Wave Hydro Dynamics

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Random and Directional Waves

Lecture No. # 02

Random Waves and Problems I

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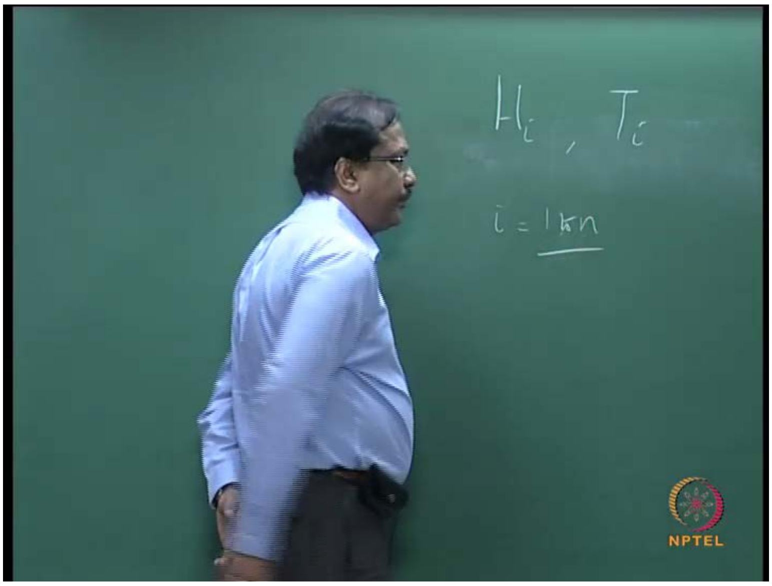

So, we have seen how the individual waves are obtained from a random record using either the do not down cross method or the up cross method. So, from this kind of an analysis you will have T and H i and I said earlier that the recording should be in such a way that, your number of waves should be at least 100. So, that means, here I will be varying into 1 to n. So, having obtained the individual wave heights and periods, we can look at other characteristics like wavelength etcetera.

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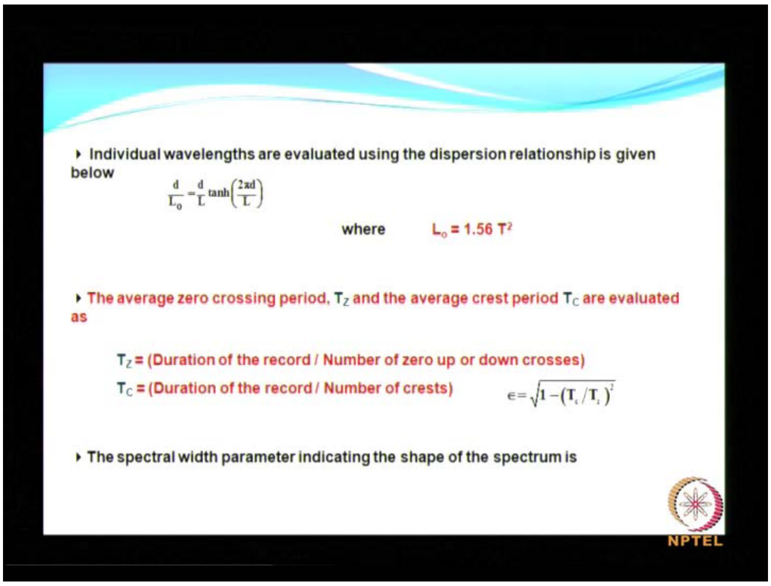

So, wavelength as you know you use the dispersion relationship, as we have indicated here and all of us know about this dispersion relationship. So, which you can however, the wavelength of individual waves.

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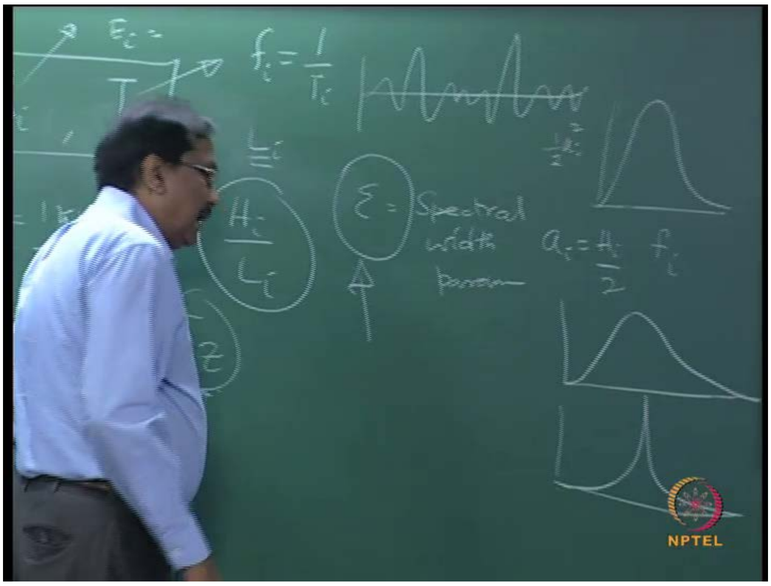

And also since H is also known L i is also known from the dispersion relationship you can arrive the wave stiffness and any other parameter also you can arrive at using this method of analysis. We usually describe the general wave characteristics in a random wave. The wave height is that usually, we normally use this significant wave height for representing the characteristics of a random wave and the frequency is referred to as peak period or the crossing period I will come to this later, but as far as our statistical analysis is concerned, we normally use what is called as 0 crossing period or average 0 crossing period.

This average 0 crossing period is just you have the average 0 up crosses, using the up crosses you have got the wave periods. So, mean of this can be just 0 up crossing period or similarly, if you have a T i from the down crossing method, we call it to the average of that is the average 0 down crossing period. So, the same T S, T z is also straightaway can be obtained as the duration of the record divided by the number of up crosses are number of down crosses, the meaning is same. We also used another period which is T c which is also called as, which is referred to as crest period. In which case, that crest period is obtained as duration of the record divided by the number of wave crest.

Epsilon is what is called as spectral width parameter spectral width parameter. The spectral width parameter, we usually have epsilon this is given as 1 minus T c divided by T z to the whole square. So, this spectral width parameter, what does what kind of information it gives. When we have a combination of a number of waves with different wave heights and wave periods, then you will see that you know that the wave period can be represented as frequency which is 1 by T, that is the frequency and you know that H i is nothing, but the energy.

Now, if you want to represent a random wave the characteristics of a random wave. Which is comprising of a number of composites, number of components with different waves and wave periods, then how do you represent this one way of representing is frequency on the x axis I can get and on the y axis, you will be drawing half into amplitude square amplitude is nothing, but H i by 2. So, this curve gives you some information about the distribution of energy. Now, whether this curve is a broadband or a narrowband, what you mean by broadband? Broadband is something like this where in you see that the energy is straight over a wide range of frequency. Narrowband is something like this, that energy is spread over a very narrow range of frequencies.

So, this can be determined well from a the analysis of ocean is, whether it is a broadband or a narrowband, you are expecting either a broadband or a narrowband, which can be determined by the value of the spectral width parameter and that spectral width parameter, in the case of the statistical analysis can be obtained as epsilon equal to square root of the ratio of a T c and T z whole square is that clear.

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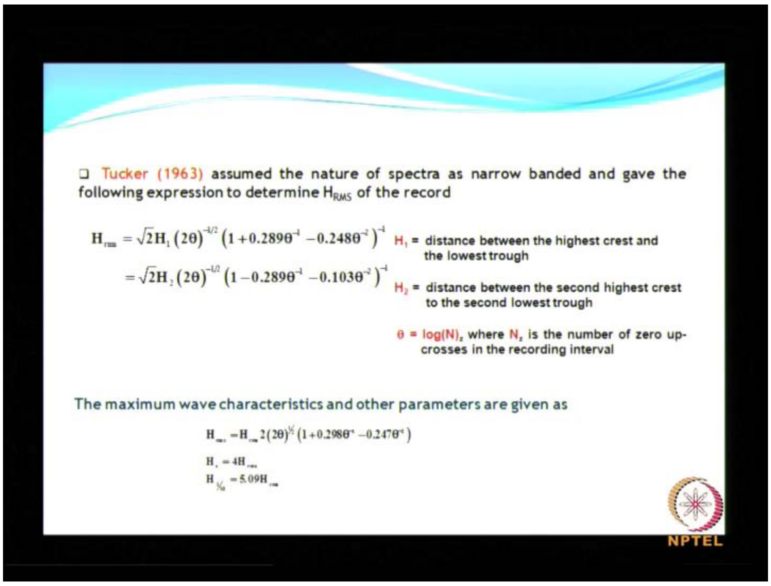

Now, all the other efforts that towards representation of random wave in terms of a single parameter, which can replace the effect of the entire random wave some way of representing some statistical measure to represent the random wave. It was the first one is the earliest paper was by tucker the method of analysis is very simple, very straight forward and is quite handy, in arriving that the value of root mean square value.

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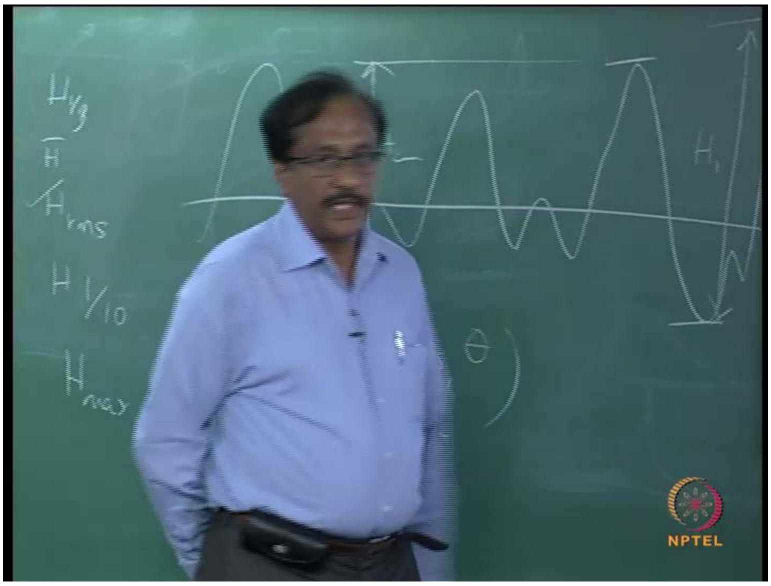

So, we normally come across H one third, H bar, H r m s, H one by ten, H max, these are the different statistical measure of the wave heights, which are usually used in the way mechanics are for the design of our structures etcetera. So, in this case you see this root mean square value can be given by that long formula, where in you see that H 1 is H 1 is the distance between the highest crest and the lowest trough. So, highest crest is, this is highest crest from a sample record I am just showing and the distance between that and the lowest crest.

So, lowest trough I will just say that this is ending here. So, the lowest trough will be; obviously this, I just make this something like this. So, obviously, this will be the lowest trough. So, this is your H 1 and the second lowest, second lowest crest is this and second lowest trough is this. So, obviously, the distance between these two is your H 2. So, this will be your H 2. So, then theta is log where N, n z is the number of 0 up crosses. Using the simple formulation, you can arrive at your H r m s and once your H r m s is obtained, then your H max, H max is given by this formula. The one just above the maximum wave characteristics and other parameters are given, as we have three different formulas there.

The first one is H max which is of course, going to be a function of H r m s and theta and H S is equal to 4 times H r m s and the final one is H 1, 10 is 5 point times 5 0.09 times H r m s, H r m s thus you see that you can arrive at the values for the different parameters that are in use.

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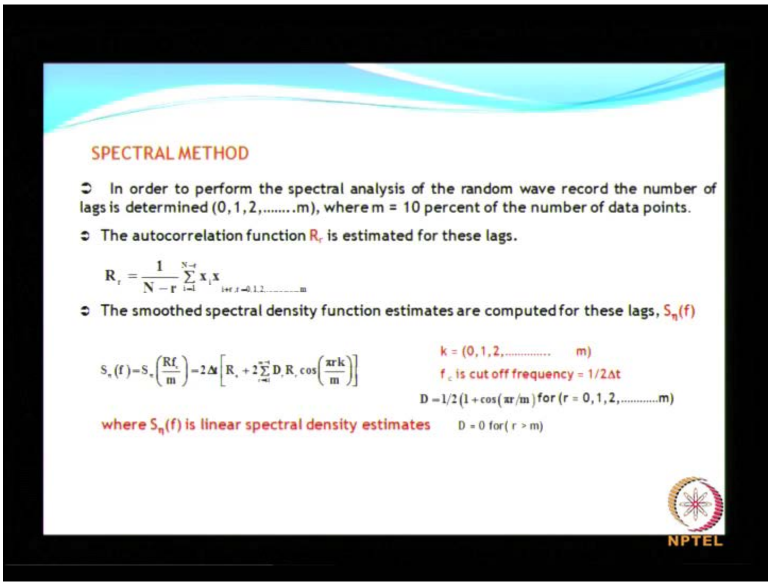

That too some extend we have covered, the different statistical methods different parameters at the statistical method.

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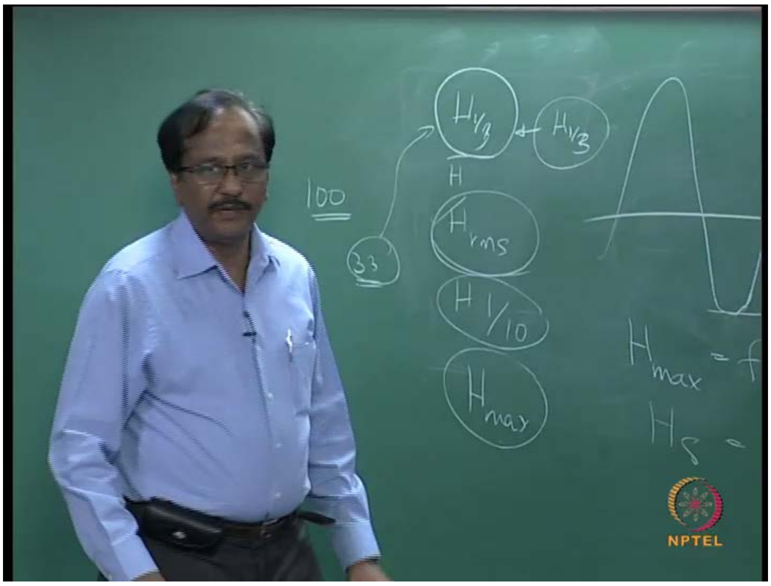

So, under the statistical method analysis, you have H one third which is nothing, but the average of highest one third of the waves. So, if you have a 100 waves you arrange them in descending order, that take the top 33 waves and take the average of that and that is going to be your H S significant wave. The significant wave height is, usually used for the design of breakwaters, because as I said earlier breakwaters are kind of flexible structures, where in the kind of failure you can anticipate is rolling down of the stones. So, the waves give us enough amount of time to rehabilitate.

So, it is enough you design the structure for significant wave height and usually the H max is avoided for the flexible type of structures and flexible in the sense I am talking about a rubble mound breakwaters. The other parameters are your H r m s which is nothing, but the root mean square value then H 1, 10 similar to H one third, we have H 1, 10, H 1, 10 is widely used in the field of naval architecture also and then finally, you have the H max which I need not have to mention what it is, but you see that there is a kind of relationship between the different kinds of different forms of representation of the wave height.

So, we have to some extend understood the statistical analysis of ocean waves or representation of the average characteristic in the statistical domain. Now, let us move to spectral method. As we have seen in the classification under spectral method, you have two different methods, which is one is via the autocorrelation function, the other one is via FFT.

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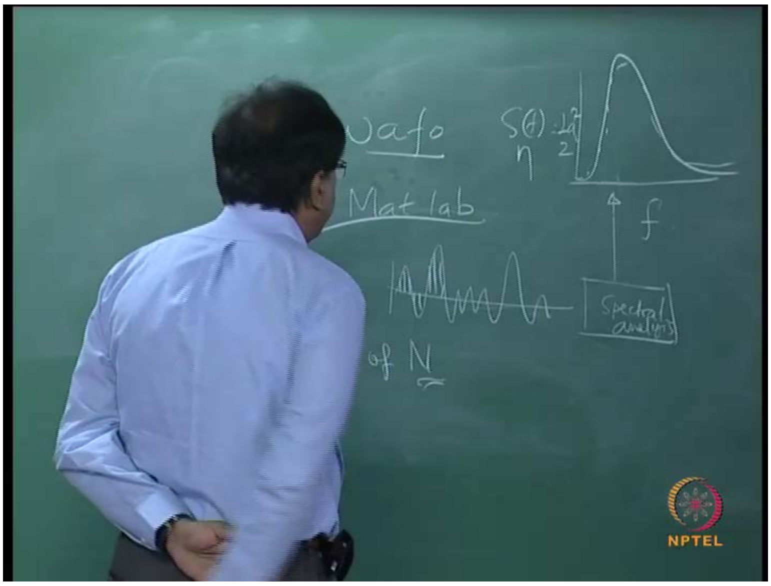

So, this says you have what is called as a Wafo. Under mat lab which gives you the distribution of energy in the frequency domain, refer to as spectral density and the shape of the curve as shown earlier, that is this will be that is nothing, but half into amplitude square usually it will be something like this. So, you will have a something like this. So, when we have an analysis a random wave, the usual way is subjected to spectral analysis or frequency domain analysis. The main purpose for this is to get the spectral density curve, that is how the energy is distributed and as a function of frequency. Before this Wafo under mat lab, we have to do it on our own right yes mark code and then try to get all the information's about the distribution of energy etcetera.

So, under the autocorrelation function in order to perform the spectral analysis, this record is assumed to have a number of lags. We usually take the number of lags or the components as 10 percent of 10 percent of N is the number of data values, what you mean by number of data values? Number of digital values. So, this will be either positive or negative, you digitized the random record at regular intervals constant sampling interval and then take 10 percent of the total number of data as the number of components.

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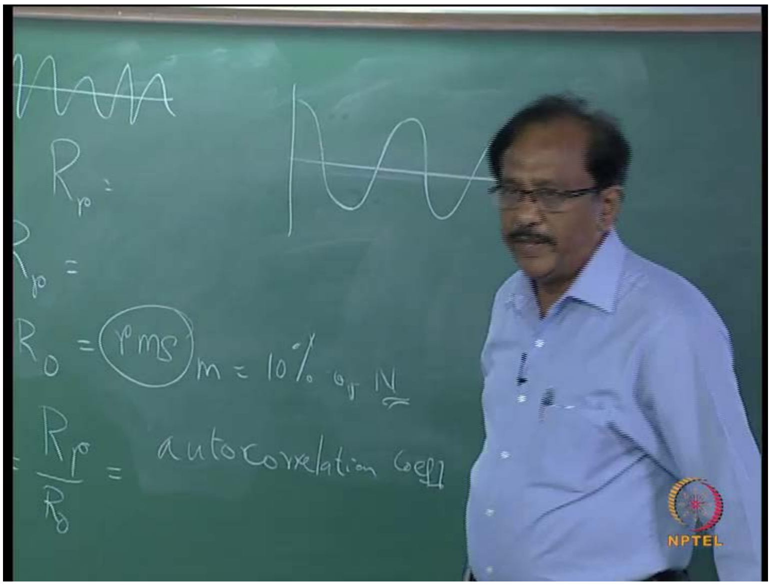

Once you are done with that, you can estimate, what is called as autocorrelation function? The autocorrelation function is given as 1 minus 1 divided by N minus R, r is the lag number and number of lags will be maximum of m which is 10 percent of the number of data. So, if the number of data is thousand you are suppose to have 10, 100 frequency components.

So, this autocorrelation function if you have a time history, it correlates within itself, as you see it is correlating within itself that is i into x of i plus R. So, you have i value here and then you multiply that with a lag and then you get the autocorrelation function. So, when R r is given by this, what will happen if R is 0, if R is 0 it is nothing, but the mean square value that is root mean square value. If R 0 is put in that expression R r suffix R r becomes 0. So, that is naturally your mean square value. So, this is nothing, but root mean square is that clear.

So, now you see that the smooth spectral density estimates are estimated for the different lags by using this expression, where d R r is given by this expression, where R is ranging from 0 to m, d equal to 0, if R is equal to greater than m that is a assumption. So, all this parameters are known to you. So, you can arrive at the variation of the spectral density curve, for further details you can refer to a number of books available on this aspect. If you draw R r divided by R 0, this is what is called as autocorrelation coefficient. What does autocorrelation coefficient indicate is, autocorrelation coefficient indicates that if you perform, if you subject are determined the autocorrelation coefficient for a sine curve may be you get the variation like this, same curve will be repeating if you are having a random time history.

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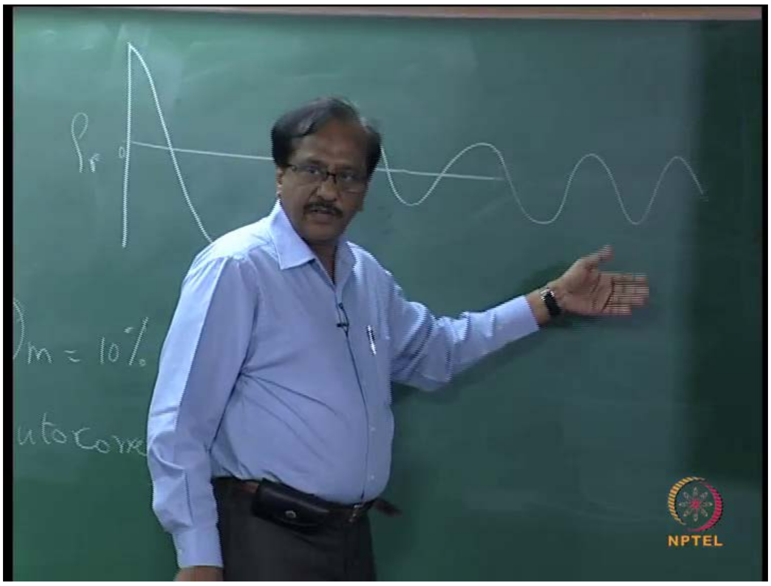

Then autocorrelation coefficient will be varying from maximum will be 1 at 0 lag, because look at this expression. So, it will be varying like this.

If autocorrelation coefficient dies down very quickly, then we say that this a really a random phenomena, a random wave sometimes what would happen is that this should not died on, but it would be oscillating something like this; that means, although the time history is looking more or less random, but it is not that random, but it is there, because of certain dominant period which is really built in the random waves. So, this is because of the existence of a particular is that clear. So, such information's you can arrive at autocorrelation coefficient.

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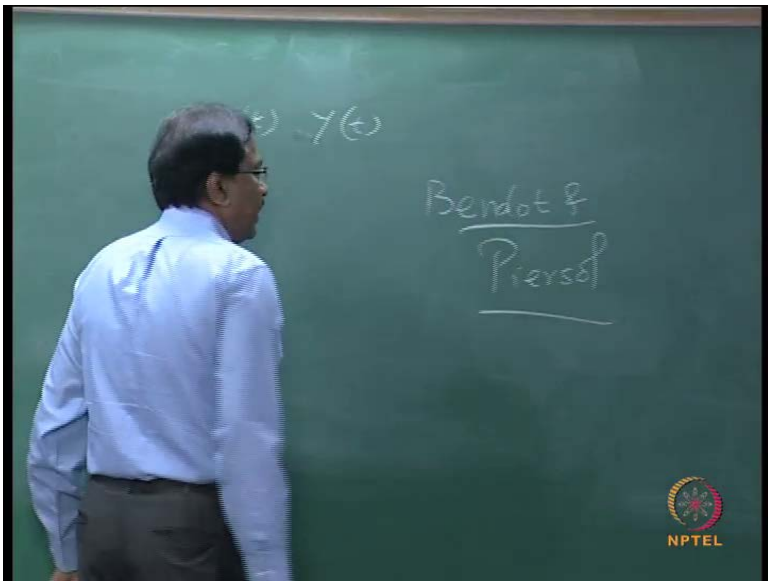

The other aspects which I am not going to cover, but you need to be aware of that, are the cross correlation and the most is the cross correlation cross correlation will be you want to correlate two variables, one is may be wave and force, you have to how is it a correlated between these two variables. So, all those information's can be obtained. So, I would suggest the book by Bendot and Piersol which is given in the end of the reference, for further details on cross correlation, autocorrelation and even all these methods which I have discussed. So, when you when you write a program for example, you want to write a program on your own to check whether the spectral density estimates are going to be correct.

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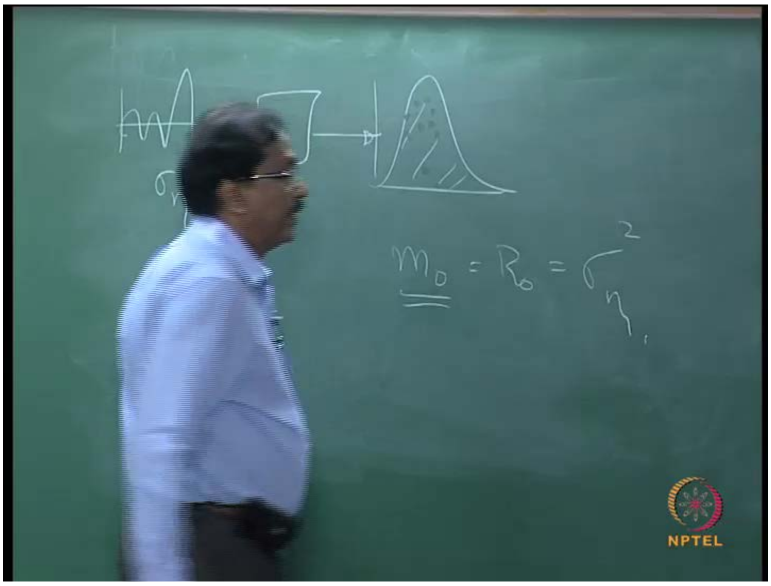

Then, you have the spectral density curve using the formulation. You just find out the area under this curve that will be m, which is refer to as 0 spectral momentum, you have to find out the area and after, you have a signal you use your sum method you get this, after getting this get the area under the spectral density curve and that is going to be your m naught.

Earlier, I said R naught that can also be calculated as I have said earlier and for this time history straightaway you can get your standard deviation, which is sigma eta. So, now m naught will be equal to will be this is see kind of a cross you can just use this as a basis for checking your results.

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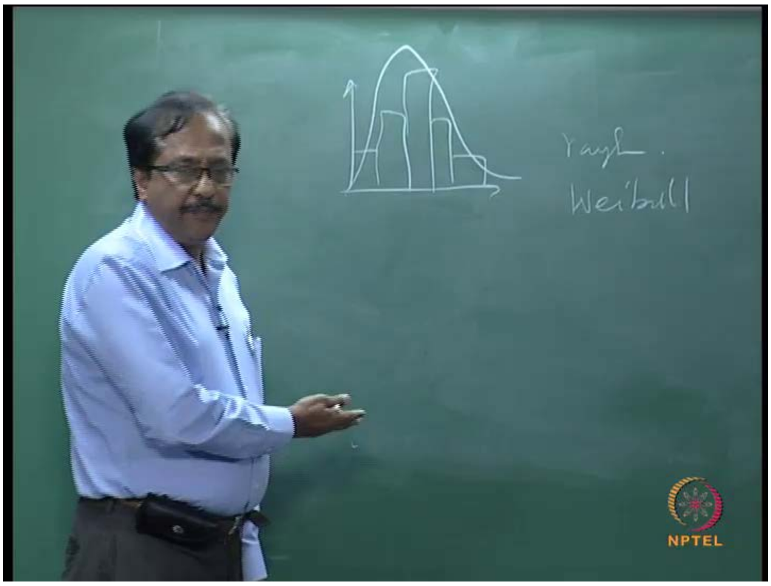

As we have seen earlier, what was done in the statistical domain, what we normally do is, we observe the probability density function and then we draw whether it follows a standard distribution for the wave heights. I mentioned a few like your Rayleigh Weibull etcetera. We saw all these distributions and I also explained the importance of comparing the probability density of measured wave height with a particular theoretical distribution, because once you have done that and once you see that it follows a standard distribution then it becomes very handy for you to arrive at lot of information about the process, likewise like I you have in that probabilistic approach in the spectral approach.

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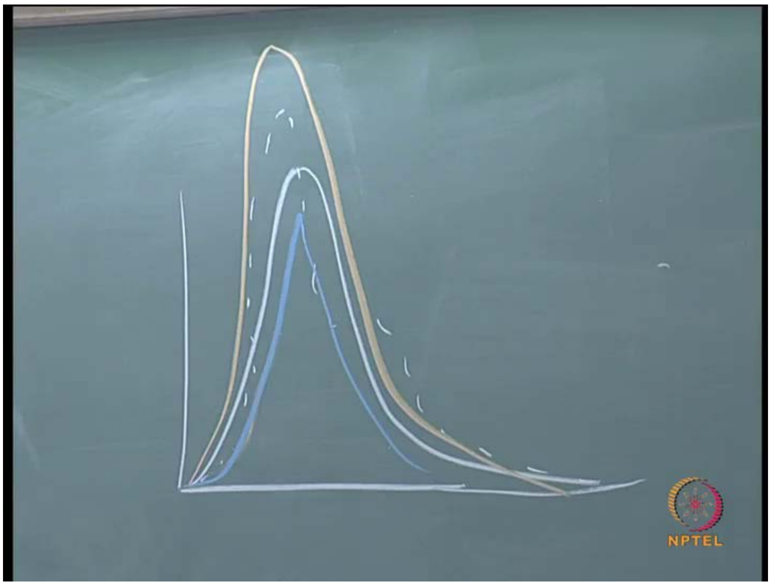

You have several standard spectra and the idea would be to draw, may be this dotted lines may be your measured spectra and then you can compare with other standard spectra's, may be this is following this or maybe there is another spectra. So, you can verify or find out which of the standard spectra it could follow, but what are the standard spectra? Similar to these standard distributions, we shall see about the standard spectra's later, after having some basic information about the spectral methods and we will also try to understand the methods using some worked out examples.

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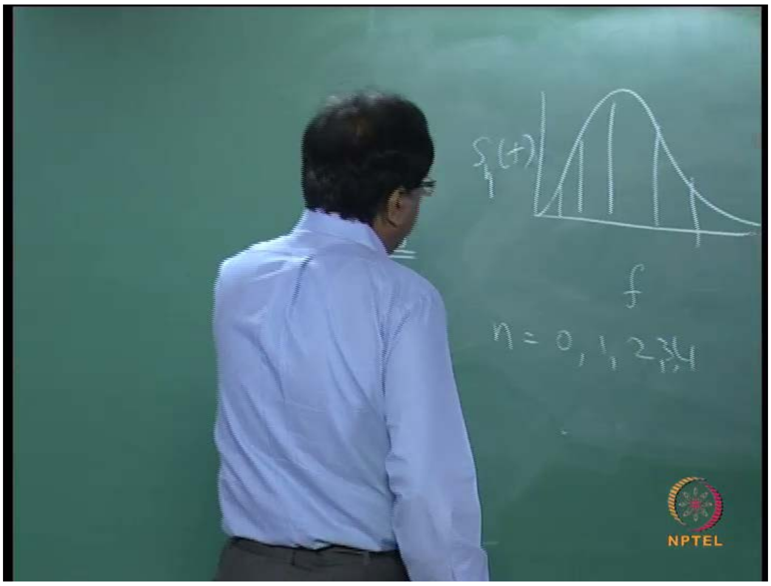

So, once you have drawn the spectral density curve. So, f and S eta of f you have this, then I know at each of the frequencies what the value of S eta of f is. So, i simply use that expression m, n, m, suffix n, where n is the order of spectral moment. So, this will be spectral density multiplied by frequency to the power n, n is the order of spectral moment, usually we have n equal to 1, 2, 3, 4.

Of this we use other spectral moments, but the most widely used spectral moment is the m naught, that 0 spectral moment is nothing, but the area under the spectral density curve which I had already explained. So, once you have got this and once this is obtained, then you can easily get values of the different parameters like your spectral width parameter. Spectral width parameter you remember under the statistical method, you saw it is a function of crust period average crust period of the average 0 classic periods. So, that was under the statistical method.

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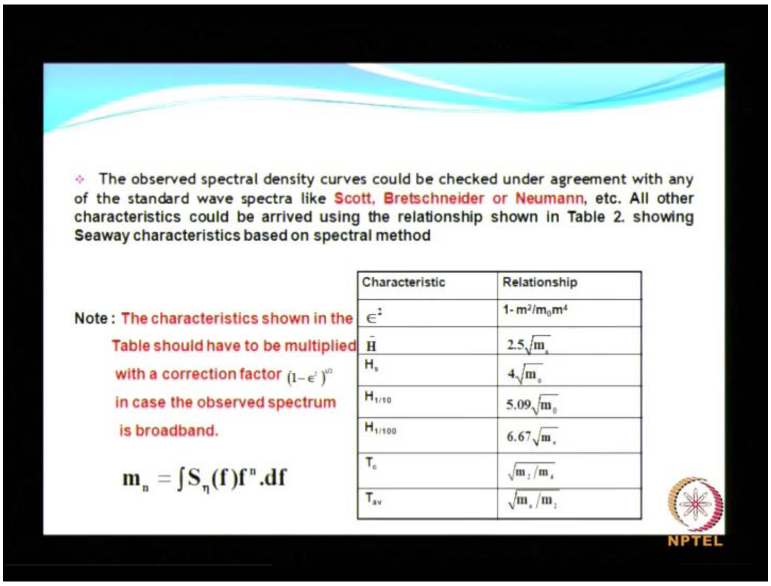

Now, this is under the spectral method, under the spectral method you see you can arrive the spectral width parameter \H bar that is the mean wave height then you have the H S significant wave height, then you have the H 1, 10, H 1, 100 and the T c average crust period and the average 0 crossing period or average period for that matter. So, that will be square root of m 0 by m 2 and m 4 is will also be coming. So, most it will be only m 0 m 2 and m 4. So, now you have a good exposure, I believe you have a good exposure now to the method of analysis of ocean waves and what you do why do you do the analysis.

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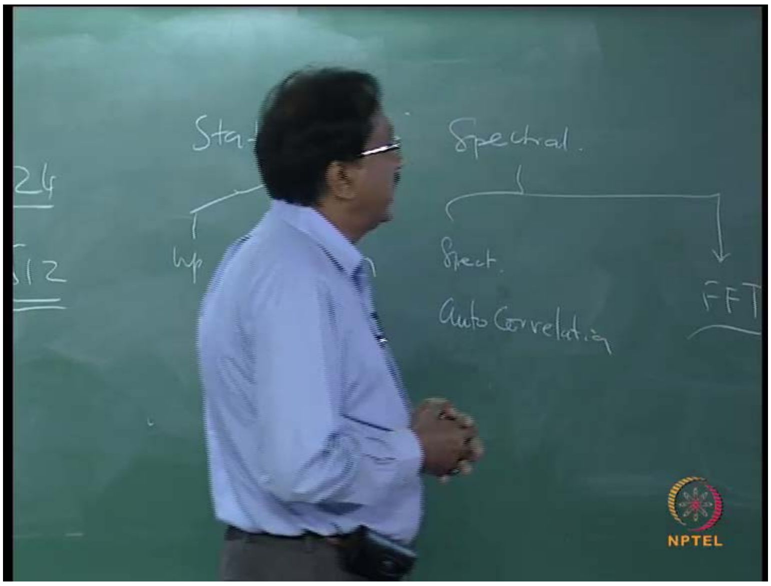

So, we have seen statistical method and the spectral method, under the spectral method here we had the up cross and the down cross which we have already seen. Now, under the spectral, we have the spectral method via autocorrelation and now much faster and easier method that is the fast Fourier transformation. Unlike the selection of the number of lags in the spectral method via autocorrelation here, the number of frequency components will be N divided by 2 that is the number of data values divided by 2.

So, if you have 1024 data values, then you will have 512 frequency components this is more accurate more simple and straight forward and you need not have to sit and write programs for this, only thing is you should understand what a fast 4ier transformation gives. Again for a detailed description you should refer to the book by Bendot and Piersol, which gives the complete analysis of this kind of random signals.

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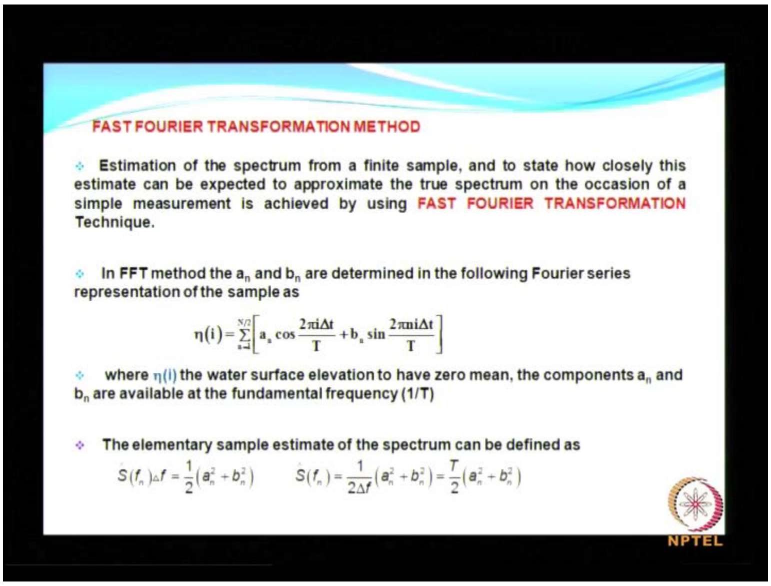

So, here it says that estimation of the spectrum from a finite sample and to state how closely this estimate can be expected to be approximately true spectrum, this is achieved by FFT this is nothing, but the basic Fourier series. So, any random signal is assumed to be formed by a number of Fourier series components and that is why the eta of i, is represented as a number of components. Where in a n and b n will be there and the one insider here Cos and sin will be the face, eta of I is the water surface elevation and that will be having a 0 mean process, that is a 0 mean process. What do you mean by 0 mean process? You have the time history, you take the average and once you have taken the average subtract it from the actual time history.

So, that will be your mean 0 process. So, the elementary sample estimate of the spectrum can be defined as the spectral density is given as here. So, you estimate the frequency components that is, a n consisting of a n and b n and then this once you arrive at this values the spectral density can be estimated. So, I suggest, because I do not want to go into the details of the mathematical aspects of all these things, only thing I suggest that you read some book, but what it gives what it conveys etcetera, we will have a discussion in the class of course, this can be removed.

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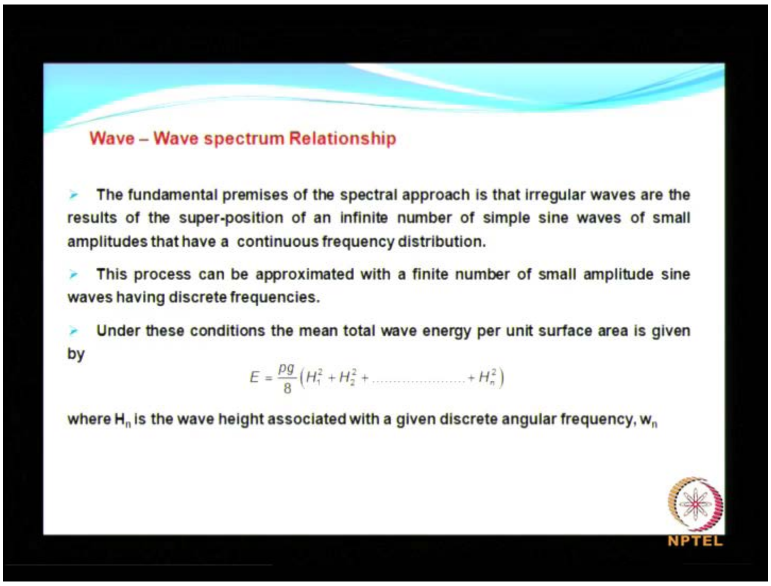

So, that is as far as the spectrum is concerned. Now, we will get into get back to this statistical method and we will understand more about statistical analysis, as well as the spectral analysis of the ocean waves. So, initially here the fundamental premises of the spectral approach, is that the random waves are the results of super position of a number of sinusoidal components or simple sine waves that, have a continuous frequency distribution. The process can be approximated with a finite number of a small amplitude with sinusoidal waves having discrete frequencies.

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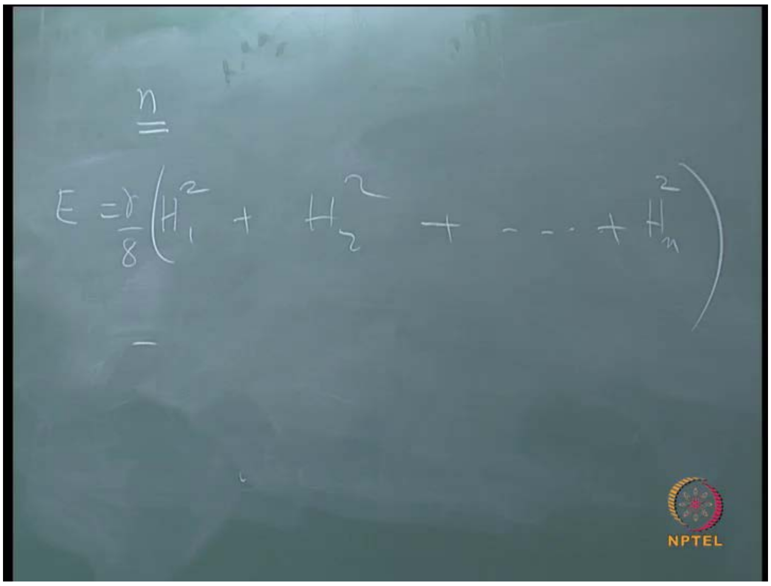

So, what is it total, suppose if you have n number of wave components, what is your energy? Energy is gamma by 8 into H square. So, here I have H and H number. So, I can write this as and each of which will be have an associated with a frequency.

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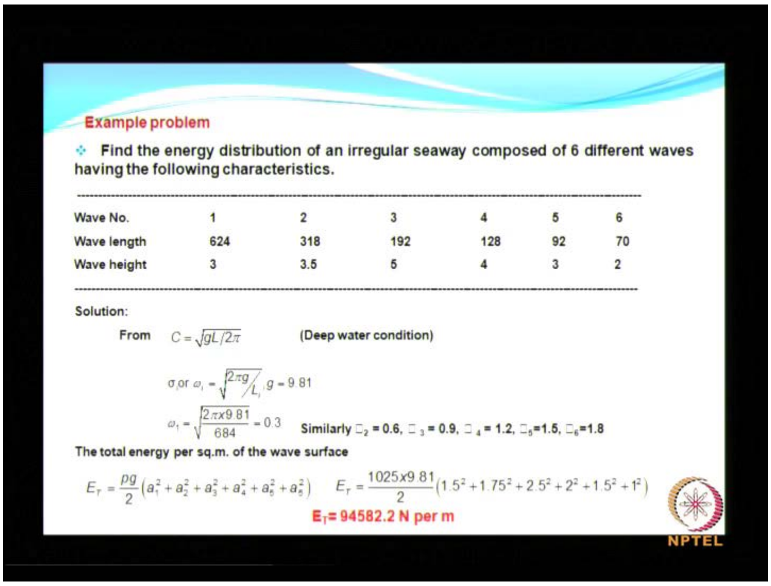

So, let us consider this problem, find the energy distribution of an irregular seaway composed of six different components. When you are rough, when you are dealing with ocean waves, actual ocean waves you will be dealing at least with 100 components that is what I have said earlier. The six number of components very less just to make you comfortable with in understanding the problem or the which are why you need to go in for, what exactly is the spectra etcetera, that is the reason I have taken only six different components.

So, for each of each I have given you the wavelength and of course, I will to make it simple I consider only deep water conditions, wave heights are given in the bottom and wave length are given in the second row. Now, you know that this is equal to g L by g L by 2 pi is your celerity hence, I can arrive at your sigma or.

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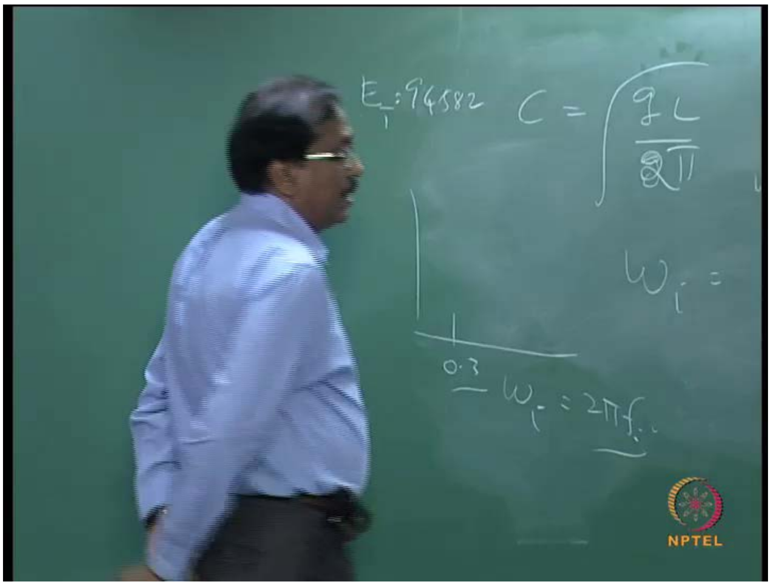

So, from this, what is w i, w 1, w i, is equal to 2 into 9.81 divided by 684 for example, for the first equation first frequency component is that clear of course, there is a pi here. So, using the relationship between sigma pi and c, you can arrive at omega 1 is that correct is it clear. So, similarly I carry out for omega 2, omega 3, omega 4, up to omega 6. So, I am having now omega i which is nothing, but 2 pi by 2 pi f, what I need is the distribution of energy. So, omega 1 is 0.3 for 0.3 that is the wave number 1 I calculate what the amplitude of amplitude square is.

So, you see that here a gamma is there and 1.5 is the amplitude, 3.5 is the wave height and similarly 3.5 here this is 31.75. So, i add up all these values and what do I get, I get the total energy as equal to 9, 4, 9, 4, 5, 8, 2 Newton per meter, what does that mean? That is the total energy contained in this 6 waves.

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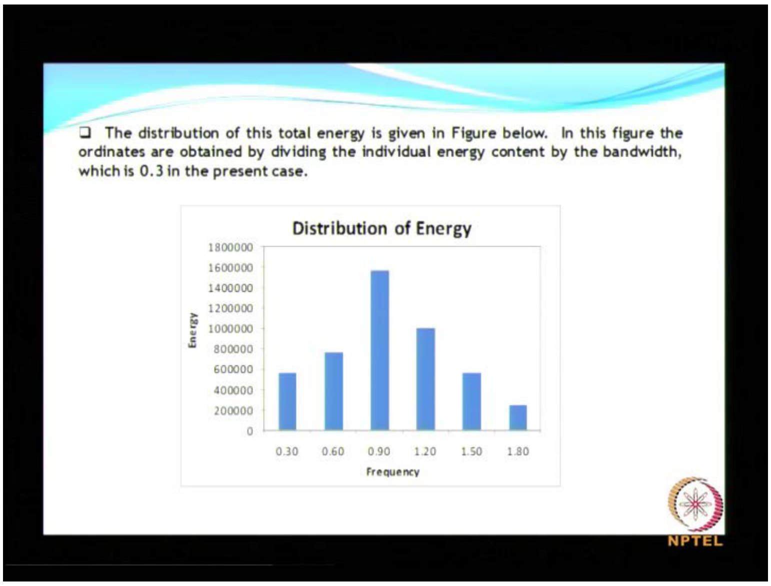

How can I draw that I can represent as shown here, that is this is omega 1, omega 2 omega 3, up to omega 6, but note that the distribution of this curve is given below, but in this curve in this figure, what we have done is the ordinates are obtained by dividing the individual unit individual energy by the band width.

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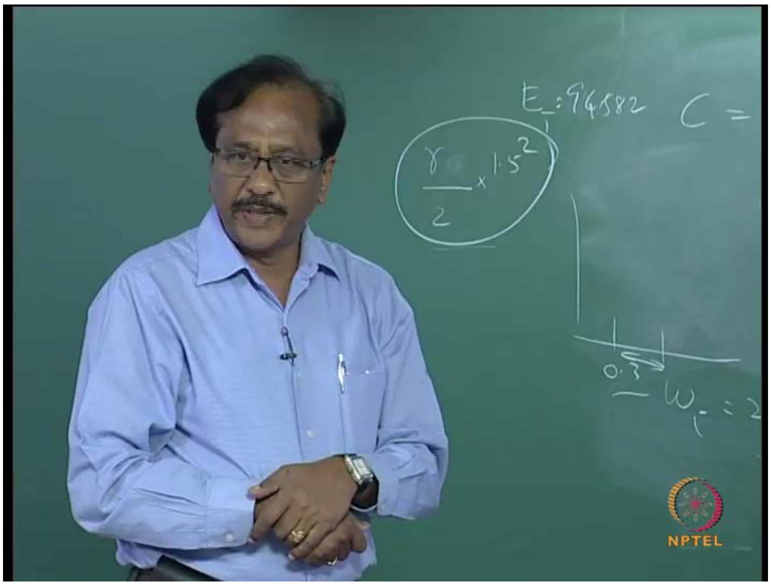

That is for example, the first one gamma into gamma divided by 2 into 1.5 square, this is the energy of the first component and that first component is divided by each of the component is divided by the class interval class, interval here is 0.3. Why do we do that, because the area under the curve should be equal to the energy total energy, that in order to make sure that total energetic is nothing, but the area under the spectral density curve and that is why we have divided the individual energy component by the band width and we have obtained the ordinates is there any doubts any of you . Is it clear?

Now, we have thus seen in the problem, that the energy in the waves and that the area under the curve gives you the same quantity range, please recollect we are calculating the individual energy contain for a particular frequency component and that energy component, we are dividing it by the class interval or the frequency interval and that will be represented in the ordinate. So, that the area under this curve gives you the total energy.

Now, instead of drawing like this we know that we can simply, because the gamma is going to be a constant. So, we remove this and we represent only half into amplitude square.

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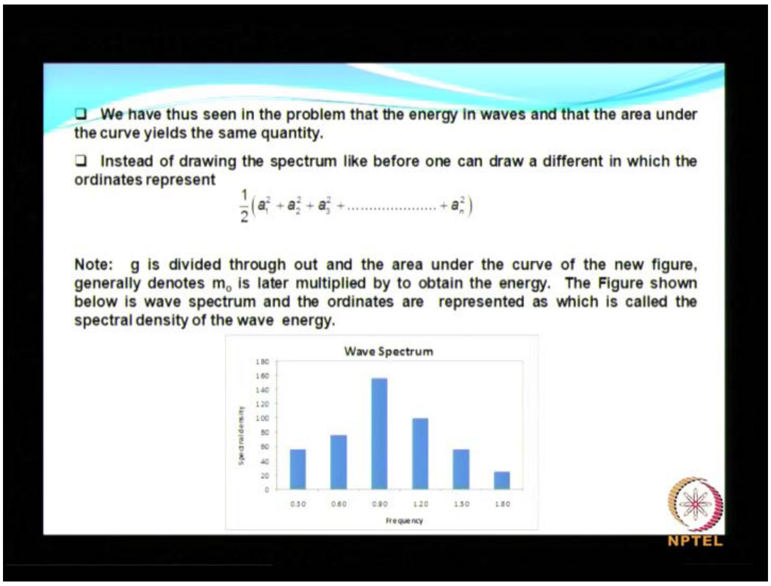

So, half into amplitude square so, when I can represent this. So, this gamma g so, I mean rho g is divided throughout and the area under this curve of the new curve now generally denotes as m naught, that is area under the spectral density is nothing, but the m naught, we have removed that constant that rho into g. Once you have calculated the distribution of the ordinates are here and this is called as the spectral density of the wave energy, very often we hear this spectral density. So, this is what it means and now you know this spectral density and area under that curve is simply multiplied by the.

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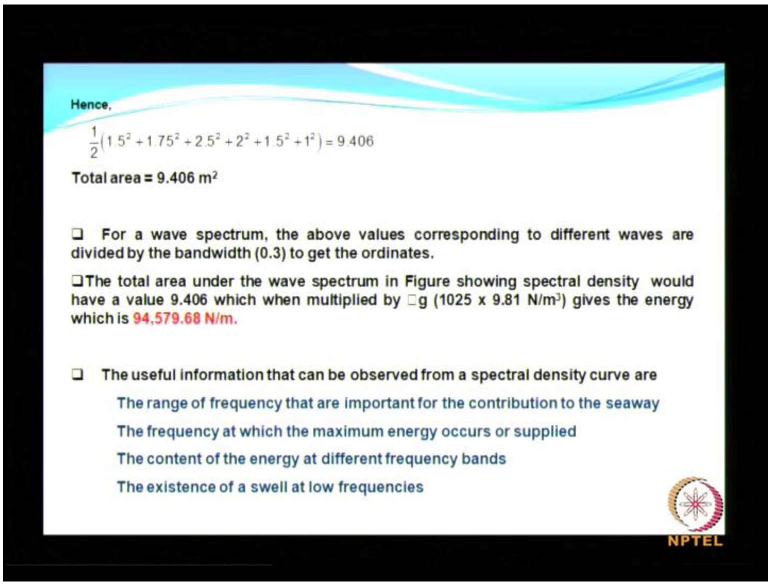

So, i just calculated the area under the curve and that will be. So, much and then you just have to multiply it by the gamma rho into g, in order to get the total energy under the curve is that clear. So, the useful information from the spectral density curve is, the range of wave frequencies, what is range of wave frequencies? That are important to the contribution of the seaway or in that particular area, what is the frequency components what you mean by frequency components?

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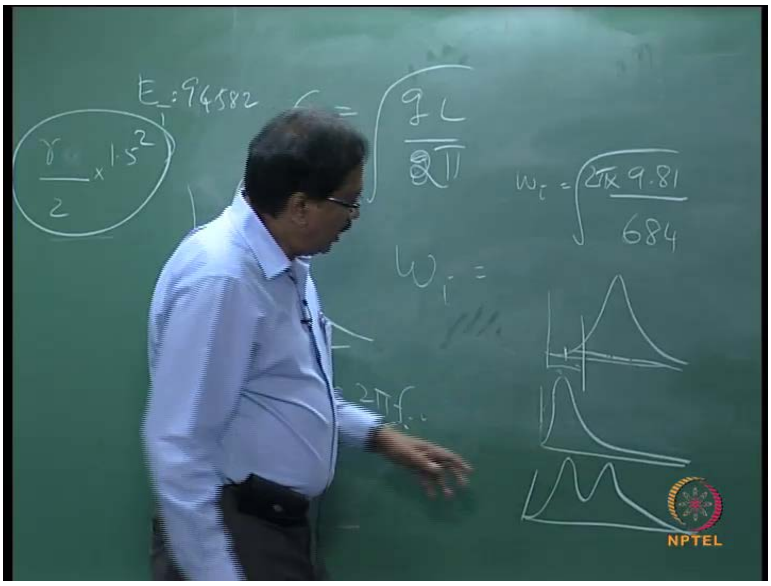

See for example, if an area as a curve like this and a spectral density like this or it shows like this, you see the difference. So, for example, if you go and stand if we have a vessel or whatever a floating structure for this frequency, only you see that it is exerting lot of energy. So, it will have more oscillations, for other frequency components when I say other frequency components, when the wave period is reciprocal of this frequency for those variable wave periods oscillations will be less and for example, the other way other aspect this.

So, this will be a low frequency component, the low frequency component is more dominant here. So, this will give you the range of frequency components, which are very important for you to design or if you want to evaluate the response of a floating structure what is more important for is the frequency range. So, the frequency I have at which the maximum energy is obtained. So, this is what will be the maximum energy, that would occur down frequency at which the maximum energy will occur the contents of energy. So, each frequency band I can get. What is the energy contained and also the existence of swell at low frequencies, if you have a low frequency component like this it is nothing, but existence of a swell what is a swell is which is outside the fetch .