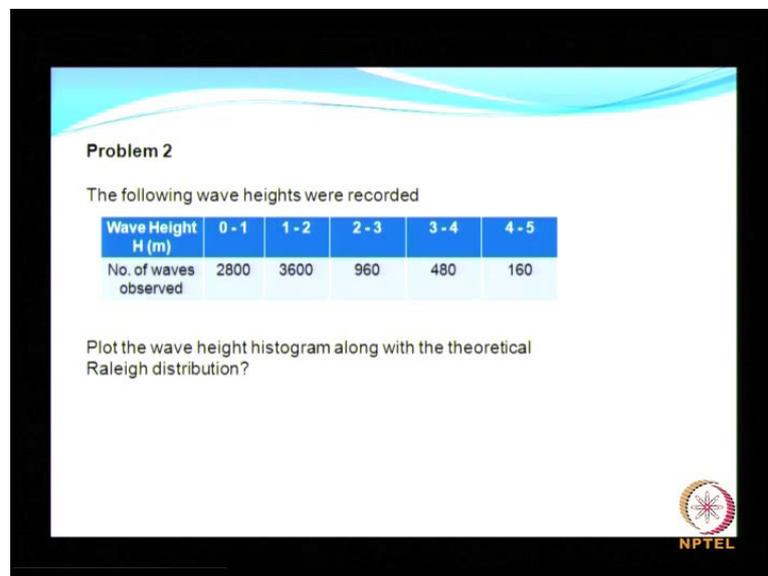


Wave Hydro Dynamics
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Random and Directional Waves
Lecture No. # 03
Random Waves and Problems II

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Problem 2

The following wave heights were recorded

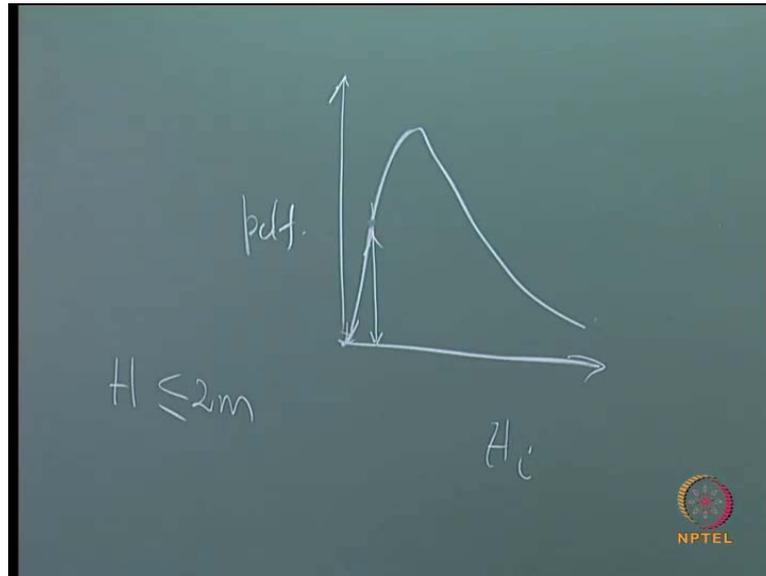
Wave Height H (m)	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
No. of waves observed	2800	3600	960	480	160

Plot the wave height histogram along with the theoretical Rayleigh distribution?



The probabilistic domain and also we will try to understand, how the Rayleigh distribution is going to help us in describing, what is called as the short term statistics. When we say short term statistics, we talk about the probability of the occurrence of a particular wave height or occurrence of a particular a wave height less than or equal to a certain specific wave height.

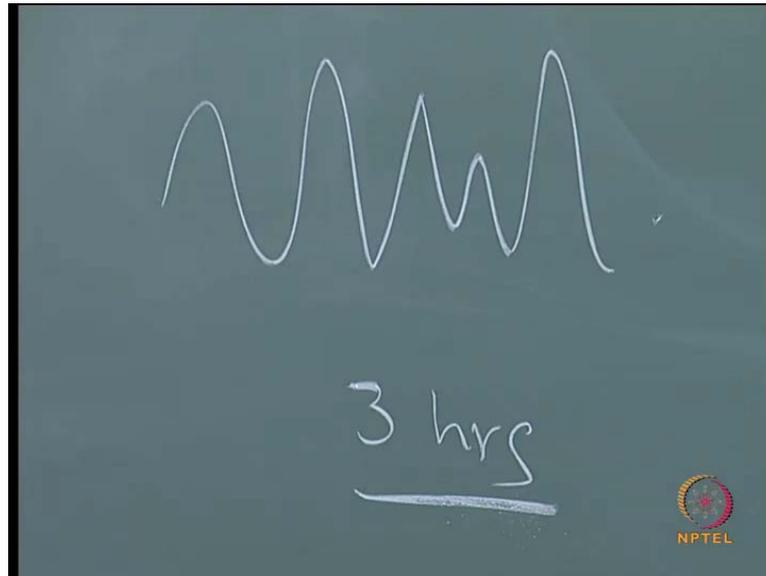
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As I have told you earlier the **probabilistic** the probability density look something like this, this will be your H_i . So, when I want to find out what is the probability of wave height less than or equal to say a specific wave height or say 1.5 or 2 meters, H less than or equal to 2 meters. Then, I can easily look at this curve and then try to obtain, what is the probability of wave height less than or equal to a specific value in this case, in this example it is 2 meters.

So, this is particularly helpful in planning of construction and also in order to assess the functionality of certain structures etcetera. Now, for short term statistics for describing the short term statistics of a particular location, short term statistics of waves of a particular location it is good enough you have wave characteristics for spread for about an year.

(Refer Slide Time: 02:12)



So, if you have measured wave characteristics remember we, when we record the standard practice is to record for once in 3 hours; that means, in a day you have about you will have 8 records. So, if you are looking for describing of describing the wave climate for a given location, it is good enough to have just 1 year record that will give you a clear indication of the variation of wave climate.

We now, let us have look at this problem, where in the range of wave height and the number of waves that has been observed in this each of these class are given here now here. So, 0 to 1 meter is less than 1 meter equal to 2800 times number of occurrences then 1 to 2 meters 3600 then 2 to 3 is 960 then, 3 to 4 is 480 and then the final one is now 160.

The custom is plot the wave height histogram along with a theoretical Rayleigh distribution, it can be either weibull or Rayleigh or any other theoretical distribution which we have already seen, but here in this example I just considered Rayleigh distribution because that is a distribution, which is most frequently adopted for describing the wave wave climate.

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The histogram data is shown in table given below

Wave Ht (m)	Mean of H _i / H	No. of waves	% occurrence = col(3)/sum x 100	% of occurrence per m of Wave Ht from record = col(4) / ΔH
1	2	3	4	5
0 - 1	0.5	2800	35	35
1 - 2	1.5	3600	45	45
2 - 3	2.5	960	12	12
3 - 4	3.5	480	6	6
4 - 5	4.5	160	2	2

Sum = 8000

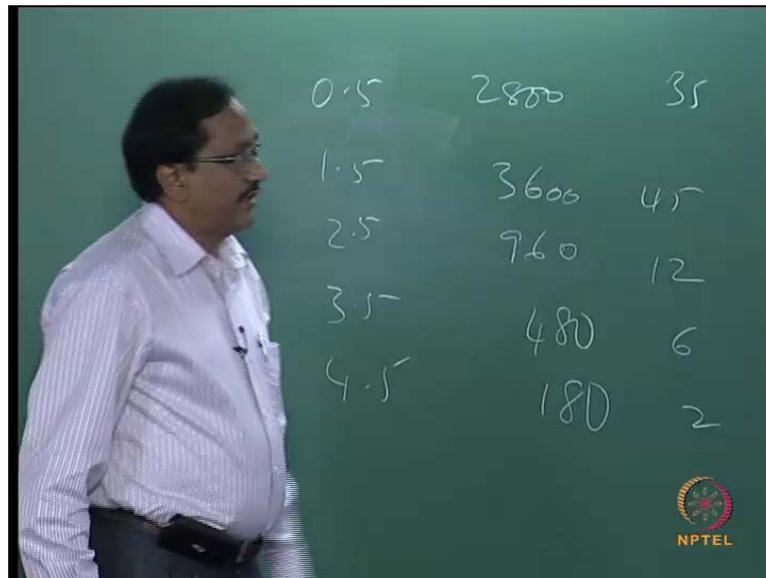
Note: The values of col (5) is obtained by dividing col (4) by 1 which is interval of H.



So, in the first column see the one which is not there in red colour is the column number and you have on in the first column, wave height ranging from a 0 to 1 and going all the way to 4 to 5 class interval is now, 1 meter in this case. So, what is the mean wave height mean of H_i by H or H is here in this case, the mean value will be 0.5, 1.5 etcetera. For this each of this class interval the number of occurrences, I have reproduce as it is given in the problem.

Now, percentage of occurrence, percentage of occurrence is quite straight forward you just have to take the column 3 and then divided by the sum. So, that is going to be 35, 45, 12 etcetera. As given in the column 4. Now, last column is the percentage of occurrence per meter of wave height from the record. So, that is that is quite simple because you have to just multiply, the percentage of occurrence by the class interval. So, in this case in the present case, the class interval is just one. So, you need not have to do anything. So, the same value will repeat here. So, I have also indicated, I have also mentioned here about a how this column 5 is obtained.

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Now, having got this, we have 0.5, 1.5, 2.5, 3.5 and 4.5 the number of occurrences is are 2800, 3600, 960, 480 then finally, 180. So, these are the values. Now, I will also write the last column which is 35, 45, 12, 6 and 2.

(Refer Slide Time: 06:05)

Rayleigh Distribution (Sample Calculation):-

$$H_{RMS}^2 = \frac{\sum H_i^2 f_i}{\sum f_i} = \frac{[0.5^2 \times 2800 + 1.5^2 \times 3600 + 2.5^2 \times 960 + 3.5^2 \times 480 + 4.5^2 \times 180]}{8000}$$

$$H_{RMS}^2 = 2.99m^2$$

$$H_{RMS} = 1.73m$$

Now according to Rayleigh Distribution

$$p(H) = \frac{2H}{H_{RMS}^2} \cdot \exp\left[-\frac{H^2}{H_{RMS}^2}\right]$$

$$p(H = 0.5m) = \frac{2 \times 0.5}{2.99} e^{-\left[\frac{0.5^2}{2.99}\right]} = 0.307 = 30.7\%$$

So, we need to first find out the H RMS value because we can either represent your even represent, the Rayleigh distribution as either a function of H RMS or H mean. So, here H RMS can be easily determined, by a formula H_i^2 into frequency of occurrence divided by the summation. So, here H_i , I am taking the the mean value is 0.5 and the

number of occurrences is 2800. The next one was a 1.5 and the corresponding value number of occurrences are given here, use that and divide by the total sum and that is going to be your H rma H RMS square.

So, H RMS in this case is 1.73 meters is that clear. So, this is a just a basic a statistics only thing is after doing after doing, the calculation using you need to understand why and where all these information are necessary. So, there are expression for the Rayleigh distribution is already given here. So, here it is $2 H_i$ divided by H RMS square into exponential of H_i negative of a H_i square divided by H RMS square. So, I can find out the probability of occurrence of wave height equal to 0.5, 1.5, 2.5 etcetera. So, this is a sample done sample calculation done for H_i equal to 0.5. So, substitute here, here it will have you will have the same H_i and here also you will have the H i and then you have your H RMS square, which is 2.99. So, that is going around equal to 0.307 which is good going to about 31 percent.

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Similarly

$p(H=1.5) = 0.473$	in terms of % = 47.3
$p(H=2.5) = 0.207$	in terms of % = 20.7
$p(H=3.5) = 0.0389$	in terms of % = 3.89
$p(H=4.5) = 0.0034$	in terms of % = 0.34

Note: The total area under the Rayleigh distribution curve should be equal to 1.
(i.e, the total probability should be 1)

Area from $H = 0.5$ to $H_i = 4.5$ (Simpson rule)

$$= (1/3) h \times [1^{st} \text{ ordinate} + \text{last ordinate} + 2(\text{odd ordinates}) + 4(\text{even ordinates})]$$

$h = \text{interval}$

$$\text{Area (from } H = 0.5 \text{ to } 4.5) = \frac{1}{3} \times 1 [(0.307 \times 1) + (0.473 \times 4) + (0.206 \times 2) + (0.0389 \times 4) + (0.0034 \times 1)]$$

$$= 0.923$$

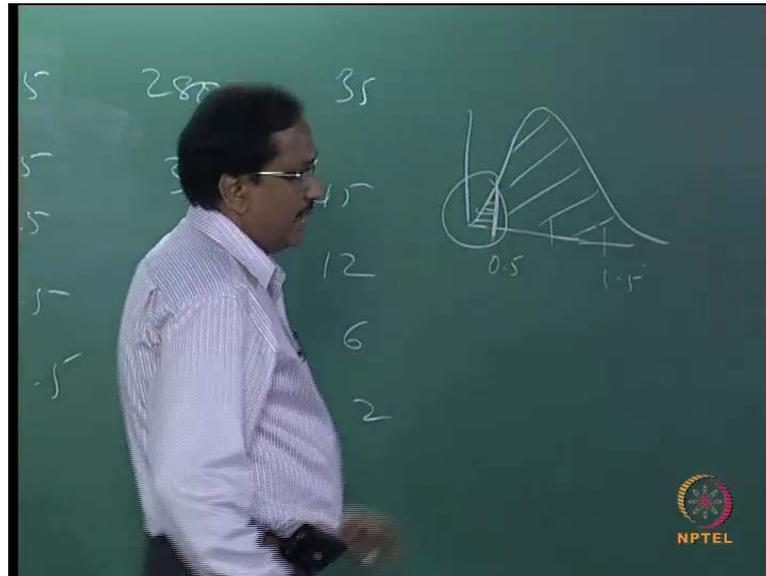
$$\text{Area (from } H = 0 \text{ to } 0.5) = \frac{1}{2} \times 0.5 \times 0.307 = 0.07675$$

Total area = $0.923 + 0.07675 \cong 1.0$



So, in the similar way you calculate for the probability of wave height equal to 1.5 meters, 2 meters, 2.5 meters etcetera and these values are provided here in the same way, as we have done for the probability of H_i , H equal to 0.5. What is the total probability? So, when i plot this when i plot 0.5, 1 etcetera and there 1.5 etcetera.

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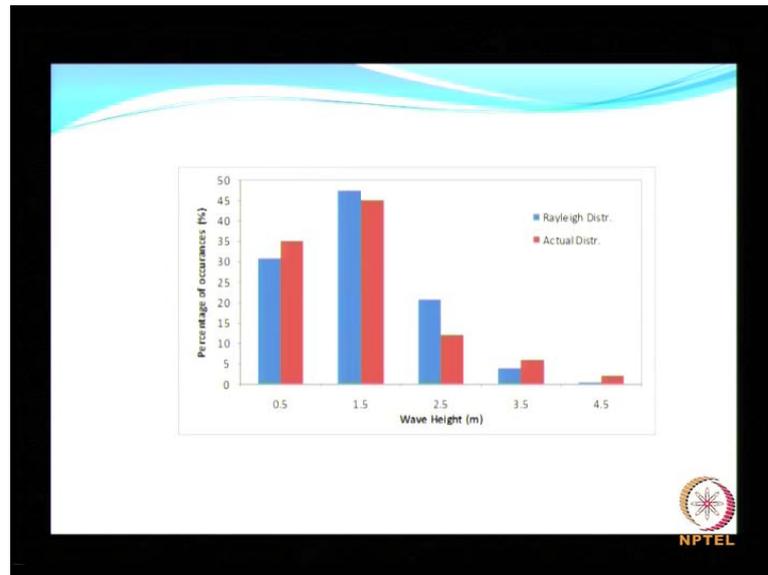


Then I get a curve like this and this is what this is nothing, but the probability density function. So, we all know that when you integrate, this this is called as the cumulative probability density function that is of course, it has to be equal to 1. So, what we do is we take the area from H . H equal to 0.5 to H_i is equal to 4.5. So, that is somewhere here, we are leaving this note that we are leaving this area. So, from here all the way down to 4.5 then we get you see Simpson rule I do not want to go into the details of the Simpson rule, but here it is shown how the Simpson rule is working out, that is first ordinate plus last ordinate plus 2 into the all ordinate then plus 4 into the even ordinate.

So, you have to find out the odd and even ordinate, within the summation area and H is the delta class interval is that clear you have any doubts. So, this is the basic mathematics. So, once you do that that is going to be 0.923. So, what we have done basically is we have taken only up to this area, but we are we have to also consider this area, that is this particular area and that is going to be just half into 0.5 into the probability of 0.5. Which we have estimated earlier that is going to be 0.307.

So, the total value will be this much and this plus this should be equal to 1. So, that plus the total area equal to 1 that is the total area under the probability density curve. So, we can plot all these values as we have got these values here. So, for 0.5, point 1.5 earlier slide then 1.5 all these values.

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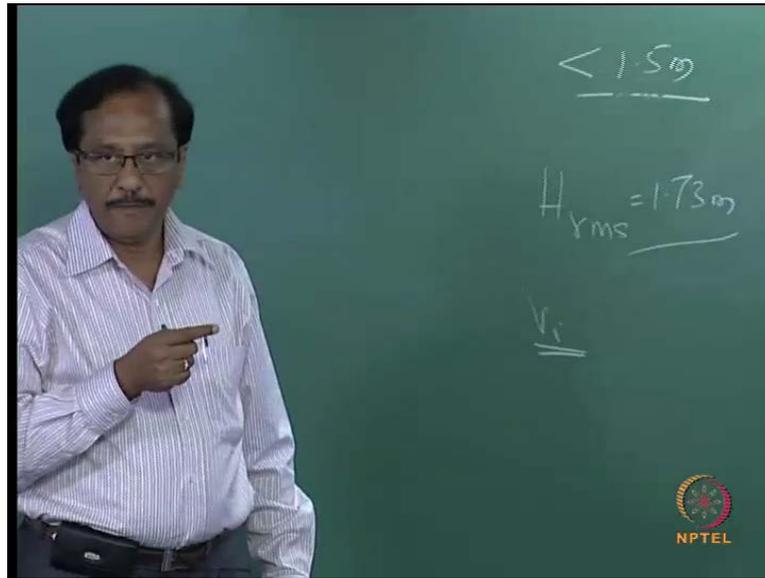


So, I can plot these values here in which and also use this. So, this values we have already got this is nothing, but here observed probability values that is for the measured data or the observed data we have got the probability density. Now, from this what did we determine, we determine the H RMS which is going to control the variation of your probability density curve and once we have done that we calculated for each of this value using the theoretical probability density. So, we have the observed and we have the theoretical and then we are now superposing.

So, the blue one shows the Rayleigh distribution, that is the theoretical Rayleigh distribution and the red one shows approx the observed probability density. So, you see that the values are more or less satisfying that is this shows that, the wave height as per these observations follow closely the Rayleigh distribution.

So, I will not go into the details of other kinds of fitting etcetera. I am just demonstrating how this examples can be adopted and how the application of this Rayleigh distribution. Now, you see that once you for example, you have a you know that all of of say east coast a certain part of the east coast of India follows, the Rayleigh distribution say for example, of Vishakhapatnam where you have a major harbor.

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So, say that it has been proved that the wave heights follow Rayleigh distribution. I can simply say that once, if someone has a is able to say that of vizag or some any any coast for that matter, it follows a Rayleigh distribution throughout the year and for month January, the H RMS is 1.25 then for month of june it is 1.75 for august it is around 1.8 etcetera. Then for each month I have I can simply rely on the Rayleigh distribution, which is. So, easy for me to calculate because it is a simple equation, where you can just use a H RMS value and obtain the probability of any kind of a wave height.

How this is useful for example, I have to carry out a specific operations. I want to mobilize and demobilize off shore in order to carry out some off shore operations, I have to mobilize and demobilize materials, then this has to be transported by maybe some barges or maybe some vessels. All this vessels will be having certain limiting wave height up to which it can really move around without much of problem.

Say assume that vessel has a limiting wave height, I mean the wave height. If the vessel is can operate only if the wave height is less than 1.5 meters and off shore operations it is very difficult, very expensive. Once you mobilize for mobilizing or even demobilizing its very, very expensive. So, a proper planning is needed. Now, he has a vessel you have a vessel, which has a operating limit that it cannot work, when the environment has a wave height greater than 1.5 meters.

Then I can easily carry out an analysis based on this and if I am lucky and if I am having a someone, who has proved that the wave height wave characteristics follow a Rayleigh distribution. I need to just have a calculator and a with the H RMS value or with wave height mean wave height value, I can easily get the Rayleigh distribution and obtain, what is the probability of wave height less than or equal to 1.5 meters, in the month of January in the month of February in the month of march etcetera.

So, I can draw the monthly probability distribution for for that location. What you get of the out of that? Once you do that you can get this helps you in planning for example, if the there is a one particular month, where the wave climate is going to be very severe for example, in the month of may and you have a probability of probability of wave height less than or equal to a 1.5 is say around 40 percent, then you have to take care of the risk involved in that within the 60 percent. So, whether is it worth while taking the risk all these kind of information is helpful in the planning stage and also we have to occurred, what is called as a whether window.

What is meant by whether window? If you want to carry out any off shore operations you have to have a continuous time because if you have continuously, about a 6 months then you will been a position to carry out the operations, suppose because if that particular task needs 6 months, then you have to search for that whether window because every time you cannot have 2 months, 3 months, 2 months, 3 months then, you are investing a lot of money on mobilizing and demobilizing your machine machines. So, in such way, it is very,very useful this kind of a calculations is that clear.

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From its definition \bar{H} or H_{RMS} being average over the entire area of the sea, should very closely represent the average energy of the sea way, i.e.,

$$\frac{\rho g \bar{H}}{8} = \frac{\rho g}{8} \left[\frac{H_1^2 + H_2^2 + \dots + H_n^2}{n} \right] = \text{ENERGY}$$

∴ If the area under the histogram curve is known, one can directly relate \bar{H} or H_{RMS} to Rayleigh distribution formula and determine from this the probability of occurrence of different wave heights. Thus the probability that $H > H_1$ is

$$p[H > H_1] = 1 - \int_0^{H_1} \frac{2H}{H_{RMS}^2} \cdot e^{-H^2/H_{RMS}^2} dH \quad \leftarrow \text{Probability of exceedence}$$

For eg. If $H_1 = 3\text{m}$ where $H_{RMS}^2 = 2.99 \text{ m}^2$

Hence, $p = e^{-(3^2/2.99)} = 0.05$



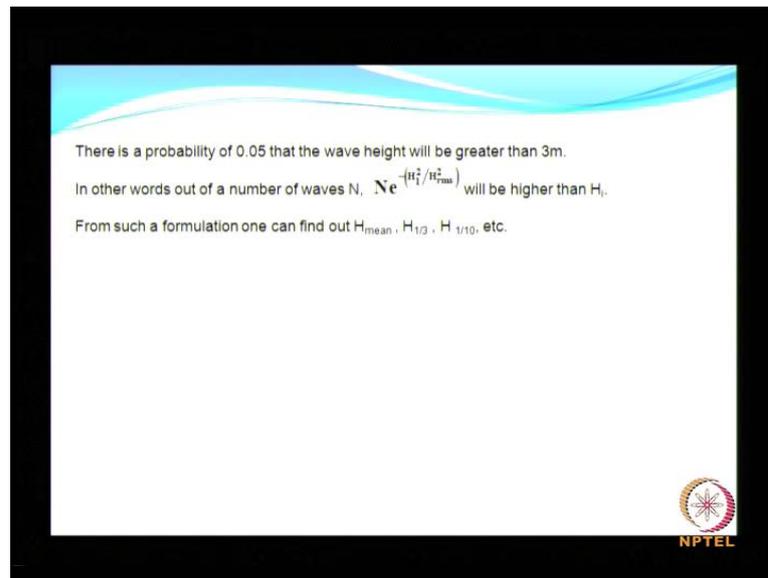
Now, what we have seen is the percentage of the probability of wave height less than or equal to a given specific value and that is what it gives the that is what is got from a probability density curve. From its definition H_{RMS} being added over the entire energy, you can also should you can also very closely represent, the average wave energy that is the right. Now, if the area of the area under the histogram curve is known, one can directly get the \bar{H} or H_{RMS} etcetera, using the Rayleigh distribution formula and we can also determine from this the probability of occurrence of different wave height that is what, I have been explaining here.

So, next one is the probability density the other thing is thus, if I want a wave height the probability of wave height greater than a specific value. So, what you get from this is a wave height less than or equal to a specific value. Now, very often you will be talking in what is known as probability of exceedence.

Probability of exceedence is nothing, but what is the probability of wave height greater than a specific value in this case, in this example. What is a probability of wave height? Greater than 1.5 meters. So, as long as that probability of exceedence is less, than the percentage of risk involved in carrying out your operation with a limited limited conditions of your vessels is what? What if? This if the probability of exceedence is very less probability of exceedence is very less means you are safe.

So, this kind of information can be obtained from this kind of analysis. So, if H_i is 3 meters and H_{RMS} equal to 3 meters, so much i get this probability of exceedence equal to is the equations, you get the probability of exceedence as equal to 0.05. What does that mean? H_i is equal to 3 meters and the probability of exceedence is 0.05.

(Refer Slide Time: 19:59)



So; that means, there is possibility probability of 0.05 or 5 percent, that wave height will be greater than 3 meters. So, off shore operation 5 percent risk is not much, you can easily digest. So, this kind of information can be obtained from the probability of exceedence in other words, out of a number of waves N . N into that value exponential of minus H_i square divided by H_{RMS} square, will be higher than H_i . So, when you have this kind of a formulation, where it is very easy for us to calculate the other wave characteristics, like your H_{mean} or $H_{1/3}$ or $H_{1/10}$ etcetera. So, is there any question.

(Refer Slide Time: 21:17)

Relation between Representative Wave Heights:

If we adopt Rayleigh distribution as an approximation to the distribution of individual wave heights, according to Longuet Higgins (1952)

$$H_{1/10} = 1.27H_{1/3} = 2.03H, H_{1/3} = 1.60H$$

Or

$$\left. \begin{aligned} \bar{H} &= 0.885H_{RMS}, H_{1/3} = 1.416H_{RMS}, \\ H_{1/10} &= 1.8H_{RMS}, H_{max} = 2.172H_{RMS} \end{aligned} \right\} A$$

These results represent the mean values of wave records taken together. Individual wave records containing less than 100 waves may give noticeable departures from these mean relation.

The above relations are valid for narrow band spectrum.

For a broad band spectrum Cartwright and Longuet Higgins (1956) suggested that Eq (A) be multiplied by $(1 - \epsilon^2)^{1/2}$, where

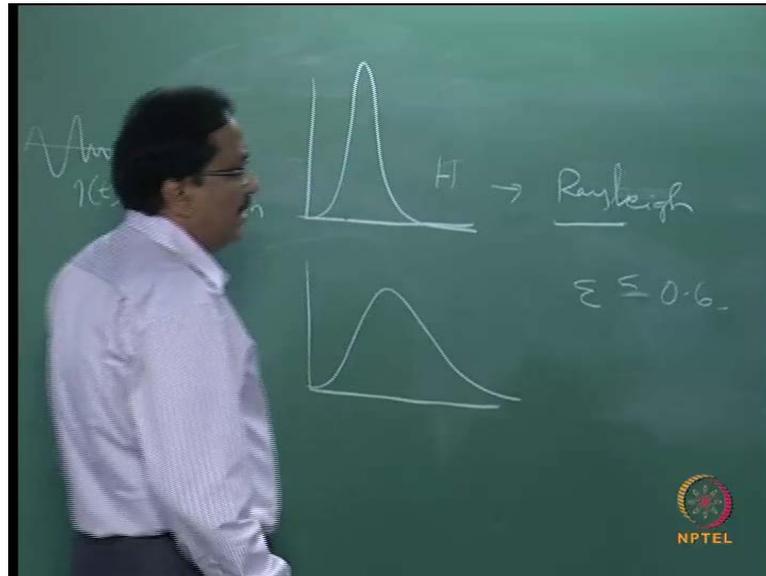
$$\epsilon = \left[1 - \frac{[T_c]}{[T_i]} \right]^{1/2}$$


So, if we say that of a sight the wave height follows a Rayleigh distribution. It was Longuet Higgins who did a considerable amount of work in this. A area where he try to prove or he try to see how the waves follow, the Rayleigh distribution and also try to establish some a relationship between the different kinds of a wave characteristics.

For example $H_{1/10}$ is equal to 1.27 into $H_{1/3}$ or that is equal to 2.2 times, almost twice the mean wave height as you can see here or the significant wave height is approximately 1.6 times, the average wave height or you have the other kinds of variations or relationships like \bar{H} that is a mean wave height is 0.9 approximately, 0.9 times the H_{RMS} and the significant wave height is 1.4 times H_{RMS} and then all these values are in terms of H_{RMS} .

So, I have just given a small comment here from Longuet Higgins this is (()) what it says, that this results represent the mean values of wave records taken together for individual, wave records containing less than 100 waves may give noticeable departure from this mean relationship this I have already indicated in the earlier lecture, where in I said that when you want to analyze a wave record, you should have a minimum of 100 waves. So, all the above relationship are valid for a narrow band spectrum.

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See usually, when. What you mean by a narrow band spectrum? Narrow band spectrum looks like this and a broad band spectrum may be looking like this that is here energy is concentrated within a narrow band frequencies whereas, your energy is spread over a wide range of frequencies. So, from a given wave elevation, if you say that it is a narrowband spectrum it usually, it will follow the η will follow a Gaussian distribution and a wave height will follow a Rayleigh distribution.

If η is a narrow band process, it will follow Gaussian process or if the wave elevation follows Gaussian its vice versa and then you can also say that it follows a you can see that it follows a Rayleigh distribution. The general guideline whether it is a narrow band or broad band, if it is anywhere between a ϵ that is the spectral with parameter is less than 0.6 less than 0.6. I would say then we say that it should be expected that the wave elevation follows a narrow band spectrum.

So, all these above relationships of the different wave characteristics having obtained by Longuet-Higgins and the assumption that it is a narrow band process, but the wave elevation need not have to follow always, the narrow band spectrum. So, in case you come across waves that are following a broad band spectrum, then you need to have some kind of a correction factor and this correction factor was introduced by Cartwright and Longuet-Higgins and that is suggested by multiplying.

You multiply all these values by a correction factor which is 1 minus epsilon square, epsilon is the spectral length parameter and the whole thing is a half I mean the square root of 1 minus epsilon square.

So, this correction factor has to be used and the spectral parameter here, I have provided the relationship with the up cross period and the 0 crossing period. So, which can be obtained using the spectral moments or by the statistical method. Herein when you are talking about random process etcetera. It is always better to use the spectral method, but there is no harm in using the statistical method. So, we have obtained we have seen the different wave characteristic now, what is that we will do next.

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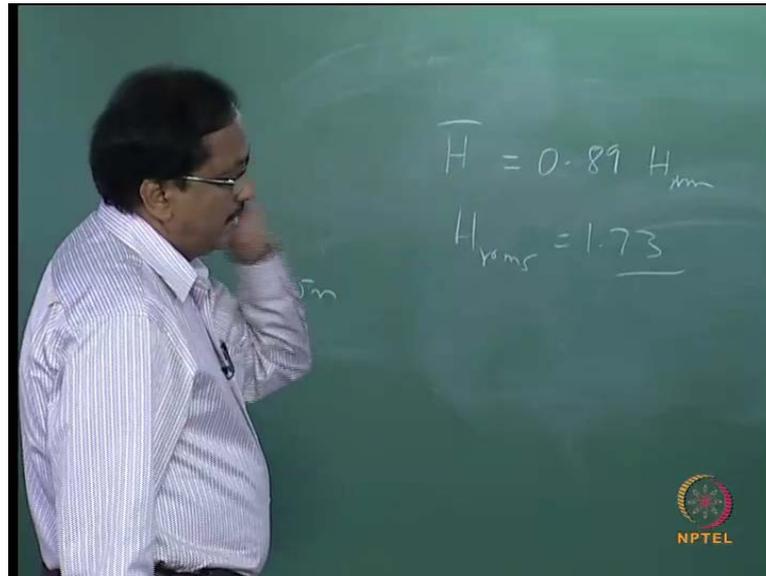
Problem 3
 Compute the wave characteristics and compare with that of prediction formula

Wave Ht (1)	No. of Waves (2)	Cumm. No. of Waves (3)	Number x Ht = (1) x (2) (4)
0.5	2800	2800	1400
1.5	3600	6400	5400
2.5	960	7360	2400
3.5	480	7840	1680
4.5	160	8000	720
	8000		11600

$$\bar{H} = \frac{11600}{8000} = 1.45\text{m}$$


Next, again we try to see if we can we are able to compute the wave characteristic and compare with that of the predicted formula that is formulas which we have given. So, we are taking the same, I mean the data and the same the values are the same as we have seen in the problem.

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And now, for which we have calculated ah H bar as equal to 1.45 meters. 1.45 meters this is easily obtained next is we use the Rayleigh distribution. Rayleigh distribution says what Rayleigh distribution says, this is equal to 0.89 into H RMS square, but the same problem we have calculated earlier and this H RMS square H RMS is 1.73 this H RMS.

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According to Rayleigh Distribution, $\bar{H} = 0.89 H_{RMS}$

H_{RMS} for this problem = 1.73m

$\bar{H} = 0.89 \times 1.73 = 1.54\text{m}$

$$\bar{H} = \frac{\sum H_i f_i}{\sum f_i}$$
$$\bar{H} = \frac{(2800 \times 0.5) + (3600 \times 1.5) + (960 \times 2.5) + (480 \times 3.5) + (160 \times 4.5)}{8000}$$

$\bar{H} = 1.45\text{m}$

It is seen that the theoretical Rayleigh distribution fits well with the wave data

$H_{max} = (1.6 \text{ to } 2.0) H_{1/2}$.

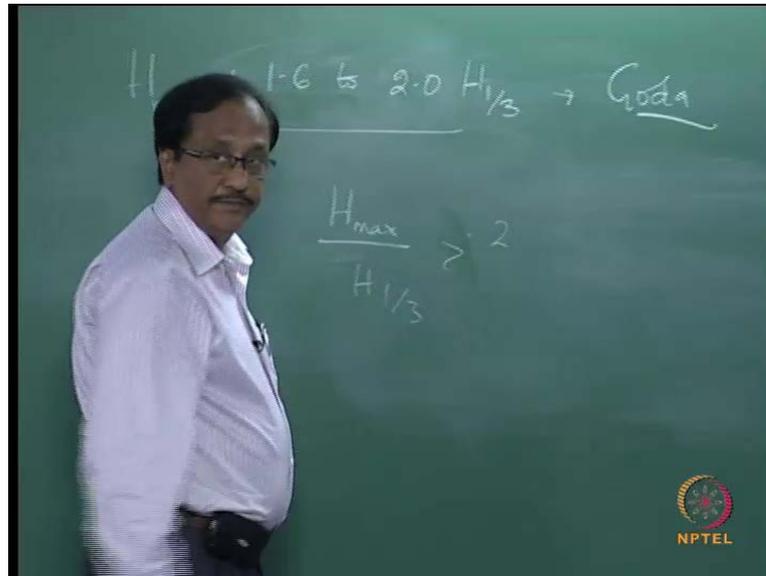
In the design of offshore structures, $H_{max} = 2H_{1/2}$ or a higher value is often employed.

For the design of a breakwater Goda has proposed the use of the relation $H_{max} = 1.8 H_{1/2}$

So, if I calculate by I mean H bar from this, I get 1.54 using this relationship because since I have already calculated my H RMS this is going to be about 1.54 meters, but by the usual calculation you get the H r, H, h bar as 1.45 that is the order of difference ok.

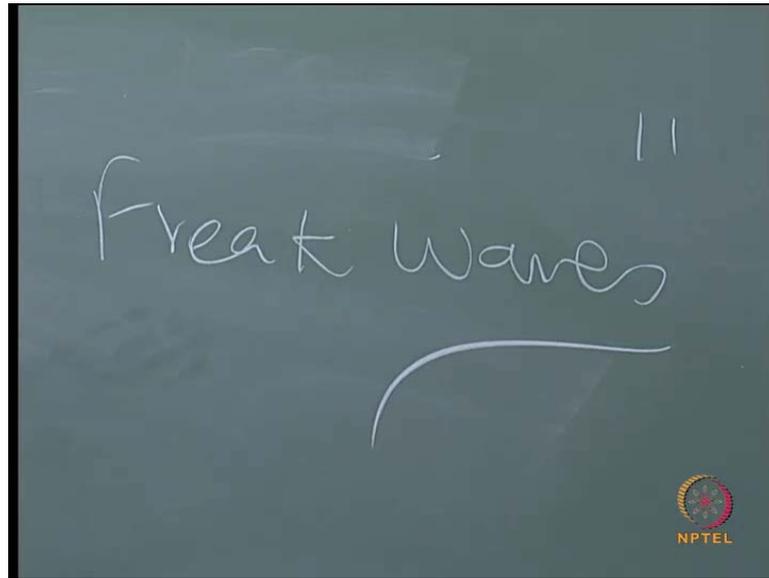
So, it is seen that the Rayleigh distribution fits well in general the wave data in general, but there are some conditions like narrow band process etcetera, etcetera which I have already explained.

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So, if it follows a Rayleigh distribution, we say that H_{max} is approximately equal to this is actually given by Goda in his a beautiful book on random waves. So, I would strongly suggest for the students to go through the book written by random seas on maritime structures. I think the reference is given at the end of the text. So, that will give you all kinds of information including this Rayleigh distribution etcetera then. So, this is H_{max} can be safely taken as. So, you see that H_{max} divided by $H_{1/3}$ should always be, should always be less than 2 for an ideal system or in general, you say that if H_{max} by $H_{1/3}$ not if you say it it has to be mostly less than 2. Very rarely it exceeds it, exceeds 2 very rarely and in the event it exceeds.

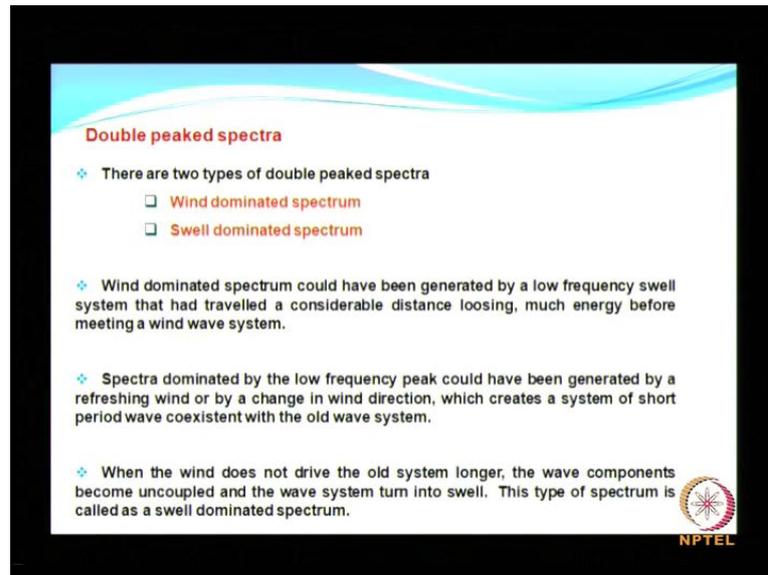
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Then the waves are called as freak waves, freak waves. So, this freak waves has a lot of energy and they can really lead to some demonstrating effects and the existence of freak waves have improved in several coast particularly, in the north sea and several other seas. So, you need to look for the word freak waves over the net. If you want to have further information on this topic its quite interesting on about these freak waves.

So, in the design of off shore structures usually H_{max} is taken as thrice the $H_{1/3}$ or slightly higher values often employed and this depends on locations sometimes say for example, in locations where you have a where it has been proved the existence of freak waves have been proved has been proved, then they have to go in for a value higher than 2. For the design of freak waters the Goda has suggested that the H_{max} can be 1.8 times the significant wave height is that clear.

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Double peaked spectra

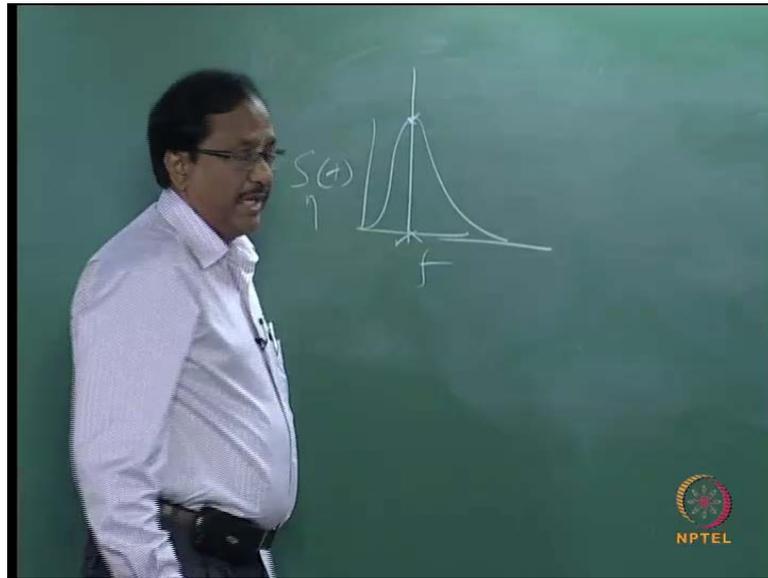
- ❖ There are two types of double peaked spectra
 - ❑ Wind dominated spectrum
 - ❑ Swell dominated spectrum
- ❖ Wind dominated spectrum could have been generated by a low frequency swell system that had travelled a considerable distance losing much energy before meeting a wind wave system.
- ❖ Spectra dominated by the low frequency peak could have been generated by a refreshing wind or by a change in wind direction, which creates a system of short period wave coexistent with the old wave system.
- ❖ When the wind does not drive the old system longer, the wave components become uncoupled and the wave system turn into swell. This type of spectrum is called as a swell dominated spectrum.



So, we have seen this statistical method, we have seen the spectral method and in this statistical method, we have what we have seen is the following of whether, the wave heights a normal distribution whether, the wave elevation follows a normal distribution or the wave heights follows a Rayleigh distribution and in what way it is useful to the field etcetera all these things have been discussed.

Now, we get back to the spectrum in order to understand more about spectrum. So, initially we started with statistical analysis, we understood we try to understand all the different methods of statistical analysis. Then we took up some, some examples and found out its usefulness in the field, the same way we will try to repeat for spectrum.

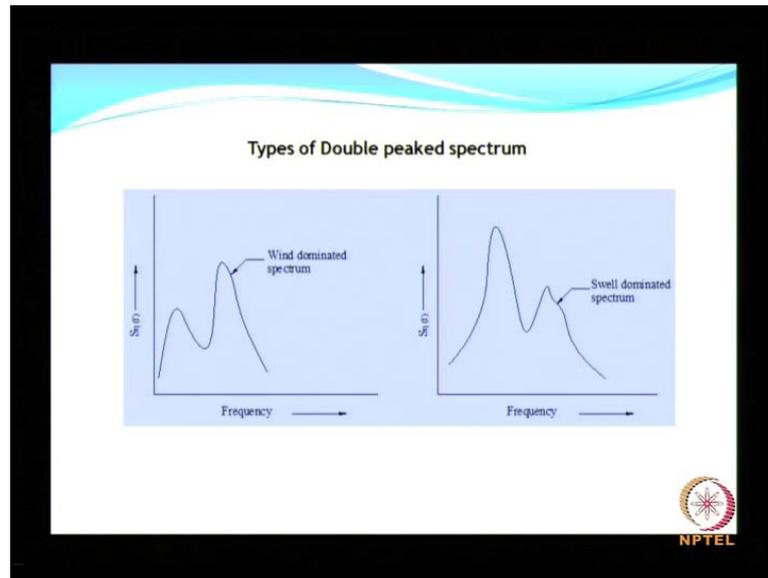
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There are two types, in the case of we have been dealing with I have already shown that the spectrum looks a single mostly, the spectrum I have already indicated what is the spectrum etcetera usually, the spectrum is a single peak spectrum. The energy is concentrated around a single frequency I mean the peak energy. Now, there are two types of double peak spectrum you can have multiplied peak or double peak. So, there are broadly there are two types of spectrum one is the wind dominated spectrum and the swell dominated spectrum. The wind dominated spectrum could have been generated by a low frequency swell system, that is low frequency swell means, swell is outside the phase area please recollect my lecture on the generation of ocean waves.

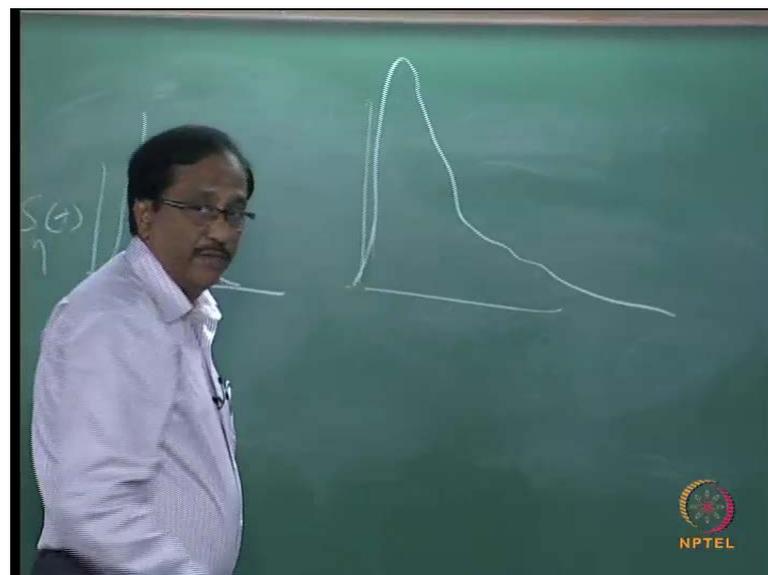
So, How can be generated by a low frequency? Swell system that had travelled a considerable distance losing much energy before meeting a wind wave system So, under the wind wave system, the energy between the wind the energy from the wind is being transferred to the growth of waves. The other one is the swell is already left I mean the fetch area. Spectra dominated by low frequencies peak low frequency components could have been generated by a refreshing wind or by a change in the wind direction, which creates a system of short period waves coexistence in a with a old wave system. So, now when the wind does not drive the old system longer, the wave components become uncoupled and the wave system turns into a swell.

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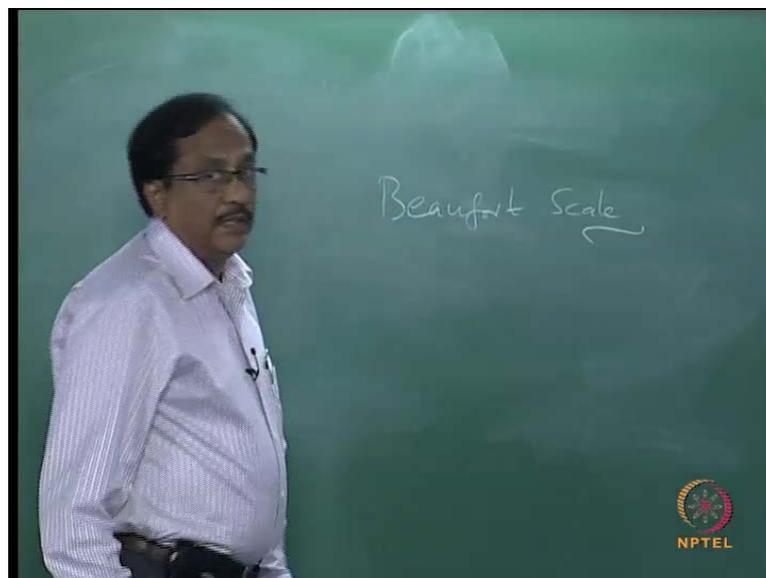
So, when you have a swell dominated spectrum you can see like this, when you draw the spectrum for a given location and if you have a spectrum on the left hand side similar to what you have on the left hand side. Then you have a low frequency component from the I mean the energy from the low frequency components is less. And then in that case you see that this is dominated by the winds, but whereas, if you have a low frequency component dominating the wave I mean, the spectral density curve then it is called as the swell dominated spectrum.

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So, when you do the analysis you may have something like this, there can be some tidal component also included in your spectrum. You can also separate the tidal component from the. So, when you say tidal component tidal component is only within the low frequency, very low frequency you can have a very long period. When I say low frequency it is a very long period which is, which is included in the wave. Wave elevation which you have already measured. So, this kind of information can be obtained the frequency component, the energy contribution from different frequency bands etcetera which I have already told in my earlier lecture.

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Next unfortunately, this is what is called as you are Beaufort scale. So, normally when you have a cyclone you get and an (()) announcement in the radio or in the T v say saying that wind speed of. So, much kilometer square hour we have a low pressure area of bay of bengal and this is going to be associated with some wind speeds of varying some values.

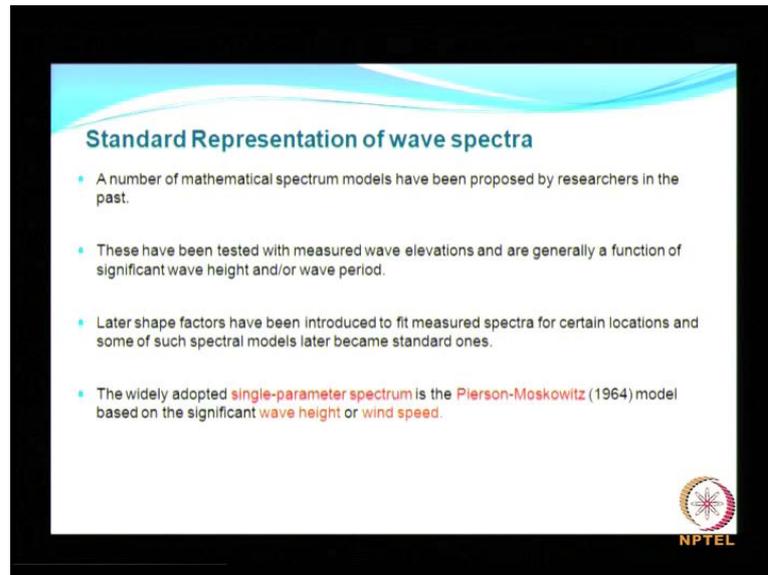
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Beaufort number	Description	Wind speed	Wave height	Sea conditions	Land conditions
0	Calm	0 knots 0 mph 0 km/h	0 m	Flat	Light breeze from vertically
1	Light air	1-3 knots 1-3 mph 0.3-1.5 m/s	0-0.3 m	Light ripples on water	Smoke drift indicates wind direction; windward waves
2	Light breeze	4-6 knots 4-6 mph 1.1-1.5 m/s	0.2-0.6 m	Small ripples; crests of glassy appearance; no breaking	Wind felt on exposed skin; leaves rustle; waves begin to move
3	Noticeable breeze	7-10 knots 7-10 mph 1.8-2.8 m/s	0.3-1.0 m	Large ripples; crests begin to break; scattered whitecaps	Leaves and small twigs consistently moving; light flag antennae
4	Moderate breeze	11-16 knots 11-16 mph 2.9-4.5 m/s	1-2 m	Small waves with breaking crests; fairly frequent white foibles	Cloud and forest appear ruffled; small branches begin to move
5	Fresh breeze	17-21 knots 17-21 mph 4.5-5.8 m/s	2-3 m	Moderate waves of some length; many white horses; small amounts of spray	Direction of a moderate sea now small waves in high light to seas
6	Strong breeze	22-27 knots 22-27 mph 6.1-7.5 m/s	3-4 m	Long waves begin to form; white foam crests are only momentary; some white spray is present	Large branches in motion; walking hard in windward areas; umbrellas not feasible; difficulty to carry objects; garbage cans to be fastened
7	High wind	28-33 knots 28-33 mph 7.7-9.1 m/s	4-5.5 m	Sea begins to curl; foam from breaking crests is blown into streaks along wind direction; noticeable amounts of white foam spray	Difficult to move in motion; effort needed to walk against the wind
8	Gale	34-40 knots 34-40 mph 9.5-10.8 m/s	5.5-7.5 m	Moderately high waves with breaking crests forming squalls; much white foam of high sea blown along wind direction; considerable white foam spray	Some high trees break; trees, cars lean on wind; progress on foot is seriously impeded
9	Strong gale	41-47 knots 41-47 mph 11.2-12.8 m/s	7-10 m	High waves whose crests sometimes roll over; distant foam is blown along wind direction; large amounts of white foam spray begin to reduce visibility	Some branches break off trees; and some small trees blow over; considerable damage to crops and farm buildings; trees lean
10	Whole gale	48-55 knots 48-55 mph 13.1-15.0 m/s	8-12.5 m	Very high waves with overhanging crests; large patches of white foam appear; the sea is a white expanse; considerable lashing of waves with heavy spray; large amounts of white foam spray reduce visibility	There are broken off or uprooted; saplings bent and propped; trees blown about; small trees and shrubs or bent; considerable loss of crops
11	Violent storm	56-63 knots 56-63 mph 15.5-17.5 m/s	11-15 m	Exceptionally high waves; very large patches of foam appear; white sea; much foam of white sea curling; very large amounts of white foam spray; visibility reduced to short distances	Structural damage to vegetation; many saplings uprooted or snapped; small trees may break; forest up and/or felled; due to age may break down completely
12	Hurricane force	64-81 knots 64-81 mph 18.0-22.5 m/s	14-18 m	High waves; sea is completely white with foam and white sea; it is blown into long spray; greatly reducing visibility	There is structural damage to vegetation; some branches may break; trees blown over; and some structural damage to crops and farm buildings; trees may be blown over

Flag number, flag number say 6, flag number 7 is hoisted in harbors. I am sure some of you must have heard. What is this flag number flag number is nothing, but the Beaufort scale that is the number. Number is given for the wind speed in terms of wind speed and the in terms of the wave height. So, for example, it is when it is 0 then you do not have any number that is going to be 0 because it is a com, then when you have about two then we say it is light breeze whereas, wherein you have the wave height of approximately point 0.22, 2.5 meters and you have the description of the sea condition and the wind, wind condition here and. So, you have other values like gentle breeze etcetera.

So, for example, if you have high wind and moderate gale etcetera. So, you see that the wave height in this case if the beaufort scale is 7, the wave height is of the order of 4.42 to 5.5 meters and if you as you go down, you see that swell let us see the swell that is during a hurricane. So, you can have a wave height of the order of about 14 meters. So, this Beaufort scale gives you the conditions of the marine environment in terms of the wave climate. So, this is quite important that you have this Beaufort scale and you can also refer to the net for much more details on the Beaufort scale.

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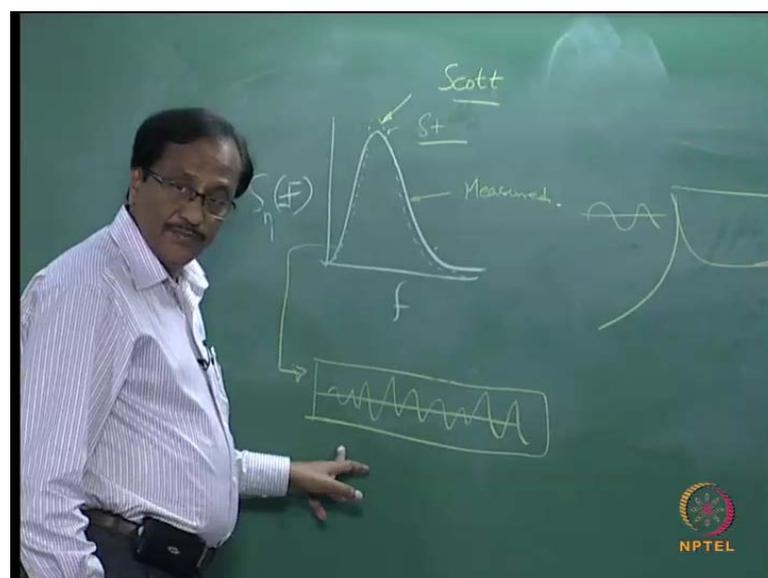
Standard Representation of wave spectra

- A number of mathematical spectrum models have been proposed by researchers in the past.
- These have been tested with measured wave elevations and are generally a function of significant wave height and/or wave period.
- Later shape factors have been introduced to fit measured spectra for certain locations and some of such spectral models later became standard ones.
- The widely adopted **single-parameter spectrum** is the **Pierson-Moskowitz (1964)** model based on the significant **wave height** or **wind speed**.

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So, as we had as we had earlier about the standard distributions like a Rayleigh distribution you now, have to go in for standard representation of waves spectra. What does that mean, is the purpose the same similar to what you did with Rayleigh distribution that is in the statistical domain. Now, you are going to do it in the spectral domain. Why do you need this first of all, why do you need the, the standardization of standard representation of of spectrum? Say for example, if you want to I will just take a few minute before we go into the detail.

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Scott
 $S(f)$
 f
Measured

NPTEL

So, naturally the spectra as I have told you varies like this that is spectrum, spectral density and the standard spectrum maybe, maybe I will come to the standard spectrum the list of standard spectrum later, but the standard spectrum if you plot may be going like this if in the event it is following.

So, now here this dotted line is your standard spectrum and this is going to be your measured spectrum for example, you have proved that the wave elevation of Indian coast of India follows a given spectrum, let me call it as Scott spectrum. So, this a particular name like Rayleigh distribution, we have Scott spectrum and we have many other spectrum. So, for the time being you assume that someone has already proved that off east coast, it follows its only an assumption that it follows Scott spectrum. And I have a and some there is going to be some experiment I mean some off shore development, which require some social kind structure to be installed and before installing maybe it is a floating structure and when you have a floating structure, you know what will happen, when it is moored.

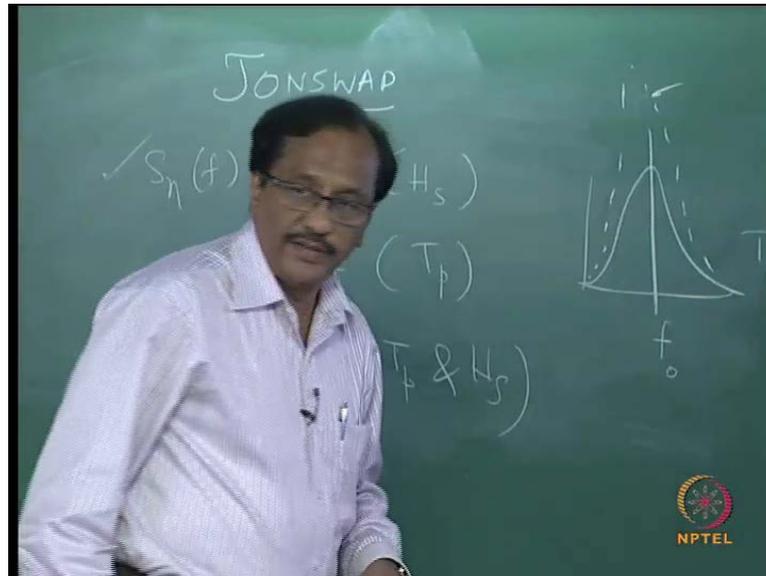
So, it will, it will undergo motions and this motions will definitely be a function of frequencies. So, the motion response of this structure will depend on the input spectrum. So, I want to find out the response of this structure for different frequencies. I need to do do that because for some frequency this this mooring can go to the last to the extreme end. So, and I also need to find out how height moves, how transferred direction moves all this information we need.

Then we try to say that. If someone has already said that it follows a standard spectrum like the Scott spectrum, I can simulate the scot spectrum in the field, in the lab, in the lab. So, this variation of the wave elevation in front of the structure can be simulated in the flume and the motion response of this structure can be adopted of course, you need to use a model scale for all this things at least. So, this once you have a standard spectrum that permits, permits us to easily handle the problem because you, you are standardizing all these results. The presentation is also quite easy straight forward and once you know that, this is a theoretical distribution then it becomes very handy. In order to show how the whole response function can look like.

So, is that clear. So, that is the, that is the reason why we have a standard spectrum similar to a standard probability distribution curve. So, a number of spectral model

mathematical spectral models have been proposed by researchers in the past. These have been tested with measured elevations and are generally, a function of significant wave height or peak period. So, you have a and or peak period.

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So, you see that the spectral density will be a function of H_s or a function of T_p . T_p is this is peak f_0 and T_p is $1/f_0$, that is the frequency of maximum energy or sometimes it can be T_p and H_s . So, here it is in this case, we call it as single parameter spectrum. The second one also is single parameter spectrum and later you can also have two parameter spectrum and multi parameter spectrum, which we will see later.

So, later the shape factors have been introduced because initially, we had only this two then later the shape factor were used and introduced to fit in order to fit, the measured spectrum because if this is a theoretical spectrum and maybe it is something like this the measured spectrum then, we use a third parameter, the third parameter was straight. So, as to pull this peak towards the measured data and that is what happened in coming up with, what is called as a Jonswap spectra which will be a discussing later in the next lecture.

The most widely adopted single parameter spectrum is the Pierson Moskowitz that was done in 64 based on this single wave height or the. It can be describe by the wind speed alone. So, we will just have a look at it later.