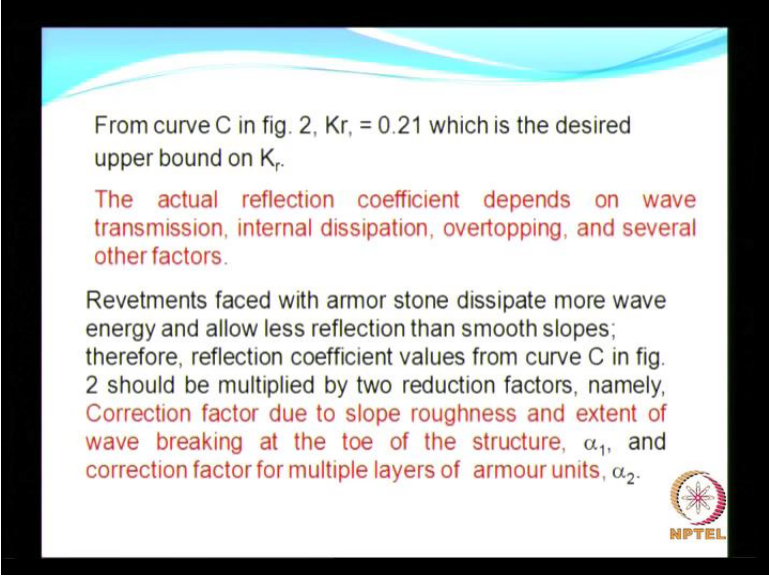


Coastal Engineering
Prof. V. Sundar
Department of Ocean Engineering
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Module - 1
Wave Deformation
Lecture - 5
Wave Deformation (Problems - III)


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From curve C in fig. 2, $K_r = 0.21$ which is the desired upper bound on K_r .

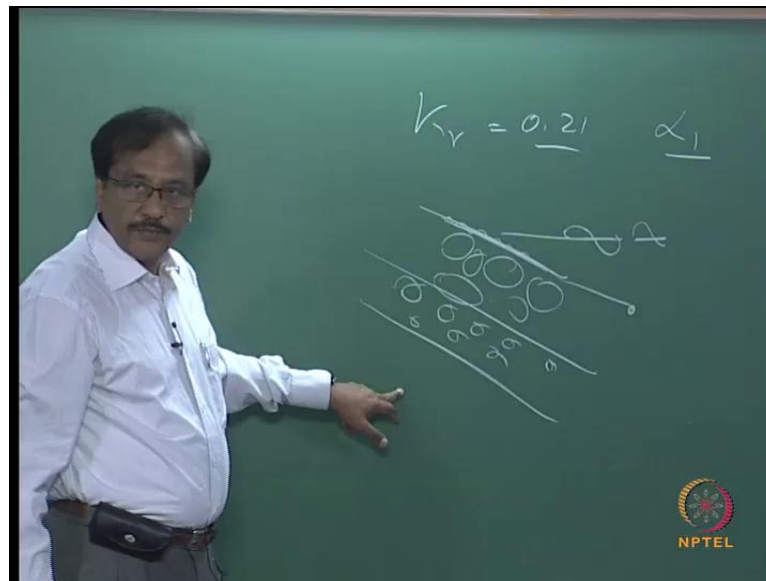
The actual reflection coefficient depends on wave transmission, internal dissipation, overtopping, and several other factors.

Revetments faced with armor stone dissipate more wave energy and allow less reflection than smooth slopes; therefore, reflection coefficient values from curve C in fig. 2 should be multiplied by two reduction factors, namely, Correction factor due to slope roughness and extent of wave breaking at the toe of the structure, α_1 , and correction factor for multiple layers of armour units, α_2 .



Not being paid anything; this is a free service you know that. So, from curve C in the earlier figure what we got?

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We got the K_r as 0.21 which is the upper bound; that is the highest value for that particular cross section which we had seen. But the actual reflection coefficient what do we expect; we have I have already told you about the phenomenon that is likely to take place. When I say upper bound, the actual reflection coefficient is bound to be smaller than the upper bound. So, hence the actual reflection coefficient is going to depend on the wave transmission, then internal dissipation, overtopping and several other factors as I have mentioned earlier.

Now, revetments faced with armor stone dissipate more wave energy and allows less reflection than smooth slopes. This point is very clear; that is why we have two coefficients. Now, since you have the upper bound or the maximum reflection coefficient in order to get more or less reflection coefficient close to reality; close to reality, because it is not so easy to get exactly the reflection coefficient. Because you know that it depends on several other parameters like modeling, the permeability, etcetera is not so easy, and we normally work with an average permeability.

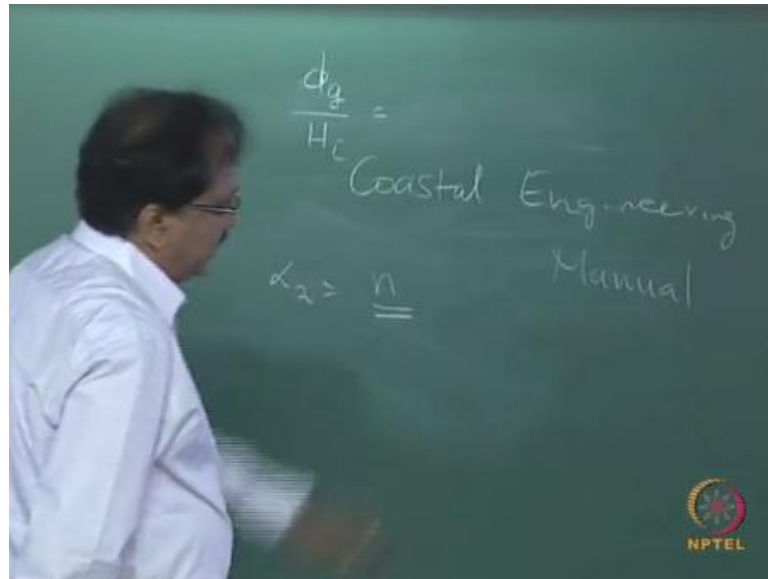
So, the maximum reflection coefficient which we got has to be multiplied by two correction factors and what are these two correction factors? One is when you have the slope, this slope when it is formed by stones, you see that one correction factor will be because of the slope roughness and also the extent of breaking there may be some

breaking near the toe of the structure. When there is a breaking of waves near the toe of the structure certain amount of energy is already dissipated.

So, naturally you see that the reflections once the energy is dissipated, then you have only lesser energy to act on this; naturally your reflection coefficient has to be small. So, that is one kind of a correction factor which is termed as α_1 and what is the other correction factor? Later when we discuss the chapter on breakwaters, there we will see how this multiple layers are going to be effective by this multiple layers. But already I have highlighted about the multiple layers earlier that the bigger size stones will be on the surface and as you go towards the core of the breakwater, the permeability or the size of the stones will reduce.

So in order to account, so you have kind of this either you can have two layers for the breakwater or for the revetment or sometimes even three layers are used. So, to account for the number of layers we have another factor which is α_2 . So, now you see there are two factors which are α_1 and α_2 . I would not go into the theoretical part because this would be easier once you know how to apply all this formulas and the procedures, etc in order to arrive that; that should be good enough for because the syllabus is quite vast covering a wide range of topics in the field of coastal engineering. So, I would like to avoid this and if you need any additional information the best book best reference would be the coastal engineering manual; coastal engineering manual I have already mentioned in the reference.

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So, book a gives a lot of information about all the aspects which I have been discussing in the class. So, let us first consider what is alpha 2?

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Table 1. α_2 correction factor for multiple layers of armour units

d_g/H_i	n			
	1	2	3	4
<0.75	1.00	0.93	0.88	0.78
0.75 to 2.0	1.00	0.71	0.70	0.69
>2.0	1.00	0.58	0.52	0.49

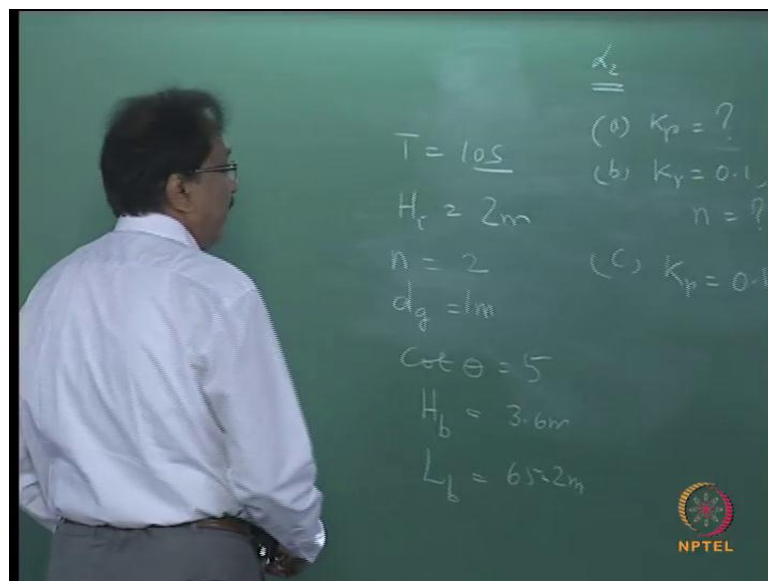


Alpha 2 now you have said it is based on the number of layers; the alpha 2 is going to be dependent on the number of layers you adopt for constructing your breakwater and that is what you have on the horizontal axis 1, 2, 3 and on the y-axis you have another parameter which is d_g/H_i ; d_g is the size of the stone which you are going to use for laying the primary layer and H_i as you know is the incident wave height. So, this coefficient factor

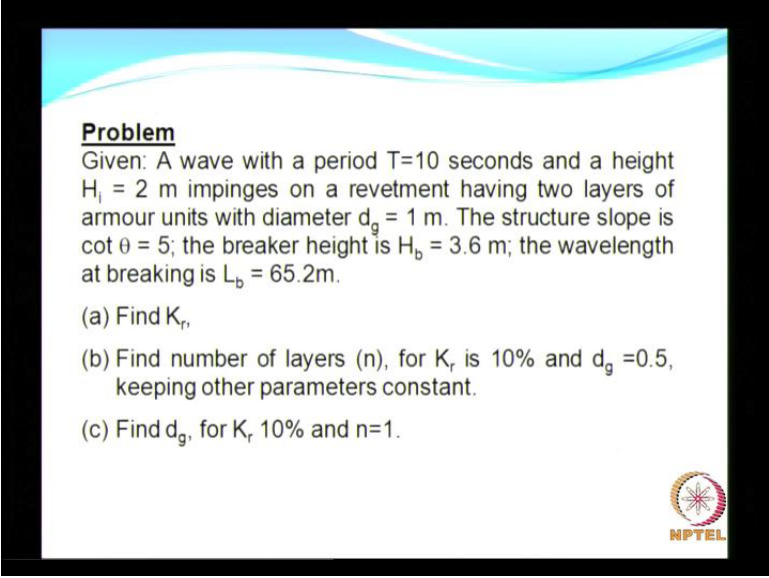
or the correction factor α_2 is going to vary as a function of number of layers and d_g by H_i . So, if you look at the horizontal axis for a particular for a constant d_g by H_i , what will you see? You see that as the number of layers increases, the coefficient decreases.

This is true because when the number of layers increases, the dissipation is going to be more and hence the reflection coefficient is expected to be less and that is what it conveys to you. So, if incase of this if d_g by H_i is increasing, what does that mean? The size is going to increase; if the size increases more amount of water is going to percolate into the medium. So, the energy is expected to be less; I mean the reflected energy is going to be less and hence you see that it is also going to decrease with an increase in d_g by H_i . So, that point is very clear. So, let us now see what I have already explained $H \alpha_2$; let us look into this problem and try to understand.

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Problem
Given: A wave with a period $T=10$ seconds and a height $H_i = 2$ m impinges on a revetment having two layers of armour units with diameter $d_g = 1$ m. The structure slope is $\cot \theta = 5$; the breaker height is $H_b = 3.6$ m; the wavelength at breaking is $L_b = 65.2$ m.

(a) Find K_r ,

(b) Find number of layers (n), for K_r is 10% and $d_g = 0.5$, keeping other parameters constant.

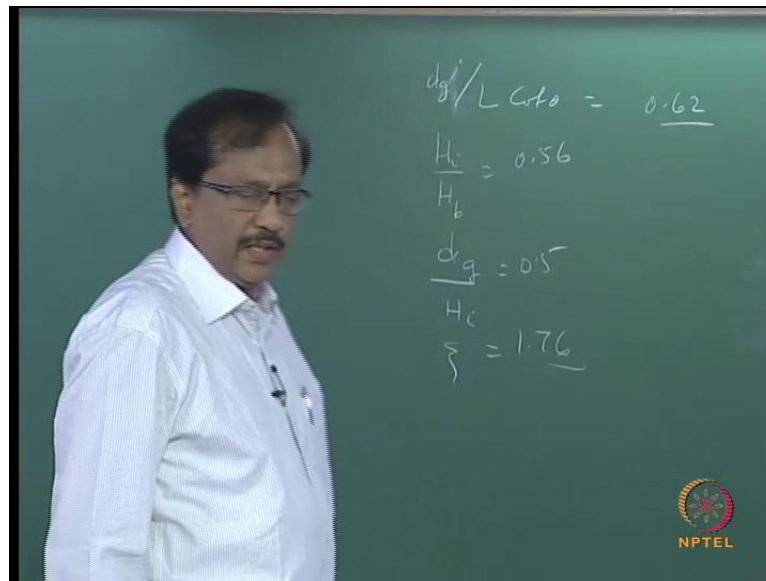
(c) Find d_g , for K_r 10% and $n=1$.



So, T equal to 10 seconds; that is the wave with a period of 10 seconds and a height H_i equal to 2 meters impinges on a revetment having 2 layers; n equals to 2 layers and d_g is also given as 1 meter. Now the structural slope $\cot \theta$ is 5, breaker wave height is also provided to you and then the wavelength at the point of breaking of the wave is 65.2 meters. What are you expected to find out? You are expected to find out K_r is how much; that is the first problem. The second problem is K_r is 0.1 or 10 percent; d_g is now 0.5 changed from 1, with this all other parameters are same. So, you are supposed to find out n . The third question is K_r is equal to 0.1, n equal to a single layer. Now you are required to find out d_g .

So, it is just playing with the variables. This makes you to understand the whole problem or the parameters that are involved and also with the help of such problems, you can also understand the effect of the parameters on your objective function; that is here in this case it will be either K_r or whatever it is, but you can easily find out the effect of different parameters; is that clear. So, shall we proceed?

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Sol: (a)
Determine the reflection coefficient.

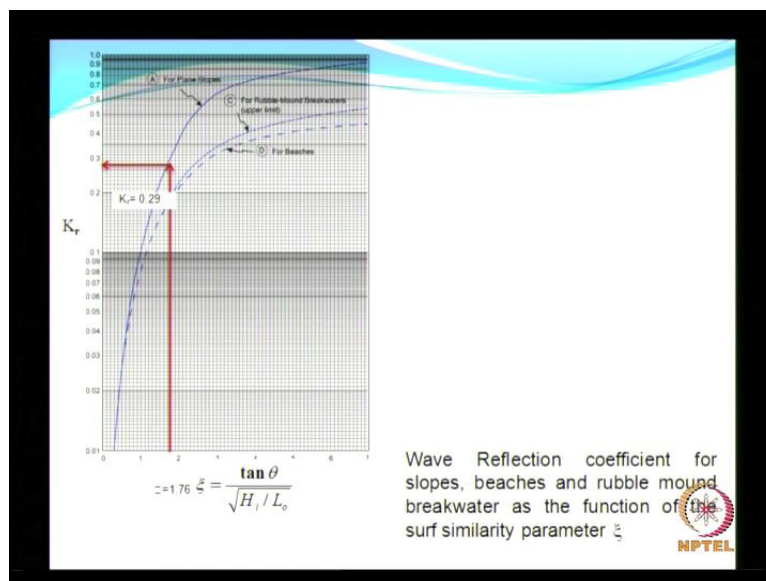
$$\sqrt{d_g / L} \cot \theta = \sqrt{1 / 65.2} (5) = 0.62$$
$$\frac{H_i}{H_b} = \frac{2.0}{3.6} = 0.56$$
$$\frac{d_g}{H_i} = \frac{1.0}{2.0} = 0.5$$
$$L_o = \frac{gT^2}{2\pi} = \frac{9.81(10)^2}{2\pi} = 156m$$
$$\xi = \frac{1/5}{\sqrt{2/156}} = 1.76$$

So, what we need to find out is this parameter d_g by L into $\cot \theta$. So, this will be around 0.62; this is 0.62 for this for this problem. Then you calculate H_i by H_b . This works out to see situation might happen that H_b would not have been given to you. But you are given T , you are given H_i , you are also given S given the slope. Now look back at I mean our discussion on the wave deformation. So, if you know the waves, b slope, wave height and wave period, you can definitely calculate your breaker height on your own. And similarly you can also calculate you are once your breaker height is known your breaker depth can easily be evaluated; you have all empirical relationships and from

that breaker depth you can calculate L_b . But here in this problem all this information's are now given to you; but if it is not given you can always calculate.

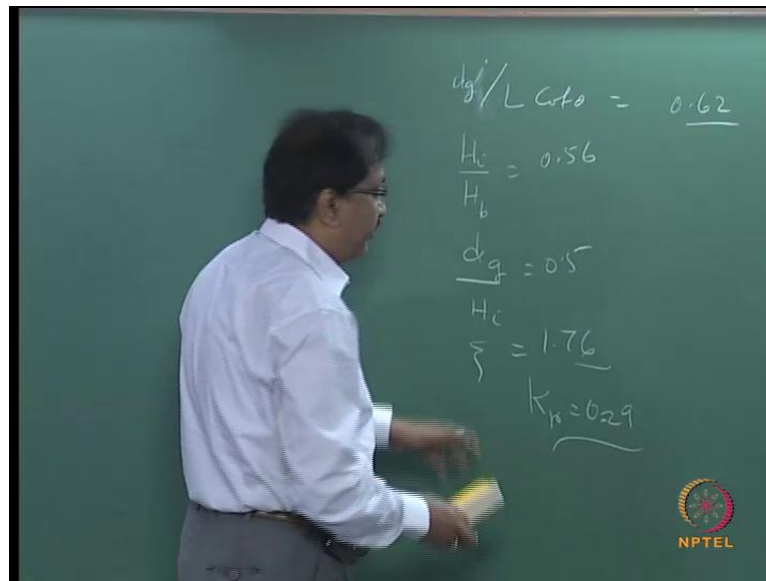
And d_g by H_i in this case will be 0.5 and epsilon that is the sub similarity parameter can easily be calculated; we have already seen enough number of problems. So, in this case this is going to work out as 1.76. Later you will see that all these dimensionless parameters are required. So, now you see when you are doing your research or when you want present your own results, it is always good to report in terms of dimensionless parameter all; the parameters which you are looking here are in a dimensionless form. So that, that will facilitate you to use the kind of nomograms which you can generate for so many field conditions for different range of wave parameters which you have tested in the lab.

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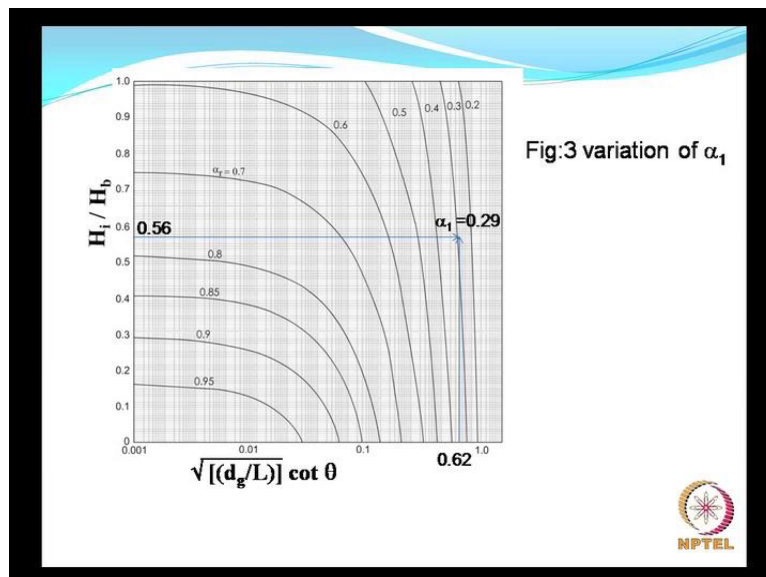
Now having calculated remember in the problem it was mentioned revetment. So, when it is revetment you use this curve the top one which is going to be for plain slopes and the maximum value for this is for the present because of sub similarity parameter is 1.76.

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So, you get the value of K_r as 0.29 from this curve.

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First let us calculate the value of α_1 . So, you have calculated this value and you have calculated this value. Use this nomogram to arrive at this gives you the lines of α_1 ranging from about 0.95 to 0.2 which depends on the two parameters H_i/H_b and the one which is given on the x-axis along the x-axis. These two parameters have been calculated. So, now, the task one of subdivision one is completed.

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$K_r = 0.29$ (for plane slopes)

From graph with $\sqrt{d_g / L} \cot \theta = 0.47$ and $H_i / H_b = 0.56$,
read $\alpha_1 = 0.29$.


From Table with $d_g / H_i < 0.75$ and $n=2$,

d_g / H_i	n			
	1	2	3	4
<0.75	1.00	0.93	0.88	0.78
0.75 to 2.0	1.00	0.71	0.70	0.69
>2.0	1.00	0.58	0.52	0.49

read $\alpha_2 = 0.93$.

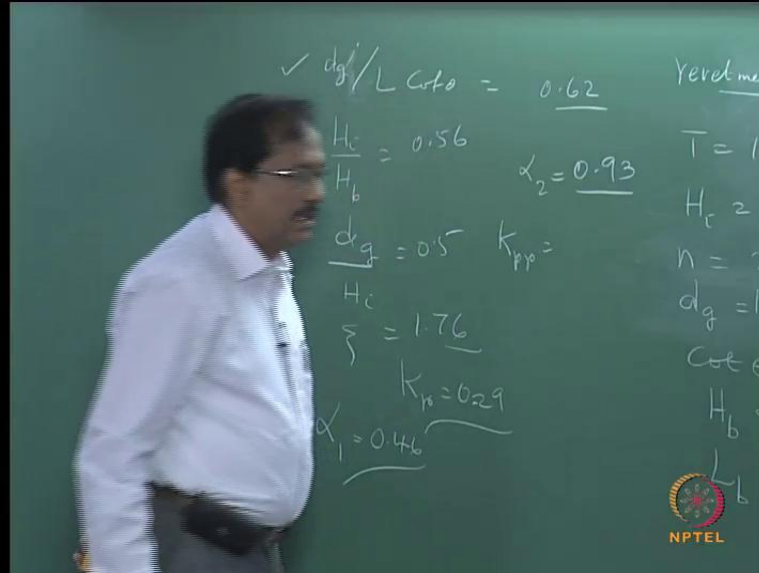
The reflection coefficient = 0.29 from the earlier example
for a smooth slope is multiplied by α_1 and α_2 so that

$K_{rr} = \alpha_1 \alpha_2 (0.29) = (0.29)(0.93)(0.29) = 0.08$



Now we need to get alpha 2. What is alpha 2? Number of layers d_g by H_i equal to 0.96, number of layers equal to 2. So, naturally from this table which we had already seen, your alpha 2 can be read as 0.9; is that clear.


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Handwritten on the chalkboard:

- $\sqrt{d_g / L} \cot \theta = 0.62$
- $H_i / H_b = 0.56$
- $\alpha_2 = 0.93$
- $d_g / H_i = 0.96$
- $n = 2$
- $\alpha_1 = 0.29$
- $K_{rr} = 0.08$

Other text on the board includes "Yevet me", "T = 1", "H_c = 2", "n = 2", "d_g = 1", "Cot", "H_b", "L_b".



So, alpha 1 and alpha 2 are found out. Now I call this as K_{rr} suffix; I mean double r I use in order to distinguish from K_r . K_r is without the correction factors, but K_{rr} is the actual correction factors actual reflection coefficient after multiplying it by the alpha 1 and alpha 2. Now you see for this kind of a problem for this kind of variable parameters,

you look at variations; what we have got for the upper bound as 0.29 and now you actually get which is 50 percent half. So, these are all extremely important when you are dealing with kind of a coastal rubble mound structures. Coastal rubble mound it is also refer to as flexible structures because the reason is when the wave is acting over the structure some of the stones get moved and it can be replenished later; any doubts, because this problem is quite simple.


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Sol: (b)
 $d_g = 0.5$, find "n"

$$\frac{d_g}{H_t} = 0.25 < 0.75$$

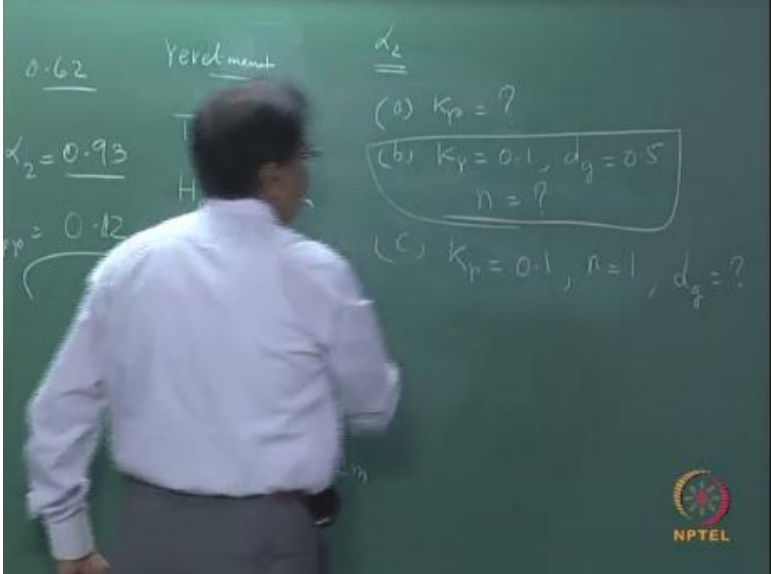
d_g/H_t	n			
	1	2	3	4
< 0.75	1.00	0.93	0.88	0.78
0.75 to 1.0	1.00	0.71	0.70	0.69
> 1.0	1.00	0.68	0.62	0.49

Therefore, $\alpha_1 = 0.38$ (from graph)
 Reduced $K_r = 0.1$ (given)
 We know $K_{rr} = \alpha_1 \alpha_2 (0.29)$ and $\alpha_1 = 0.38$, therefore,
 $\alpha_2 = 0.1 / (0.29 * 0.38) = 0.907$;
 For $\frac{d_g}{H_t} = 0.25 < 0.75$, and $\alpha_2 = 0.9$; **n = 2 (no. of layers)**



Now, let us move into the next problem.

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0.62 Verdiment

$\alpha_2 = 0.93$

$K_p = 0.12$

T


H

α_2

(a) $K_p = ?$

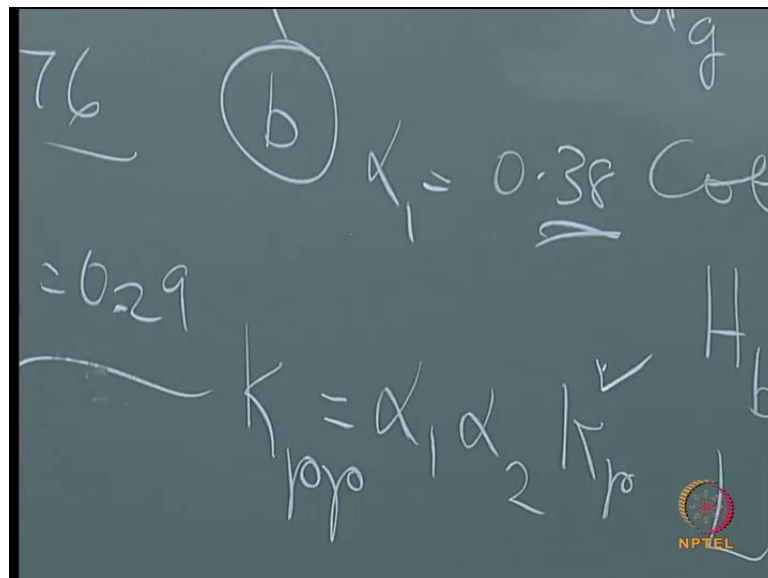
(b) $K_r = 0.1, d_g = 0.5$
 $n = ?$

(c) $K_p = 0.1, n = 1, d_g = ?$



Next problem is we are having now K_r is 0.1, now we assume instead of earlier problem of d_g equal to 1 meter; now d_g is considered as 0.5 meters. So, will this K_r change whatever we have evaluated because K_r if you look at the sub similarity parameter, it is only the function of b slope, then H_i and T ; all these things I have retained same. So, this value will be same; only the values for the correction factor will change which needs to be evaluated from the figures. So, let us follow the same procedure; I do not want to repeat. So, for this you calculate d_g by H_i ; d_g by H_i in this case works out to 0.25 compared to 0.5 in the earlier case and α_1 is picked up from this picture from this slide for the corresponding values.

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And in this problem we have for b we have α_1 equal to 0.38. So, what is K_r ? K_r has to be multiplied by the two correction factors with the one without any correction factors. So, this can be this is actually refer to as this is anyway you know and α_1 is 0.38 and K_r is now 0.1; you understand, it is given K_r is given as 0.1. So, use this equal to 0.1 and this is known and you need to calculate what; you need to calculate α_2 and in this case α_2 will be 0.907. So, then once you have the d_g by H_i which is less than the first column and this value is coming to around close to 0.9 and hence what is this? This means that the number of layers to be adopted is 2; naturally you cannot have two and half or 2.5 or 2.25 and all, it has to be one layer, two layers, etc. So, this is the second problem. So, you see that the sub division b is answered.

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Sol: (c)
For $n=1$ and $K_r = 0.10$, find " d_g "
 $K_{rr} = \alpha_1 \alpha_2 (K_r)$;
For $n=1$; $\alpha_2 = 1$;
Therefore $\alpha_1 = 0.1/(0.29 \times 1) = 0.344$
For,
$$\frac{H_i}{H_b} = \frac{2.0}{3.60} = 0.56, \text{ and } \alpha_1 = 0.344$$


from graph
$$\sqrt{d_g / L} \cot \theta = 0.54$$



Now we will move on to the third one; what is the third one? Third one is n is equal to 1
 $K_r K_r r$ in fact, this should be $K_r r$ is equal to 0.1. So, use the same equation. So, you
see the same equation. For this case α_2 can be read as equal to 1 and hence use the
above equation to get α_1 . α_1 now for this particular problem we will work out
to 0.34, but what is H_i by b . So, use this curve; you have determined α_1 , you know
the value of H_i by b , you use this particular curve to get your value of d_g by L .

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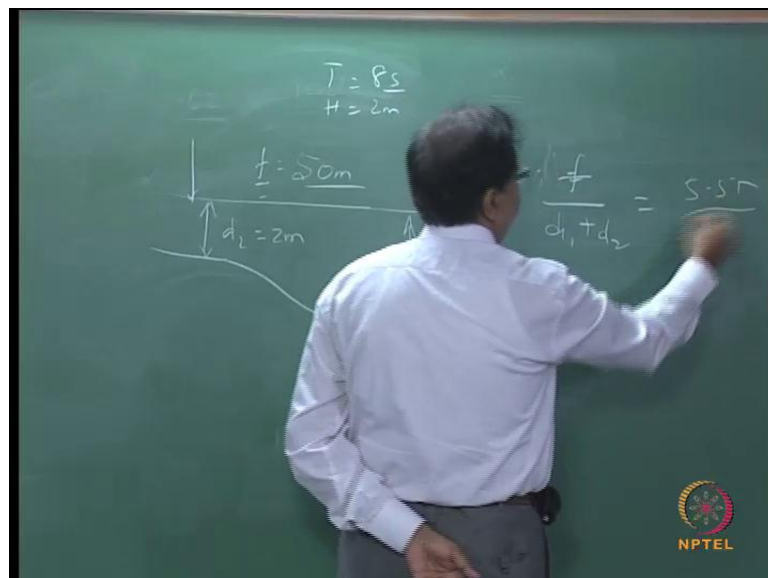
Therefore, $d_g = 0.76\text{m}$
From the problem we can state the part
(a) has over predicted, Since, for $d_g = 0.5$, the number of
layers (n) required is 2, and for single layer, the diameter
of stone (d_g) required is 0.76m



So, which will work out to 0.54 from which you can easily arrive at the value of d_g and here d_g in this case would be around 0.76 meters; is that clear. So, what did we understand; if you want what you should do is you should go through the slides again, if you have any doubts go through the slides again in order to understand what we have proved; that is through problem one we have seen that it is over predicted; that is if you are using the those curves it is going to be over prediction of reflection. And we also found out since for d_g equal to 0.5, the number of layers required is two and for a single layer the diameter required is 0.76 meters.

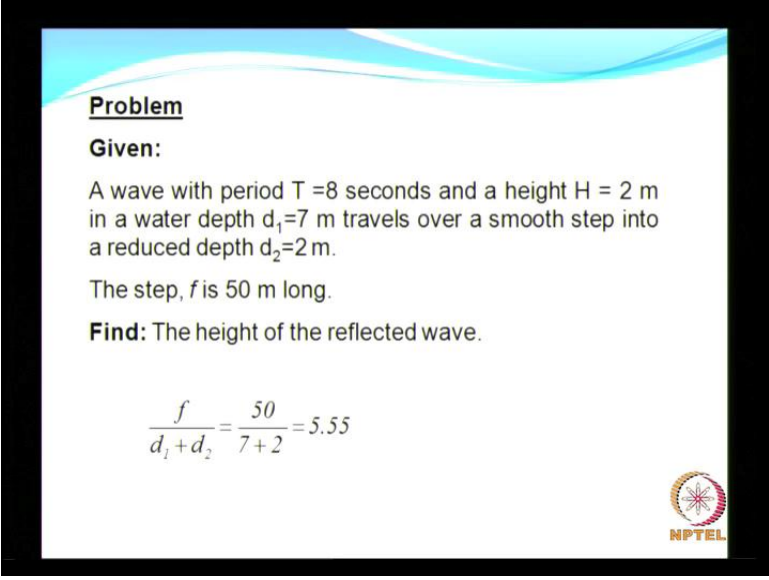
So, if you are using a single layer the size of the stone might be bigger also. From stability point of view, it is always preferred to have more than single layer and never in such kind of structures you should never be greedy and also trying to be a miser like you just want to save money and then why should I go for two layers; we will just have one layer. It might happen one single flood everything will be the whole thing can get washed off; is that clear. So, that explains the problem related to reflection when you deal with multiple layers, etcetera.

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Now very often you would have a wave propagating from a water depth which is larger to shallower water depth. This change in the bathymetry can also result in reflection; how do you evaluate this.

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Problem

Given:

A wave with period $T = 8$ seconds and a height $H = 2$ m in a water depth $d_1 = 7$ m travels over a smooth step into a reduced depth $d_2 = 2$ m.

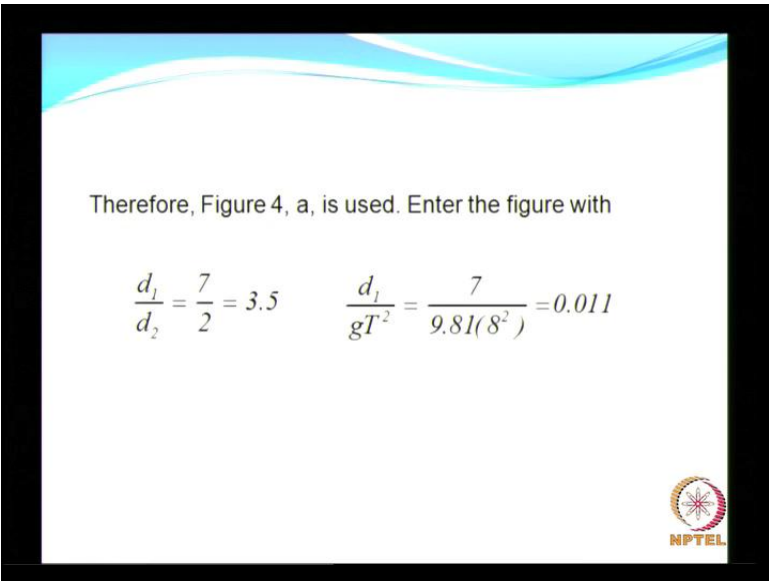
The step, f is 50 m long.

Find: The height of the reflected wave.


$$\frac{f}{d_1 + d_2} = \frac{50}{7 + 2} = 5.55$$


So, here you have a wave T equal to 8 seconds, wave height is equal to 2 meters and it is travelling from a deep water depth of 7 meters to water depth of 2 meters; may be the step is approximately 50 meters long and we need to get the reflection coefficient. So, the first step will be if I call this as f or whatever it is, then I calculate the parameter and this will be 5.5.

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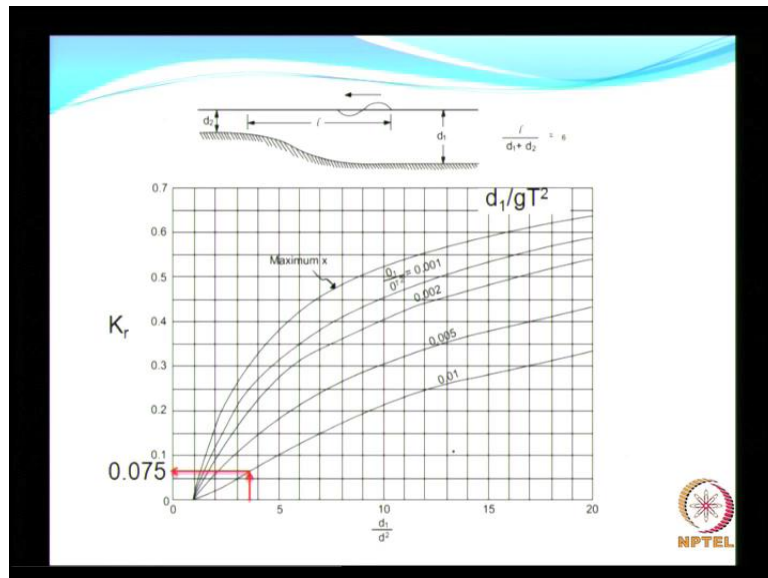


Therefore, Figure 4, a, is used. Enter the figure with

$$\frac{d_1}{d_2} = \frac{7}{2} = 3.5 \quad \frac{d_1}{gT^2} = \frac{7}{9.81(8^2)} = 0.011$$


And d_1 by d_2 is going to be 3.5 and d_1 by gT^2 for the present problem will be 0.011. Now simple dimensionless parameter you need to calculate.

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Once you calculate this, then need to enter into this nomogram and use that for d_1 by d_2 .

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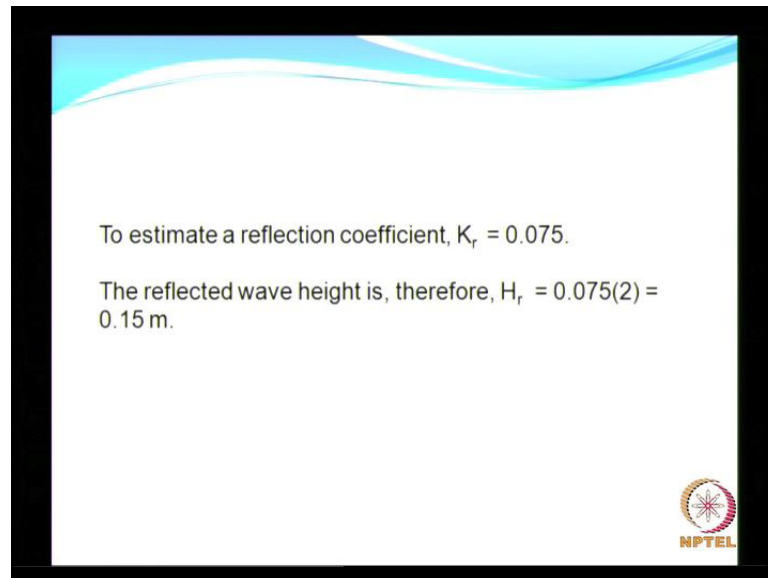
$$\frac{d_1}{d_2} = 3.5$$
$$\frac{d_1}{gT^2} = 0.01$$

What was d_1 by d_2 is 3.5 and d_1 by gT^2 square equal to 0.01. With this values you get into the figure d_1 by d_2 equal to 3.5 and you see the lines of d_1 by gT^2 square lines of constant d_1 by gT^2 square, pick up the value for your 0.01 which is the bottom most curve and then this is what you will get as the reflection coefficient.

See this is a basic course wherein some of the phenomena and what are all some of the parameters, etcetera which we need to consider when you want to go in for real large

scale problems involved in the field. It may be either solving through experimental work or solving through numerical work, that is the reason why I have explained with the help of worked out examples and I hope this will be forming a good base to get started if you are really interested in specializing in this area. Any questions; no questions, do you have any questions?

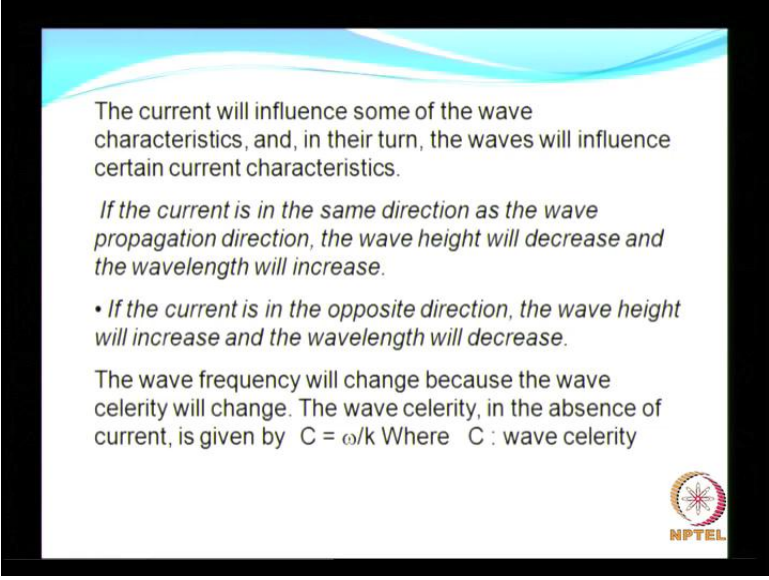
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So, this is what I wanted to show that the estimation of a reflection coefficient is this much. So, the reflected wave height is 0.15. We will get started now with waves and currents. So, we are done with reflection, diffraction, refraction and I have also said for practical problems you need to consider the effect of combined reflection, refraction and diffraction; wherever all this kind of all this phenomena occur you have to consider all the phenomena. And similarly when you are dealing with waves, in locations where you do have the propagation of waves over currents, you need to consider them.

Current can flow in the same direction as that of waves in which we say following currents we refer to that condition as following current and we can also have the wave in this direction and the current in this direction in which case it is referred to as opposing currents. I think we saw in brief what happens to when you have in the same direction or when it goes in the other direction when we discussed about wave loads on cylinders; when we took the super position of currents and waves we found what was happening to the wave loads.

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The current will influence some of the wave characteristics, and, in their turn, the waves will influence certain current characteristics.

If the current is in the same direction as the wave propagation direction, the wave height will decrease and the wavelength will increase.

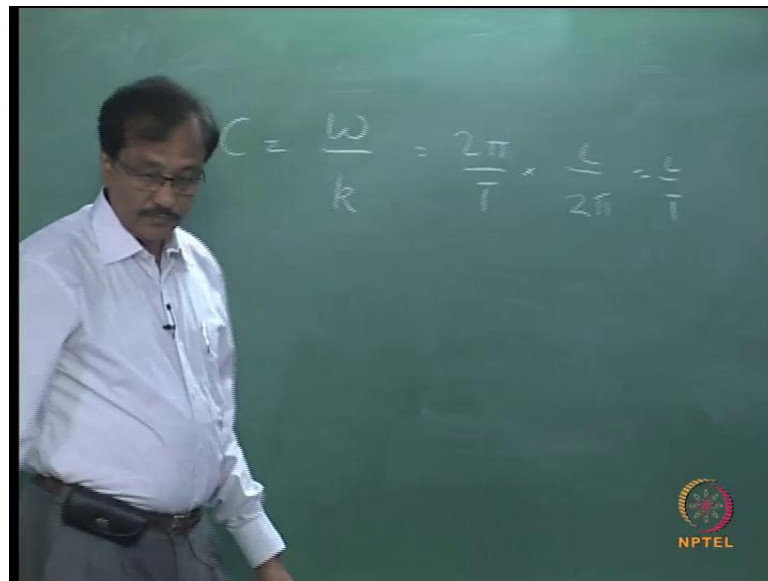
- *If the current is in the opposite direction, the wave height will increase and the wavelength will decrease.*

The wave frequency will change because the wave celerity will change. The wave celerity, in the absence of current, is given by $C = \omega/k$ Where C : wave celerity



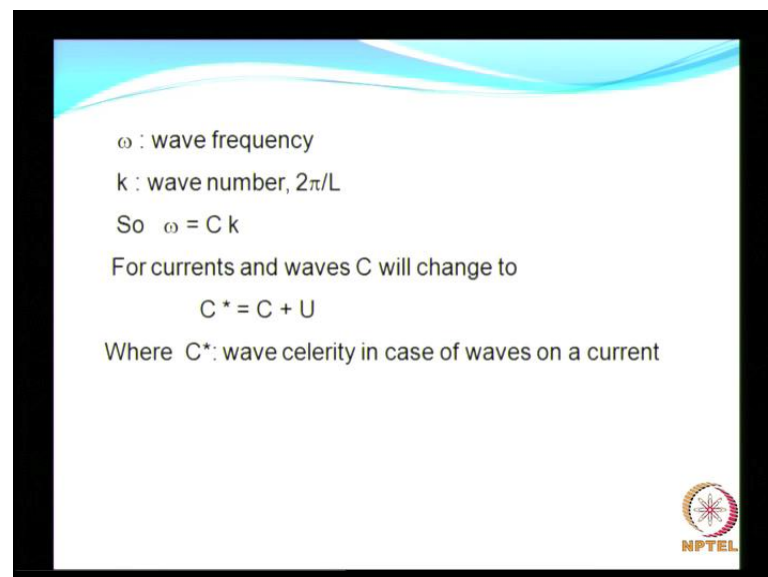
The current will influence some of the wave characteristics and in turn the waves are certain to influence the characteristics of the current; it is vice versa. Now if the current is in the same direction, I mean the following current, then what would happen? The wave height will decrease and the wave length will increase. Try to recollect when we worked out a problem on wave loads on cylinder, for a following current you found that the wave force was increasing. I clearly explained the reasons for this increase in wave force for the following current; if you do not remember please go back and check my lecture material on wave loads on structures. If the current is in the opposite direction, what will happen? The wave height will increase and the wave length will decrease. The wave frequency will change because the wave celerity is going to change.

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The wave celerity in the absence of current is C equal to ω by k . What is ω ? Wave frequency, 2π by T ; that is wave angular frequency and this one is. So, you have L by T .


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Now for currents and waves celerity is expected to change; celerity is expected to change.

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$$C^* = C + U$$
$$\omega^* = C^* k = (C + U) k =$$
$$k(C + U) = \omega + kU$$




Let me call that as C^* equal to C plus U where C^* is the wave celerity in presence of the current; whenever I use a star that indicates that waves and currents are propagating together. Now, frequency will change from ω^* which can be written as but C^* actually is into k as it is indicated here which can further be simplified as ω plus k into capital U . What is ω^* ? ω^* is the wave frequency in the case of waves and current. The component that is superposed now which is referred to as kU is referred to as Doppler effect.

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U : is the current velocity component in the direction of the wave propagation. Frequency will change to ω^* , given by:

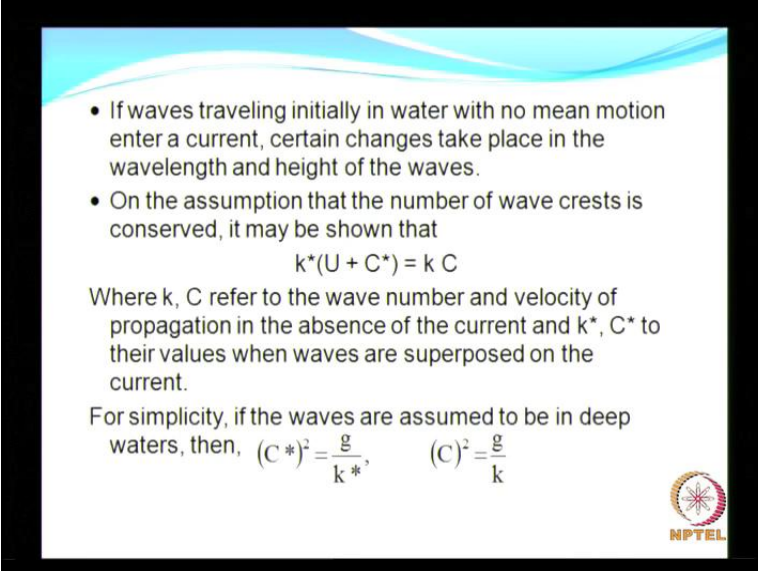
$$\omega^* = C^* k = (C + U) k = \omega + kU$$

Where ω^* : wave frequency in the case of waves and a current This component, kU , is often called the Doppler effect



So, this is the additional component due to the waves propagating over the current. The current is zero, naturally w star is going to be equal to w , w or ω ; is that clear, shall I proceed.

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- If waves traveling initially in water with no mean motion enter a current, certain changes take place in the wavelength and height of the waves.
- On the assumption that the number of wave crests is conserved, it may be shown that

$$k^*(U + C^*) = k C$$

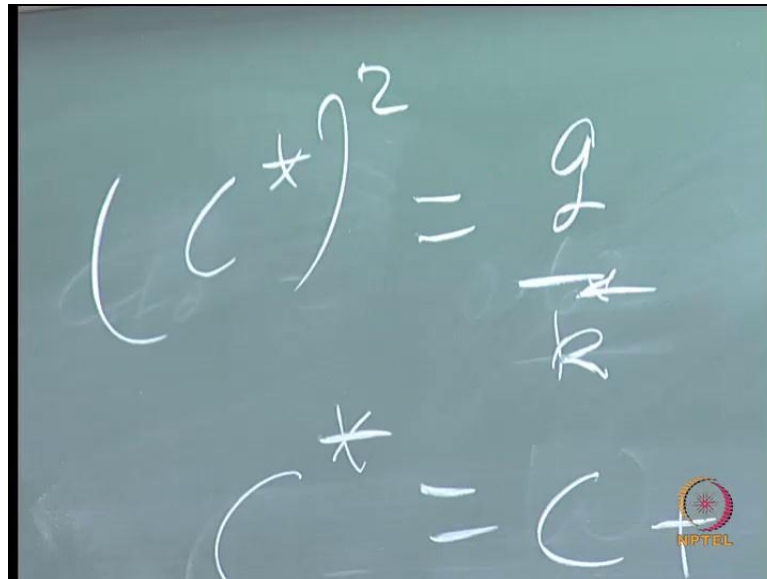
Where k , C refer to the wave number and velocity of propagation in the absence of the current and k^* , C^* to their values when waves are superposed on the current.

For simplicity, if the waves are assumed to be in deep waters, then, $(C^*)^2 = \frac{g}{k^*}$, $(C)^2 = \frac{g}{k}$

So, if waves traveling initially in water depth with no mean motion enter a current, certain changes take place in wavelength as well as wave height. If we assume that the number of crests are conserved; you understand the number of crests are conserved, on that assumption we can deduce k star equal to U plus U star equal to k into C . One is with the absence of current and another is in presence of the current. So, now for simplicity in order to understand the characteristics of the waves, we will consider deep water conditions. Always deep water condition is very easy compared to shallow water conditions because the effect of depth will not come into picture in deep waters.

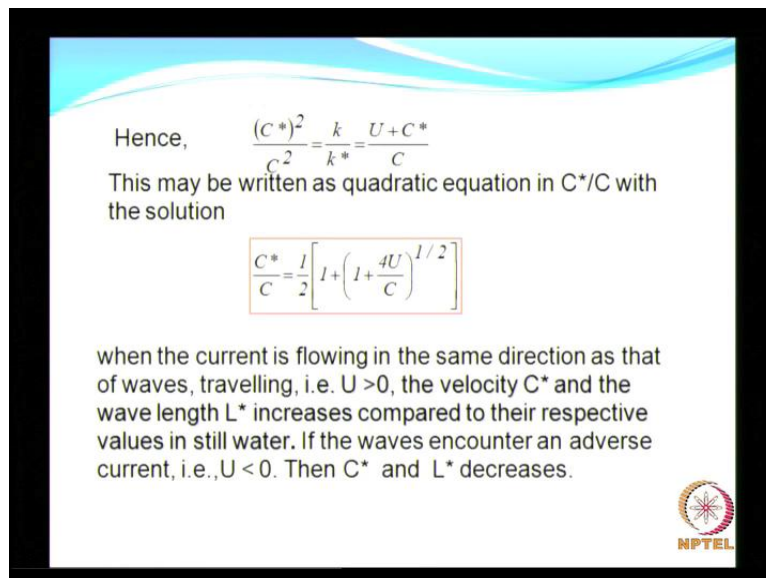
Just imagine in all this things if you have the $\tan h k d$ which is going to take care of the effect of water depth. So, every time you need to deal with that variable; understand. So, here in we are trying to understand what is the effect of current on the wave characteristics. It does not matter if we consider just the deep water condition for simplicity. So, in that case we can assume that it is in deep waters and hence you will have the C star square will be g by k .

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$$(C^*)^2 = gR$$
$$C^* = C$$

So, I will also write this C star equal to g by, so that you need not have to keep on referring your slides back and forth.

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Hence, $\frac{(C^*)^2}{C^2} = \frac{k}{k^*} = \frac{U + C^*}{C}$

This may be written as quadratic equation in C^*/C with the solution

$$\frac{C^*}{C} = \frac{1}{2} \left[1 + \left(1 + \frac{4U}{C} \right)^{1/2} \right]$$

when the current is flowing in the same direction as that of waves, travelling, i.e. $U > 0$, the velocity C^* and the wave length L^* increases compared to their respective values in still water. If the waves encounter an adverse current, i.e., $U < 0$. Then C^* and L^* decreases.

Using these equations, this all this, you can write or you can get this expression as C star square divided by C square equal to k by k star which is going to be equal to. What is this? This is a quadratic equation in C star by C. It is just a quadratic equation. So, now I can remove all this things because you have arrived at the final expression which is going

to give you the relationship of the variation in the celerity in presence of currents with that of in the absence of current.

Now this is my final expression which gives that. Now what is happening with this expression? When the current is flowing in the same direction as that of the waves that is U will be greater than 0; right, the celerity C star and when I say C star then naturally the wavelength increases compared to the respective values in the still water; that is without the current. If the waves encounter an adverse current that is current in the opposite direction, then your wavelength is decreasing. This is what we have tried to prove; is that clear. So, I will stop here.