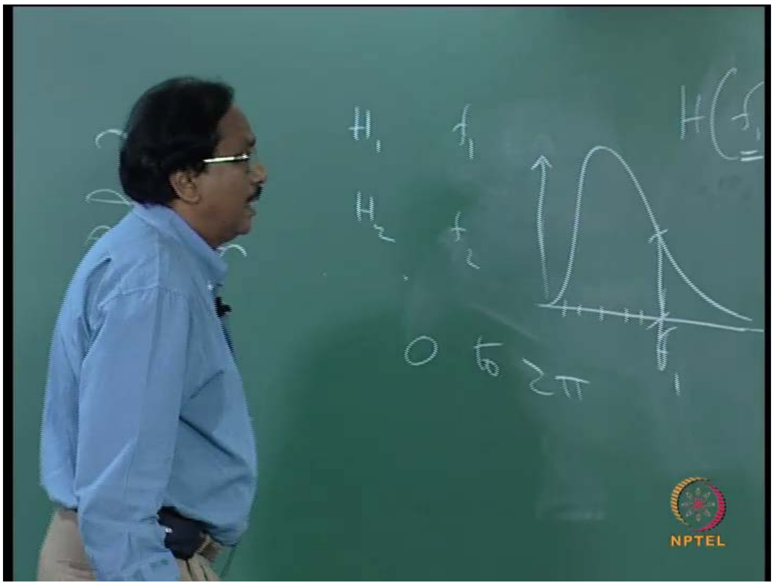

So, that phase is chosen as uniformly distributed from zero to 2 pi. And this random number can be generated by a random number generator it is quite straight forward and this random phase is for example here, it can define as R n s e random number.

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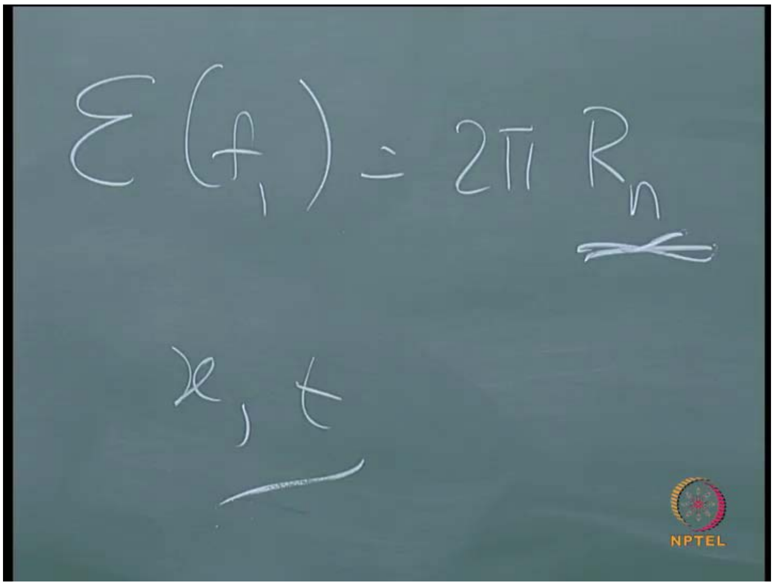

So, at any given location see we are working with the two variables, one is x and another is t. Later we also be working with x, y, z and t now, at this point we are considering a horizontal axis x at any given point, at which the wave profile is defined decide. So, for example, I have a tank and I want to define at this location, the variation of the wave elevation. So, my wave maker is here so, my desired location is x at that particular location now x can be say t n zero no problem.

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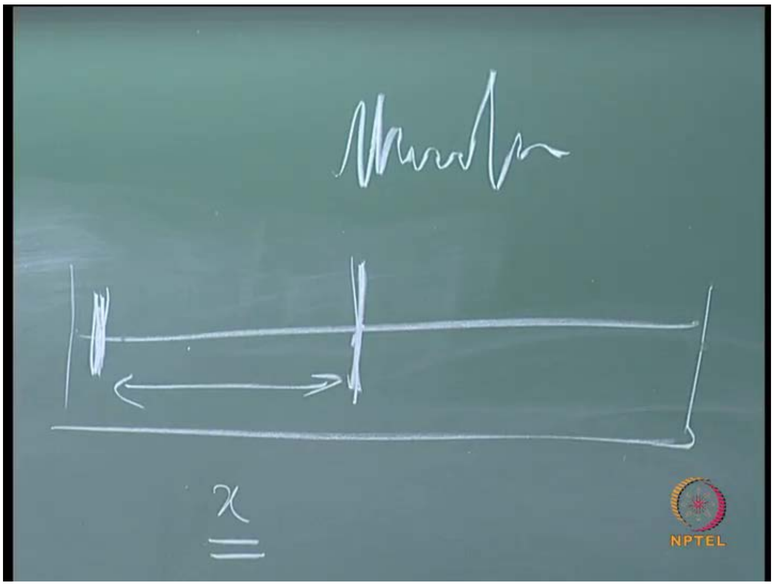

So, then I use this expression at a given point x as summation of n to capital N, this N to capital N is nothing, but the number of frequency commons. And it is a summation that is what I have been telling that you add all the components this number of components is up to you.

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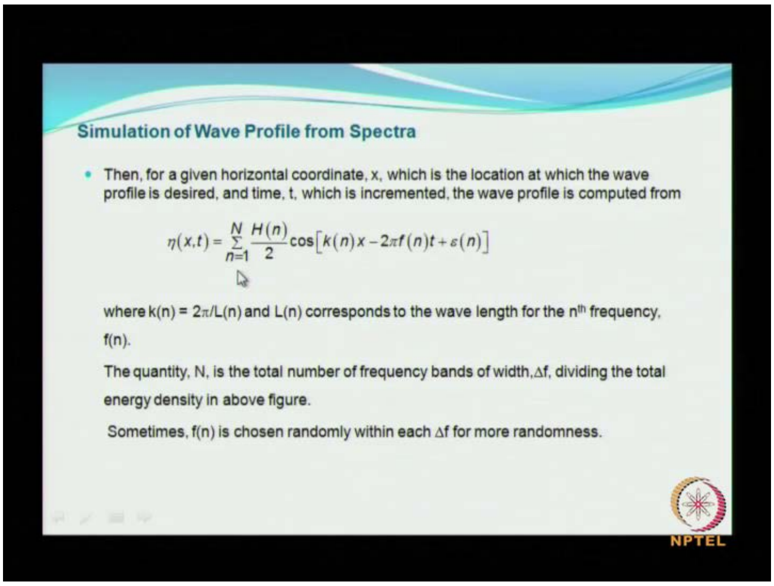

So, the quantity n is the total number of frequency bands of the width delta f dividing the whole total energy density in the above figure I mean you take the total figure. And then you can slice into number of frequency bands, and then you can do the summation. Now, sometimes f n is chosen randomly within each delta f for more random number. So, within the delta f itself the frequency, see within delta f is this delta x itself the frequency is changing right. There is a very small interval, within that itself if you want to have, you can still do that by introducing that randomness within that delta f.

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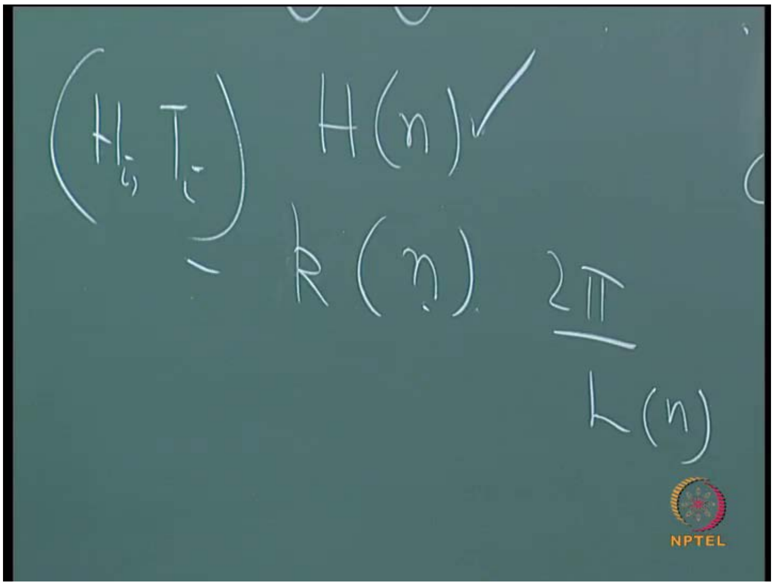

So, basically you see that you have wave height which is H wave height of nth component where n is equal to 1, 2, 3, 4 etcetera up to n and then k of n that is nth component that is k is 2 pi by L. So, each of this is when I say H of n and k of n this is this is clear. And this is 2 pi by L of n so, that is this is a pair H of this is what we are trying to represent in terms of n number of. So, n will be raining up to number of n equal to one to capital N. Capital N will be total number of components that are going to be superposed this are clear?

So, that is a very simple way of simularating the time history. So, once you have you are the mathematical expression, you can simulate any kind of I mean revaluation any kind of a spectrum and some of the examples we have seen just now. The methods outline earlier, request a given spectrum may be pear some scot or whatever, it is or at actual input wave energy density function. What does it mean? It can be a theoretical spectrum may be p m spectrum or it can be measures spectrum.

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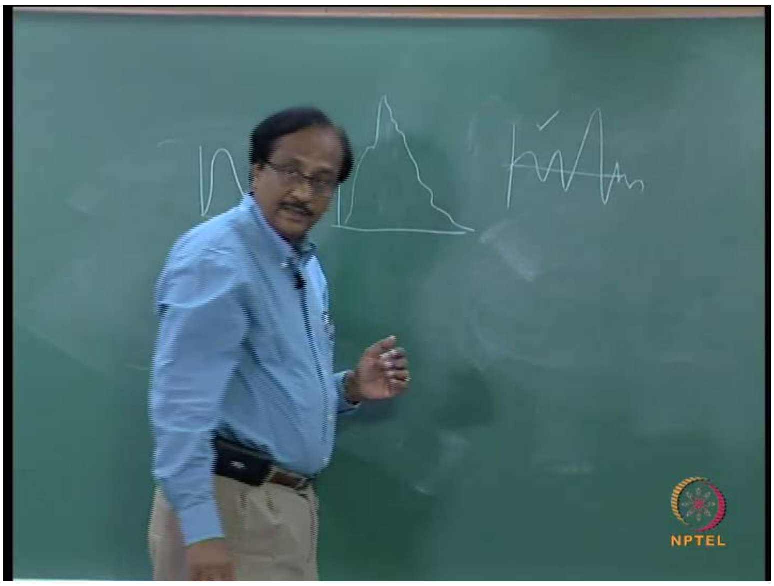

For example, you have initiates some spectrum of Bay of Bengal and you want to use that spectrum in lab, you can still do that the principal is same. So, you have you can always say that, I have already measured time history from the Bay of Bengal. That is also, possible you can still use this time sheet we have make around do the work or if someone gives you, only the measures spectrum, are not the time histories. Then you are forced to simulate your time history on your own, for simple reason you do not have this. So, it can be either the theoretical spectrum or they actually measure spectrum from the ocean.

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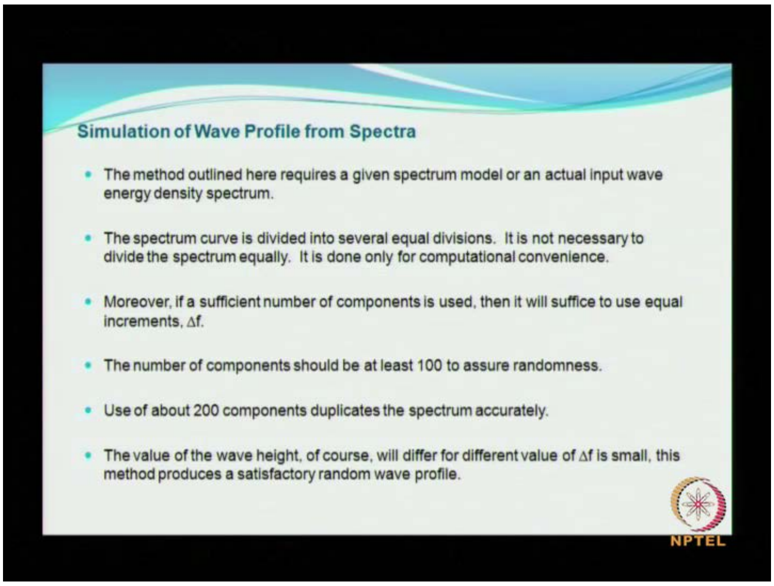

The spectrum curve are the spectral curve is divided into a number of equal divisions is not necessary to divide the spectrum equally. It is done only for computational convenience. All these guidelines are have been taken by from the book of professor Chakrabarti because it so clearly explained. So, I do not want to make any changes.

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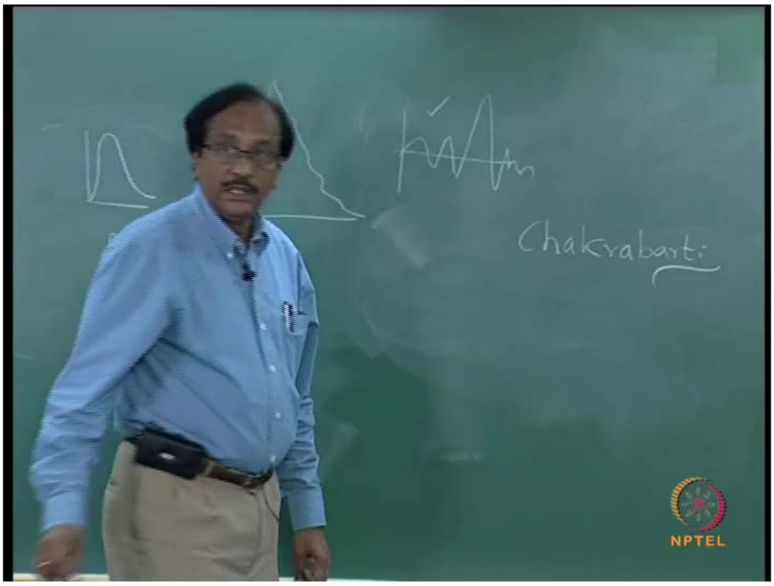

So, moreover if a number of sufficient number of components is used, then it will be suffice to use equal increments of delta f. U shall it have to refer equal increment of delta f however delta f is always constant.

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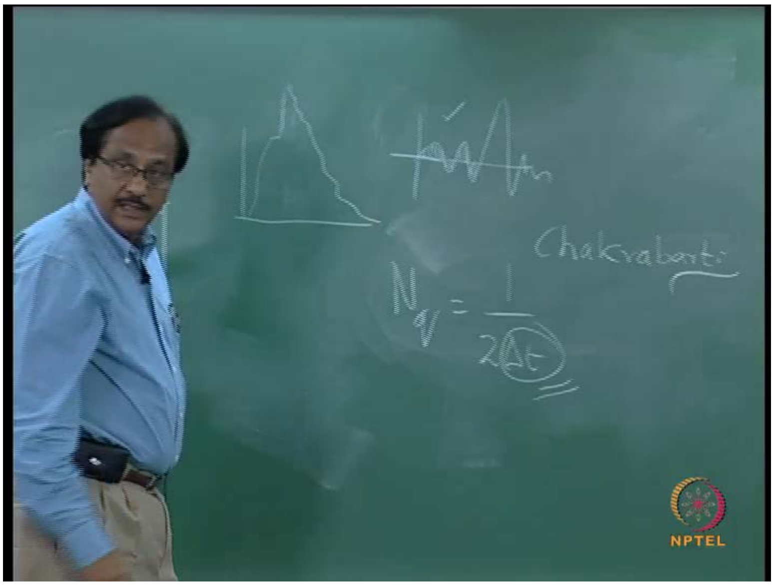

At this point of time I would also like to say, that there is what is called as requisite frequency which is 1 by 2 into delta t. So, or cut off frequency, beyond, which we say that up to the requisite frequency we have at least 90 percent of the, suppose if you have the cut off frequency somewhat here. Which is defined by this and this delta t is what? Delta t is the digital values of the alias signal taken from the mean line. So, this sampling interval is delta t.

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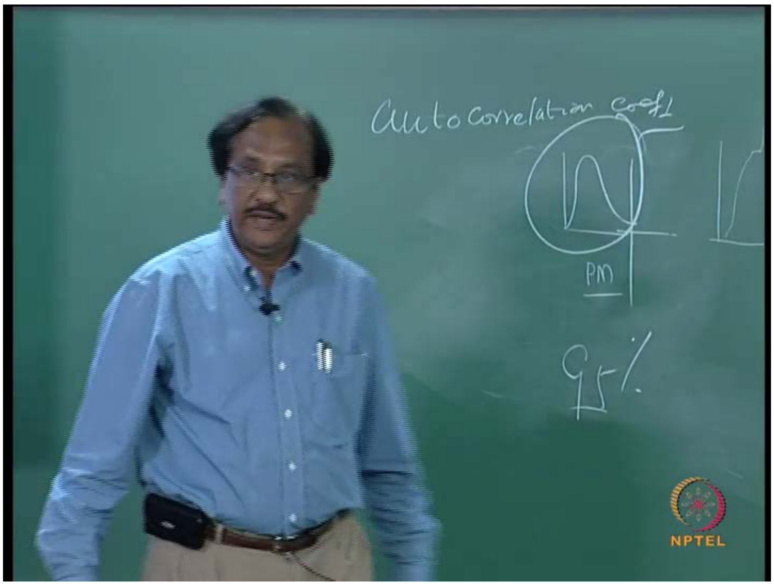

You have to be careful in taking this choosing this delta t. So, we say that cut off frequency, beyond which within this area at least 90 percent 95 percent of the total area is already accommodated. You have considered 95 percent minimum of the 95 percent of the energy content. Now, the number of components should at least, be hundred to assure this randomness. So, if you take just very few number of components and try to do that it is not going to, you do not have the randomness coming into picture at all. When I say? Randomness also, try to have this autocorrelation coefficient.

How this would look like? I will explain in earlier class. So, it should be at least 100 use of about 200 components duplicate this spectrum accurately. So, the number of components can be around 100 so, that is based on this model. The value of wave height of course, will differ for different value of delta f. Different values of delta f is small this method, if delta f is small this method will produce the satisfactory randomly wave profile.

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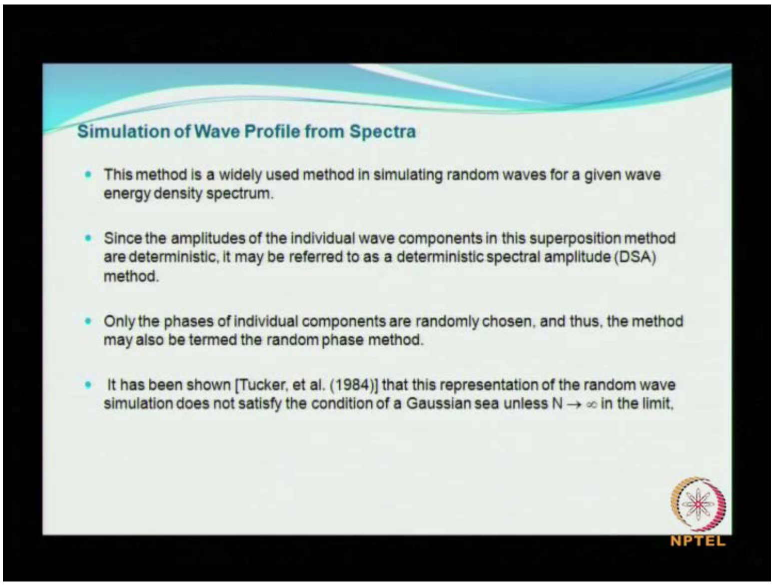

Now, next let us look at this method is widely used method it is a widely used method in simulating random waves, for a given energy spectrum, we have seen already and it is also, called as deterministic spectral amplitude model or method.

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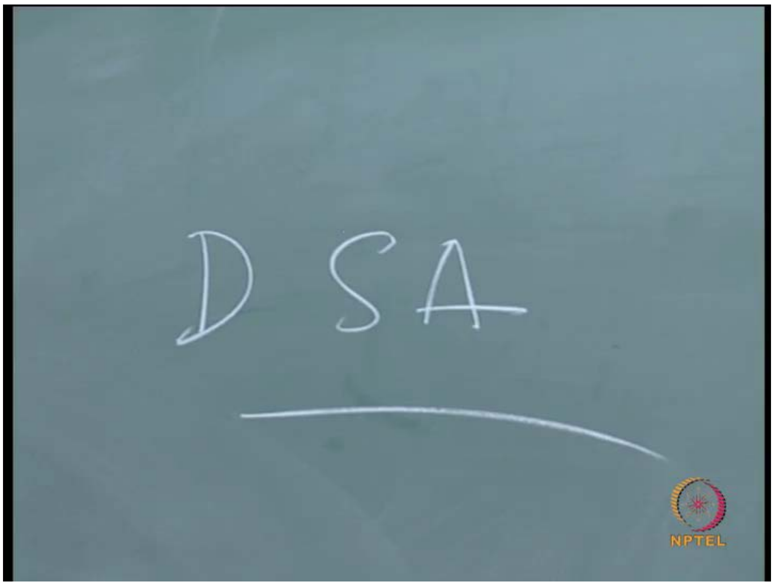

See, since hearing the amplitudes of individual wave components in super positional method are deterministic. So, you can get the amplitude of each of this spectrum, if we consider a particular spectrum the amplitude can be easily be estimated. So, just now, I have this expression here, few minutes back. How do you get at this particular frequency? How do you get the amplitude as a function of spectral density? So, they quite (()) this reason why it is called as deterministic spectral amplitude model? Only the senses of individual components are individually chosen.

And thus the method may also, be termed as random phase method. So, I will just the last one is it has been shown by tucker et al, that this representation of the random simulation does not satisfy a condition of a Gaussian sea unless N is infinity in the limit.

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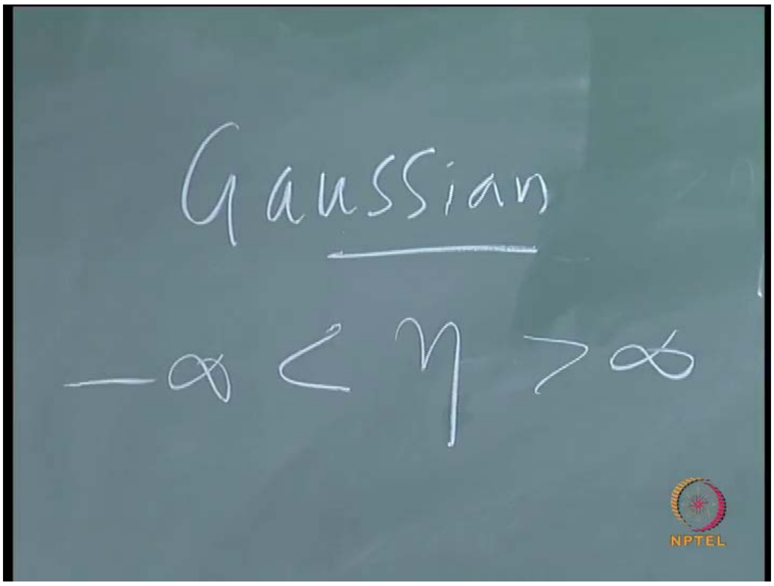

What is mean by the Gaussian process? And I said the wave elevation usually, follows a Gaussian process. That is, it follows the normal distribution eta is minus infinity to plus infinity this is the variation.

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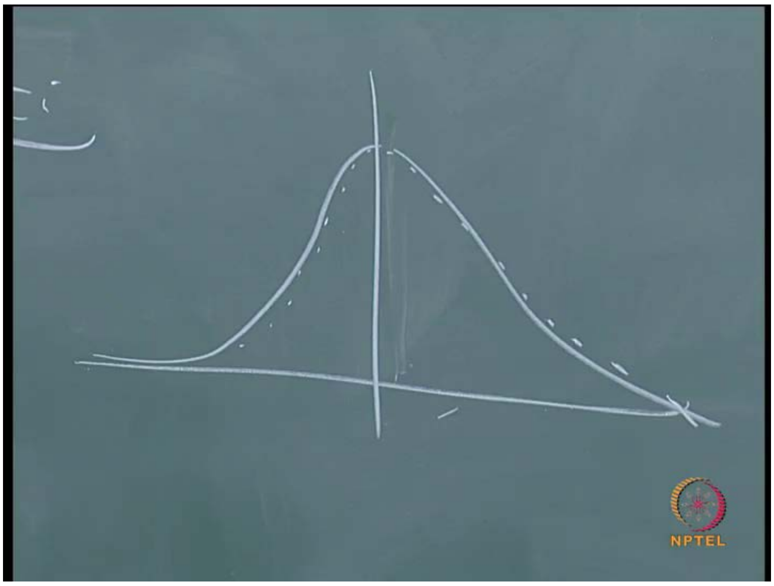

So, for this, the when you draw the Gaussian distribution, it will be something like this. So, your measured points will be… So, then you say that it follows the Gaussian distribution. I also has, I have mentioned that if eta follows a Gaussian distribution, then wave height would follow Rayleigh distribution.

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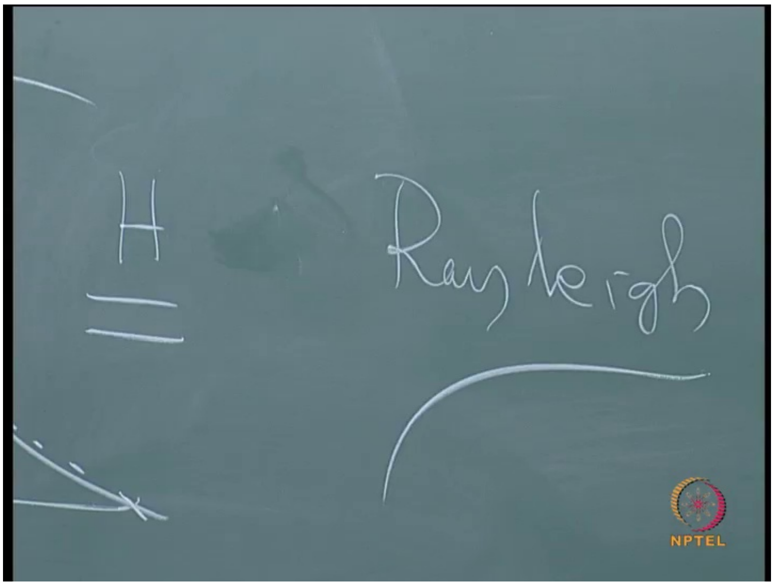

So, if you want to have some kind of a condition that you want to simulate this, that follows Gaussian distribution. Then you have to be careful take necessary actions, be careful using this method.

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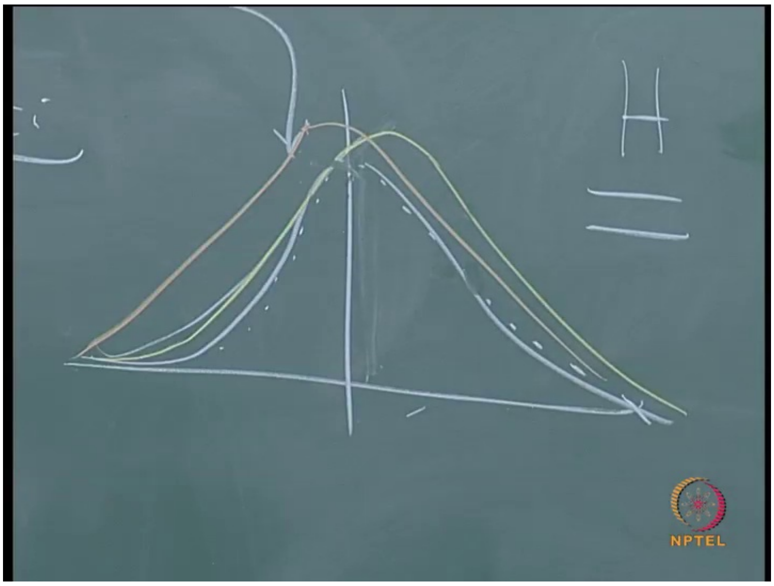

Sometimes, you may be interested not to generate a Gaussian distribution you may be, more interesting in generating a non Gaussian distribution. That is if it can be either measured the spectrum measured the probability of your, may be positively screwed cube. So, this is measured and this is the theoretical Gaussian or it can be negatively screwed so this is what is called as non Gaussian distribution.

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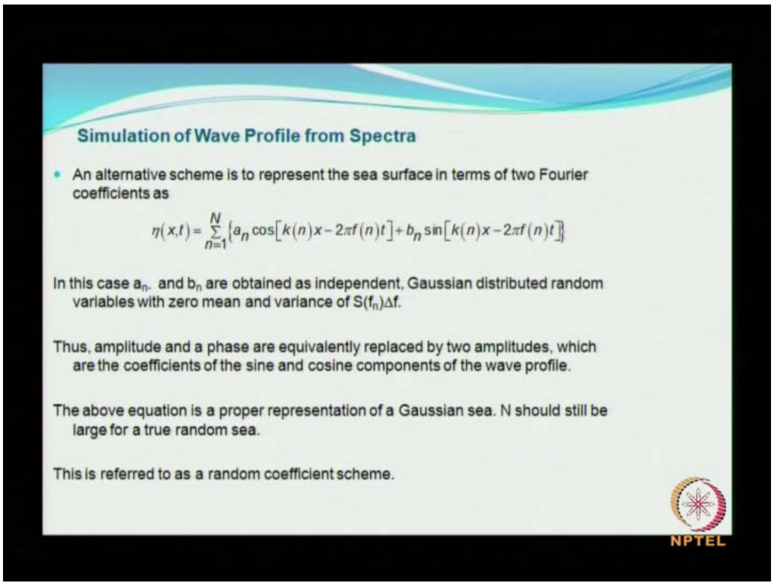

Now, an alternate scheme to represent the sea surface would be in terms of two Fourier components. In terms of (()) two Fourier component I am sure, that all of you have basic knowledge of fouross, I will not go into all those aspects. Now, eta of z, t can be represented as here, you have the summation something like, Fourier series a n, b n amplitudes and then other all the other things that take care of the phase variation. So, in this case a n and b n are obtained as independent, Gaussian distributed the random variable zero means a variance of so much, variant we have already seen.

Thus, amplitude and phase are equivalently replaced by two amplitudes, which are the coefficients of sine and cosine components of the wave profile. So, you see that this above equation is proper representation of a Gaussian sea. So, N should be still be large, if you want to really, to simulate a true random sea this I belief it is also, refer to us random coefficient scheme. So, the entire variance is now represented as a function of a form of two Fourier components coefficients.

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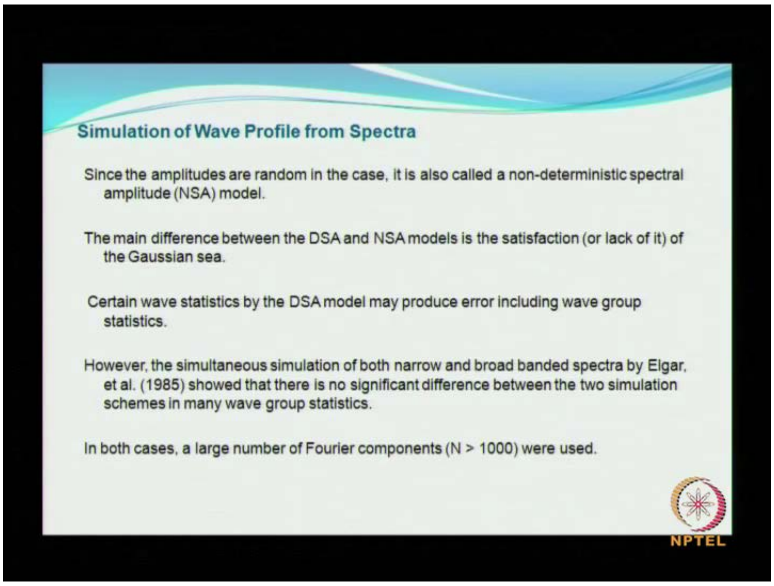

So, one is the earlier DSA model and now, since the amplitude and the amplitudes are random in the case. This is now, called as non deterministic spectral model. Now, since both amplitudes the amplitude is random in this case. The amplitude is random in this case, when you are defining with the elevation unlike the earlier method, unlike the DSA model now, the main difference between the DSA and NSA model the satisfaction of the Gaussian sea. If you use, NSA model you more or less have a Gaussian process and any way you have to use more number of large number of superficial of large number of components.

But in DSA, it is somewhat doubtful whether, you are going to land of the Gaussian process. But chance you might land up with Gaussian process but, it was Elgar at al where, he showed that there is no significant difference between the two simulation schemes, in many of the group statistics in both cases, whether it is DSA model or the NSA model, it always better to use more number of components. When we use the inverse a f t, the number of components? You are going to superpose you will be n by two components so where the n is the total number of data values that is total number of data values.

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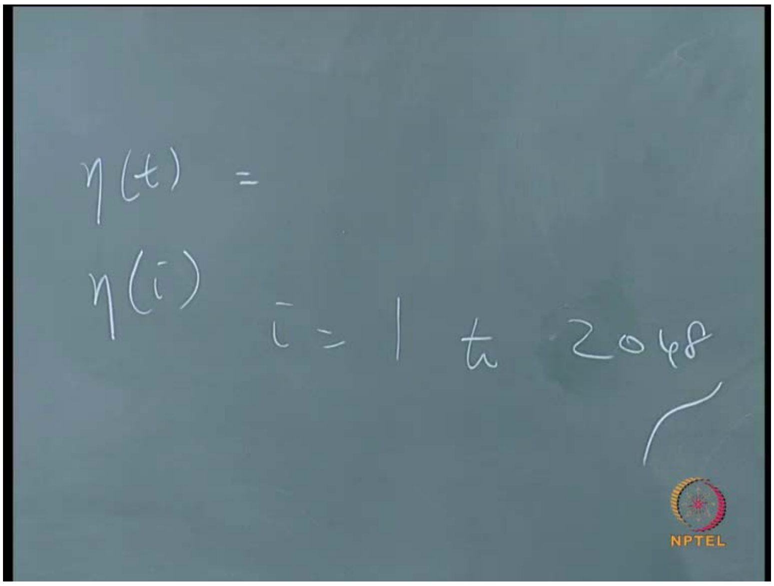

I mean so, if I say, eta of i so, i equal to one to may be 4000 something may be 1024 or 2048 normal is this is what they do for lab experiments? So, then the number of components will be 1024.

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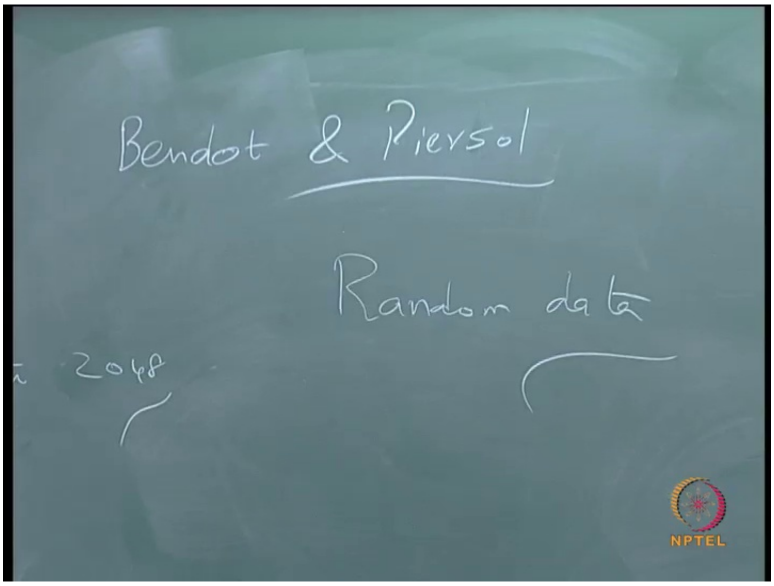

So, I suggest you also, Bendot and Piersol, the title is something like the random data the reference is given. What I suggest that? I just make a mention here, Bandot and Piesol that gives a lot of information about the basic statistics and how do you treat the random phenomena. Now, having an exposure to the simulation of predefined spectral characteristics, using the both the DSA model and the f a model the NSA model. We now get into what is meant by wave atlas for the Indian Ocean it is not the Indian Ocean, it is Indian Ocean plus along the Indian coast.

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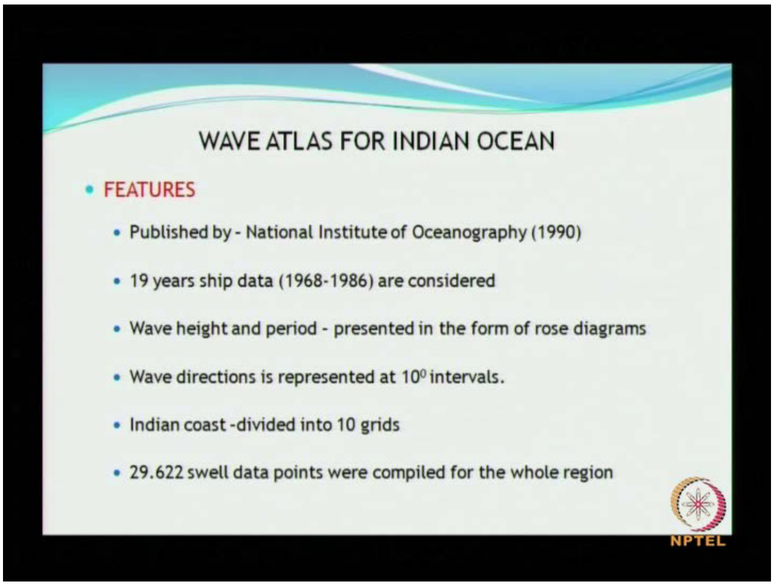

This was published this is the old atlas, but still, it gives us a same kind of indications concerned in the wave climate. There are so many sophisticated models in this days and so much of abundance of information on web data. But one, why the reason, why I have considered the wave atlas? Is the way you present the results the way, the results are reported to you or the wave characteristics. So, this is published in national institute of by national institute of oceanography. And nineteen years of ship observed data was considered and wave height and wave period was presented in the form of rose diagrams. This is called as either wave rose diagrams or wind rose diagrams.

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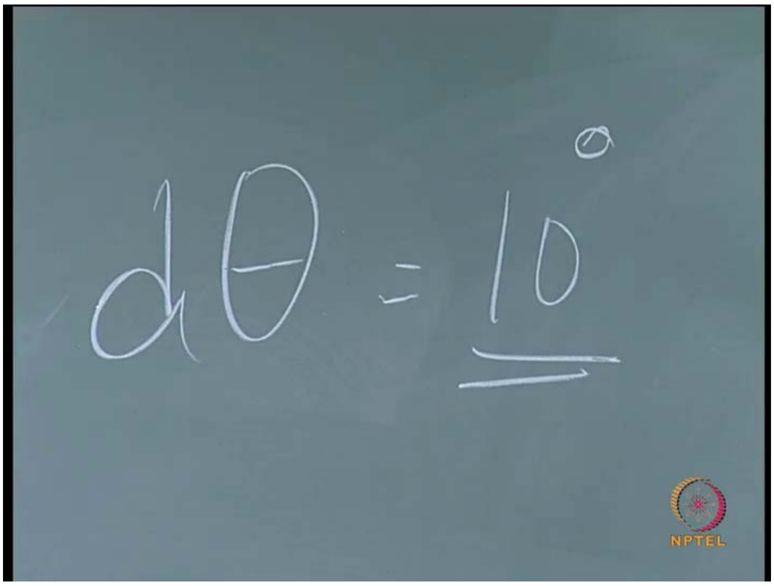

A wave height rose diagrams or wind speed rose diagrams. So, wave directions are represented in 10 degree interval d theta I will say, the Indian cost is divided into 10 grids and about 29000 swell data files were complied with the whole region.

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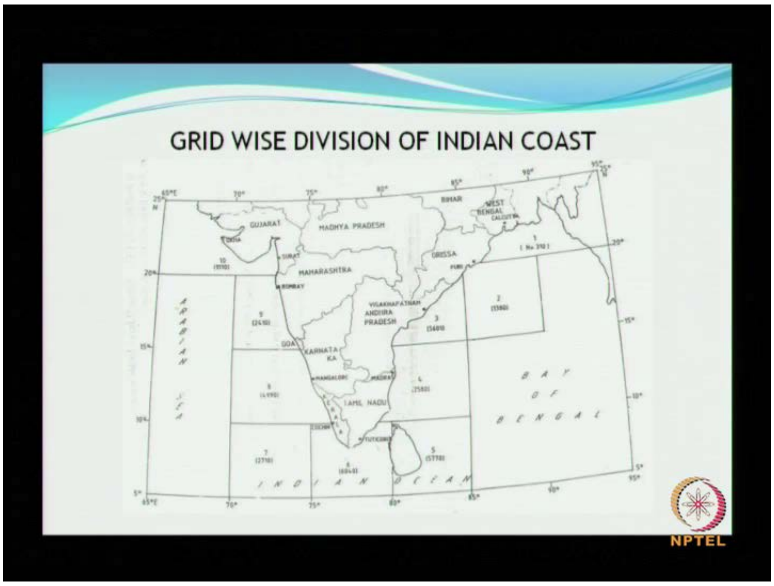

So, this is the Indian map, where in the whole area is divided into the number of grids as mentioned earlier. And then the wave height roses matter of representation percentage frequency of occurrence of each class of wave height in 10 degrees intervals as for each month for every month. So, starting from January February etcetera for each of those grids.

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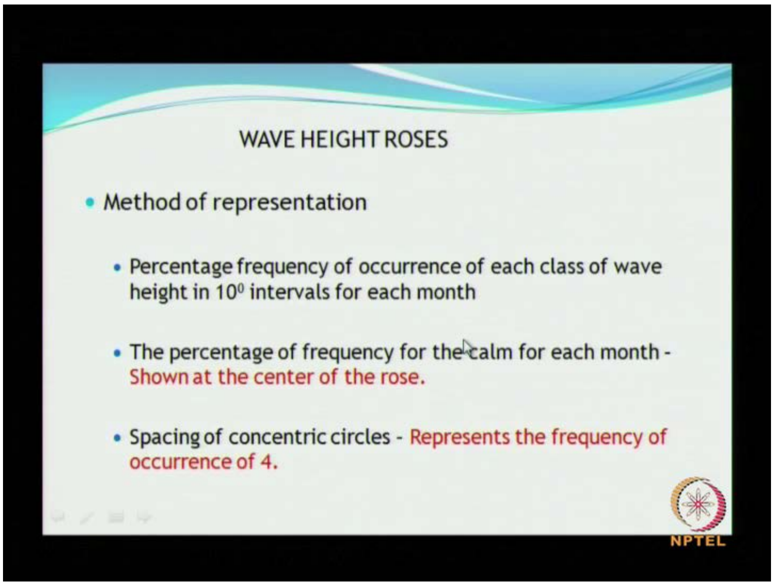

For example, if we take this grid it is available January, February, March all the 12 months, compiling all the 20 years of data. And it gives the problematic description of the wave height and wave period, coming from different directions at that particular grid.

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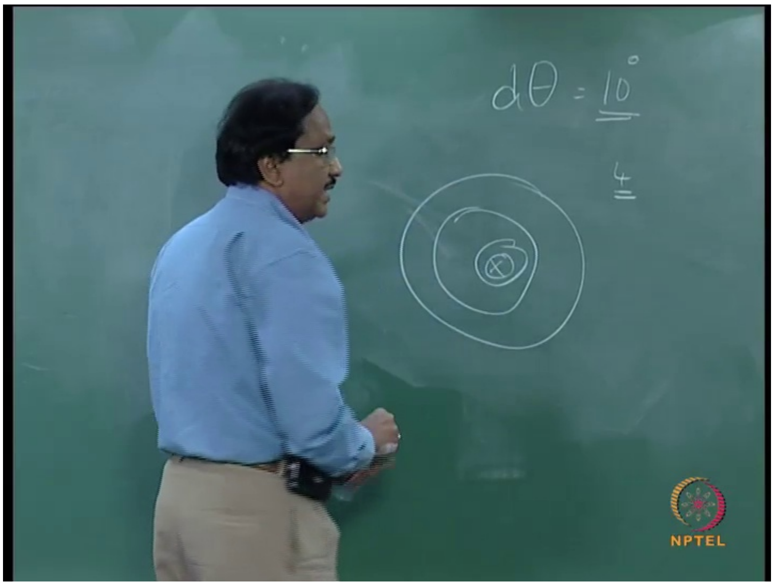

So, the percentage of frequency of calm is that given in the center, I will come very quickly, you will be seeing the rose diagrams and spacing of concentric circles represents. The frequency of occurrence of 4 so, you will have concentric circles and you will see here, the concentric circles represent the as number four as, I have indicated here.

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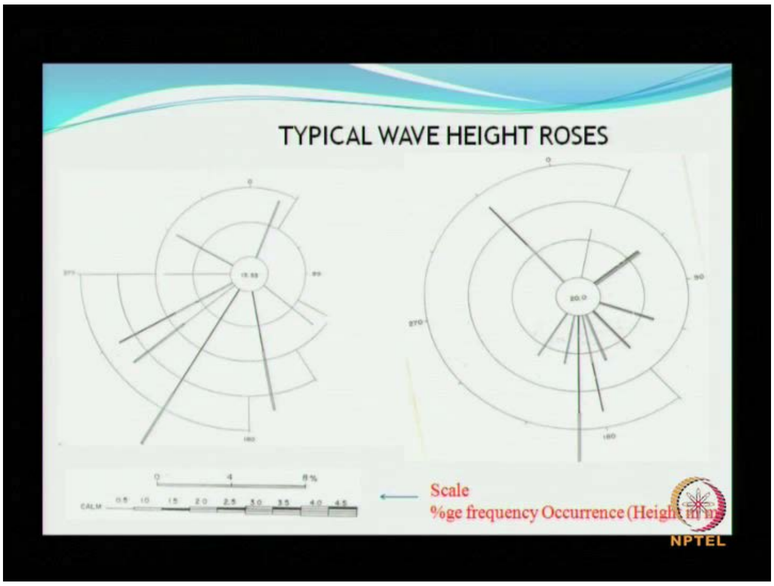

I mean the frequency of occurrence as 4 then you see the lines, each line indicates the direction so, you take a particular location, a particular grid. So, in the particular grid, what you have in the center is the percentage of occurrence of the calm the calmness. All other things are given here, for you have a scale with top percent occurrence of 4 in terms of 4. And here, is the percentage of occurrence of wave height in meters. So, each of these indicates this values, of the values indicates the how much is the what is the percentage of occurrence of wave height ranging between 2.5 meters to 3 meters or this information can be obtained.

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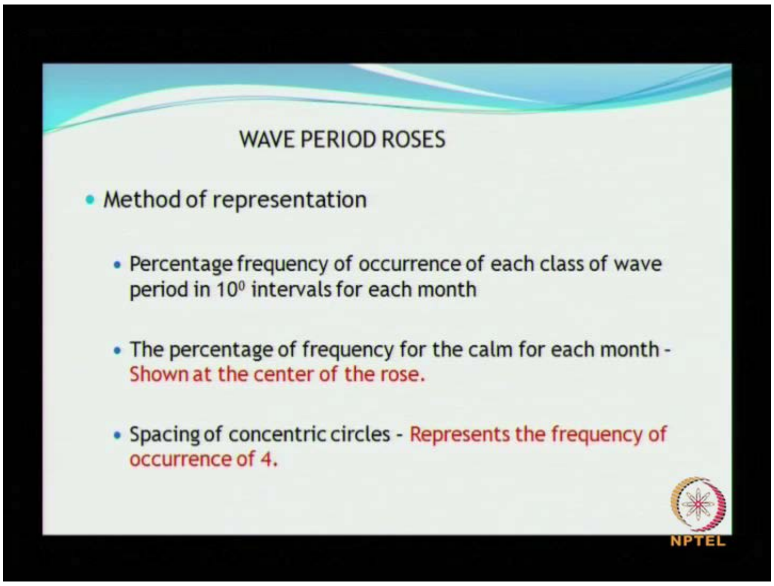

Similarly, you have the wave period and rose diagram the same procedure is followed and you have the period n for example, here percentage of occurrence is the same thing.

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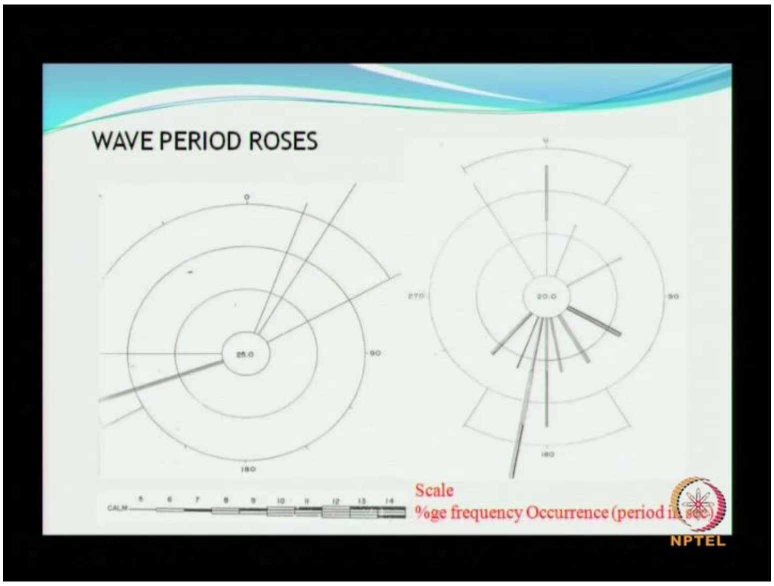

And you see that these how this period diagram is presented.

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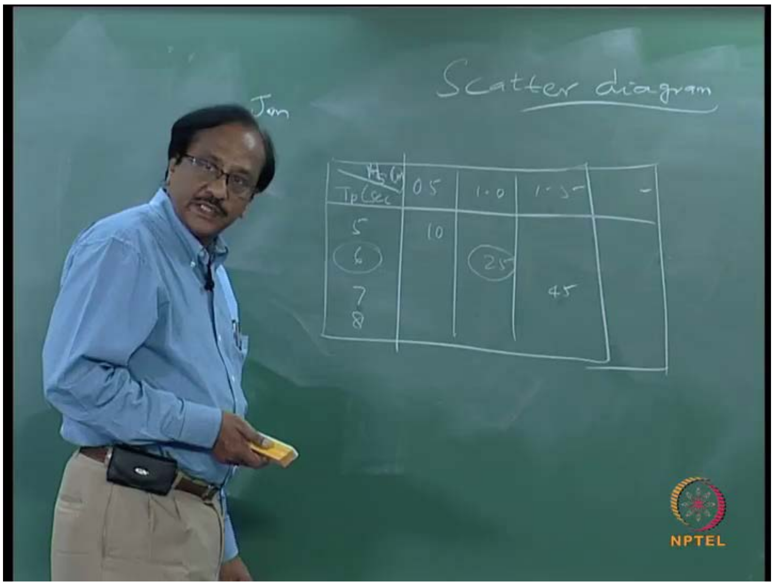

So, before that there is another aspect that is scatter diagram. This also, is used for a given location say, for the month of January data has been acquired it may be measured data. So, you will have H s in meter and T p in seconds. So, you may have 5, 6, 7, 8 etcetera and then you are may be 0.5, 1 meter, 1.5 etcetera. So, here in this 10, 25, 30, 40 etcetera so, what does this means? This gives scattered diagram this represents the scattered diagram which gives, 25 percent of the time. The wave height is approximately 6 seconds with an associated wave height of one meter.

Similarly, here the most frequently occurring combination is wave height of 1.5 meters with a wave period of 7 seconds, this kind of information is very useful for the design of structures. In addition there is also, what is called as persistence diagram, so, you have to look for how long as storm can last in a particular location. This also, used important this also being used, I think I will stop with this, and then we will see in the next class.