Wave Hydro Dynamics

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Module No. # 04
Random and Directional Waves
Lecture No. # 04
Random Waves and Problems III

Earlier, we have seen how important it is to represent the measured wave spectra, as a kind of whether it follows any of the standard spectra.

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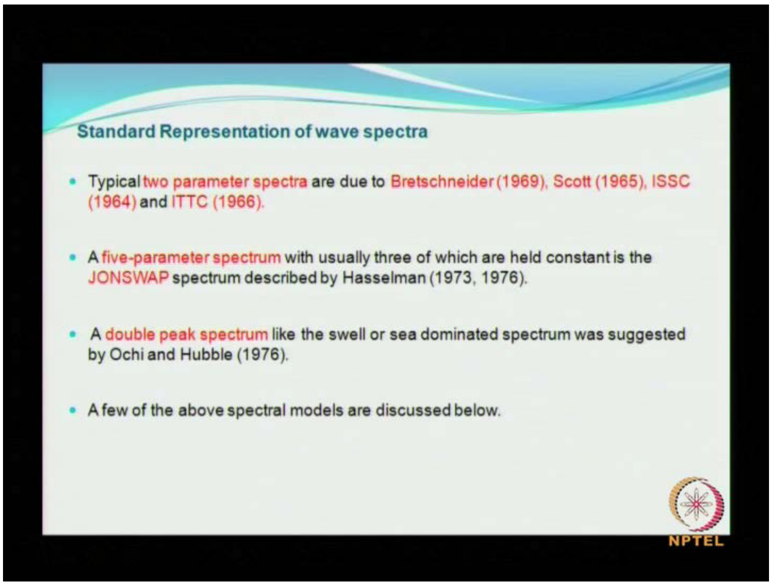

So, it is always important to report the results in a standardized form, that is one aspect the other aspect is I have also told you, once you are able to prove that of a side, if the spectra is following a standard spectrum. Then it boils down to be very straight forward, and it is very easy to get other kinds of information from the spectra, so earlier what we have seen was a some of these aspects.

Now, let us look the standard representation of this spectra, typical two parameters spectra are that due to Bretschneider the years are given within the brackets, Scott ISSC that is International Shifts Structure Conference that was in 64. And then international towing town conference, that is during this conference this parameter had this spectra has evolved; and that has taken place in 66, of five parameter spectrum which usually three of which are held constant is what is called as JONSWAP spectrum JONSWAP is join north sea wave project, that has been described by Hasselman 73 and 76.

So, these are all some of the widely used spectrum earlier, we have also seen Pierson-Moskowitz spectrum, so this JONSWAP spectrum is actually a modification of Pierson Moskowitz spectrum, we will come to that again. A few of the above spectrum models are now going to be discussed, as much as possible I will give all the information, but I suggest you refer to the book on hydrodynamics off shore structures, by S. K. Chakravarti, he is a very good book which gives, complete description about the standard representation of standard spectra.

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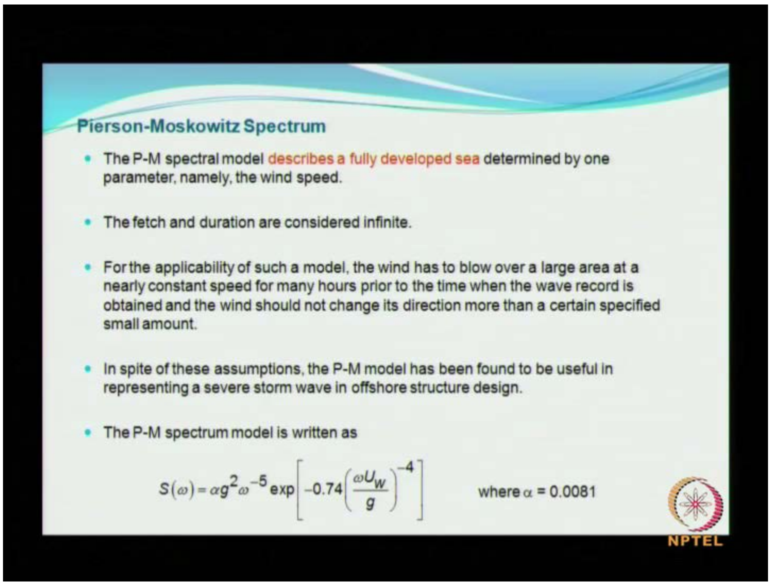

Now, let me start with the Pierson Moskowitz spectrum, the Pierson Moskowitz spectrum is one of the most widely adopted spectrum, and usually researches when they are trying to simulate random waves in the lab, may be to measure forces are pressures on structures due to random waves. The resort to waves that follow Pierson Moskowitz spectrum, for the simple reason that it describe a fully developed the sea, and it is a quite straight forward, because it is defined only by a single parameter; that single parameter is nothing but, the wave speed sorry wind speed.

Later you will see that the wind speed, can also be represented as a single parameter that is the significant wave height, here in in this formulation the fetch and the duration, are consider to be infinite. For the applicability of such a model, the wind has to blow over a large area, at a nearly a constant speed you cannot say 100 percent constant speed; for several hours prior to the time and the wave record is obtained. And the further, the wind should be not changing its direction more than a specified small amount.

These are some of the conditions, where you can conditions for the applicability of wave elevation following a Pierson Moskowitz spectrum. In spite of all this assumption the Pierson Moskowitz spectrum model, has been found to be quite useful in representing serious storm wave in the offshore structural design, this is what is claimed, by people who have a really adopted the Pierson Moskowitz spectrum for, considering the storm wave in offshore structure design, is that clear.

Now, this is written as I said all this parameters, now you see that this is the spectral density, as a function of angular frequency and here, it is now function of only the wind speed and frequency (Refer Slide Time: 05:30)

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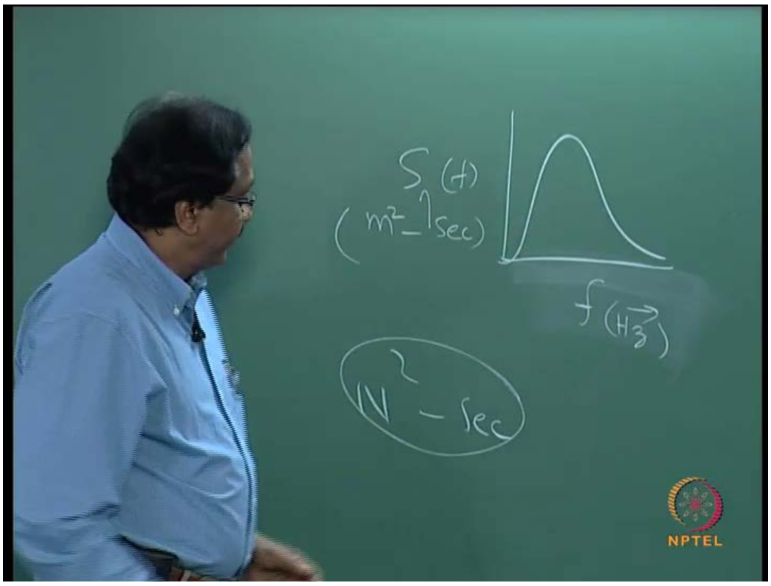

So, remember when you draw the spectrum on the x axis, you will have frequency on the y axis it will have you will have S eta of f, and the unit for this is meter square second, this we have already seen with the help of a problem. Similarly, when you draw the four spectrums on the y axis, you will have Newton meter second, remember the units, and recollect what we have discussed in the earlier problem.

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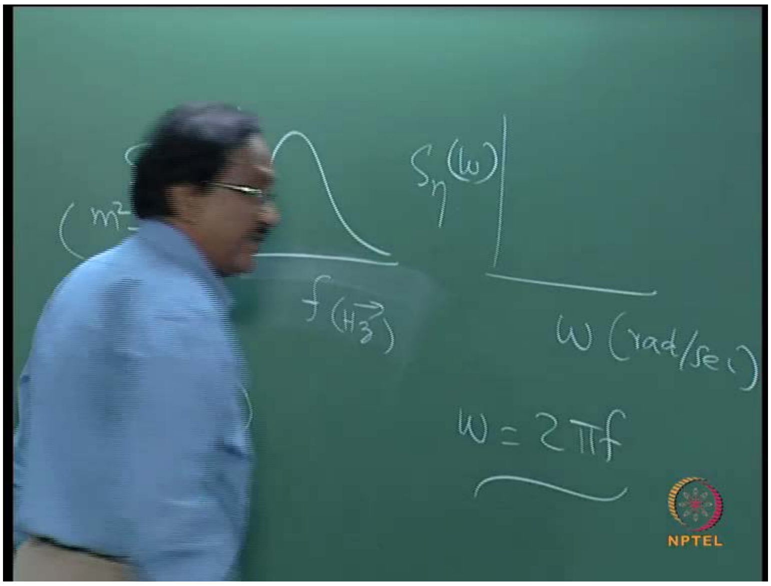

So, it is also it is also represented as omega, which is radiance per second and w, w that is radiance per second is related to frequency, linear frequency as 2 pi f, so alpha is a constant in this.

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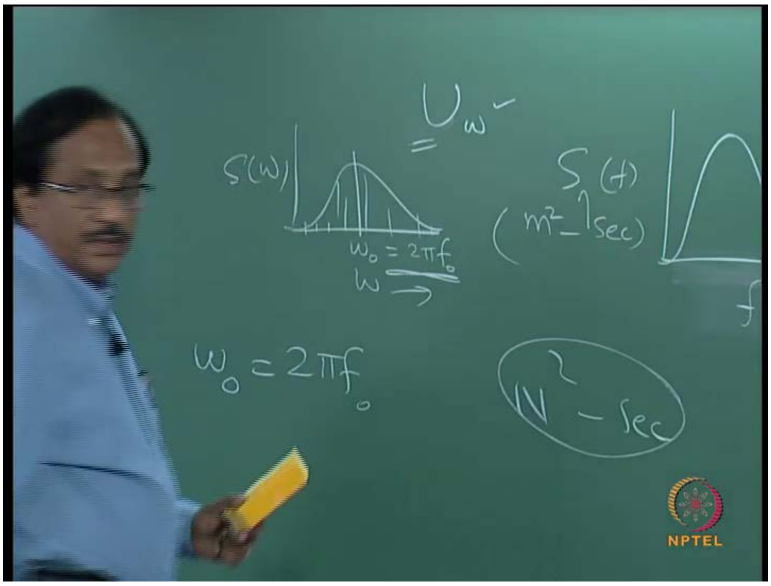

So, what I do this is the simple relationship, so I the moment I know the omega range at there is cross intervals I can draw this for a particular wind speed for a particular wind speed U, is that clear for a particular wind speed U, I can draw a spectrum using this mathematical expression.

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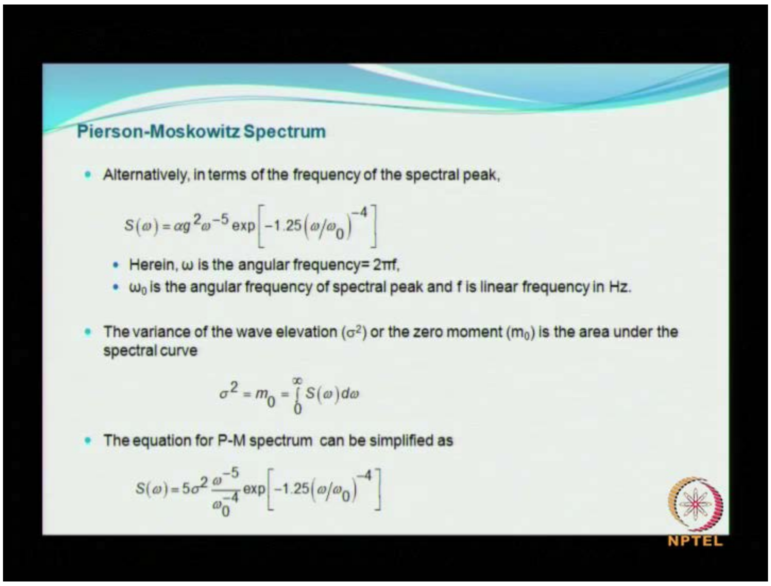

Now, alternatively in terms of very often we do not refer to wind speed, although the whole process depended on wind speed, we refer to either the significant wave height or the peak frequency. So, the Pierson Moskowitz spectrum can now be represented as shown in this expression, where in omega naught is nothing but, where f naught is, this is your omega naught.

Angular frequency at which the peak occurs, the variance of the wave elevation that is nothing but, sigma square will be equal to m naught as we have seen, when we when I we when we were introduced to random waves. So, sigma square that is the variance is equals to m naught, that will be nothing but, the area under the spectral density curve (Refer Slide Time: 08:42).

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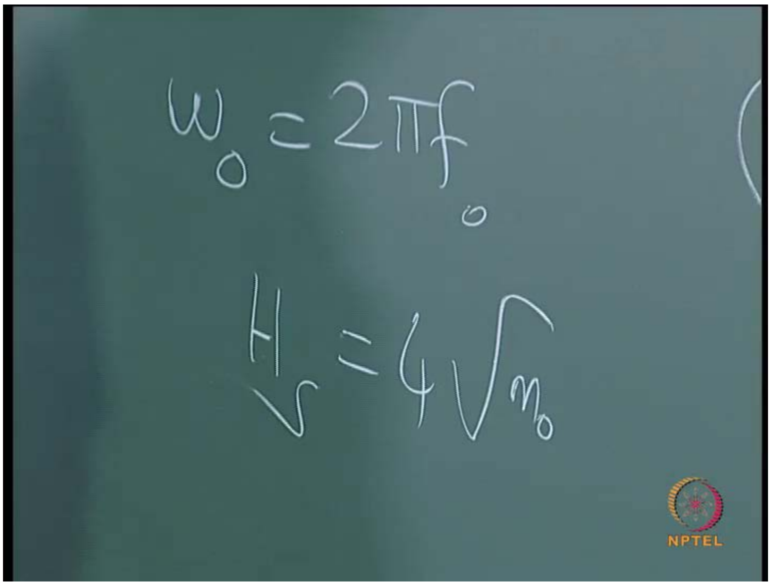

The area under the spectral density curve is nothing but, m naught and you also know that square root of m naught into 4 is what H s be louder, so this also we have seen earlier in the lecture (No audio from 09:42 to 09:59).

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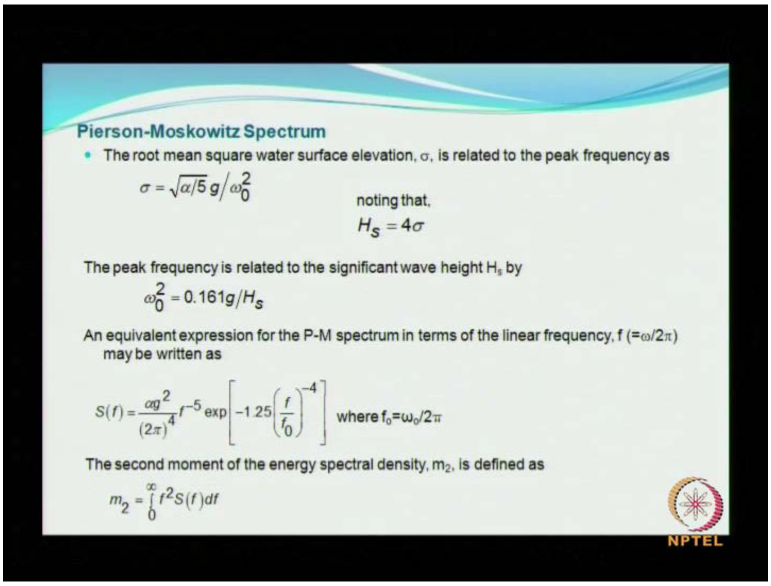

Using the above the equation can further be written, in terms of your sigma as indicated here, which can this is what I had mention here, that H s equal to 4 times sigma (Refer Slide Time: 10:09). So, using this simple site I mean, the root mean square value it can be related to the peak frequency that is what I have explained earlier, because H s equal to 4 times square root of m naught or 4 times sigma, sigma is nothing but, the variance is that clear. Now, the peak frequency can also be related to the significant wave height as shown here, so it is a function of only single parameter.

So, now, if you represent you can represent this as a function of linear frequency, some people have the habit of representing in terms of linear frequency or angular variance, there is no harm in that. So, in this case you have as a function linear frequency, the expression is shown here, I have not deriving this, all this six can be easily be derived, refer to some of the notes which you have been provided or checked with the hydrodynamics of offshore structures, by Chakravarti or any other standard books, and its all quite straight forward (Refer Slide Time: 11:19).

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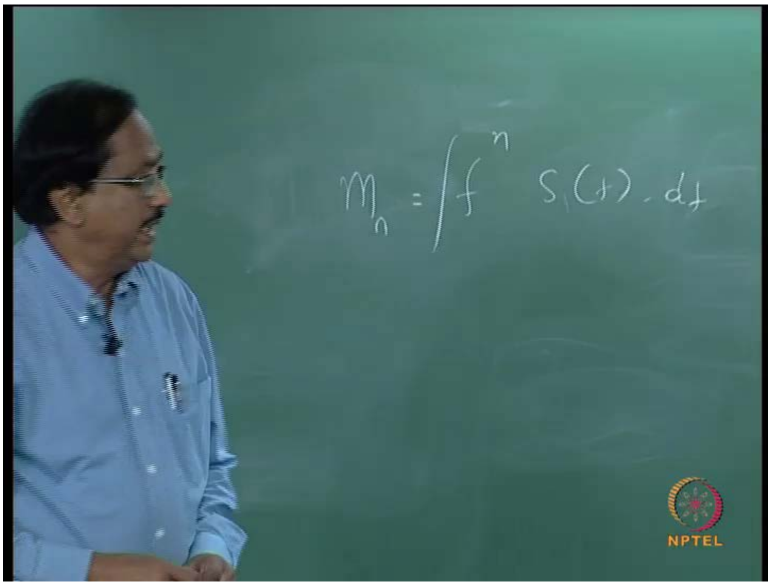

So, the second moment is defined as we have already seen, what is a a m moment, that is nth moment right, we discussed about zeroth moment, second moment, fourth moment. When we were interested in arriving at the different wave periods etcetera, are different wave height under the spectral method.

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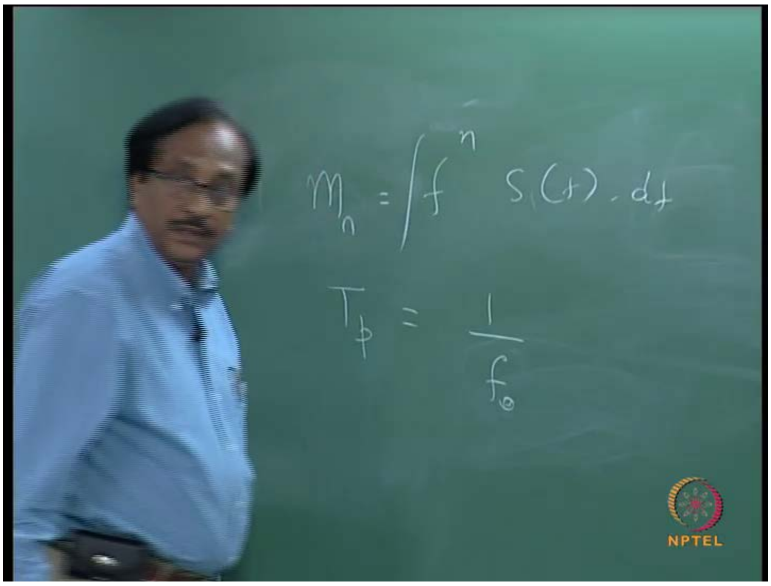

So, the second moment is obtained as f to the power n this is nothing but, f to the power n of course, you have the integration. So, what I what do I need to draw a Pierson Moskowitz spectrum, to draw a Pierson Moskowitz spectrum I need only the T p, which is nothing but, 1 by f naught (Refer Slide Time: 12:37). So, let me assume, so for different wave periods, peak periods I have different spectra, is that clear.

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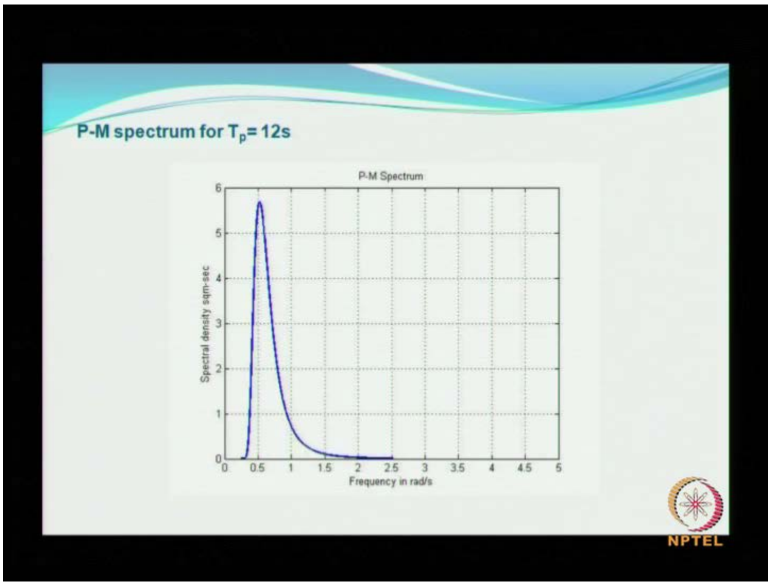

So, now, in this case and we have selected T p equal to 12 seconds, so this shows the variation of the spectral density, as a function of frequency in the in the x axis, so this shows how the distributions looks like. So, you can gather all other kinds of information, which we have already discussed about the range of frequencies frequency, they had which the maximum energy occurs etcetera.

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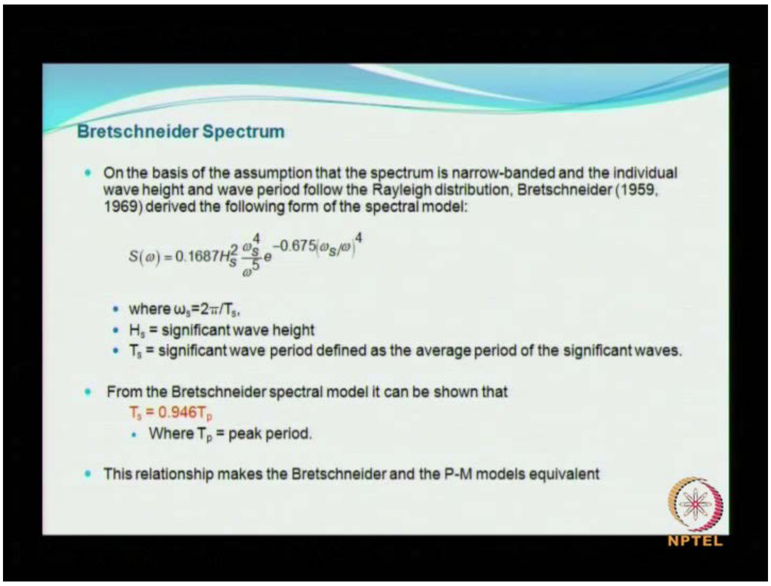

Now, we move on to Bretschneider spectrum, so this is also been used widely, so there on the bases that the spectrum is a narrow banded, and the individual wave and wave period, wave height and wave period follow there is a condition that if, it follows that Rayleigh distribution, then he derived Bretschneider derived a formula as shown here.

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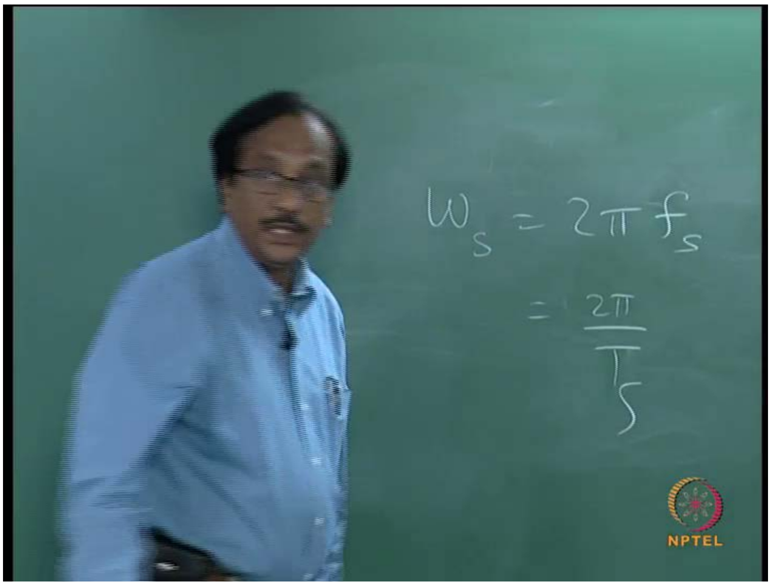

And here, note that you have H s here, plus you have omega s, omega s is nothing but, the significant I mean 2 pi into 2 pi 2 pi, 2 pi f, so it is now usually we use 2 pi f s. So, now, you are T s is nothing but, the significant wave period defined by the average period of the significant waves (Refer Slide Time: 15:14). Now, from Bretschneider spectral model, it can be shown that t s is approximately equal to point, it is equal to 0.946 T p.

So, again this spectral model also, can be equivalent as a function of peak period, now the above relationship make the Bretschneider spectrum, and the Pierson Moskowitz model almost same are equivalent, is that clear. So, Pierson Moskowitz spectrum is a single parameter spectrum whereas, Bretschneider spectrum is a two parameter spectrum.

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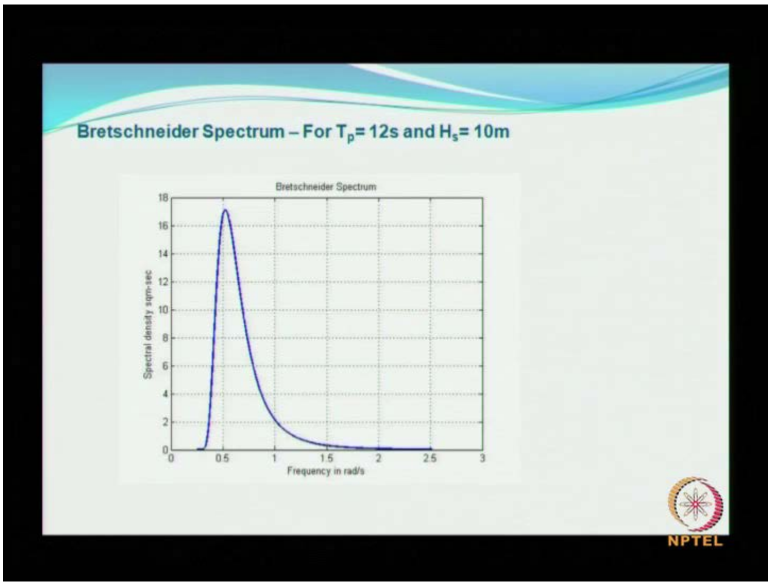

So, when you are defining Bretschneider spectrum, you need to have both the T p as well as H s. So, all through my discussion, I am going to take the T p as a 12 second wave period, and H s as 10 seconds, is that clear?.

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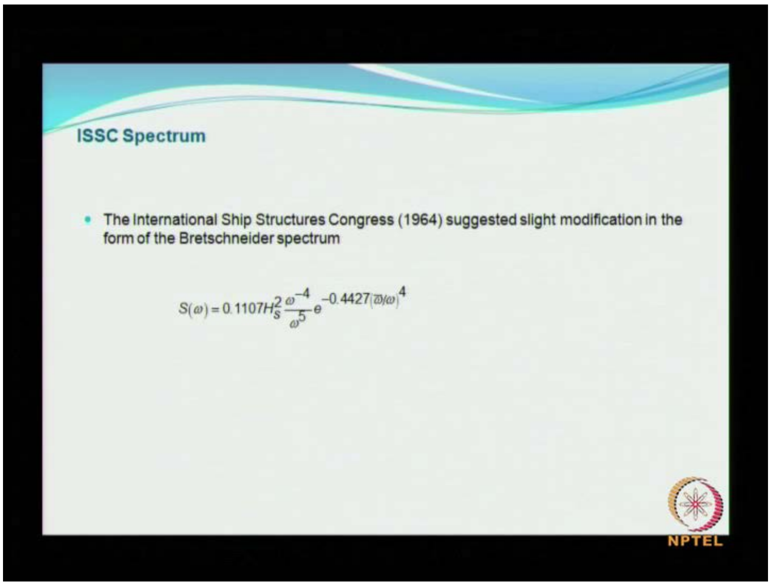

Now, yes ISSC spectrum this international shifts structures congress, suggested slight modification with in the form of Bretschneider spectrum, as indicated here. So, you have a mean angular frequency and and the form is slightly different than your Bretschneider spectrum, then we will try to the formulations are slightly different, but it is all two parameters spectrum.

And then for a given wave height, given significant wave height and given peak period, if you try to plot together and then super pose all these figures, then you really see what is the kind of difference you have, between the different spectrums. This is basically, all this theoretical spectrum are being tried to fit some of the measured spectrums; so that the measurements from any locations can be accommodated or can be theoretically described, that is the idea.

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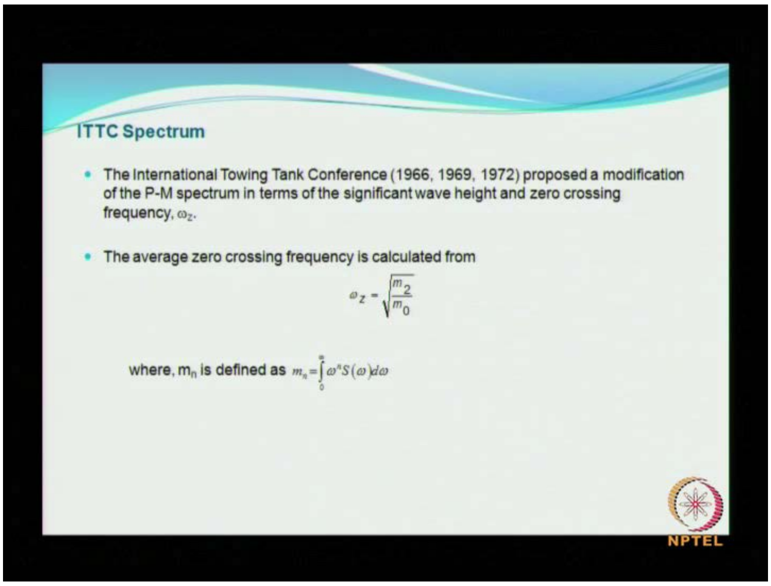

So, then you have the ITTC spectrum, that is this was formulated right from formulated improved etcetera, from 66 to 72 and this was the, this considered a modification of the Pierson Moskowitz spectrum, as the function of significant wave height and zero crossing period. So, the average zero crossing period can be calculated by the square root of m naught m 2 that is the second moment, divided by zeroth moment, and how do you get the moment we have already seen, its also indicated here order of n, so 0 second usually we use 0 second moment or fourth moment.

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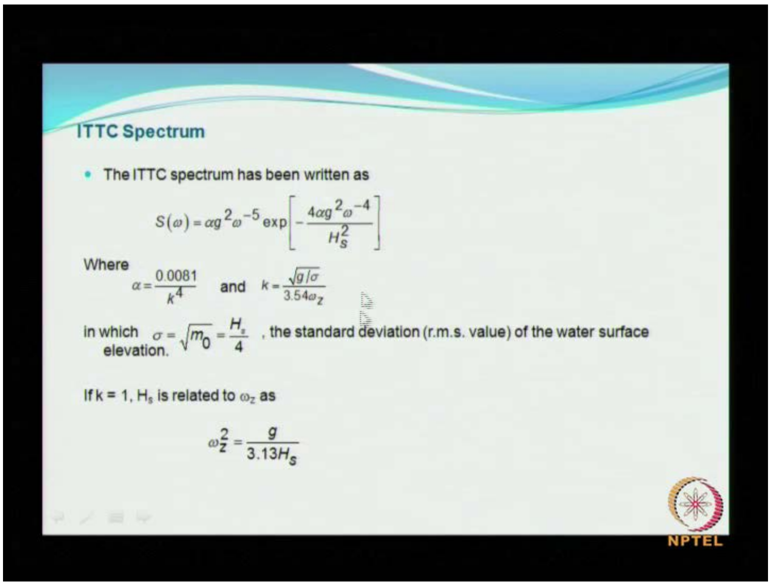

So, the ITTC spectrum is given by this expression, and all this sign which this we have already seen that is nothing but, the standard deviation, than if K all this things you try to reduces, this is going to be your final expression.

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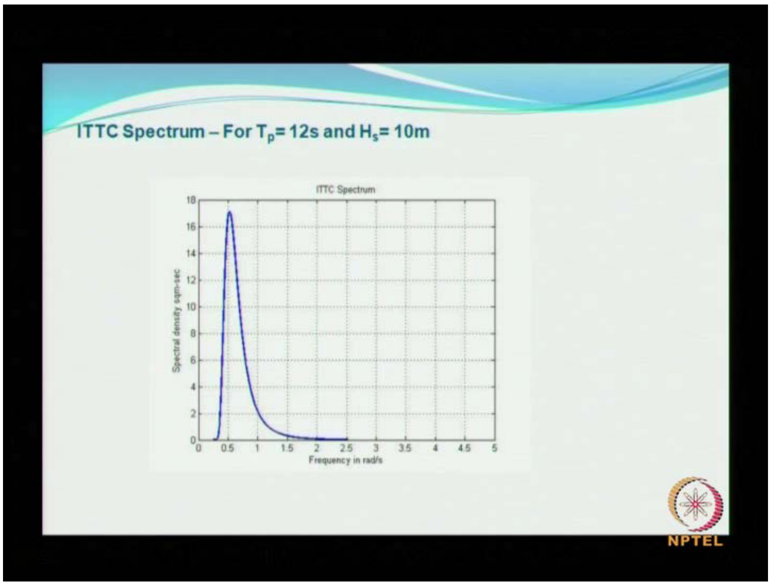

And then, when you plot for T p equal to 12 second, and H s equal to 10 seconds, 10 meters into look like this. So, now it will it looks as if it is almost same as ISSC spectrum or Bretschneider spectrum, but later you see some of the differences.

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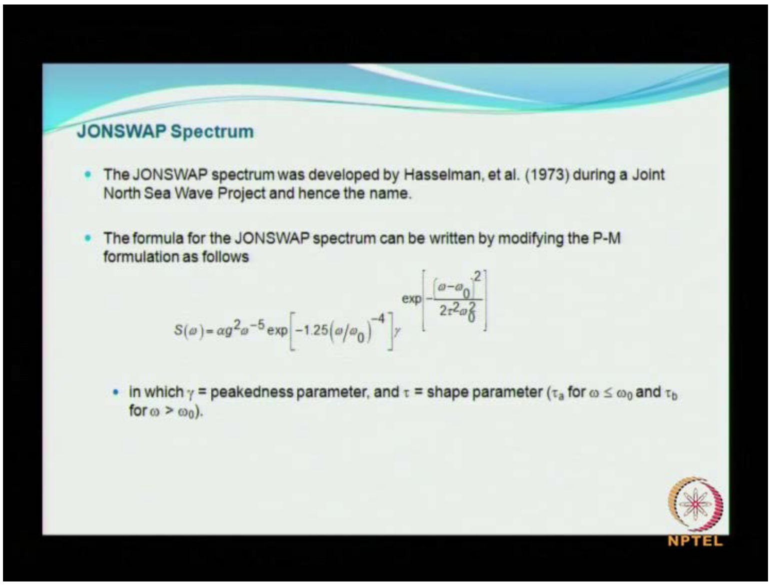

Now, comes JONSWAP spectrum, what happen was JONSWAP there was a tremendous increase in the offshore oil production exploration, and as well as exploitation in the north sea, several locations in the north sea.

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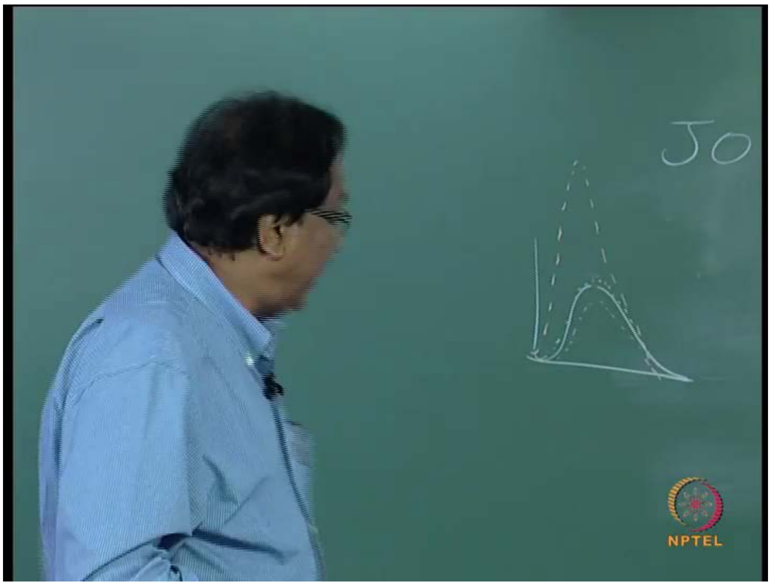

And they were always trying to compare with Bretschneider I mean Pierson Moskowitz spectrum, when they started comparing with the measured spectrum, so the if the measured spectrum is something like this, we can as well accept or it is something like this, with some amount of degree of a resolution, we can still accept. But, if the measured spectrum was always like, most or instances it is something like this, then they found that it is a kind of a different kind of behaviour of the waves, that is somewhere around the peak frequency, there is a peakening.

The spectra is the the energy is getting, it is increasing near the peak frequency more than what could be an anticipated, when it is a likely to be assumed that it follows a Pierson Moskowitz spectrum. So, then this was a thought of peakedness, this was a consider as peakedness, but they wanted to retain the Pierson Moskowitz spectrum, so what what was done was the the Pierson Moskowitz spectrum was retrained, but they had an add on that is gamma to the power this factor, exponential of all these things (Refer Slide Time: 21:56).

Here, gamma is not known and the tau is not known, these are the two new parameters now, now what are, so in which this gamma is going to be your peakedness parameter; in order to take care of the peaking of the energy around the peak frequency. And then, tau is defined as shape factor, and tau was defined as tau a for frequency less than omega naught, for this range you have tau a, for this range you have tau b, is that clear (No audio from 23:00 to 23:15).

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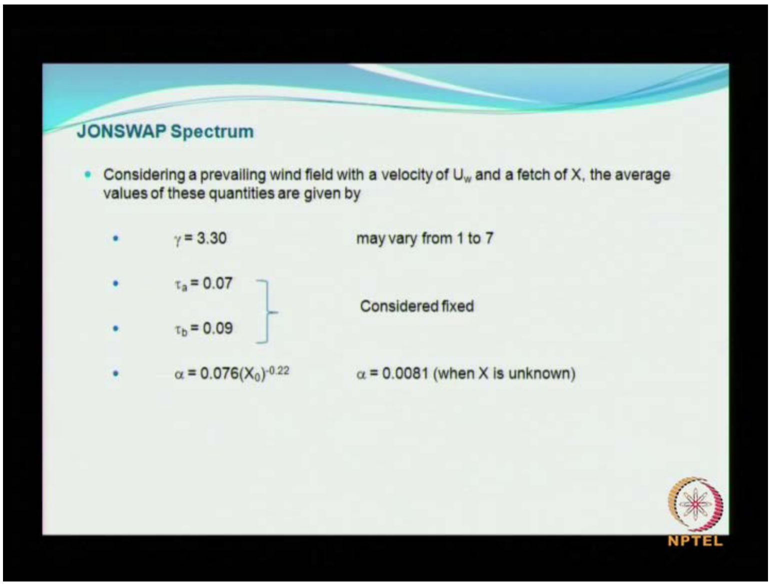

So, considering the prevailing wind, wind field with the velocity of U w etcetera, use of X w and X X the average values of theses quantities, can be assumed to be gamma is 3.5 and it can vary anywhere between 1 and 7. So, this take takes care of the peakedness, but this 3.3 is an average value that has been assigned, but what about this tau a and tau b, tau a is a 0.07 and tau b is 0.09. And alpha is equal to 0.0081, if x naught is not is a, if the fetch is not known, if the fetch is known, then you put that fetch as used that appropriate expression, to get the value of alpha, Is that clear?.

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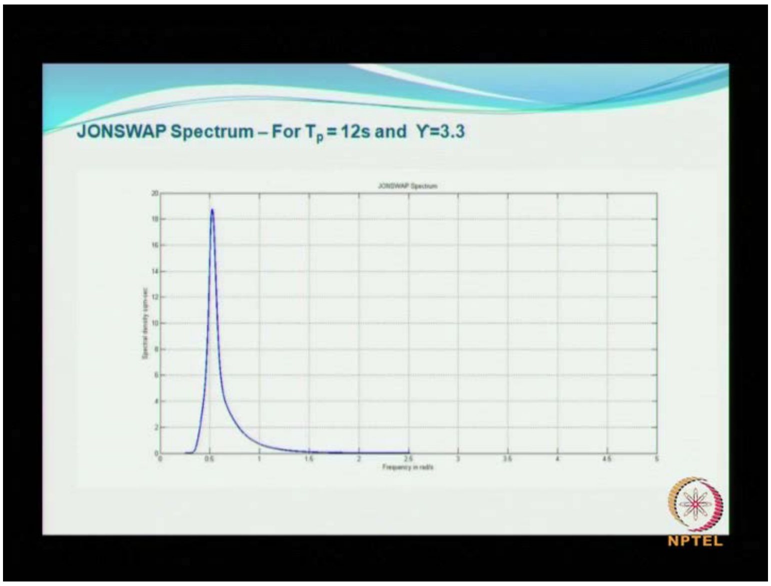

So, once you do this, you get you look at this shape of this spectrums, the shape of the spectrum is like this, it is pickening, if gamma is equal to 1, what happens Pierson spectrum.

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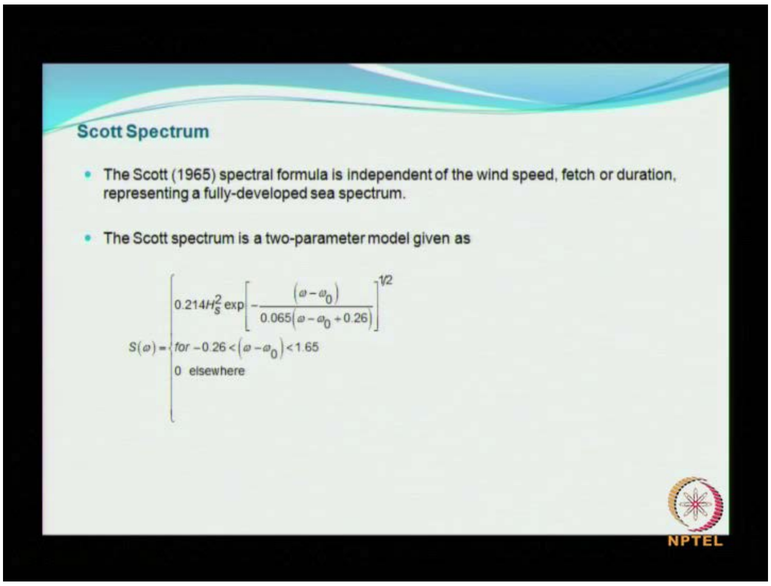

Then, comes Scott spectrum (No audio from 25:06 to 25:16), Scott spectrum the spectral is independent of a wind speed fetch and duration, and it again represents a fully developed C spectrum, and the expression given here is in terms of H s and you have a omega naught.

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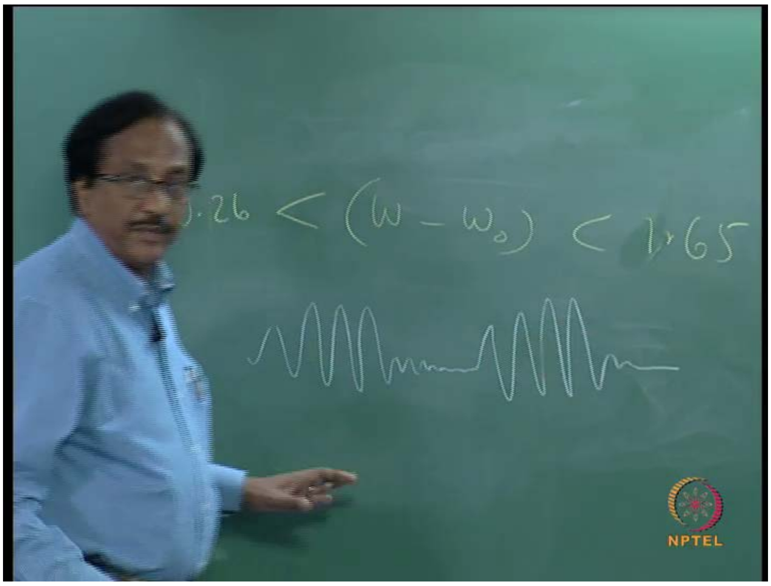

But, there is a frequency range, the above expression is valid the expression is valid only for omega minus omega naught equal to, so of course, but all other location it has to be having value of 0. (Refer Slide Time: 26:21). When you are try to stimulate the wave elevation from a Scott spectrum usually, you will see that it is a narrow band spectrum. So, narrow band spectrum would look, I think we have some that is what will happen is, so you can see some kind of a wave groups, that is you have waves of higher magnitude, then it will slow down and then again you will have the same kind of waves.

So, this are called as waves groups which again I will come back to this later, but then in the case of Scott spectrum mostly the eta, the wave elevation will be following the, it will be a narrow band process.

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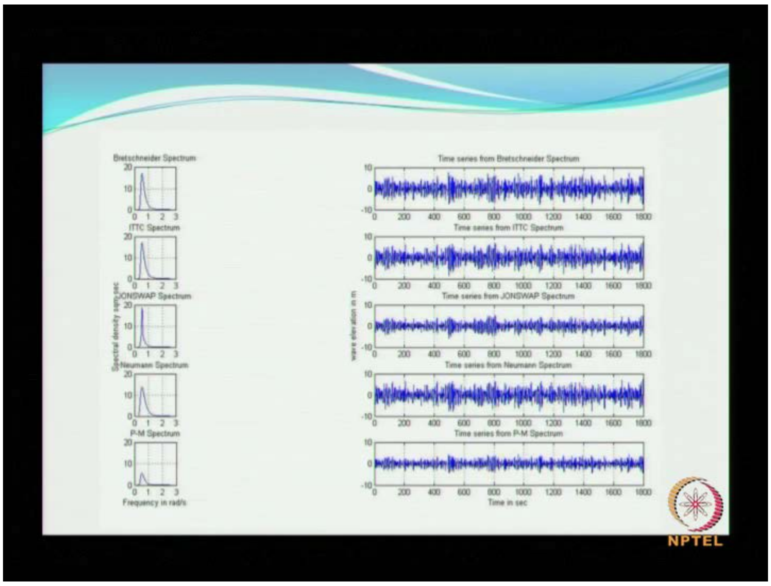

So, this figure shows on the left hand side the Bretschneider spectrum, the ITTC spectrum, JONSWAP spectrum, Neumann spectrum, Pierson Moskowitz spectrum, etcetera, on the right hand side you see is corresponding time speeds, Is that clear?

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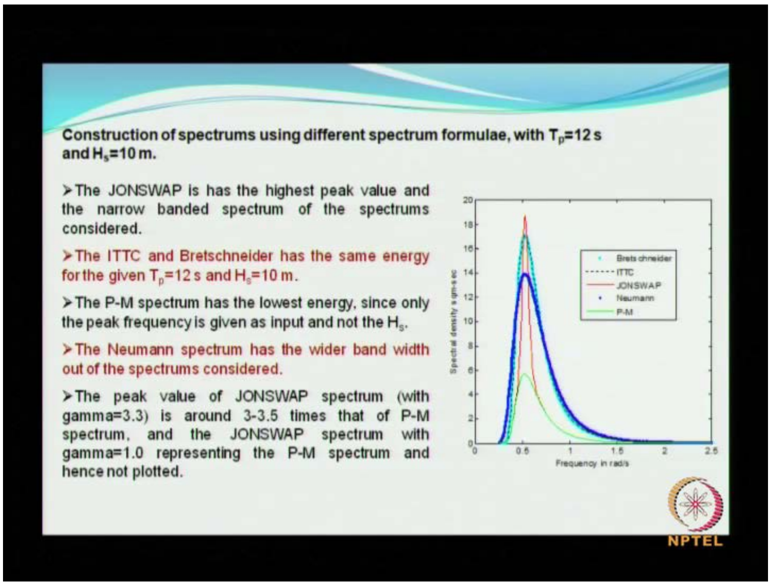

So, this is the picture, which shows the construction of spectrum using different spectra formulation, for T p equal to 12 second and H s equal to 10 meters, so have a close look at the, try to spend some time, and try to understand what the figure gives you. For example, try to identify the JONSWAP, JONSWAP is a the brown colour the

JONSWAP is perhaps highest peak value, and the narrow banded spectrum of the, narrow banded bandedness is considered.

ITTC and Bretschneider has the same amount of energy, Pierson Moskowitz spectrum as a lowest energy, since only the peak frequency is consider is given as the input, and not the H s. Then the normal spectrum, which is blue colour has a wider band width finally, JONSWAP spectrum, the peak value of JONSWAP with gamma that is the peakedness factor is a 3.3 is around 3.5 times that of P-M spectrum.

So, you look at this picture, compare basically the P-M spectrum is something like this whereas, something like this. So, that shows, that gives you information about mostly the single parameters, I mean single peak spectrum; all these spectral models you will see that there is only one spike, only one peak.

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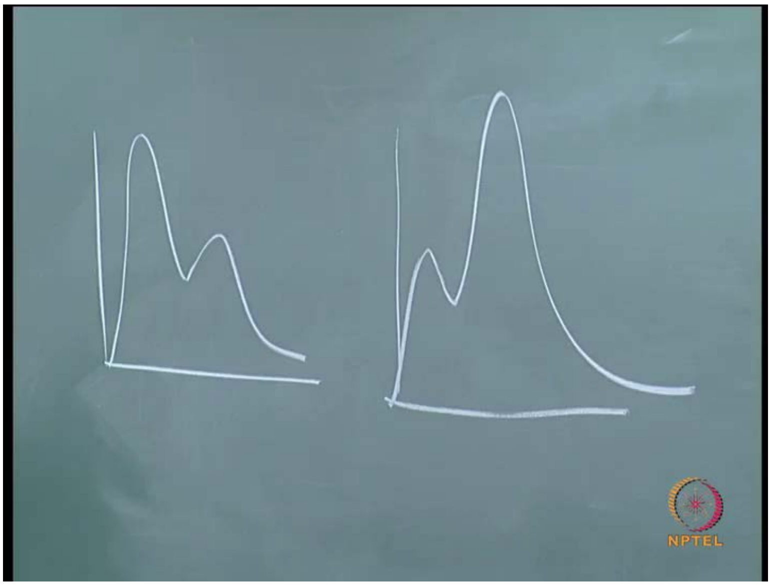

But, remember I also told you about the swell dominated spectrum, and the wind dominated spectrum, you can also have a multiple peaks.

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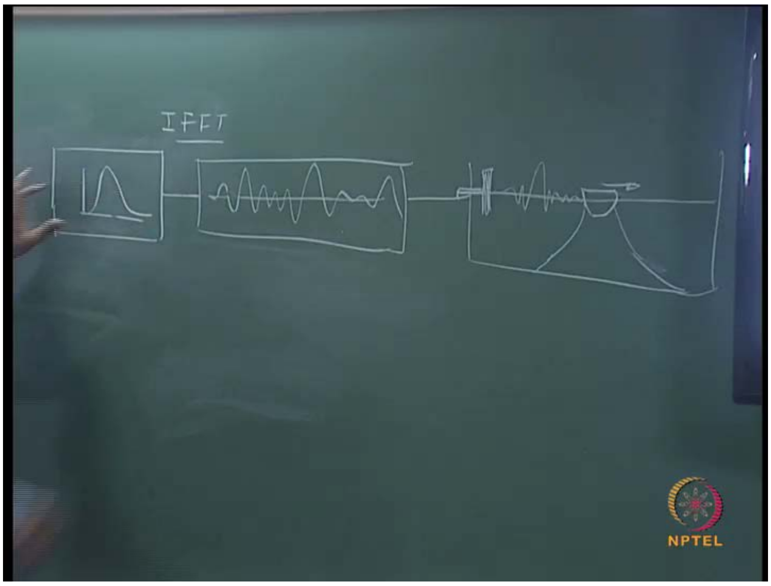

As I said earlier, there are two reasons why we are having the standard spectrum, one is you select a standard spectrum, from which you can stimulate a time series, and the method is usually refer to as Inverse Fast Fourier Transformation, I have already mention this to some extend. Then this can drive a way maker, in the tank is that clear?, but proper calibration etcetera has to be done proper trans function has to be established, all those things are not going to go into the details, that needs additional study.

There are number of reference books, I can suggest the book by Goda, which is referred in the in the lecture material, so this gives a very good method, and there are other relevant papers which you can, see how this can be done. Because, this is quite vital for the simple reason, if you want to test a structure it is response it is response to a particular sea state, I can model it and also, if I am interested generating almost the similar kind of a spectrum, I can still do it or if someone has proved that it follows a Scott spectrum.

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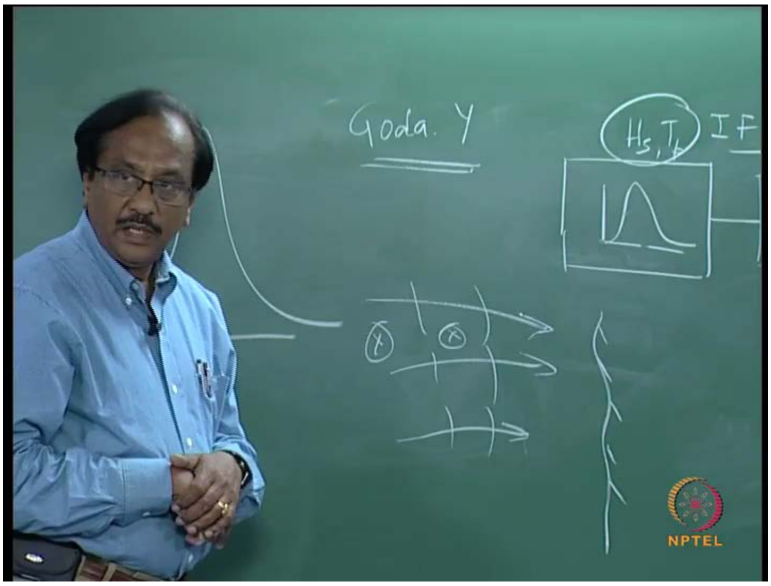

I can use this methodology, generate this is the spectrum with a particular H s and T p given in the field, I will apply a model scale, and simulate the same kind of time history, and then subject it to the waves and then measured the motions etcetera, may be the more in forces there are, so many things pressures forces etcetera, so this is one aspect. So, you go from a standard spectra simulate and then generate, the other thing is (No audio from 33:54 to 34:06), you have waves coming from the ocean, I have measured the waves here and I have measured the waves somewhere here etcetera.

Along the location, I want to what do you do with this, I draw the time history this is what you have measured, so I use this fast fourier transformation, to get my spectral density. Here, also when you have measured the motion responses etcetera, I use the time history of the motion etcetera, to I adopt a fast fourier transformation to get the spectral density.

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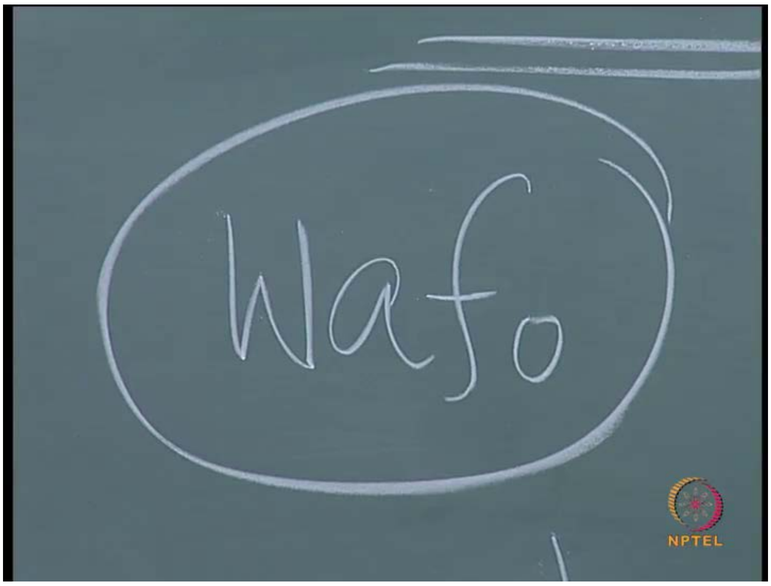

I would suggest wafo, important useful tool in the mat lab, which can be use for drawing the spherical density, this also uses the Inverse FFT am I right?, so I suggest you look into this tool, to understand more about this information. Now, so far so good, we are seen all this things using a single parameter spectrum.

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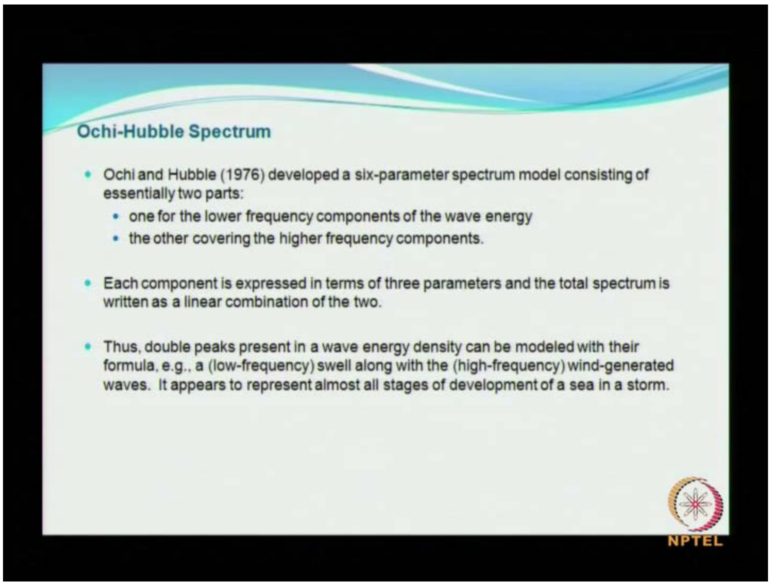

Other any spectral models for two parameter, two pics spectrum Ochi-Hubble, develop the sixth parameter spectrum, they develop a sixth parameter spectrum, consisting of essentially of two parts, one for the lower frequency component of the energy, and other covering higher frequency components. Each components is now expressed, in terms of three parameters and the total spectrum is written as a linear combination of two, so each spectrum will have three parameters, so they both will be linearly added, so that would be that it is a sixth parameter spectrum.

So, thus double pics present in the wave energy, density can be model with with a formula that is example, a low frequency its well, along with a high frequency wind generated waves. Please recollect our my lecture on wind generated waves how the waves are generated etcetera, now it appears to represent almost all stages of development of sea in a storm.

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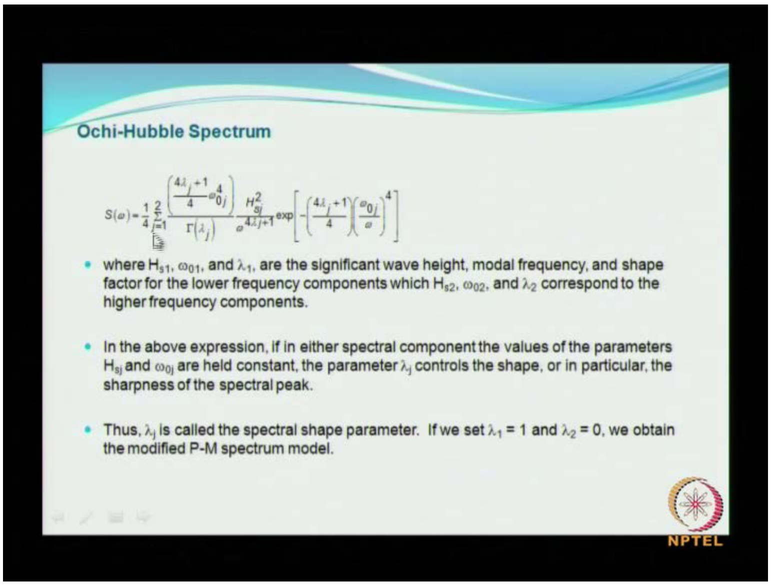

The spectral density here is double submission as you can see here, 1 by 4 then there is gamma j omega naught you already know, and then you have gamma j where H s 1, this will have H s 1 for this swell spectrum or for the H s 2 for the wind spectrum wind dominate spectrum.

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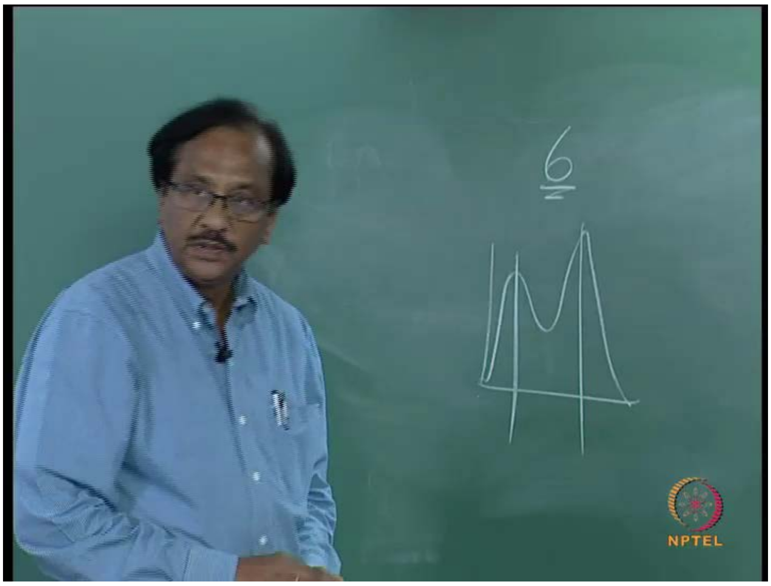

Then here similarly, here omega naught you will have two value, one for the low frequency zone and another for the high frequency zone, so when you have. So, this will be a f naught and this will be this is f naught 1 and f naught 2, Is that clear?. So, H s 1, alpha 1 are the significant wave height model frequency, and the shape factor, so gamma 1 and lambda 1 and lambda 2 are the shape factors.

So, in the upper wave expression, if in either the spectral components, the values of the parameters H s j or held constant, that is the significant wave height or the and the peak frequency or the held constant. The parameter lambda j will naturally control the shape, or the or in particular the sharpness of the spectral peak, we have already seen this lambda the effect of lambda, in the case of a JONSWAP spectrum, is that clear.

So, now, you see that, this will be called as the spectral shape parameter, and if we set alpha 1 equal to 1 lambda 1 equal to 1, and lambda 2 is equal to 0, we obtain a kind of a modified spectral Pierson Moskowitz spectrum model, Is that clear? (Refer Slide Time: 39:20).

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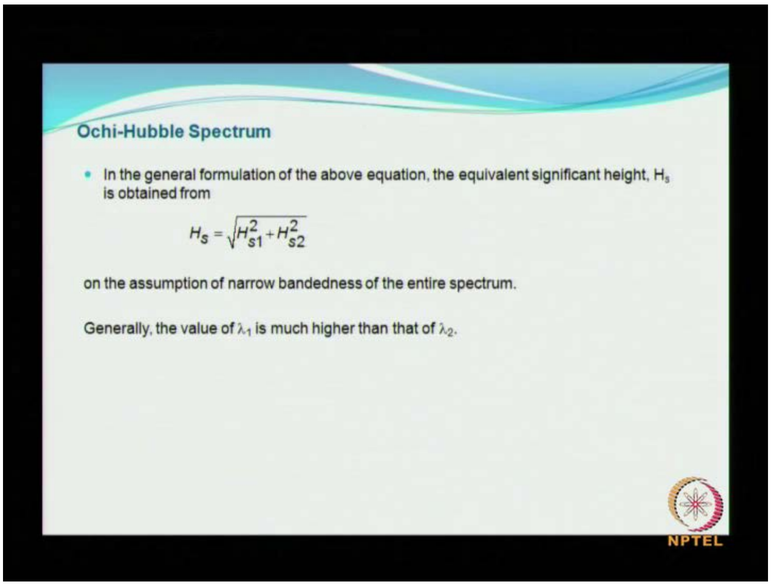

I think we are almost done, the here and the in the general formulation of the above equation, the equivalent talking about an equivalent H s, because it is a linear summation right, you can use it as H s square H s 1 square plus H s H s 2 square whole whole, I mean the square root, Is that ok?. So, generally the lambda 1 is a much higher than, that of lambda 2, I think I will stop here.