Wave Hydro Dynamics

Prof. V. Sundar

Department of Ocean Engineering

Indian Institute of Technology, Madras

Module No. #04

Random and Directional Waves

Lecture No. #01

Random Waves

So, today we will have some details about the random waves, how it is described, what is meant by random waves, how it can be characterized. And how does the wave profile look when you take the measurements in the open ocean, and what you do with all these measurements. So, this will be covered right from kind of fundamentals going up to up to the analysis of the kind of waves.

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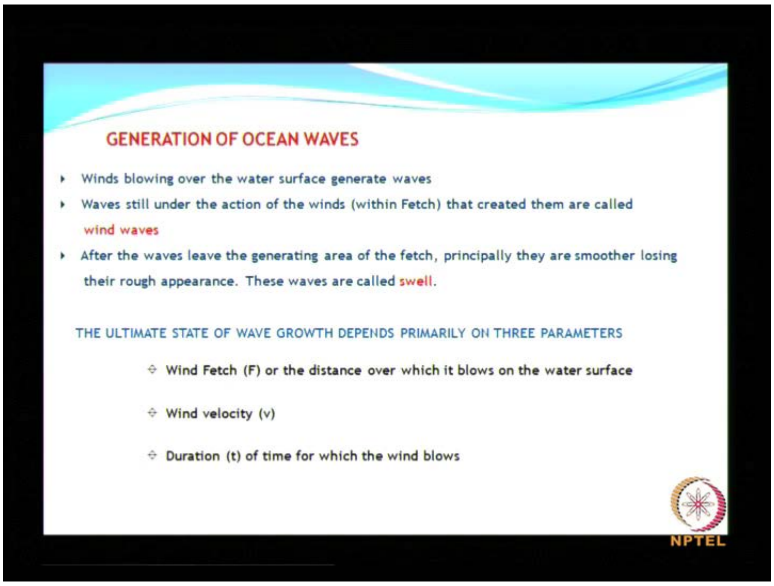

As we have seen earlier class, generation of waves in the ocean that is, because of the wind blowing over the water surface, and it basically depends on the growth always, basically depends on 3 parameters - wind velocity, wind the fetch, the distance over which the wind blows over the water surface, and then the duration of the wind.

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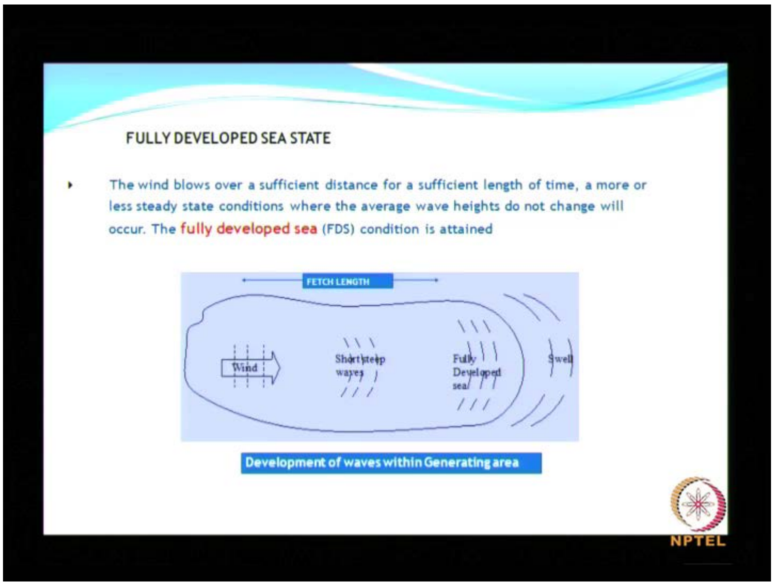

This we have already, I have already mentioned about this, and this also however mentioned, but I will just touch upon it again, so that it is refresh. So, somewhere in the deep ocean, the wind starts blowing; and when the wind starts blowing you have short steep waves generated. And as long as the wind is continuously blowing over the water surface, the generation of the waves or the growth of waves continue, and the propagation will be towards the shore.

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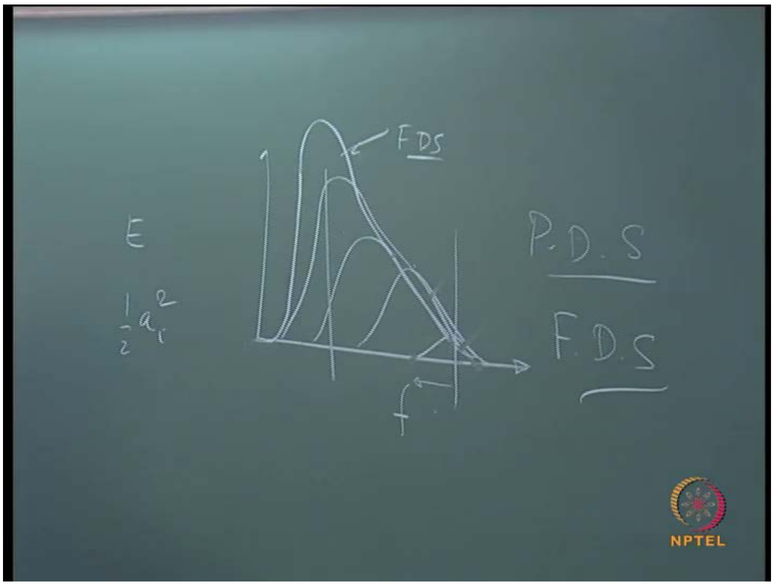

So, when you talk about the waves, so let me say on the x axis it is a frequency. On the y axis let us have the energy; energy is characterize by half into amplitude square so, initially short steepness will generate. When the waves, when the winds starts blowing then it will continue so, you see the peak, the shifted towards the lower frequency area. So, that is within this shifting of the frequency, the energy this is how the energy is being transmitted. So, initially you have the short steep wave generated and slowly you will see that the frequency lower frequency components dominate and you see that the energy will be being shifted towards the low frequency and finally, you will have more or less this is in general you will have something like so as the waves are growing you call this as partially developed sea.

But how long it can keep on growing? It will reach a stage when the action of the wind no longer helps in the growth of the waves. When such a situation reaches then we say that the sea is a fully developed sea. So, we can have this partially developed sea somewhere here, characterized by lesser wind speed for each of these will be lesser wind speed or lesser duration. So, you can have a figure this will be either for lower duration, lesser duration or for lower wind speed. Either of these 2 increases, you see that the shifting of the energy pattern move towards the low frequencies get started and then it will be a gradual progressive towards the low frequency region, until you get a situation where you feel it all of the sea is attained.

So, in this figure we can say that this is partially developed sea and this is where you have the fully developed sea.

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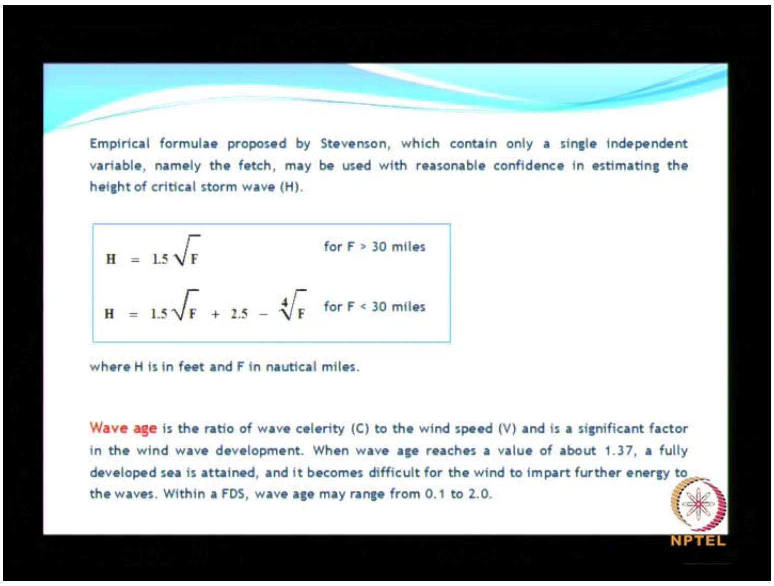

This is in general, the growth of the waves and the growth of the waves these are all age old formulas, very old formulas where they were using it as a function of just a empirical relationship to get the wave height as a function of just a single parameter that is the fetch. A fetch when it is a greater than 30 miles and when there are and when it is less than 30 miles and please note that this is a adjacent feet. This is one of the most oldest formula which is a which has been widely used earlier for the prediction of a wave height from directly from the fetch, it is not a function of any other parameter.

Wave age although we do not come directly from research, I mean the design point of view, but wave age is the one it says it is the ratio of wave celerity to the wind speed. And is this is wave age is a very significant factor in the phenomena of wind wave development and we are talking only about wind waves. Then wave age reaches a value of about 1.37, then we say that a fully developed sea is attained. There has been a number of studies by several investigators who contributed to this wave growth, wind wave growth. I suggested if you are interested read some of the books given at a references. And it becomes very difficult for the wind that to impart further energy to the waves. As I said there is always a an optimum limit up to which the wind is enhancing the growth of the image way on which there will not be any effect.

Within the fully developed sea, the wave age may be ranging anywhere between 0.1 to 2. So, 0.1 more towards, 0.1 would be the partially developed sea. (No audio from 08:00 to

08:08) There are other empirical relationships also and these are all again very old formulas. For example, the maximum wave height is just controlled by if a formula you have a formula, just controlled by wind velocity. The 0.026 into v square the second 1 is a included the fetch and the third 1 gives the wave period that is a function of velocity and the wind velocity and the fetch.

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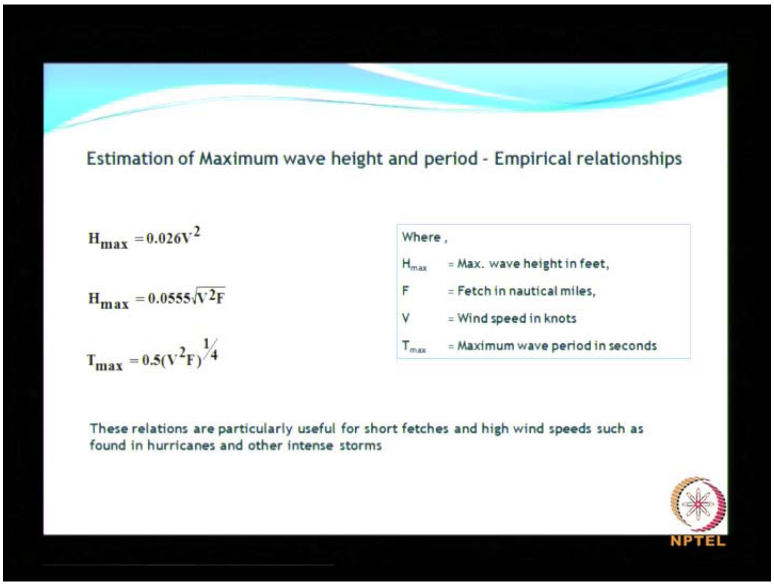

Now, please note the units for the different variables. Of course, this says that this not in much use, but nevertheless it is better we have all these formulas. And our, when we deal with this kind of study is better we aware of formulas that exist. These relationships are particularly useful when you are talking about short fetches. And with dominating wind speed that is very high wind speed such as that you can find in that can occur due to hurricanes and storms, Indian storms.

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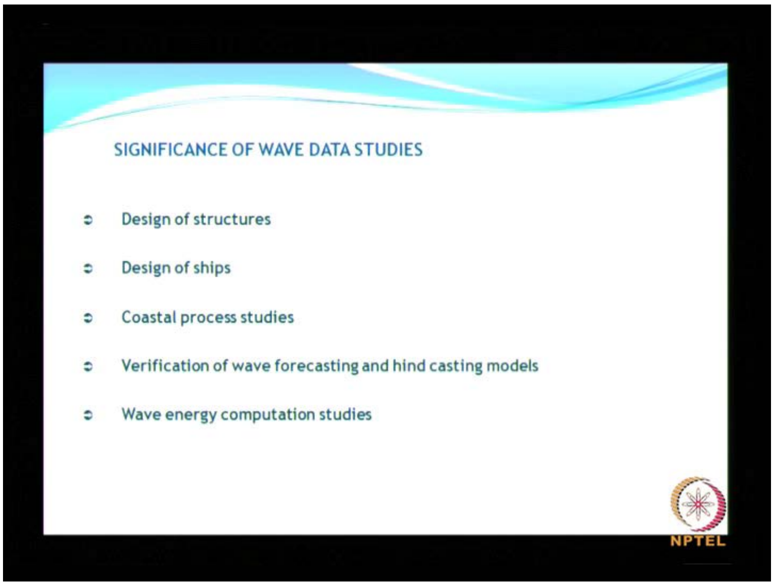

So, in general why do you need to have a information about wave studies, wave data studies? We have, I have already given some kind of introduction in my introduction lecture, but again when you go into the final aspects of wave data these are exclusively important for the design of structures, design of ships, coastal processes of studies, verification of wave forecasting hind casting models, wave cum energy computation and there are variety of other problems. As I say design of structures it covers a host of structures in the coast along the coast as well as in the harbors structure, I am referring to all kinds of structures that is in coastal on the coast or in the deep ocean.

Say for example, it is also wave study data information is needed for obtaining what is called as a scattered diagram.

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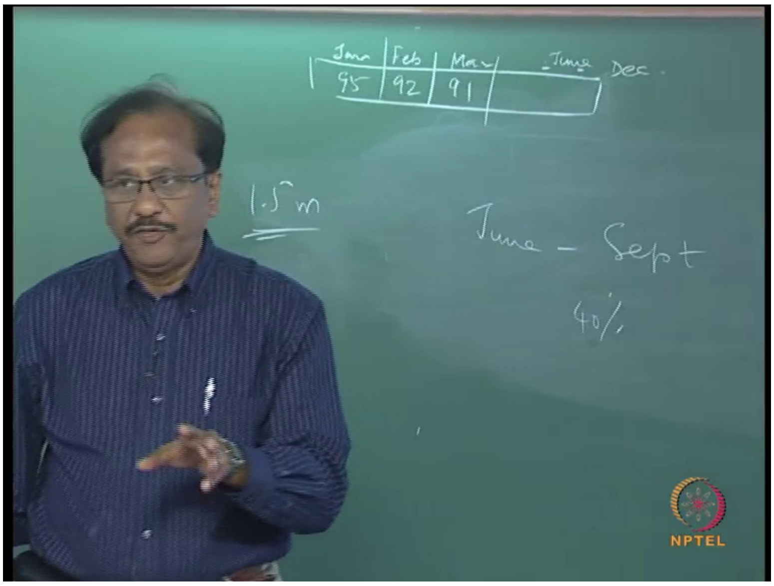

What is a scattered diagram? Scattered diagram is for example, you have H s and T p for example, you have a number of class intervals for H s. Let me say that you have a class interval of 1 then 1.5, 1.5 to 2 etcetera and then here may be 6 to 7, 7 to 8 like this and like this. With the measure data you get all the values. How many waves with same a bit 10, 15 etcetera say this is 9? So, from this you get, how many number of waves as occurred in the month of January or in the month of February or in the month of march etcetera with a combination of wave height varying from 1 to 1.5 meters and wave period associated wave period ranging between 7 and 8 seconds.

So, like this you can have a a scattered diagram which gives the percentage of occurrence of wave height and wave period. Why such information are necessary or we can talk about percentage of occurrence of wave height. For example, what you see percentage of occurrence of wave height less than or equal to 1.5 meters? If someone says 80 percent then in the month of January if someone is able to give me with a help of a, with the data analysis after getting some some figure like this either a scattered diagram or a probability diagram which we will see later.

Then I will be in a position to and if he says that 80 percent of the time the wave height is less than 1.5 meters in the month of March. And then if I am able to carry out specific operations in the off shore in the month of January in such a way that wave height if it is less than 1 .5 meters all my operations can be done. If the wave height is greater than 1.5

meters, I cannot carry out any operations in the off shore then I will prefer January. And suppose in case in the month of February you have wave height percentage of occurrence greater than 1.5 meters, I mean less than 1.5 meters equal to about of the order of say 50 meter 50 percent. Then you see that February is not so helpful compared to January, that is an element of risk involved.

So, this kind of a probability description of waves are in terms of scattered diagram helps us to also assess what is called as the weather window. The weather window in the sense I will try to have say January February march like December. And my limiting wave height for the operations for carrying out any kind of operations in the off shore is again as I said you see around a 1.5 meters. And if it happens for a particular side this is 95 February 92 and March say 91, something like that and may be up to June it is something less than 91.

And suddenly from June to September as in the case of east coast of India you have a percentage of a 1.5 meter less than 1.5 meter is say around 40 percent only approximately or even less than 40 percent. Then I would prefer either I would be prepare to take a risk of for that 40 percent because there is a always an element of risk. Because the percentage of wave height less than 1.5 meters decrease, then you had be careful in planning your off shore operation. So this kind of information is only a an example and we might be able to see in detail when we are working out some problems. And this kind of information are extremely important and for which the wave data measurement and analysis are essential.

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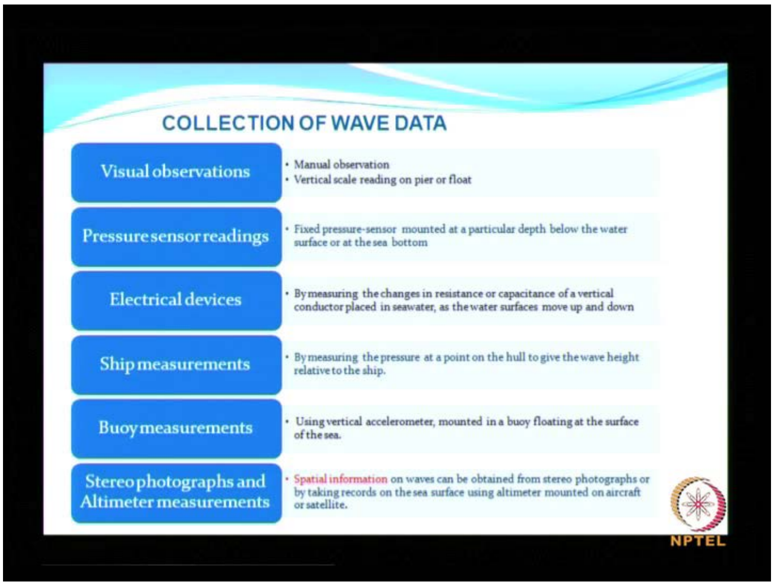

Collection of wave data, collection of wave data you can broadly classified as indicated in the dark blue colour. That is based on either visual observation, pressure sensor recording or electrical devices, ship measurements, (( )) measurements, buoy measurements as well as stereo photographs, and Altimeter measurements. The first one is manual observation, that is visual observation. So, you have a person standing along the coast, looking into the ocean, and then using his eye judgment he will be able asses what is the kind of order of wave height. So, he can only give us the order of wave height, and usually this is done in harbors, where they have some little bit of infrastructure for the person to stand along the coast, and that is what they do. Although it is not reliable measure, and it also depends on the person who is doing this exercise, still it gives some kind of information about the variation in the wave climate.

The other slightly better way of doing it is to have a kind of a marker, guided with a scale along an existing pair which is quite common. May be an existing (( )) and then you have a person who keeps on measuring this difference in the levels as the waves are moving up and down. And of course, make sure that you need to include the tidal range you have apply the correction for the tidal range. That is one way of simplest, but not accurate, but anyway it can give you some amount of information.

The next one is a pressure sensor reading. Where pressure sensors mounted in the flume I mean in the seabed near the coastal seabed can since the pressure when the waves are propagating. And this pressure signal are sent to the on shore stations and then it can be analyzed later. And remember earlier I had told you when we looked at the pressures, dynamic pressures and a progressive waves under the wave dynamics course.

Therein I had mentioned the importance of application of pressure sensors because the basic tsunami pre warning system is based on the pressure sensors. Because it will the moment the pressure sensor sensors a long period wave like that of a tsunami it is quite unusual and immediate it can be made to trigger to the on shore station. So, that is that is a basic kind of a principle on which the tsunami pre warning system are working. And it is also very easy to have the pressure sensors mounted on some kind of a small point in the near shore in order to get the information about the near shore the climate also.

It is quite a handy and also the pressure sensors are also very useful for measuring the pressures not only on in Open Ocean, but in the lab also you can use the same thing. And when it is mounted on structures you use those pressures I mean those reading as a pressure exerted on structures due to waves, which we will be seeing later. Then comes electrical devices, electrical devices by machine which change in the resistance or capacitance of a vertical a conductor placed in sea water. So, sea water is an electrolyte, all of us know. So, when we place a gauge which is going to change is either it is resistance or capacitance, when it is placed in water. The change in the resistance is being transferred to the data associated system and then from that you get the displacement of the water surface about the mean water line.

So, this principle is very, this is the most widely used principle for laboratory measurements of ocean waves and laboratory measurement of waves. So, it is either based on the principle of change in resistance or change in capacitance. Then comes ship measurements by measuring pressure at a point on the hull to give wave height relative to the ship, this is 1 way of doing it. And also in the under the ship measurements you also can have some kind of observers either trained observers or untrained observers to continuously, not continuously at least on a daily basis. Few times in a day they keep finding out what is the approximate wave height etcetera, wave characters in general.

But these wave data may or may not be directly useful to us over a simple reason these wave characteristics are available only in the ship route, along the ship route. So, and then comes the buoy measurements, buoy measurements is the most widely used particularly for major project they go in for buoy measurements. They this is just using the vertical accelerometer mounted in a buoy floating at the sea surface. You can have either directional wave rider buoy or directional wave rider buoy.

So and there are other host of parameters that can be that can be measured with the help of a buoy apart from the displacement of a water surface. And this is small commonly used the buoy measurements are mostly commonly used and more accurate and there is absolutely no problem, there is a problem with the buoy measurements because you have this particularly when it is mounted in shallow water the cutting of the mooring liners. Because the buoy will be something like this it will be more to the sea floor. So if this is cut then you know what happens? Your buoy disappears and the entire exercise of trying to measure the wave climate is lost. Apart from this problem does not much of other technical problem with the measurement of the wave climate using a buoy.

Finally, the special information on waves can be obtained by stereo photographs and also taking records on the sea surface using altimeter mounted on air craft or through satellite. This has become very common these days that the lot of institutes which have been working in this area because you do not have a, you do not need a big laboratory or etcetera to investigate to work on this work in this area. And there lot of things to be done using this and this are more or less I mean more or less quite good the research what the information you get is quite useful. A number of studies have been carried out in order to prove that the results coming out from the altimeter measurements etcetera is quite genuine and quite reliable.

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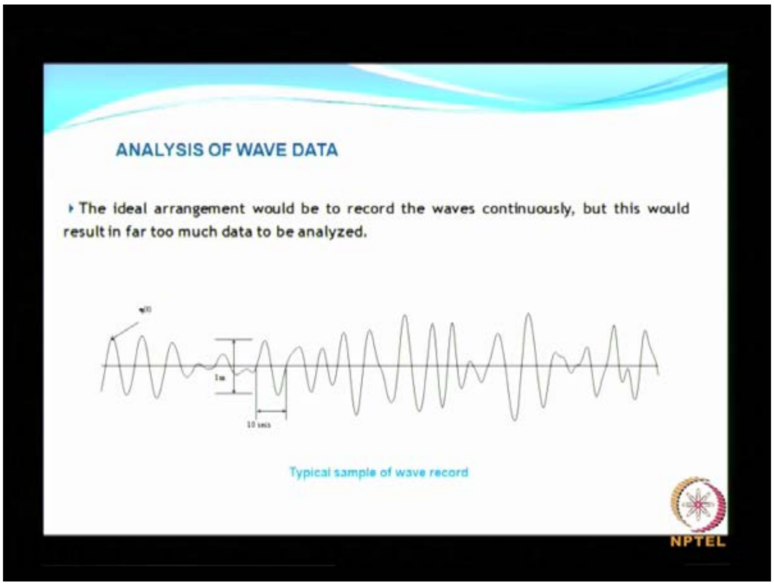

So, that gives us a overall impression about the wave data. The importance of wave data and how you can what all the measures means of measuring the wave data. But before coming into the regular waves, before coming into the random waves, what did we see? We took a regular wave and try to analyze what happens, when a wave with a particular wave height and wave period propagates in a water depth d. So, we looked at the pressure, celerity and then the power, energy etcetera. And then we looked at the whole thing.

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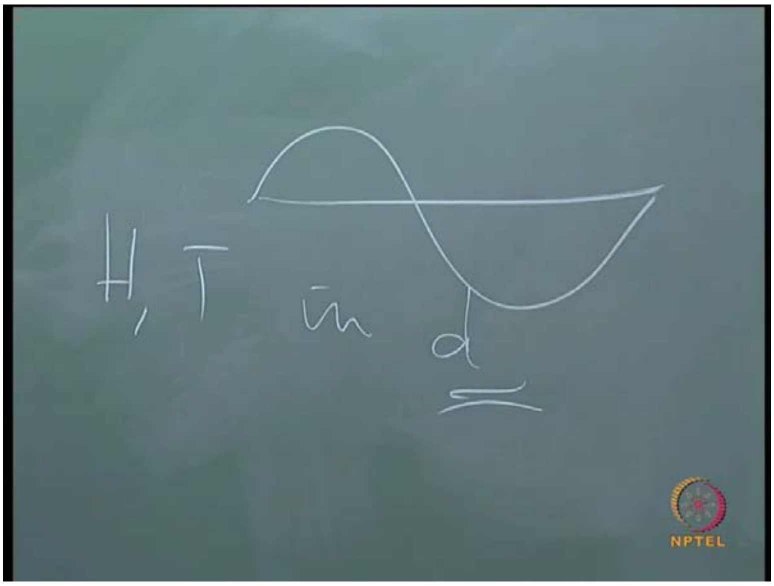

This is in order to get the information of the wave characteristics by just considering one single frequency wave, but in the open ocean it is not like this and try to recollect when we when I presented the basic wave dynamics. The ocean surface will not look like this; you can never have a regular sinusoidal wave moving in the ocean. But instead the surface elevation will be like what you look on the screen. And this is a random wave and this is how it look it would look like. And when you have a random wave like this what you do you cannot have a such a long record and just how do you describe? There should be a way or means to express the characteristics of such a long a random.

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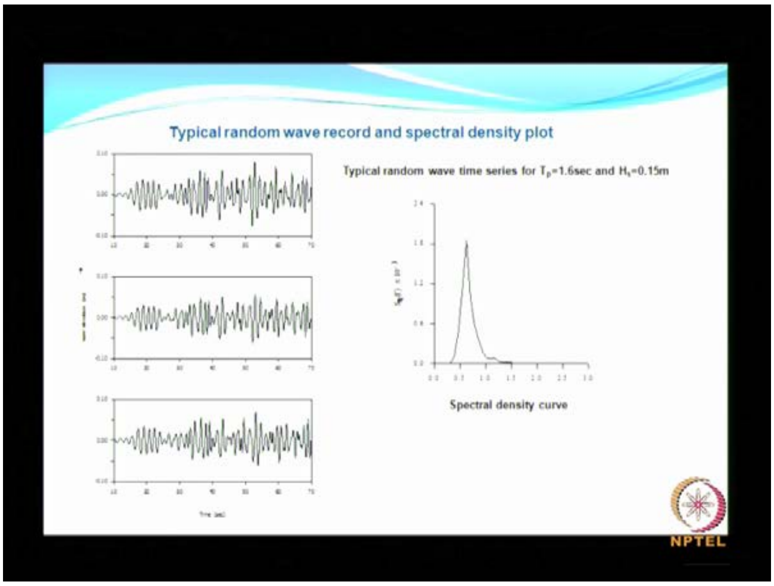

But they use a common way common method is to obtain what is called as the spectral density or the distribution of energy.

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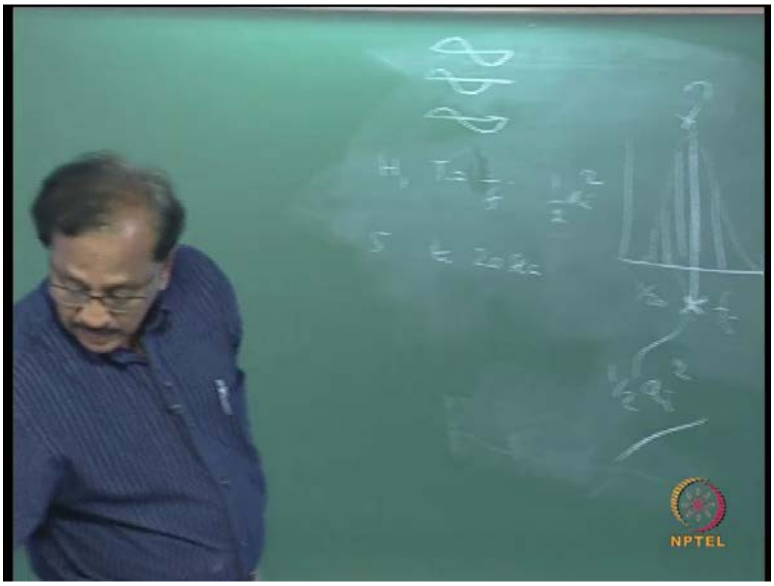

So, I should say as you know the wave has 2 parameters 1 is wave height and wave period or frequency or 1 by f frequency of the time period. So, this is how it would look like and on the right hand side you have see that, if you subject the random wave you should be able to get what is known as a spectral density plot. What exactly is the spectral density? Spectral density gives nothing but the distribution of amplitude square as a function of frequency.

So, for example, you have a wave period has a set say for 5 seconds, from 5 seconds to 20 seconds. So, this will be 1 by 20 to somewhere 1 by 5 and this is the area where you are likely to have your distribution of energy. So, at each frequency you will now know what is the amplitude square? Half amplitude square is nothing but say energy. After all the time history on the left hand side which you see is due to the super position of a number of sinusoidal components of different frequencies and maybe same amplitude or different amplitudes like this there are different frequencies. You please refer to of my earlier lecture where I had shown some slides by superposing a number of sinusoidal components.

Now, here this is how you try to generate a random wave, but now, if you have random wave which is measured in the field, what you do is? You try to arrive at the information as shown here, that is in a through a plot called as spectral density plot. So, if you have this, I can always say what is the frequency of maximum energy? This is the maximum energy. What is the maximum energy, what is the frequency at which the maximum energy could occur? What is the range of frequencies?

Say for example, this is the range of frequencies over which you will have the energy being distributed. So, all this kind of information can be obtained more we will see later. So, how long should be the measurement? Just because you have an instrument that can measure the wave climate or the displacement of the water surface about a mean, can you keep on continuing the, I mean the measurement. So, what will happen? You have a data.

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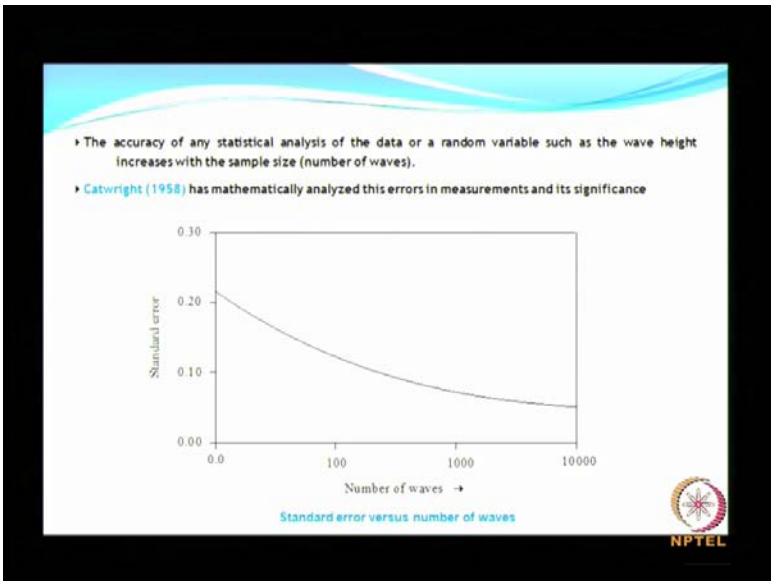

So, there is a kind of compromise you should need so the accuracy of any statistical analysis of the data or a random variable like eta increases with the sample size. That is right. If you want to have the more number of this, the sample size will increase. But the competition time also I mean the number of ways we are going to deal if also going to you will be having lot of data. So, Catwright he mathematically as analyzed and he has said this is how the standard error will look like by increasing the number of waves. So, based on this it was suggested by several investigators in the past that a minimum of hundred waves could be considered for the analysis. A minimum of hundred waves could be considered for the analysis in order to obtain the average wave characteristics or the energy distribution in the random wave, is that clear.

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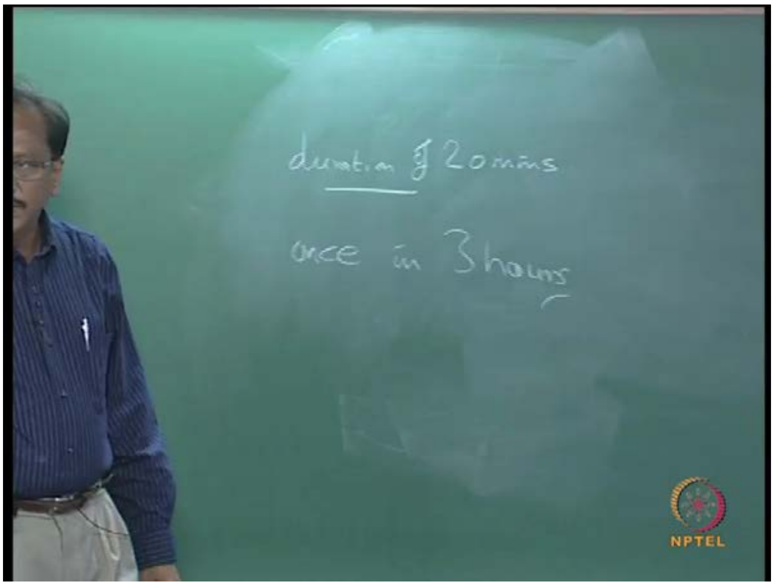

So, the standard practice is to measure the wave climate for a duration of 20 minutes duration continuously. How frequently you need to measure? Again, just because you have a continuous recording equipment, you cannot keep on recording. So, you measure once in 3 hours. What is that once in 3 hours? The assumption is that the basic statistics in the ocean in particular for the waves will remain more or less same, more or less same at least for a duration of 3 hours. So, you consider 3 hours duration. So, for 24 hours that is for 1 day, you will have 8 records typically, 8 values of wave height and period.

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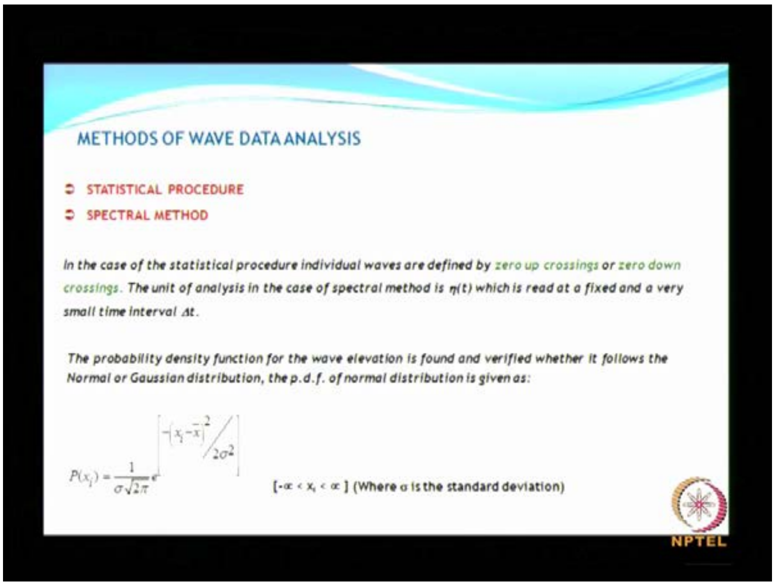

(No audio from 32:24 to 32:39) Having seen the basic information about the random waves, let us move on to methods of wave data analysis. Broadly you have 2 procedures 1 is the statistical method the other 1 is the spectral method. Both these methods are quite straight forward, both of them give lot of information about the wave characteristic that are needed for the design of structures. In the case of statistical procedure the individual waves are defined by 0 up crossing or 0 down crossing. The unit of analysis, in this case in the case of spectral method is trying to extract, the values of eta at a fixed time interval.

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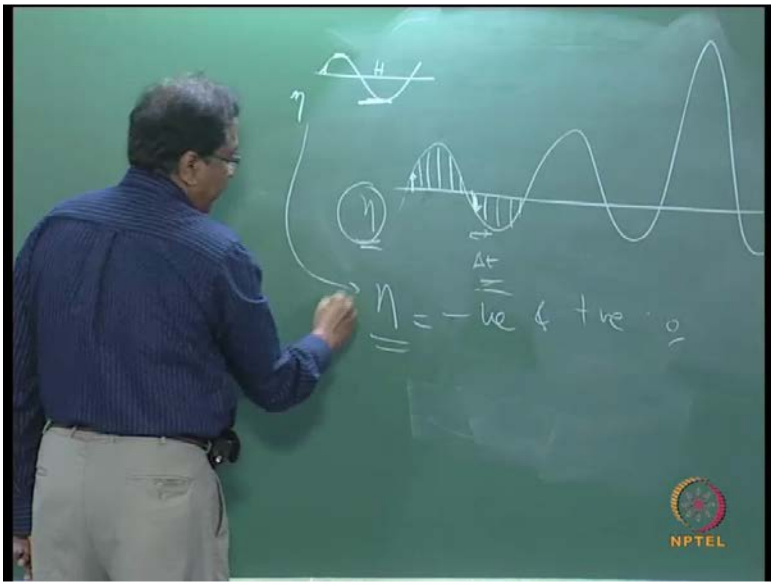

So, you digitized this is how a random wave look like a portion of the random wave. So, at regular intervals so, this is delta t which should be maintained the constant at delta t you digitize the displacement of the water surface about a beam line. So, you will have positive 0 as well as negative values. See usually, most of the cases the probability density of the wave elevation that is probability density, I am sure many of you know. If not, you need to look into the basic statistics book. The probability density function of a wave elevation is found to be mostly following a normal distribution or a Gaussian distribution.

If you refer to books they will also use that wave elevation is a Gaussian process. That means it follows a normal distribution or it is a Gaussian process. This is more or less true when you are having the measurements in slightly deeper waters. But when as you come closer to the shallower waters, as you know the waves becomes slipper it starts filling the bottom and becomes slipper there is always a kind of asymmetries in the wave profile and hence the probability distribution of eta may slightly deviate from normal distribution.

So, when I say eta, eta will have positive as well as negative. Please, remember that, here we are talking about a Gaussian distribution where sigma is the standard deviation and x bar is the mean and x will be taking a value anywhere between minus infinity to plus infinity. When I say eta or the wave elevation, wave elevation can be both negative and positive and of course, 0. But when I say wave height, wave height is usually, what is the definition of wave height?

When we talk about a regular wave, this is the distance between the crust and the trough. So, wave height always has a positive value. But here also we have eta, where eta can have positive negative or 0 value. So, it is more or less most of the locations in the off shore, more or less it will be following Gaussian distribution. So, when you have a measured, wave elevation you are suppose to check whether it follows a Gaussian distribution or otherwise? Because later you see that in literature you should refer to some additional literature. More about the formulas that are applied, applicable that can be applied once you know that eta follows a Gaussian distribution.

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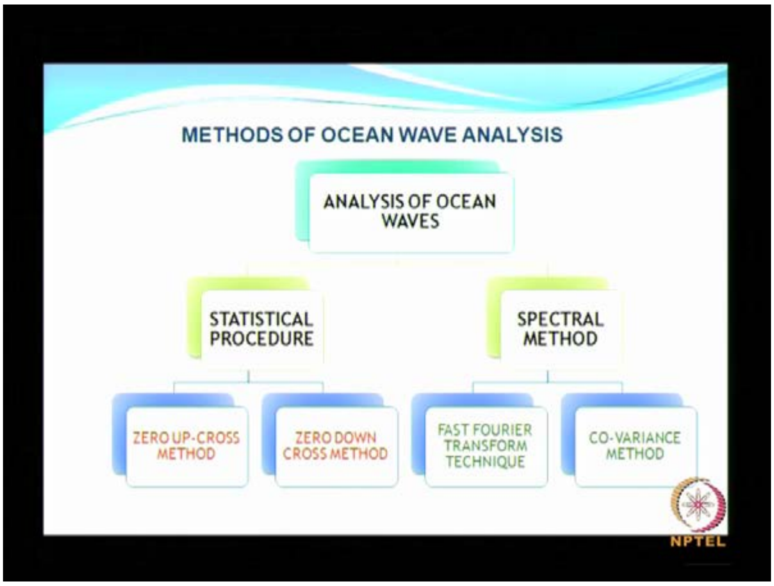

So, methods of analysis of ocean waves statistical method and the spectral method, as I have mentioned earlier. The statistical method can be characterize as categorize as 0 up cross method and 0 down cross method, Whereas, the spectral method can be classified as that obtained through fast Fourier transformation are covariance method.

Earlier it used to be bit tricky in order to get the spectral density. But these days we have a number of software's, standard software's or even in the lab mat lab you can easily get all these information. So, the software that uses, that is widely used for the spectral analysis the wafo. And those who are working on mat lab please think of this wafo and that gives you that is quite handy for solving the problem related to fast fourier transformation, I mean to get these spectral density. The covariance method was used earlier, but these days they are not using it because that is a bit cumbersome.

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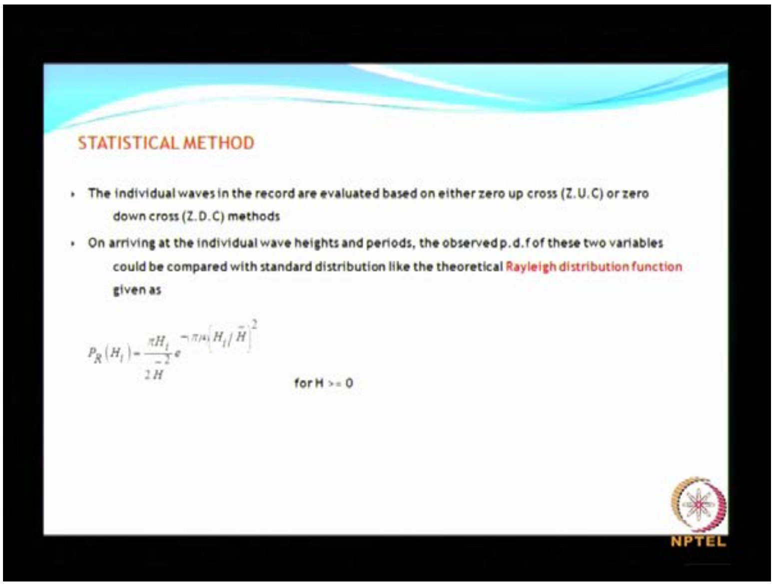

So, statistical method under, the statistical method the waves individual waves are can be obtained using the 0 up cross method or the 0 down cross method. (No audio from 39:36 to 39:43) How do you get that? (No audio from 39:45 to 39:53) How do you separate that? Because I have a, I have another slide which shows the same thing, but I feel that it is better I explain with the help of this. So, let me explain this 0 up cross method and the down cross method with the help of a portion of the wave record which we have got. So, wave recorder may be a long 1. So, I am just taking a portion. So, this is where, this is the mean water line and this is where it is moving in the upward direction and the next point where it crosses the mean water line is somewhere here and that the next cross section is somewhere here.

So now, we see that this is your first wave for example, and this is the wave height 1 using the up cross method, this is the wave period using up cross method similarly, here I will have this as H u 2 and T u 2. So, this is how you can do for the entire record and obtain the and get the individual waves where characteristics of the waves like wave period and wave height through the up cross method. Now, this is where it crosses in the downward direction and then we call this as down cross method.

So, in this case this will be your H d 1 and this will be T d 1. Similarly, here this will be H d 2 and this will be T d 2. So, like this as I said earlier for a good representation of the wave characteristics from a given record you should cover at least a minimum of hundred waves. So, I have the individual waves which can be H 1, H I 3, i equal to 1 comma m H i and T I, is it clear. So, I can have as I said earlier n will be should be greater than or equal to hundred. So, you have enough number of data, then from which you can obtain the statistical measure of the waves. And also we can talk about of the Rayleigh distribution, as the wave profile, wave elevation follows the Gaussian distribution.

And in most of the cases if it follows a Gaussian distribution I mean the wave elevation the wave height is also is expected to follow a Rayleigh distribution for Rayleigh distribution is given. So, I will remove this so it is also available in a the lecturer material lead us to a slide. So, once you have got the individual wave height and the wave period, you can arrive at the probability density of the of the waves. That is when you have the hundred or more than hundred values of the wave height using the simplest statistical ray you can obtain the probability density of the measured wave illusion or the percentage of occurrence of different class intervals for the wave heights for the measured values.

So, now, you have a Rayleigh distribution which is given as shown here it is a single parameter single parameter distribution which is just a function of H bar. What is advantage of this, what is the advantage of saying that I have found that of Chennai the wave height is following Rayleigh distribution with an average of 1.5 meters in the month of June. So, what am I talking about? In the month of June, if I am able to say I am able to say only after proving that the wave height is going to has following a

Rayleigh distribution. If I am, if I have already proved, then I have the liberty of saying if the June month mean wave height is 1.5 meters, I can always say that in the month of June, the wave height follows a Rayleigh distribution of Chennai coast.

So, then the whole lot of information I can get. So, like what is a per percentage of occurrence? I mean the, what is the probability of wave height less than or equal to 1 meter? What is the probability of wave height less than or equal to 2 meters? All this information you can get once you have proved that is follows a Rayleigh distribution, otherwise you need to do that only with the measured probability density. For doing this, for understanding this kind of a information it is enough you have 1 year data. These are also referred to as short term wave statistics for short term wave statistics 1 year data is good enough. And mind you, we are not taking about the design wave height; we are not talking about the design wave. This is for may be operational wave conditions, you can say as operation wave conditions.

So, if you draw the probability density, you will generally know what is the kind of wave climate that exist of a particular site during the different months.

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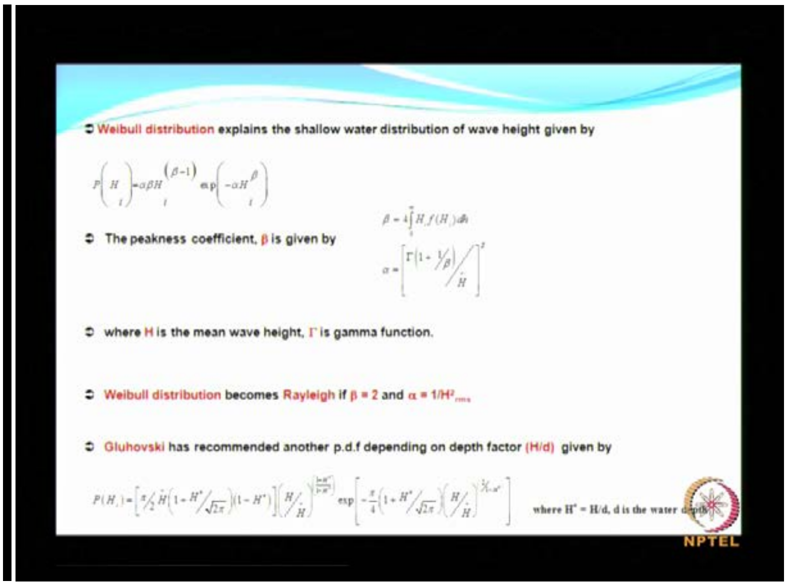

Apart from the widely used Rayleigh distributions, there are other standard distributions for explaining or for discussing about the distribution of wave height. One such distribution is the Weibull distribution, what does the Weibull distribution? This Weibull distribution is very important or explain the distribution of waves, wave heights mostly in shallower waters. As I said the wave slipper near the coastal waters this kind of this distribution has been found quite useful.

Here you are provided here this here all this parameter is nothing but the wave height and you have the beta parameter is nothing but the H i into the frequency of wave height the whole thing that is going to be theta. And below is the 1 which is alpha, the gamma function of 1 plus alpha 1 by beta and divided by the H bar. So, once you have to once you evaluate the beta and gamma use this distribution to get the probability density function. Weibull distribution becomes a Rayleigh distribution, if beta equal to 2 and alpha is equal to 1 by H r m s square. H r m s nothing but the root mean square by u, with a help of a problem we will be able to understand all this things.

There are other types of distributions which are called one is called as Gluhovski distribution, which takes into effect the water depth. The expression is quite long, it is not that widely adopted, it is a compared to Weibull distribution or the Rayleigh distribution. This is only for general information that this expression can be used, where you are talking about this very coastal, I mean in the near shore area. So, here x star which is included there is h by (( )) into picture. So, with this I conclude the basics of the wave statistics and then we will proceed again.