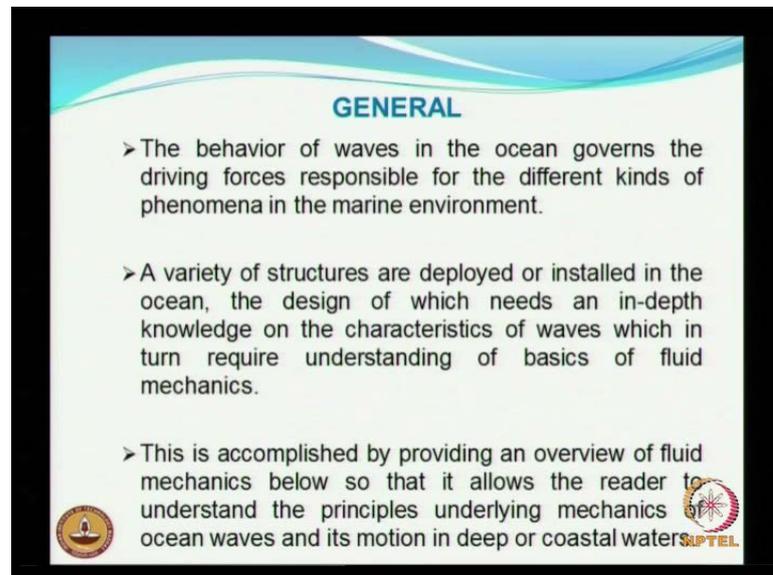


Wave Hydro Dynamics
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Module No. # 01
Basic Fluid Mechanics
Lecture No. # 01
Basic Fluid Dynamics I

Before we get started with the subject on wave mechanics, I just would like to take some time to introduce you to the basics of fluid mechanics. You have a number of textbooks, internet facilities, where you have a number of lectures, PDF files, chapter wise and I am sure that all these things, all these material would be quite helpful in understanding the subject fluid mechanics. Before we get into the wave mechanics, what I have tried to do is just to bring out some of the essential, most essential aspects of this subject, which actually is needed for the bigger topic that is the wave mechanics.

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GENERAL

- The behavior of waves in the ocean governs the driving forces responsible for the different kinds of phenomena in the marine environment.
- A variety of structures are deployed or installed in the ocean, the design of which needs an in-depth knowledge on the characteristics of waves which in turn require understanding of basics of fluid mechanics.
- This is accomplished by providing an overview of fluid mechanics below so that it allows the reader to understand the principles underlying mechanics of ocean waves and its motion in deep or coastal waters.

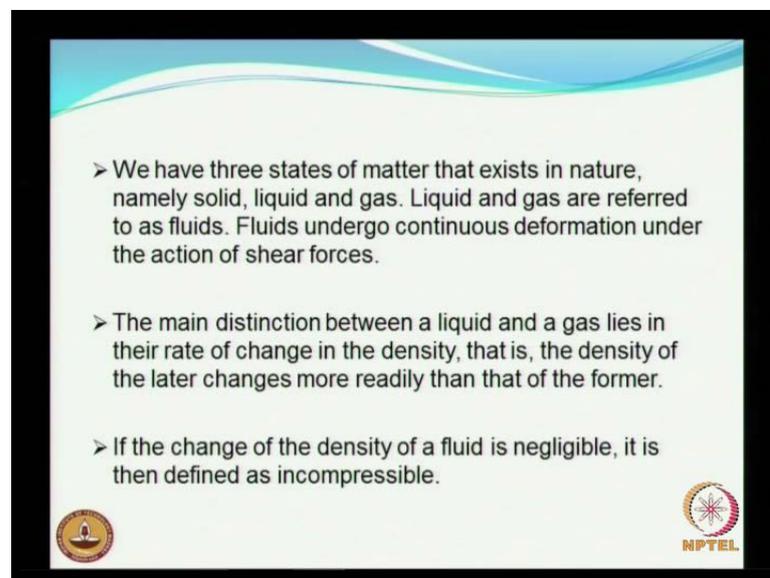
So, I start with the general aspect the behavior of ocean waves or the behavior of waves in the ocean governs the driving forces responsible for the different kinds of phenomena in the marine environment. The different kinds of phenomena that is starting with how

the waves are generated, how they propagate, how they form, how they form and then how they propagate and when they propagate, what are all the kinds of deformations or phenomena it would undergo in the absence or in the presence of any obstructions. All these things will be discussing later.

So, in general, in order to understand all those phenomena the driving forces are very important and the driving forces are governed by the basic fluid mechanics. So, a variety of structures are deployed or installed in the ocean, the design of which needs an in-depth knowledge on the characteristics of waves which in turn require understanding of basics of fluid mechanics.

So, this is accomplished by providing an **overview of fluid mechanics** overview on the fluid mechanics aspects. So, that it allows the reader to understand the principles underlying in the mechanics of ocean waves and its motion in deep as well as in coastal waters. So, the purpose for going through recourse to the fluid mechanics has been highlighted so we get started.

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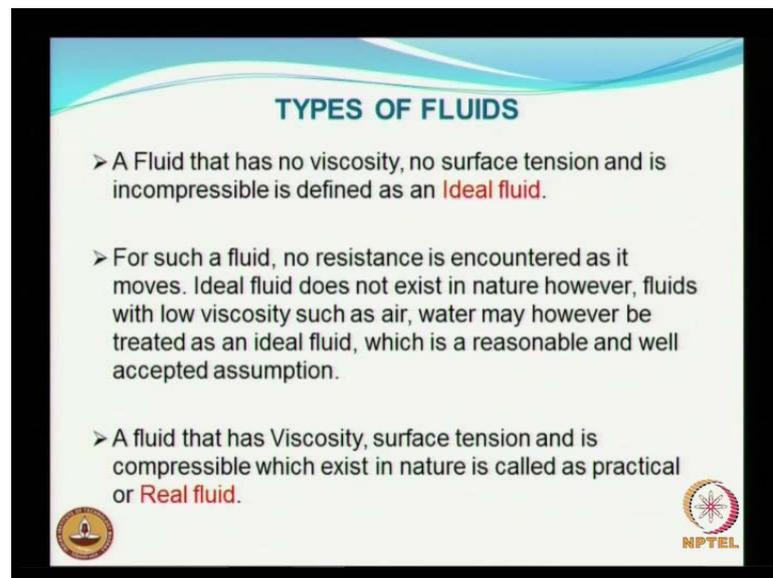
- > We have three states of matter that exist in nature, namely solid, liquid and gas. Liquid and gas are referred to as fluids. Fluids undergo continuous deformation under the action of shear forces.
- > The main distinction between a liquid and a gas lies in their rate of change in the density, that is, the density of the latter changes more readily than that of the former.
- > If the change of the density of a fluid is negligible, it is then defined as incompressible.

We have three states of matter that exist in nature, namely solid, gas and liquid. So liquid and gas are termed as fluids, which undergo deformation under the action of shear stresses or shear forces. Most of them are self-explanatory although they are a self-explanatory, I just briefly discuss about the some of these aspects. Now, the main

distinction between a liquid and a gas lies in the rate of change of density, that is, the density of the gas changes more rapidly or more readily than that of a liquid.

So, that is, the rate at change, rate at which the change of change in the density take place that really governs, whether the fluid is a liquid or it is a gas. It is a change of a density of a fluid is negligible, then we say that it is incompressible, it is defined as incompressible.

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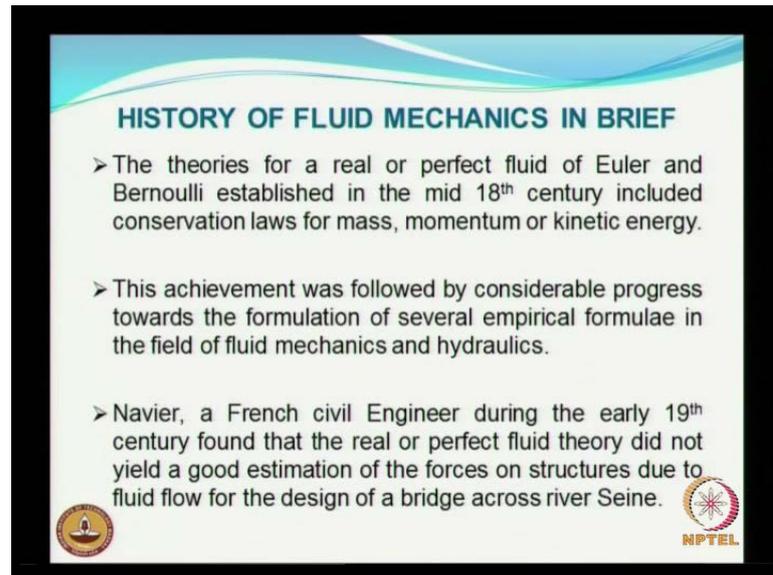
The slide is titled "TYPES OF FLUIDS" and contains three bullet points. The first bullet point defines an ideal fluid as one with no viscosity, no surface tension, and being incompressible. The second bullet point explains that while ideal fluids do not exist in nature, fluids with low viscosity like air and water can be treated as ideal fluids. The third bullet point defines a practical or real fluid as one with viscosity, surface tension, and being compressible. The slide also features a small logo in the bottom left corner and the NPTEL logo in the bottom right corner.

TYPES OF FLUIDS

- > A Fluid that has no viscosity, no surface tension and is incompressible is defined as an **Ideal fluid**.
- > For such a fluid, no resistance is encountered as it moves. Ideal fluid does not exist in nature however, fluids with low viscosity such as air, water may however be treated as an ideal fluid, which is a reasonable and well accepted assumption.
- > A fluid that has Viscosity, surface tension and is compressible which exist in nature is called as practical or **Real fluid**.

So, let us go to the next slide. We basically have two broadly classified fluids, types of fluids. A fluid that has no viscosity, no surface tension and is incompressible is termed as ideal fluid. For such a fluid, no resistance is encountered as it moves. Ideal fluid does not change in nature however, fluid with low viscosity such as that of such as air, water however be treated as in ideal fluid, which is reasonable and well accepted assumption. A fluid that has viscosity, surface tension and it is incompressible which do exist in nature is termed as practical fluids or real fluids. These are the basic definition of a real fluid and an ideal fluids which not very often, some of the students they make a mistake of calling an ideal fluid as ideal flow. Flow is not ideal; it is only the fluid which is ideal. Flow can be irrotational, rotational.

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HISTORY OF FLUID MECHANICS IN BRIEF

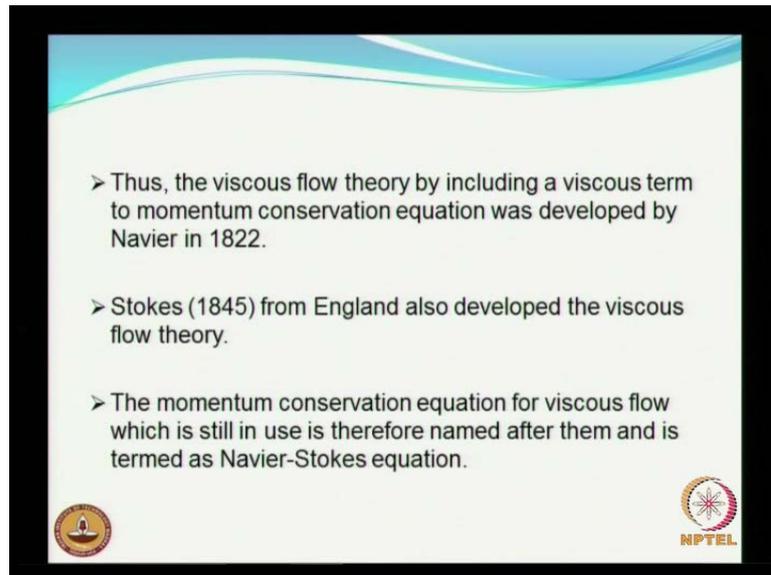
- The theories for a real or perfect fluid of Euler and Bernoulli established in the mid 18th century included conservation laws for mass, momentum or kinetic energy.
- This achievement was followed by considerable progress towards the formulation of several empirical formulae in the field of fluid mechanics and hydraulics.
- Navier, a French civil Engineer during the early 19th century found that the real or perfect fluid theory did not yield a good estimation of the forces on structures due to fluid flow for the design of a bridge across river Seine.

So next, History of fluid mechanics in brief: There are a few books which has beautifully described about the history of fluid mechanics quite in detail and which is quite interesting also, but what I have done is I have just discussed in brief. The theories of a real or real fluid of Euler and Bernoulli, these are the scientist establish, they established in the mid eighteenth century on these theories included the conservation of law, laws of for mass and momentum or kinetic energy. Then this achievement was followed by considerable progress towards the formulation of several empirical formulae in the field of fluid mechanics as well as in hydraulics.

Many of you might have heard the Navier and Stocks. It was Navier; a French civil Engineer during the early 19th century found that the real fluid theory does not yield a good estimation of forces on structures due to fluid flow. When the design of for a bridge across river Seine was made so, this was realized by Navier later and this allowed the viscous flow theory in which a viscous term to the momentum conservation equation was developed by Navier in 1822.

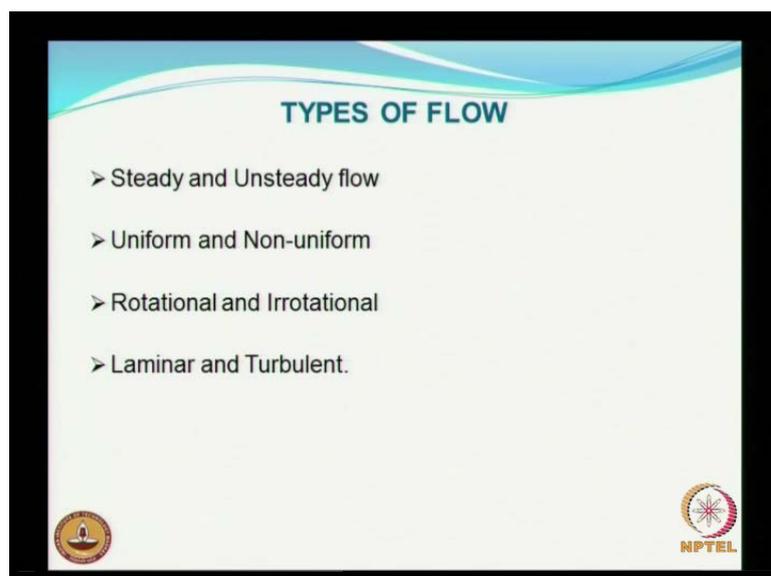
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- Thus, the viscous flow theory by including a viscous term to momentum conservation equation was developed by Navier in 1822.
- Stokes (1845) from England also developed the viscous flow theory.
- The momentum conservation equation for viscous flow which is still in use is therefore named after them and is termed as Navier-Stokes equation.

I am not showing any equation, I am just giving you some of them background. Then just few years around the same time or may be 20 years later it was Stokes from England who also developed viscous flow theory. The momentum conservation equation for describing the viscous flow which is still a use is therefore, named after them and is termed as Navier-Stokes equation. Very often you see what is Navier Stokes many of the equations, many of the phenomena can be well describe by Navier-Stokes equation.

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TYPES OF FLOW

- Steady and Unsteady flow
- Uniform and Non-uniform
- Rotational and Irrotational
- Laminar and Turbulent.

Having gone through the basics of fluid mechanics, very brief we will just look into the types of flow. We have broadly classified eight types of flows: Steady and Unsteady, Uniform or Non-uniform, Rotational or Irrotational, Laminar and Turbulent. Remember these are flows what we had seen earlier is the fluid, ideal and real fluid. Now, flows are given here 8 forms, but there are some combinations that are also possible.

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STEADY FLOW & UNSTEADY FLOW

- > Fluid characteristics such as Velocity, u , pressure, p , density, ρ , temp, T , etc., at any point do not change with time. **E.g. Flow of water with constant discharge rate**
i.e. At (x, y, z) $\left(\frac{\partial u}{\partial t}\right) = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0.$
- > Fluid characteristics do change with time
i.e. At (x, y, z) $\left(\frac{\partial u}{\partial t}\right) \neq 0, \frac{\partial p}{\partial t} \neq 0, \frac{\partial \rho}{\partial t} \neq 0, \frac{\partial T}{\partial t} \neq 0$

Most of the practical problems of engineering involve only steady flow conditions and is simpler to solve than problems of unsteady flow. **E.g. behavior of ocean wave**

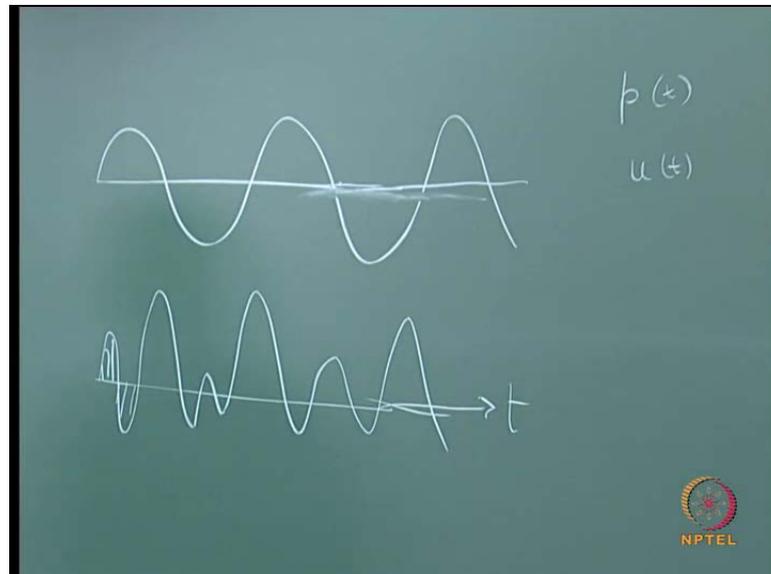
 

So we will just examine one by one, the types of flow, the steady and unsteady flow. Fluid characteristics such as velocity, pressure, density, temperature etcetera, at any point do not change in time. For example, water moving with a constant distance discharge rate so, at any point when you consider the flow of water with constant discharge rate or in general that is only an example, in general first, this kind of a flow at any point x, y, z the rate of change with respect to time, the rate of change of the property which we are discussing u, p or ρ with respect to time is zero that is do not change with time.

If, it is a changing with time if, it changes with time what is it? Unsteady flow. So, for example, if it changes with time then for example, here I have taken the behavior of ocean waves. What do you mean by behavior? Why ocean waves are unsteady? You will see that later how a wave varies, In the case of a regular wave, if you measure the waves in the deep ocean so, this is the surface elevation and you see that the surface elevation is going to vary with respect to time. When you measure the open ocean or if you later you

will also see that the waves are can be this kind of a wave can approximately said to be represented can be represented as a sinusoidal wave.

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So, these waves you see that when they are moving on the surface it will have its particle velocities and other characteristics that will be changing within the fluid medium. So, all those changes like the change with the pressure etcetera or velocities will all be changing with respect to time. So, that is why we say that it is an unsteady flow.

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UNIFORM FLOW

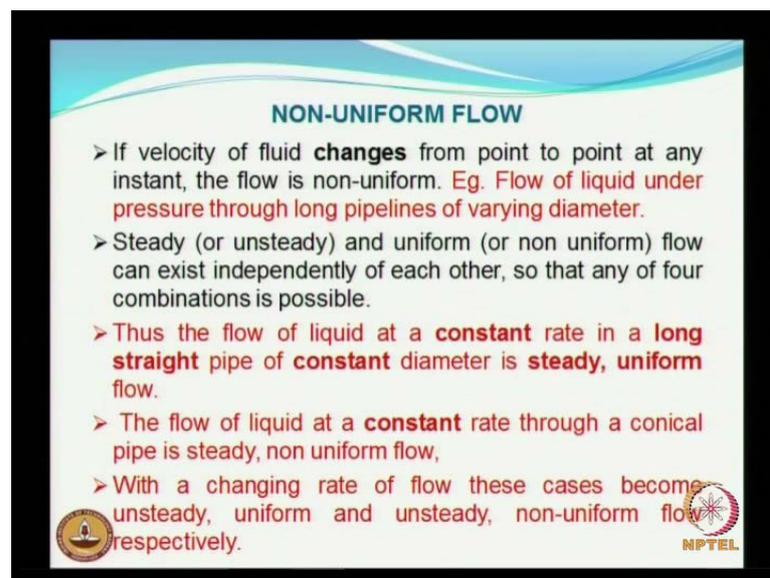
At any given instant of time, when fluid properties **does not change** both in magnitude and direction, from point to point within the fluid, the flow is said to be uniform.

$$\left(\frac{\partial u}{\partial s} \right)_{t=t_1} = 0$$

The NPTEL logo is visible in the bottom right corner of the slide.

Now, Uniform Flow: At any given instant of time, when the fluid particles as stated earlier u, pressure, density etcetera does not change in both magnitude and direction from point to point not from time to time from point to point within the fluid, the flow is said to be that is why we are showing it as $\frac{du}{ds}$. S is with respect to space. So, the variation of velocity with respect to space at any given of point of time is going to be equal to 0.

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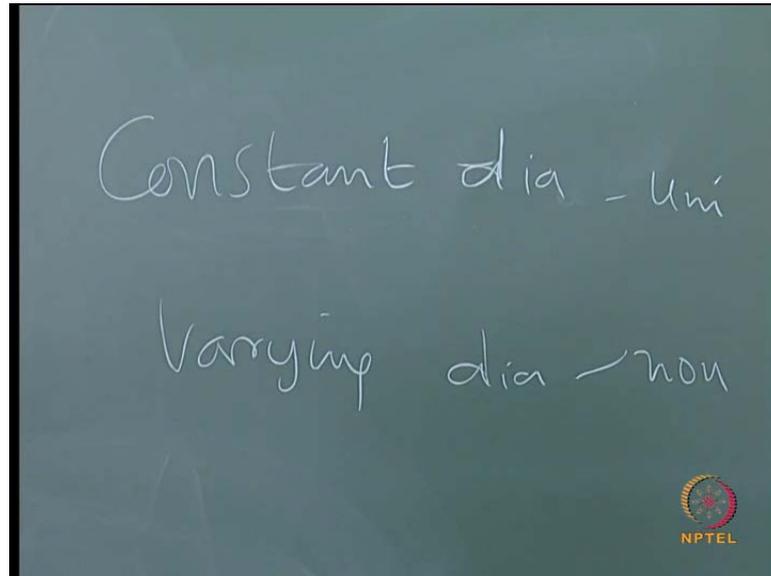
NON-UNIFORM FLOW

- > If velocity of fluid **changes** from point to point at any instant, the flow is non-uniform. Eg. Flow of liquid under pressure through long pipelines of varying diameter.
- > Steady (or unsteady) and uniform (or non uniform) flow can exist independently of each other, so that any of four combinations is possible.
- > Thus the flow of liquid at a **constant** rate in a **long straight** pipe of **constant** diameter is **steady, uniform** flow.
- > The flow of liquid at a **constant** rate through a conical pipe is steady, non uniform flow,
- > With a changing rate of flow these cases become unsteady, uniform and unsteady, non-uniform flow respectively.

So, Non-Uniform Flow: If the velocity changes, if the velocity of fluid, when I say velocity it also includes the other properties of the fluid. If the velocity of fluid changes from point to point at any given time instant, then it is called as non-uniform flow the ones which are having the red color font that shows only the examples for the different kind of flows. Here in the case of non-uniform flow, flow of liquid under pressure through long pipe lines of varying diameter. Suppose, if the diameter is constant, **if this diameter is constant**, the same flow of fluid under pressure will be uniform flow.

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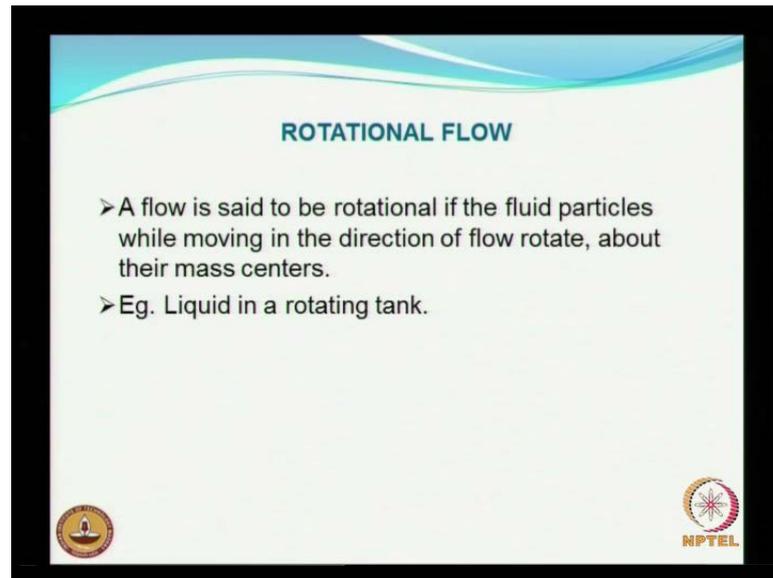
So, now this is constant dia and now, we have varying dia. That is going to take care of whether the flow is going to be non-uniform or uniform. Now, under steady or unsteady and uniform or non-uniform, flow can exist independently of each other that is you can have steady or unsteady, uniform or non-uniform, but they can also have combinations. Combinational are also possible.

So, for example, if you have a flow of liquid under a constant rate and it is in a long pipe line constant diameter. So, you see that constant rate, constant diameter. So, here I have said varying diameter, constant diameter. So, if it is a constant rate in a long diameter, but the flow is steady, I mean the diameter is constant, then you have steady uniform flow. Then the flow of liquid whatever I have highlighted that is very important. So, the flow of a liquid at a constant rate through a conical pipe that is steady, because constant rate is there so you have a constant rate here, but the shape of the pipe is different.

So, the shape of that pipe dictates whether the flow is going to be uniform or non-uniform and the rate at which it is pumped that is going to be controlling whether the flow is going to be steady or unsteady. So, you have a control of the types of flow you want to generate. So, with the changing rate of flow these cases become unsteady uniform and unsteady become unsteady uniform, unsteady or non-uniform flow all these cases are possible.

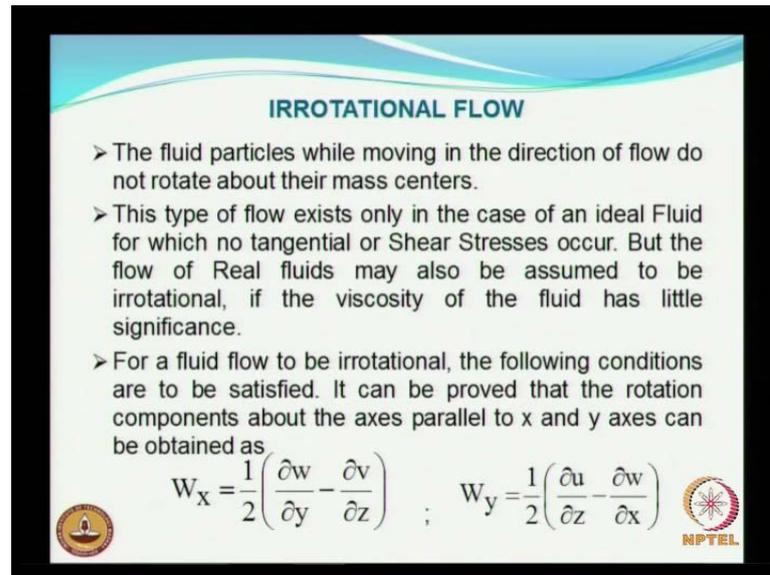
So, although these are very fundamental I have noticed that there is a kind of confusion some of the students express when they are asked for some question they fumble and they get very easily confused between uniform, steady, unsteady, non-uniform. So, please remember the kinds of examples, the examples which is if you remember the example that will clearly indicate the type of flows etcetera.

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You have what is meant by rotational flow: A flow is said to be rotational, if the fluid particles while moving in the direction of flow rotate, about their mass centers that is natural so it has to rotate about its mass centers so, for example, liquid in a rotating tank so, that is a classical example for **anirrotation[al]**- for a rotational flow very often we deal with irrotational flow and some of the derivations you will start the flow is ideal, fluid is ideal, flow is irrotational. for example, when we derive the velocity potential in order to understand the wave mechanics that how the wave propagates etcetera. we will be forced to derive a velocity potential.

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IRROTATIONAL FLOW

- The fluid particles while moving in the direction of flow do not rotate about their mass centers.
- This type of flow exists only in the case of an ideal Fluid for which no tangential or Shear Stresses occur. But the flow of Real fluids may also be assumed to be irrotational, if the viscosity of the fluid has little significance.
- For a fluid flow to be irrotational, the following conditions are to be satisfied. It can be proved that the rotation components about the axes parallel to x and y axes can be obtained as

$$W_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) ; \quad W_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

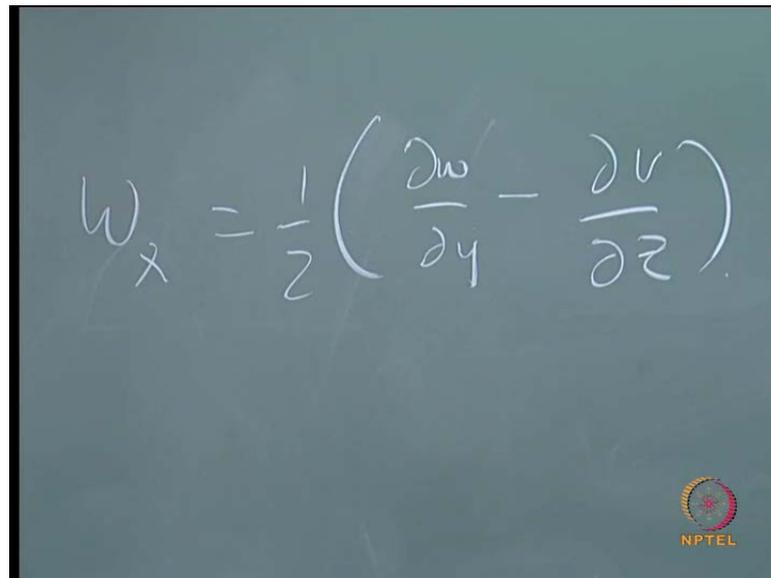
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The basic assumption there is the fluid is ideal, flow is irrotational. So, the fluid particles while moving in the direction of flow **do not** do not rotate about their mass centers. This type of flow exist only in case of ideal Fluid. Why ideal fluid? Only in ideal fluid there is no tangential or Shear Stresses that would occur. Although the subject is quite simple you should remember all these definitions forever.

So, it says that the flow this type of flow that is irrotational flow exist only in the case of an ideal fluid for which no tangential or shear stresses occur, but if the viscosity of the fluid is very small or very less or does not have much of significance then in that case even this kind of flow can be assume even in the case of real fluids. For example, when I told in the beginning air and water are assumed as ideal fluid, because it has very low significance, low viscosity etcetera.

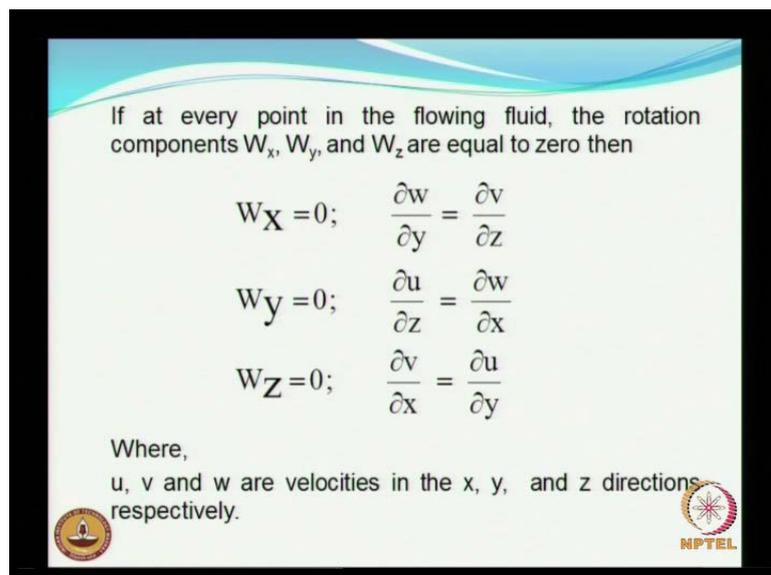
So, it is only an assumption and the assumption is quite acceptable. Now, for a fluid flow to be irrotational, the following conditions are to be satisfied. What are the following conditions? That is it may easily be proved that the rotational components about axes parallel to y and x or x and y axes can be are said here. dou y that is W_x half into similarly, for W_y along the y axes and this with respect to the x axes.

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$$W_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

The image shows a chalkboard with the equation $W_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$ written in white chalk. In the bottom right corner, there is a small circular logo with a star and the text 'NPTEL' below it.

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If at every point in the flowing fluid, the rotation components W_x , W_y , and W_z are equal to zero then

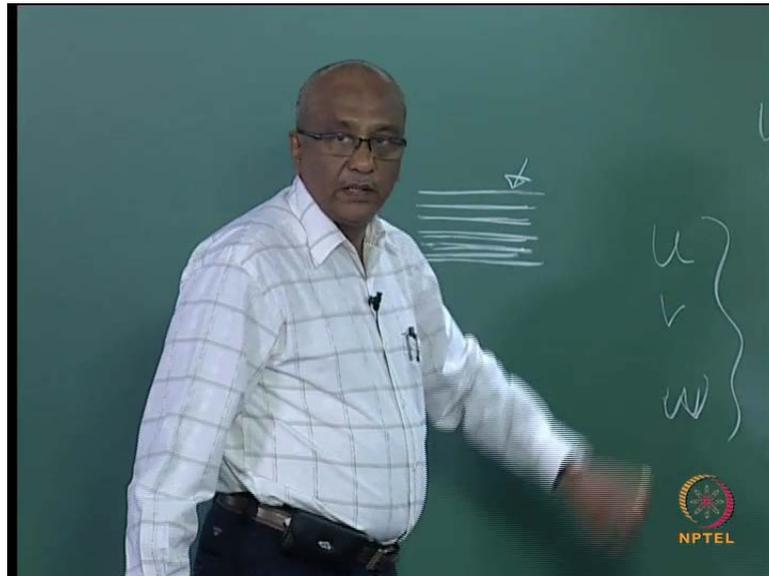
$$W_x = 0; \quad \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$
$$W_y = 0; \quad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$
$$W_z = 0; \quad \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Where,
u, v and w are velocities in the x, y, and z directions respectively.

The slide has a light blue header with a wavy pattern. It contains the text and three equations. At the bottom left is a circular logo with a lamp, and at the bottom right is the 'NPTEL' logo.

So, if at every point in a fluid which is flowing, the rotational components as indicated here W_x equal to zero, W_y is equal to zero, and W_z equal to zero and then automatically you will get these expressions on the right hand side. That is $\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$. So, if when you have a particle motion or the velocities of the particle in a fluid moving in the x direction and y direction.

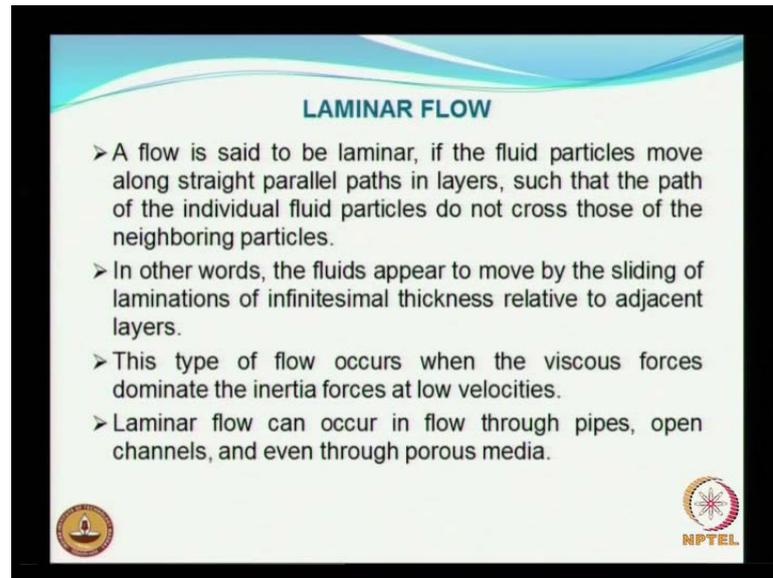
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So, you have u and v , u v and w then with this using this differentiation etcetera this relationship you will be in a position to find out whether the flow with that kind of velocities will that be following will that can be can it be termed as an irrotational flow or otherwise by simply trying to get the dou this relationship through... So, u v w are the velocities in the x y z directions.

Now, we move on to laminar flow. A flow is said to be laminar if the fluid particles move along straight parallel parts in layers that is when you we surely see the particles will be moving in layers. This can easily simulated in a lab, by having a controlled discharge over glass tube and this is the classical example that is classical exercise done in under graduate course.

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LAMINAR FLOW

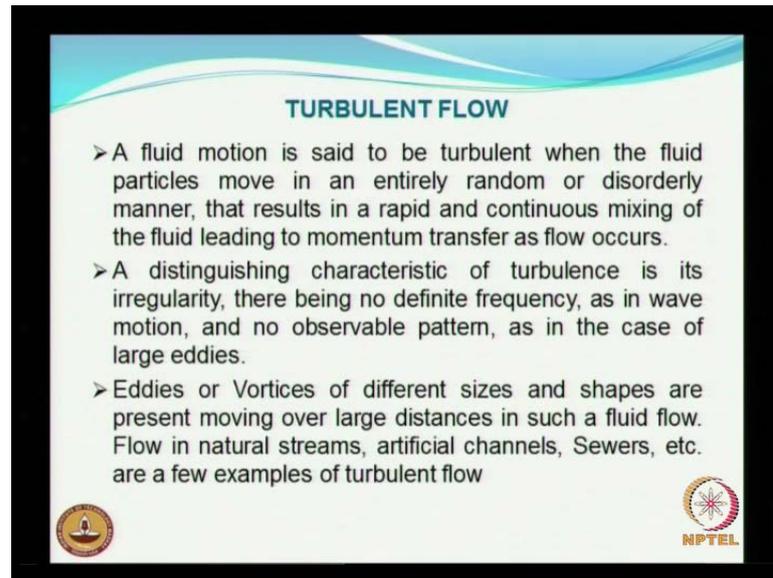
- A flow is said to be laminar, if the fluid particles move along straight parallel paths in layers, such that the path of the individual fluid particles do not cross those of the neighboring particles.
- In other words, the fluids appear to move by the sliding of laminations of infinitesimal thickness relative to adjacent layers.
- This type of flow occurs when the viscous forces dominate the inertia forces at low velocities.
- Laminar flow can occur in flow through pipes, open channels, and even through porous media.

So, you can see that if the fluid particles are move along straight parallel path in layers, such that the paths of the individual fluid particles do not cross the neighboring particles. So, each particle will be moving in layers or in laminate in other words fluids, the fluids appear to be moving or to move by sliding laminations, something like sliding over. So, this type of flow occurs when viscous flows dominate the inertia force and particularly at low velocities. So, what you do you have a tube and you inject some die so, that you can see the path and you control your reynold's apparatus this is what I am trying to explain.

So you can just try to control your discharge and if the discharge is very less and you control it in such a way that you can easily visualize the flow that is moving. In kind of you can generate a kind of a laminar flow. So, laminar flow can occur in flow through pipes or open channels or even through porous media. Of course, laminar flows are much easier for us to solve compare to turbulent flows.

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TURBULENT FLOW

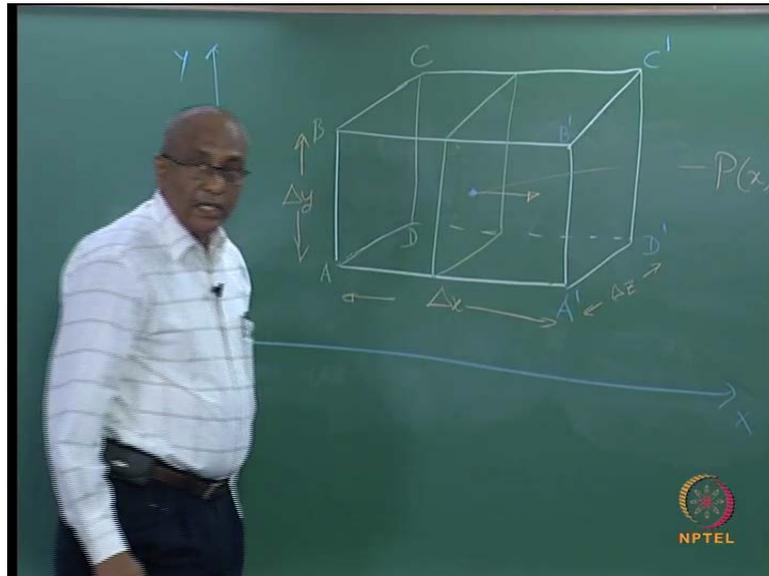
- A fluid motion is said to be turbulent when the fluid particles move in an entirely random or disorderly manner, that results in a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs.
- A distinguishing characteristic of turbulence is its irregularity, there being no definite frequency, as in wave motion, and no observable pattern, as in the case of large eddies.
- Eddies or Vortices of different sizes and shapes are present moving over large distances in such a fluid flow. Flow in natural streams, artificial channels, Sewers, etc. are a few examples of turbulent flow

Turbulent Flows: This is just an opposite to the laminar flow. A fluid motion is said to be turbulent when the fluid particles move in entirely random or disorderly manner, that results in a rapid and continuous mixing of fluid leading to momentum transfer as flow occurs. See as far as this turbulent flow is concerned, it has a kind of a direct connection with our kind of studies in particularly when it is a either wave mechanics or coastal engineering. So for some of this turbulent flow you will be a coming across I will we are discussing well while I am describing about some of the phenomena that we come across in both the subjects like wave mechanics as well as in the field of coastal engineering.

A distinguish characteristic of turbulence is its irregularity, **there is** there being no definite frequency, as in the case of wave motion, and no observable pattern, as in the case of large eddies. So, eddies are what is this a different shapes and sizes are present over large distances in such a fluid flow. Flow in natural streams, artificial channels, Sewers, etcetera.. are all few examples of turbulent flow. It is quite difficult to deal with turbulent flow compare to laminar flow, because it is laminar flow is a kind of a flow which can be termed whereas, turbulence flow it is very difficult to term the type of flow because it is going to be moving in random.

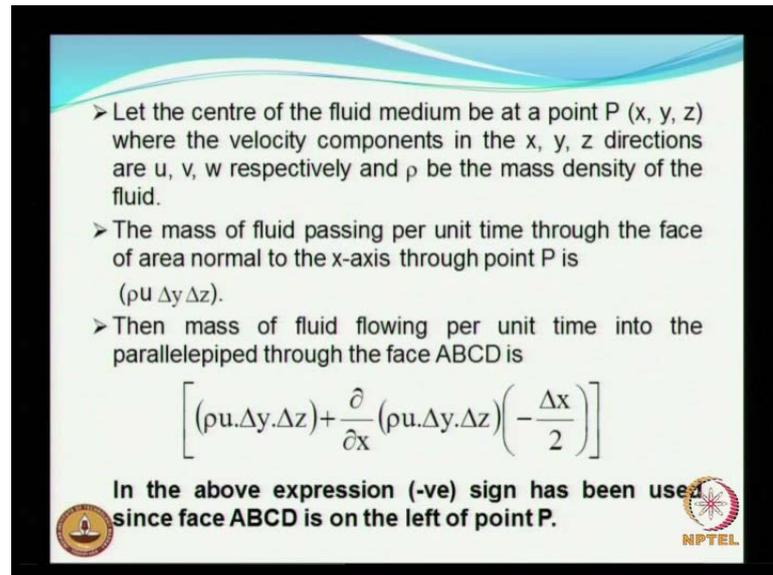
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So, having seen the types of flows in brief we will just look at one of the important equation that is the continuity equation and continuity equation that is its speaks or it tells about the conservation of mass. Now, let me so I am considering a point A B C and D. Now, this is A dash B dash C dash and D dash and I am identifying an element a point at this location at the center and my axes is x and y and of course, you have the z.

Let me take this distance as delta x and this will be delta y and I have delta z. So, my point here is defined as P of x y z. Now, when you consider an elementary rectangular parallelepiped with sides as indicated, the planes are also indicated here which we are going to consider the flow is from left to right and it starts at the center point. Consider a plane running at the center point. So, through this plane, at this point center point the flow is taking place and the mass of the fluid will be as indicated here.

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➤ Let the centre of the fluid medium be at a point P (x, y, z) where the velocity components in the x, y, z directions are u, v, w respectively and ρ be the mass density of the fluid.

➤ The mass of fluid passing per unit time through the face of area normal to the x-axis through point P is $(\rho u \Delta y \Delta z)$.

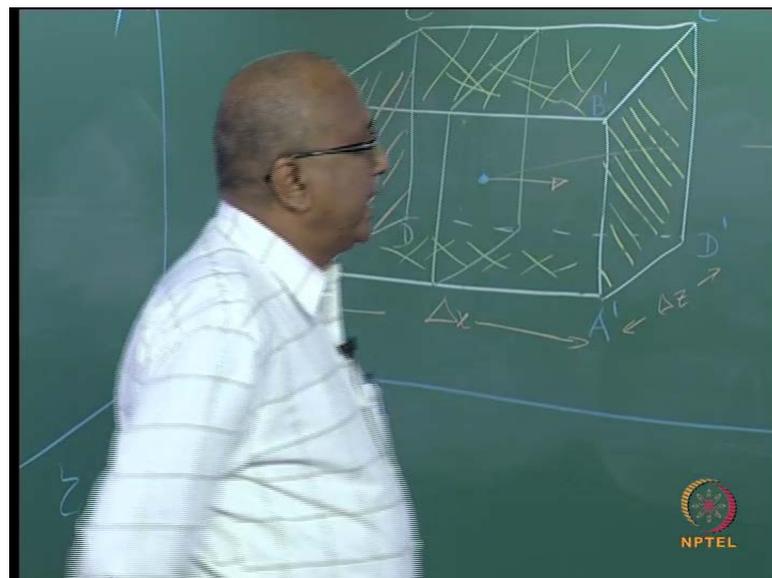
➤ Then mass of fluid flowing per unit time into the parallelepiped through the face ABCD is

$$\left[(\rho u \Delta y \Delta z) + \frac{\partial}{\partial x} (\rho u \Delta y \Delta z) \left(-\frac{\Delta x}{2} \right) \right]$$

In the above expression (-ve) sign has been used since face ABCD is on the left of point P.



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So, now this as we consider that u v and w are the velocities in x y z direction and rho is the mass density so, the mass of the fluid that is going to pass per unit time through the phase normal to the x axis. At point p will be rho into u into delta y and delta z that is the area, this is the area. Now, what is the mass? What is the then the mass of the fluid that is flowing per unit time into this through the phase A B C D. So, we are coming we are talking about this phase this phase will be the mass that is rho u delta y and delta z that will be entering but, what about the rate of change, rate of change will be with respect to the x direction that has to be added.

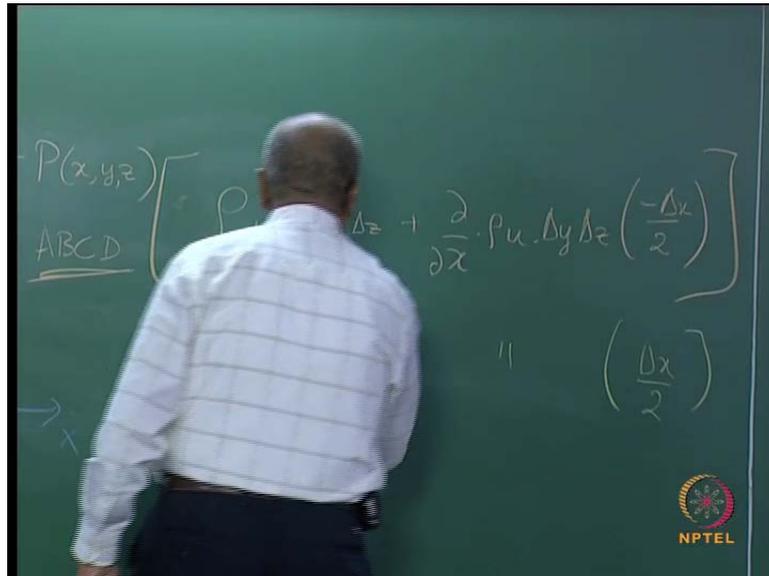
So, we add that as Δx , ρu by Δx with respect to x . This is the mass, but over what distance over? because the point is on the in the center the distance will be Δx divided by 2, but it will have a negative sign because the reference point which we have taken is a center and the phase which we consider is the on the left hand side, that is on the other side of the opposite side of with that with that of the direction of flow and that is the reason why we have minus Δx by...

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$$\rho u \Delta y \Delta z + \frac{\partial}{\partial x} (\rho u \Delta y \Delta z) \left(\frac{-\Delta x}{2} \right)$$

So, I write this as into plus, ρu divided by Δx . So, this will be the mass fluid flowing per unit time through the phase A B C D. Now, you consider you repeat the same thing by considering this area, this phase, this phase will be practically the same except that **except that** because it is on the on the side towards a positive direction of the flow so you will have the same thing that is what is written here.

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Similarly the mass of fluid flowing per unit time out of the fluid medium through the face A' B' C' D' is

$$\left[(\rho u \Delta y \Delta z) + \frac{\partial}{\partial x} (\rho u \Delta y \Delta z) \left(\frac{\Delta x}{2} \right) \right]$$

Therefore, the net mass of fluid that has remained in the fluid medium per unit time through the pair of faces ABCD and A'B'C'D' is obtained as

$$= \frac{-\partial}{\partial x} (\rho u \Delta y \Delta z) \Delta x$$

$$= \frac{-\partial}{\partial x} (\rho u) \Delta x \Delta y \Delta z$$

So, you have a fluid flow per unit time in this over this phase and over this phase is that clear we have established that. Now, having established that we need to get the net mass of fluid that has remained in the fluid medium, after all we are considering this as a fluid medium and what is the net mass of fluid that has remained in this fluid medium per unit time through this pair of phases, because we are taking only these two pair of faces. So, that will be this minus this then you will get as shown here.

But then delta x is not going to be a function of x, because we have considered a an element of pure fluid. When we consider an element of fluid the sides are the lengths are fixed. Hence the delta x can be removed out, but we retain the variation of u and density with respect to space not with respect to time. Here, with respect to space, it is retained.

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> The area $(\Delta y \cdot \Delta z)$ has been taken out of the parenthesis since it is not a function of x .
 > By applying the same procedure the net mass of fluid that remains in the cube per unit time through the other two pairs of faces of the cube may also be obtained as

$$\frac{-\partial}{\partial y}(\rho v) \cdot (\Delta x \cdot \Delta y \cdot \Delta z)$$

(Through pair of faces AA'D'D and B B'C'C)

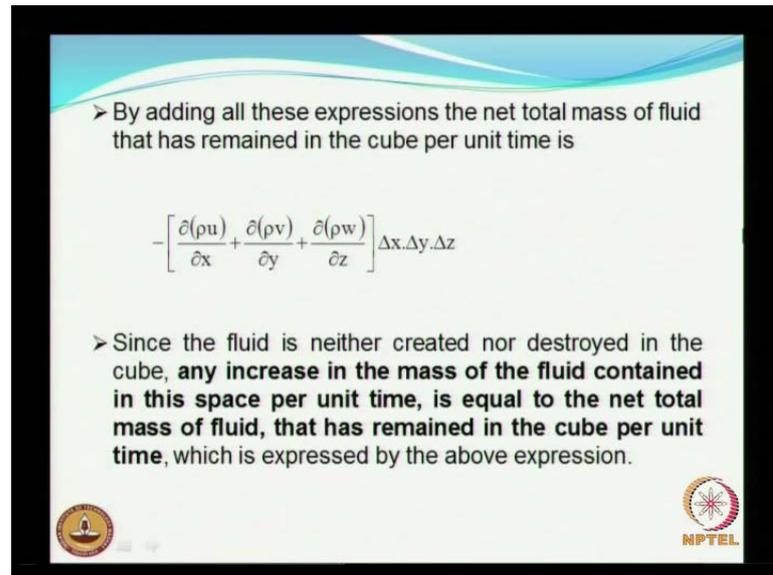
$$\frac{-\partial}{\partial z}(\rho w) \cdot (\Delta x \cdot \Delta y \cdot \Delta z)$$

(Through pair of faces D D'C'C and AA' B'B)




Now, by applying the same procedure the net mass that remains through other two phases can also be obtained instead of here we have u so, you will have v and w that is all. So, we consider this phase and this phase **so, this phase and this phase and** similarly, the other two phases that is this phase and the other phase behind the board. So, by adding all these three, we get the net total mass of fluid that has remained in the cube per unit time, which is going to be as indicated here not indicated as calculated or as derived.

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➤ By adding all these expressions the net total mass of fluid that has remained in the cube per unit time is

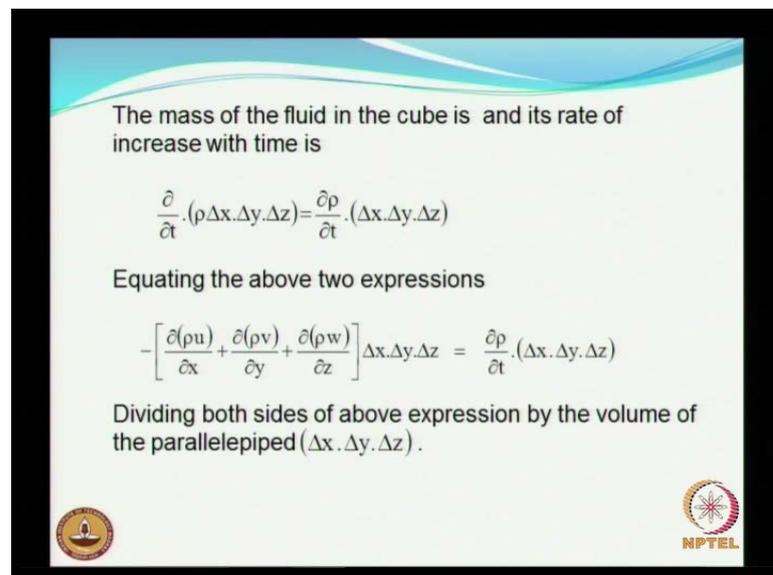
$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \Delta x \cdot \Delta y \cdot \Delta z$$

➤ Since the fluid is neither created nor destroyed in the cube, **any increase in the mass of the fluid contained in this space per unit time, is equal to the net total mass of fluid, that has remained in the cube per unit time**, which is expressed by the above expression.



So, what does a conservation say conservation law since the fluid is neither created nor destroyed in the cube, any increase in the mass of fluid contained in this space per unit time, is equal to the net total mass of fluid, that has remained in the cube per unit. So, there is no creation or removal, no addition or reduction or removal.

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The mass of the fluid in the cube is and its rate of increase with time is

$$\frac{\partial}{\partial t} (\rho \Delta x \cdot \Delta y \cdot \Delta z) = \frac{\partial \rho}{\partial t} (\Delta x \cdot \Delta y \cdot \Delta z)$$

Equating the above two expressions

$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \Delta x \cdot \Delta y \cdot \Delta z = \frac{\partial \rho}{\partial t} (\Delta x \cdot \Delta y \cdot \Delta z)$$

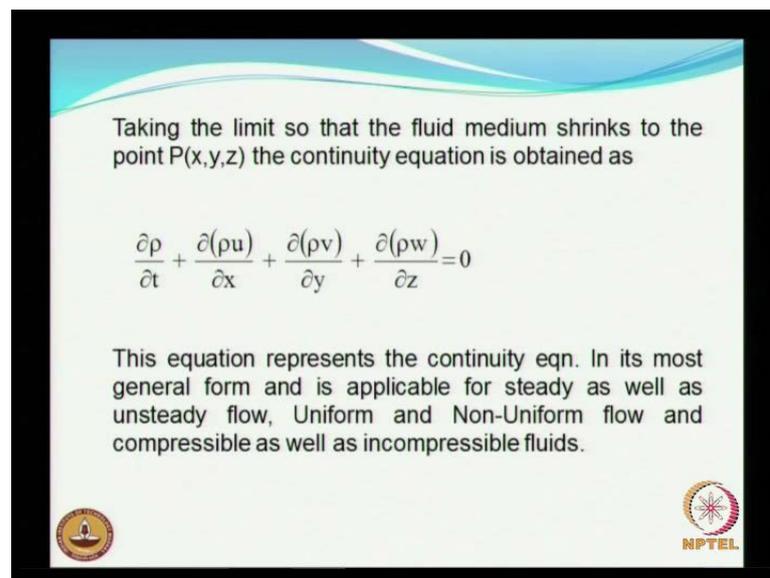
Dividing both sides of above expression by the volume of the parallelepiped $(\Delta x \cdot \Delta y \cdot \Delta z)$.



So, now the mass of the fluid in the cube and its rate increase with respect to time, the total mass that is you have rho into rho t that is with respect to time and this is going to be a mass. So, the mass of the fluid in the cube and its rate of increase with time can be

easily expressed as shown here. Now, we need to equate in order to have the conservation laws satisfied, we have to equate this with the earlier expression which is now going to be written as this that is summation of the net mass that has remained in the flow, remained in the cube considering all the six phases I mean three pair of phases.

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Taking the limit so that the fluid medium shrinks to the point P(x,y,z) the continuity equation is obtained as

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

This equation represents the continuity eqn. In its most general form and is applicable for steady as well as unsteady flow, Uniform and Non-Uniform flow and compressible as well as incompressible fluids.



So, now this is the final expression which we have arrived now divide both sides of the above expression by the volume. Once you do that you are going to get taking element that fluid medium shrinks to the point P, the entire fluid medium that we have now expanded the whole thing just to understand how this different components act? now if you assume taking the limits so, that the entire fluid medium shrinks to point P the continuity equation is obtained as we have seen earlier and this is going to be the continuity equation.

This equation now represents the continuity equation. In its most general form is applicable for steady as well as unsteady flow, Uniform flow, Non-Uniform flow, compressible as well as incompressible. See, you the variation with respect to time is considered there, variation with respect to space is considered there so, all these factors are included so you can apply it for compressible as well as for incompressible fluids and it is applicable for all types of flow which we had discussed so far.

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For Steady flow

$$\frac{\partial \rho}{\partial t} = 0$$

i.e., the above eq. Becomes

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0.$$

Further for an **incompressible fluid** the mass density ' ρ ' does not change with x, y, z and t , hence, the above equation simplifies to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

So, in the case of steady flow, we know that **do rho** or do rho by do t has to be equal to zero. So, the above equation then becomes as shown here. So, this will be applicable for both compressible and incompressible flow, incompressible fluid but, but only if steady flow is applicable, it can be applicable only for so, now you remove for example, for an incompressible fluid, the mass density will not be changing with respect to x, y, z and t and hence the above equation can be simplified as do u by do x plus do u by do y and do w by do z equal to zero.

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FORCES ACTING ON FLUIDS IN MOTION

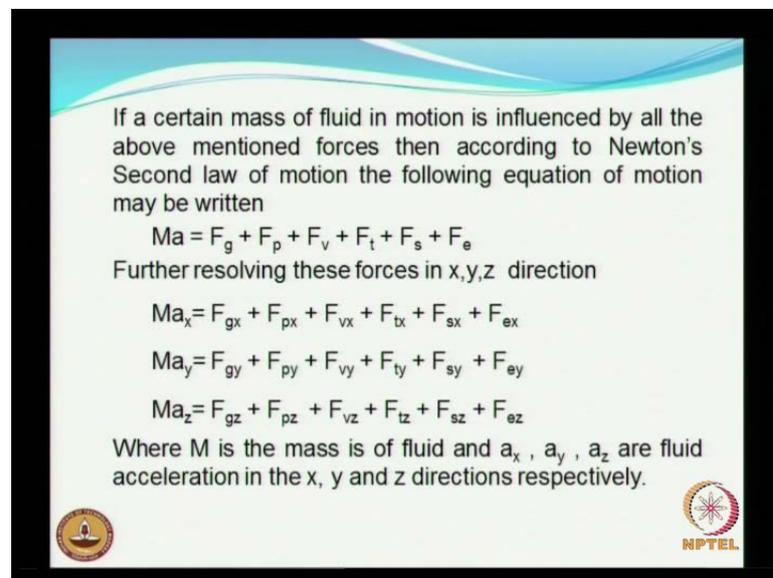
The different forces influencing the fluid motion are due to gravity, pressure, viscosity, turbulence, surface tension and compressibility and are listed below.

F_g (Gravity Force)	→	Due to Wt. of Fluid
		= (Mass * gravitational constant)
F_p (Pressure Force)	→	Due to pressure gradient
F_v (Viscous Force)	→	Due to Viscosity
F_t (Turbulent Force)	→	Due to Turbulence
F_s (Surface Tension Force)	→	Due to Surface Tension
F_e (Compressibility Force)	→	Due to elastic property of the fluid

So, this is the basic equation which is called as a continuity equation which is widely used for describing the flow in a medium. A force consider acting on fluids in motion, that different forces influencing the fluid motion are due to gravity, pressure, viscosity, turbulence, surface tension and compressibility and these are all listed below. F_g is the Gravity Force due to weight as a fluid, is a product of mass and gravitational constant then pressure force due to pressure gradient, viscous force due to the existence of viscosity then turbulence force due to turbulence, surface tension force due to surface tension and due to the elasticity property of the fluid you can have the compressibility force.

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If a certain mass of fluid in motion is influenced by all the above mentioned forces then according to Newton's Second law of motion the following equation of motion may be written

$$Ma = F_g + F_p + F_v + F_t + F_s + F_e$$

Further resolving these forces in x,y,z direction

$$Ma_x = F_{gx} + F_{px} + F_{vx} + F_{tx} + F_{sx} + F_{ex}$$

$$Ma_y = F_{gy} + F_{py} + F_{vy} + F_{ty} + F_{sy} + F_{ey}$$

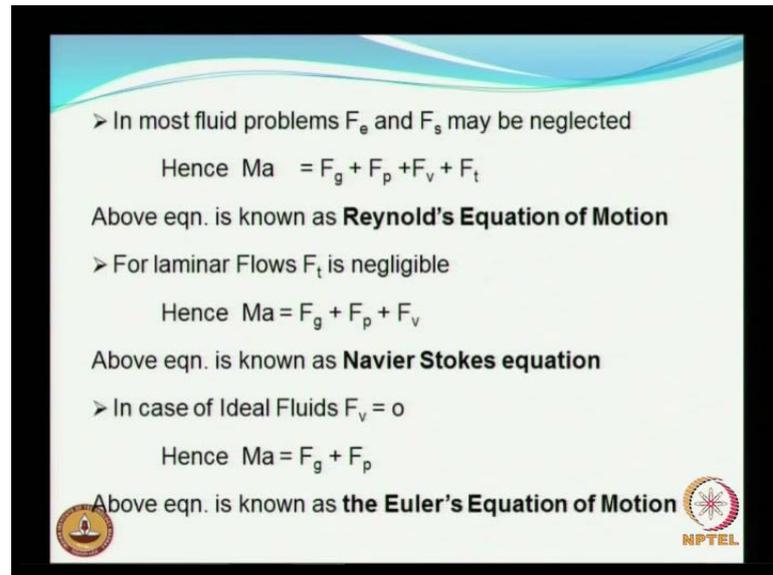
$$Ma_z = F_{gz} + F_{pz} + F_{vz} + F_{tz} + F_{sz} + F_{ez}$$

Where M is the mass is of fluid and a_x , a_y , a_z are fluid acceleration in the x, y and z directions respectively.

Now, if a certain mass of fluid in motion is influenced by all the above mentioned forces, all the above mentioned force components then the Newton Second law of motion according to which we can write mass into acceleration will be the summation of all these force components. Further, you have a respective components in the x y z direction as we have seen here. It is all the first one I mean x direction then z direction then you have the y direction. So, all other things a_x , a_y and a_z are the fluid accelerations in the x y z directions.

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> In most fluid problems F_e and F_s may be neglected
Hence $Ma = F_g + F_p + F_v + F_t$
Above eqn. is known as **Reynold's Equation of Motion**

> For laminar Flows F_t is negligible
Hence $Ma = F_g + F_p + F_v$
Above eqn. is known as **Navier Stokes equation**

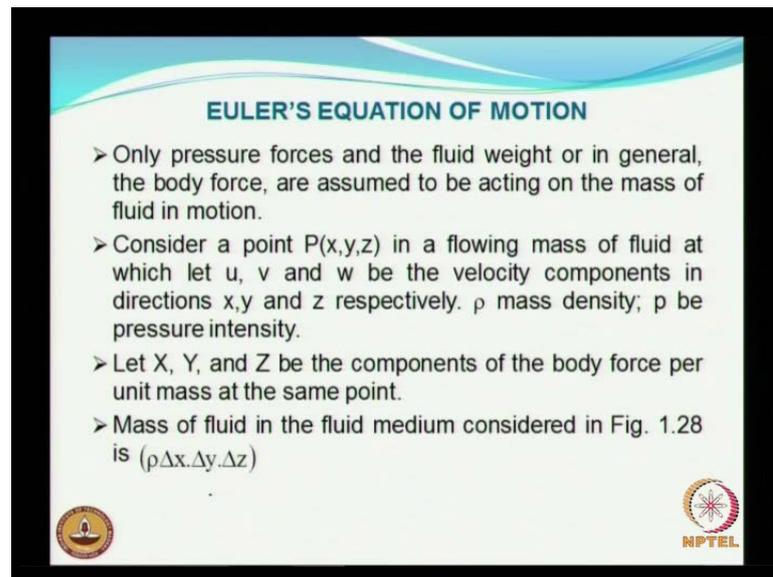
> In case of Ideal Fluids $F_v = 0$
Hence $Ma = F_g + F_p$
Above eqn. is known as **the Euler's Equation of Motion**



Now, in most fluid problem we, neglect the elasticity force as a force to elasticity and the surface tension. So, hence the equation can reduce to as shown here gravity, pressure, viscous and turbulent and this force can be called as is known as the Reynold's Equation of Motion and for in the case of turbulence flows, in the case of laminar flows, turbulence, effect of turbulence are neglected. So, that would result into the summation of gravity, pressure and viscous force and this is called as the Navier Stokes equation which is widely adopted.

Then in the case of ideal fluid, then viscous force is neglected and hence you have only the summation of gravity forces and the pressure forces resulting in what is called as the Euler's Equation of Motion. Which will try to derive in the next class probably.

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EULER'S EQUATION OF MOTION

- Only pressure forces and the fluid weight or in general, the body force, are assumed to be acting on the mass of fluid in motion.
- Consider a point $P(x,y,z)$ in a flowing mass of fluid at which let u , v and w be the velocity components in directions x,y and z respectively. ρ mass density; p be pressure intensity.
- Let X , Y , and Z be the components of the body force per unit mass at the same point.
- Mass of fluid in the fluid medium considered in Fig. 1.28 is $(\rho\Delta x.\Delta y.\Delta z)$

