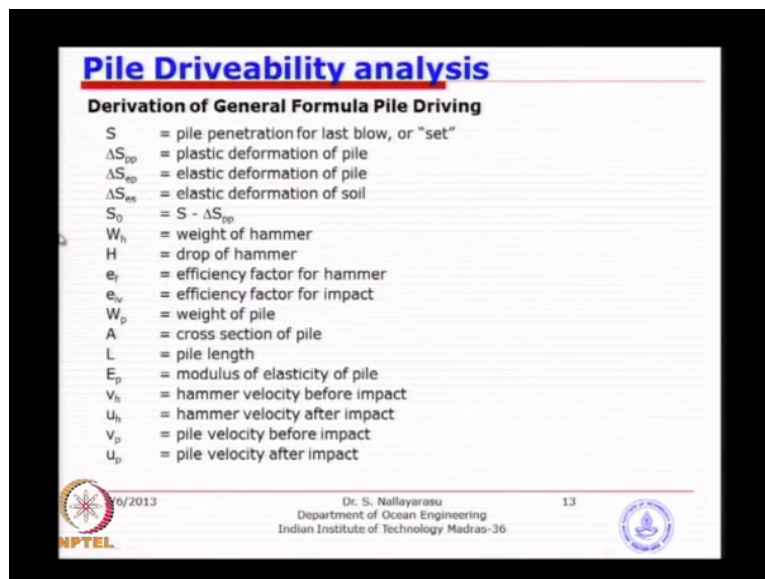


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**Module 1**  
**Lecture 19**  
**Pile Driveability Analysis 2**

So this pile driving formulas the dynamic formulas we were looking at of the simplest form and also we were looking at various modifications done by the past study work one of the important aspect of this engineering news formula is trying to incorporate as much parameters that represent the real driving. So we will just quickly look at how it could be actually incorporated based on various you know the energy losses as well as the pile driving format itself.

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**Pile Driveability analysis**

**Derivation of General Formula Pile Driving**

$S$	= pile penetration for last blow, or "set"
$\Delta S_{pp}$	= plastic deformation of pile
$\Delta S_{ep}$	= elastic deformation of pile
$\Delta S_{es}$	= elastic deformation of soil
$S_0$	= $S - \Delta S_{pp}$
$W_h$	= weight of hammer
$H$	= drop of hammer
$e_f$	= efficiency factor for hammer
$e_v$	= efficiency factor for impact
$W_p$	= weight of pile
$A$	= cross section of pile
$L$	= pile length
$E_p$	= modulus of elasticity of pile
$v_h$	= hammer velocity before impact
$u_h$	= hammer velocity after impact
$v_p$	= pile velocity before impact
$u_p$	= pile velocity after impact

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So  $S$  is the pile penetration or we call it set, sometime the notation for each blow or could be consecutive several blows in reality we normally use 300 blows as a reference, you know if you look at in offshore pile driving we do not look at actually every blow what is the set value for consecutive set 300 blows you just keep on driving and then find out what was the previous 300 and the current set value or displacement of the pile that we call it set.

But when you look at the last few blows for example before the pile refused to go any further then we may actually have to look at instead of few hundred blows may be 5 blows or 10 blows, whether it is going down and that value needs to be reviewed carefully because the

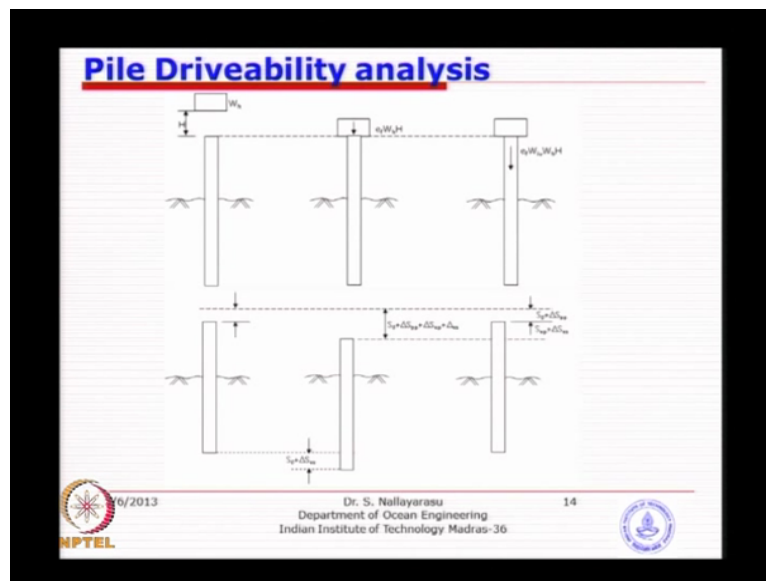
last 300 blows and if the penetration is very small you do not want to go to that level for example continuously you hammer for 300 or 500 blows and the total penetration is only 1 inch which means you might have damaged pile or damaged the hammer because 1 inch over 300 blows is too small for a pile driving hammer.

So that is why initial starting time you monitor using larger blow count but when you really reaching near refusal or penetration is not going down you just look at each blow as how much is going down. So that is basically the pile penetration and you look at the  $\delta$  S which is the deformation of the pile and deformation of the soil, either elastic or plastic. So you got two components for example for the pile itself these notations you have to understand P is pile and S is soil, elastic and plastic.

Hammer weight, hammer drop the drop height from which it is falling down and then efficiency of the hammer, efficiency of impact you know basically the at just before impact and after impact what is the kind of energy loss that means during the impact process and then section properties of the pile, length, modulus of elasticity these are known parameters and then you see here the hammer velocity and the pile velocity at the point of impact how the pile goes down, if it is a rigid material not having any spring stiffness then that means there is no movement.

But here you have got a pile which has got its own properties and supporter on a soil medium which is going to go down. So there will be a velocity of the pile head. So this is basically the before and after the notation is given you know basically  $v$  and  $u$  so you got just remember because we are going to calculate the co-efficient of restitution and using these velocities.

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So what you can see in these picture is starting from the pile is into the ground at certain level after through you are just analysing it or you could actually have the pile just resting on top of the sea bed and trying to drive and dropping from a height of  $H$  with a weight of  $W_h$  which gives you the impact energy is basically the weight times the height and energy efficiency of the hammer itself the internal system, how much it is 90 percent or 80 percent depending on what.

And when it is impacting on to the pile it transfer the weight as the impact energy and transmits to the pile material having elastic elasto plastic compression if it is so high and then plastic deformation for sure if it is in elastic stage it is going to bounce back. So in order for you to drive you need to see the soil is failing and then finally you have a set value which includes the elastic compression of the pile, we do not want the pile to go into plastic compression that means the pile is failed that means we cannot use.

So one of the purpose of this analysis why we are going and doing is to make sure that the stress levels on the pile does not go beyond the yield value or at least less than the yield value so that the pile is in good condition and that is why the selection of hammer is also going to be according to what we are going to analyse and you see here is the composition of elastic and elasto plastic and the soil compression.

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**Pile Driveability analysis**

$g$  = gravitational acceleration  
 $R_u$  = load capacity of pile (just after driving)  
 $E_1$  = energy reaching pile  
 $E_2$  = energy left after impact

The energy reaching the pile is

$$E_1 = \eta_h W_h H = \frac{W_h v_h^2}{2g}$$

The energy efficiency of impact is

$$\eta_i = \frac{(W_h / 2g) u_h^2 + (W_p / 2g) u_p^2}{(W_h / 2g) v_h^2 + (W_p / 2g) v_p^2} = \frac{E_2}{E_1}$$

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The energy reaching the pile is basically the original energy multiplied by the efficiency as you know very well multiplication of efficiency factor and energy can be calculated or equal into kinetic energy half mv square W by g will give you the mass of the dropping part it is not the whole hammer, you know this hammer has got say several components the outer shell and you got anvil and you got ballast specially the hammer going under water remember we saw one hammer which is going to drive and going to go down it has to have positive weight, otherwise what will happen the hammer will not sit on the pile.

So you will have to have additional ballast to make sure that when you are driving the stress waves coming back and reflecting it should not push the (pile out) the hammer out of the pile because the reverse force. So you will have additional ballast by means of filling water in the compartments. So most of the steam hammers or in the hydraulic hammers where they need to go do work under water will need to fill up more water so that the additional weight is increased we call it ballast weight though the hammer has got its own weight but it needs to be heavier enough that even in under water condition the displaced weight will be compensated by additional ballast.

So basically this  $W_h$  is only the moving component going up, coming down and that you can calculate  $W$  by  $g$  will give you the mass and then and calculate the kinetic energy and the energy efficiency during the impact is taken as the ratio of the velocity before impact and after impact. So basically that is why you see here  $u_h$ ,  $v_h$ ,  $u_p$ ,  $v_p$  which is the ratio of just prior to impact and just at the end of impact, whereas the energy of the hammer itself is due to system components how much effectively it is stable to generate the energy basically it

depends on the type of hammer like diesel engine, rated energy could be something which you will not be able to produce the same amount of energy. So there is one part is the hammer efficiency this one is the efficiency of the impact during that particular period.

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**Pile Driveability analysis**

The law of impulse gives

$$\frac{W_h}{g}(v_h - u_h) = \frac{W_p}{g}(v_p - u_p)$$

Thus, the coefficient of elastic restitution,  $n$ , is

$$n = \frac{u_p - u_h}{v_h - v_p} = \frac{\text{Velocity of pile after impact} - \text{velocity of hammer after impact}}{\text{Velocity of hammer before impact} - \text{velocity of pile before impact}}$$

Assuming  $v_p = 0$ , and eliminating,  $u_h$ ,  $u_p$ , and  $v_h$ ,

$$\eta_i = \frac{W_h + n^2 W_p}{W_h + W_p}$$

The energy left after Impact is

$$E_z = \eta_i \eta_h W_h H = \eta_h W_h H \left( \frac{W_h + n^2 W_p}{W_h + W_p} \right)$$

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**Pile Driveability analysis**

**MODIFIED ENGINEERING NEWS (ENR) FORMULA**

$$R_u = \left( \frac{\eta_h W_h H}{s + C} \right) \left( \frac{W_h + n^2 W_p}{W_h + W_p} \right)$$

Where

- $R_u$  = Ultimate pile capacity ( $R_s + R_E$ ).
- $\eta_h$  = Hammer efficiency.
- $W_h$  = Weight of Ram.
- $W_p$  = Weight of pile including pile cap, cab block.
- $H$  = Height of fall of ram.
- $s$  = Set, amount of point penetration per blow.
- $n$  = Co-efficient of restitution.
- $C$  = Temporary compression allowance.

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**Pile Driveability analysis**



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And that is what this engineering news formula finally incorporates is like this, so what we are just looking at is the various components in it. So the co-efficient of restitution is defined as the velocity of the pile or relative velocity of the pile and hammer before and after impact that is the. So what is co-efficient of restitution if the co-efficient of restitution is 0, what happens and if the restitution is greater than 1 or less than 1 there could be three values, most of the time you will see restitution will be less than 1, if it is higher than 1 something must be terribly wrong.

You know basically after impact it goes even higher which could actually happen if you actually take a projectile or a gun. If you actually impact perpendicular to a wall which might have some amount of elastic absorption which might then it will be retarding the velocity. But if you impact the same gun in an angle to a for example 40 degrees, 45 degrees it could actually after the impact it could take a higher velocity very rarely of course.

So if it is equal to 0 that means is fully no more you know reflection. Most of the materials that we are talking about is steel, timber, wire or wire mesh, concrete will have considered restitution definitely less than 1, some amount of energy will be absorbed. So what we are looking at is during the process of impact we want to find out what kind of changes the driving system goes through and that is what the engineering news formula is taking into account so that is the derivation that is you put all these velocity components  $u_p$  and  $u_h$  and  $v_h$  and  $v_p$ .

And assuming that the velocity of the pile you know basically the velocity of the pile just before impact that means still has not strike at the top of the pile that means it is going to be

definitely 0 and you substitute the parameters you will get a relationship that  $E_2$  which is what we were looking at in here and with respect to hammer weight, hammer fall and your efficiency of hammer which is already substituted, the efficiency of the striking itself is  $\eta_1$  is just the formula what you calculated from here and the energy available at the end of the strike is available in terms of hammer efficiency and the weight of the hammer and height of the fall multiplied by the co-efficient of restitution.

Suppose if you have no cushion is provided between the pile and the hammer striking then that value will be steel to steel. So you may take 0.5, 0.6 depending on what type of pile head you have. If you put timber or you put some other material that could actually taken according to the material I will have a table which gives you a typical values of co-efficient of restitution.

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### Pile Driveability analysis

The work done during impact is approximately

$$E_2 = R_u \left( S + \Delta S_{pp} + \frac{1}{2} \Delta S_{ep} \right)$$


Neglecting the elastic deformation of the soil, and introducing Hooke's law for the pile

$$\Delta S_{ep} = C \frac{R_u L}{AE_p}$$

Where C = ratio between actual displacement at pile top and that given by Hooke's Law. From the above equations, the following equation is obtained

$$R_u = \frac{\eta_h W_h H}{S + \left( \frac{CR_u L}{2AE_p} \right) + \Delta S_{pp}} \left( \frac{W_h + n^2 W_p}{W_h + W_p} \right)$$



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**Pile Driveability analysis**

**OTHER SIMPLER FORMS SET Vs RESISTANCE**

Sanders	$R_u = \frac{W_h H}{S}$
Engineering News	$R_u = \frac{W_h H}{S + C_1}$
Eytelwein	$R_u = \frac{W_h H}{S + C_1} \frac{W_h}{W_h + W_p}$
Hiley	$R_u = \frac{\eta_h W_h H}{S_u + \frac{1}{2}(C_1 + C_2 + C_3)} \left( \frac{W_h + n^2 W_p}{W_h + W_p} \right)$


$C_1$  = Temporary Compression allowance for pile  
 $C_2$  = Temporary Compression allowance for pile cushion / follower  
 $C_3$  = Temporary Compression allowance for soil



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And then you have work done during the impact is you can take you know average value of that elastic compression because initially it is 0 and at the end up the strike is it is some amount of the compression is going through. Plastic deformation we definitely do not want to actually go to that stage you do not want the pile to be permanently deforming but in theoretical case you can have but ultimately when we do the Driveability analysis we will remove this from and limit the stresses in such a way that we will not go into plastic and select the hammer accordingly we do not want to put a bigger hammer which will make the pile to compress plastically.

And then the final soil settlement or the pile tip settlement and  $R_u$  is the resistance given by the soil in total in could be skin friction or it could be skin friction plus end bearing. And neglecting the elastic deformation of the soil which is going to be very small and basically you can rewrite this equation like this. So that means you take this one out, you take this one out so you are just rewriting that.

And substitution of this and basically that  $S$  is substituted by  $S$  plus this parameter the total  $\Delta S$  is your file settlement plus the elastic pile compression put it into this formula what you are having here is very similar to what was proposed earlier three parameters here like this you know you have this total compression and the elastic and plastic compression of the pile. It is very similar only thing is we are just trying to get that form there multiplied by  $\eta$  which we just derived based on the co-efficient of restitution.

So this is the resistance offered by the soil if the hammer parameters are like this, so forward or backward calculations can be made. If you know all these parameters of hammer and the



pile head, pile restitution and the efficiency and the set value that you want to set. Now if you want to set value of say 1 inch I want to see 1 inch then I will have to calculate this value because these are known values of the pile and I know what is the plastic compression I want to allow if I do not want to allow I can remove it.

And minus 1 inch minus this will give you the actual set value then I substitute here and then I can find out what will be the maximum capacity achieved by the pile at that time of lost strike. So you keep on driving and you stop when the final penetration is say 1 inch is going not beyond that. So you stop there so you can calculate back what capacity it would have achieved or vice versa you know what is the resistance calculated by you using your soil mechanics calculation during driving this is the estimated resistance.

And I want to find out what is the hammer size required. So I can keep these two parameters separately and I know historically I would like to restrict my set value to 20 mm then I can substitute back, I know the value of resistance, I know the value of set I want to set for the final driving then I can calculate I require the hammer of this height and the weight.

So that is exactly the idea dynamic pile driving formula is very simple as long as you know the weight of the hammer and the fall height which is the energy form and you have the resistance computed from your soil mechanics calculation of pile diameter is given, soil layers are given only thing is you need to find out at the time of driving what could be the resistance which I think we discussed about this in the last class clay may actually behave differently than the sand, sand may actually increase the resistance during driving because of densification, whereas clay if it is a soft clay it might actually break down but come back after few hours, if it is a hard clay it will break down but it will stay for a longer period of time setting back will not happen.

So you need to understand the  $R_u$  is not exactly same as the static capacity you calculated, I think last few classes we were doing the static pile capacity calculation based on API code you know alpha method, beta method and so on we have been doing that you know very well that is not exactly this because that will be a long term capacity after driving the pile several years later the pile soil interface has set up some resistance and the capacity so much.

But this  $R_u$  is at the time of driving how much resistance is offered by the soil against the driving. So this will be definitely different it could be lower, it could be higher. So this list we established after understanding of the long term static capacity. Normally you know in the

starting of the project what we do is you estimate the static capacity take 50 percent extra if it is a sandy material or if it is a very much soft clay you take only 50 percent of the total capacity as the driving resistance which you can easily understand. If it is a soft clay for sure is going to offer lesser resistance during driving then the long term capacity.

So that is how if you are not sure for sure if do for full capacity. For example if it is a soft clay and you also do not know what could be the resistance offered by the pile during driving the best thing that you can do is do it for the full capacity which could actually happen you know you start driving, stop driving for 6 hours, 8 hours or may be 15 hours because of some problem, come back and restart at that time the soil may actually offer you the full static capacity.

So in any case we may have to do the driving analysis for restart case which is because of pile failure or hammer failure or environmental conditions may be not allowing us to do work. So establishing this  $R_u$  is a very big challenge because that is going to be a deciding factor if you do a calculation which is not correct or if you underestimate or you over estimate when you go offshore you may face problem either way, either your hammer is too big when you place it the hammer because you selected a very big hammer the pile is going very easily that normally has happened in several projects you do an overestimate of resistance and big a big hammer, you expect a hard driving but when you place the hammer without even impact the pile is going down very easily.

Then you have a suspicion differently that whether your original calculation of  $R_u$  is correct and original calculation of even static capacity is correct then you will have a doubt whether the pile has sufficient capacity to take the super structure load or not, in many cases it has happened in that way that we expect a hard driving but because the hammer is too big or the soil would not be understood carefully by the geotechnical engineers, you got the site driving become very very easy instead of expecting a higher blow count you expect one or two blow counts, pile is going very easy.

That also can put the whole project in problem because once the capacity is not sufficient it is as good as the jacket cannot be used because you have a suspicion that whether the super structure load can be taken by the substructure and the pile, if it is not there then you will not get a certification. So that is exactly why the pile Driveability can put you in either way trouble if it is early refusal because the soil is too hard compare to what you understood or if it is too soft or the incorrect selection of hammer would put you into a lot of trouble.

So that is why this analysis so approximate is normally not used because the approximations that you make has got a higher end effect. So that is why dynamic formulas are not recommended for offshore pile driving we go into slight more detail that is the final formula of just rearranging this S this becomes slightly and delta S p has been removed because of the plastic deformation is not.

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**Pile Driveability analysis**

**MODIFIED ENGINEERING NEWS (ENR) FORMULA**

$$R_u = \left( \frac{\eta_h W_h H}{s + C} \right) \left( \frac{W_h + n^2 W_p}{W_h + W_p} \right)$$

Where

- R<sub>u</sub> = Ultimate pile capacity (R<sub>s</sub>+R<sub>e</sub>).
- η<sub>h</sub> = Hammer efficiency.
- W<sub>h</sub> = Weight of Ram.
- W<sub>p</sub> = Weight of pile including pile cap, cab block.
- H = Height of fall of ram.
- s = Set, amount of point penetration per blow.
- n = Co-efficient of restitution.
- C = Temporary compression allowance.

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And rearranging this you will get this kind of formula this is the formula for quite some time even now onshore pile driving people are using it because it is quite easy you do not need to understand much what you need is a hammer weight and the set value that you want to determine as a final penetration and then resistance against driving as ultimate capacity.

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**Pile Driveability analysis**

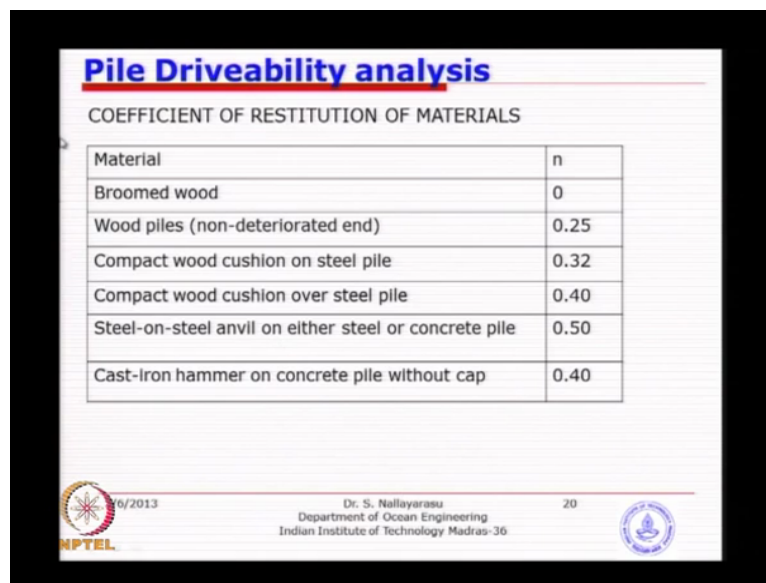
**EFFICIENCY OF HAMMERS**

Type	Efficiency
Drop Hammers	0.75-1.00
Single-acting hammers	0.75-0.85
Double-acting or Differential	0.85
Diesel Hammers	0.85-1.00

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Typical hammer efficiency I have not listed down the hydraulic hammers because you can theoretically speaking hydraulic hammers can go to 100 percent because the efficiency of the hydraulic system even if you look at some of the hydraulic power packs we can operate upto 100 percent pressure but normally that the owners do not want to design any pile driving system to 100 percent about 95 percent we use, whereas you can see here mechanical drop hammers or diesel hammers in the range of 70 to not 100 definitely for sure you will not be able to take it to 100 many of these diesel systems over the age it becomes substantially lower, in fact some of the diesel hammers in use we use even only 60 percent or 70 percent.

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**Pile Driveability analysis**

COEFFICIENT OF RESTITUTION OF MATERIALS

Material	n
Broomed wood	0
Wood piles (non-deteriorated end)	0.25
Compact wood cushion on steel pile	0.32
Compact wood cushion over steel pile	0.40
Steel-on-steel anvil on either steel or concrete pile	0.50
Cast-Iron hammer on concrete pile without cap	0.40

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The next table will give you the idea of co-efficient of restitution of different materials interface you know for broomed wood like a soft material almost 0 and the remainder is less than half just for using your.

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**Pile Driveability analysis**

**MINIMUM PILE WALL THICKNESS**

Pile Diameter		Nominal Wall Thickness, t	
in.	mm	in.	mm
24	610	1/2	13
30	762	9/16	14
36	914	5/8	16
42	1067	11/16	17
48	1219	3/4	19
60	1524	7/8	22
72	1829	1	25
84	2134	11/8	28
96	2438	11/4	31
108	2743	13/8	34
120	3048	11/2	37

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Earlier revision of API has given some recommendations depending on the pile diameter what should be the minimum thickness that you should be looking at in order to avoid pile failure by buckling or pile failure by you know (20:37) but in the recent revision of the code they have removed this because there was quite a few cases where you know it was misunderstood that if I satisfy this thickness it could be sufficient enough to drive, in many cases it is not because this is only a minimum but based on your selected hammer and the soil behaviour you will have to determine what thickness is required to avoid pile failure.

But some of the cases I think in last few years there was arguments between contractors and the owners that if I satisfy minimum (21:12) you should have apply this because no one knows actually pile has overstressed or not overstressed during driving because it is underground and it is very hard for you to determine whether the pile has actually overstressed.

Of course failed means it could easily come out because the pile started buckling and the direction could change. But there is a over stress for example the stresses are beyond yield specially if you have not used pile driving analysis like what we are going to learn, if you have used the dynamic pile driving formula there is no way that you can find out what is the stress levels at various locations below seabed.

So that is why this recommended values have been removed from the code but still we could use it as a guidance just to understand what actually we are looking at for example if you look at 60 inch pile, 1 and a half meter diameter what was wall thickness is recommend is

something like 7 by 8 inch or 22 mm. So the D by T ratio if you calculate you know basically is 60 divided by 22 you will get around around 60, you know 1 in. So you can see here 72 inch pile, 1 inch wall thickness, D by T ratio is 25.

Now we remember in the design course we were looking at D by T ratio for structural members from 20 all the way to maximum 60, we do not want to have D by T ratios greater than 60. The obvious reason is we do not want local buckling to govern the member stresses is it or not, that is why we are so between 20 and 60 and of course we do not want to have D by T ratios less than 20 because you will not be able to fabricate the bending strength during fabrication itself will be very large value and which is going to be a residual stress.

So the 20 to 60 you could see most of the values of D by T ratios are around lower bound, so that means during driving you do not want to have the pile to have local buckling failure especially large axial value coming as a dynamic stress, you do not want to have a buckling failure. So that is the meaning that we want to understand the D by T ratio suppose you see here D by T ratio will be even 600 by 10 will be reaching about 58, whereas if you look at 72 and 60.

So you do not want to have values beyond 60 which will put you into trouble and that is why most of the pile selection that we are doing nowadays in actual situations we keep somewhere around 35, 40, or 45 maximum. So in this using this lower bound values if you try to argue, you definitely will not be correct that is why the course have removed this particular recommendation and it is between 20 and 60 you select a middle value and start doing your pile driving analysis and then later if you find you require to increase the thickness you can increase.

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### Pile Driveability analysis

**GUIDELINE ON HAMMER ENERGY**

Pile Outside Diameter mm	Guideline Wall Thickness, mm					
	36	60	120	180	300	500
610	13	13	22	—	—	—
762	14	14	18	—	—	—
914	16	16	16	22	—	—
1067	18	18	18	19	32	—
1219	19	19	19	19	29	44
1524	22	22	22	22	22	35
1829	—	—	25	25	25	29
2134	—	—	—	29	29	29
2438	—	—	—	32	32	32
2743	—	—	—	—	35	35
3048	—	—	—	—	38	38

Consider a hammer of 100 Tonnes Ram weight and a stroke height of 1.5m will give a hammer energy of 150 Tm or 1500 kNm or 1500 kJ

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Similarly there was another guidance given earlier in the previous revision, now it is no more because of again quite a few arguments that what type of hammer or what is the kind of energy requirement you should select for a given pile size, as you can see larger the diameter of course why we go for larger diameter, the larger diameter we go because the smaller diameter we are unable to get the capacity and that means larger diameter you might have already gone for larger penetration also.

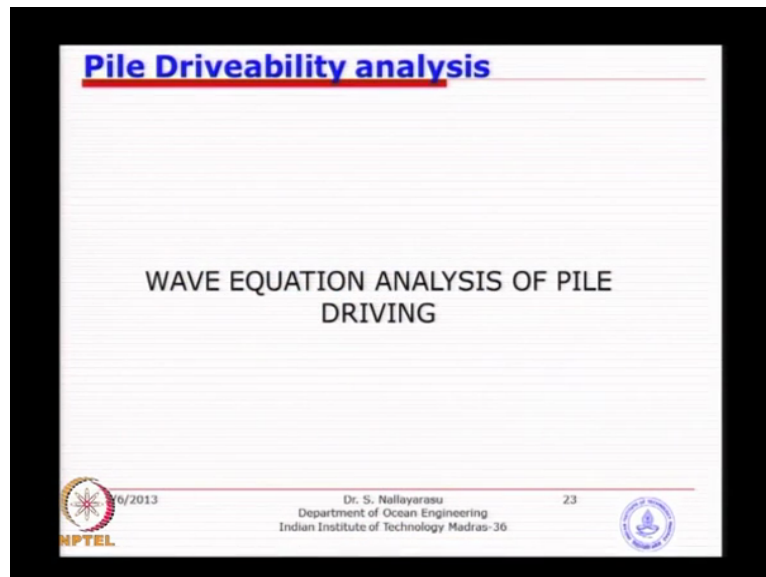
So that is why depending on the diameter there was a recommended values of energy kilo joule which is what we were looking at 500 kilo joule is the reasonable hammer commonly used in onshore or coastal pile driving. But 500 is too small for actually any offshore pile driving of jacket type of structures where you have the penetration of 100 meters. So that also have a reasonable issues in the recent times and has been removed but you can see here as the piles size increased definitely the most of the hammers that normally we use is about 1000 kilo joule most of the pile driving hammers.

If it is a 1500 hammer it is definitely a very big size which you do not want to put it on to a pile of smaller size which will make the pile stresses very very large for both reasons one is the weight itself, because as the energy content is the energy of the hammer is larger the RAM size is larger and once the RAM size is larger thus the total weight of the hammer itself is going to large so static stresses will be very large.

For example if it is a 200 ton like this one is 100 ton RAM weight will make the whole weight of the hammer its 200 tons. So 200 tons when you place it may be it is not a very big

stress because your pile is designed for 1000 tons but it will be very large because is a cantilever, after final installation may be it can take 1000 tons, so that is where you have to see that and because of the dynamic impact of the 100 tons, the dynamic stress also will be quite large we will be looking at that.

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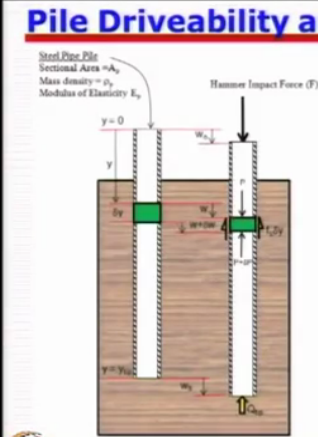


So quickly we will look at what we will do it for offshore driving system, one of the thing is you might have already taken the hydro dynamic course I think might have seen the wave equation in 2 dimension I think at least are at least in 2 dimension you might have derived and solved using method of separation of variable I think we have been introduced (( ))(26:59) theory. So similar derivation only thing is in this one it is not a surface wave it is going to be a stress wave propagating in a material which is metal or soil.



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**Pile Driveability analysis**



Steel Pipe Pile  
Sectional Area  $= A_p$   
Mass density  $= \rho_p$   
Modulus of Elasticity  $E_p$

Hammer Impact Force (F)

Notation

- $w_0$  = Displacement of pile top
- $w_{tip}$  = Displacement of pile tip
- $w_0 + \text{elastic compression of pile}$
- $w = w_0 + w_c$
- $w$  = Displacement of the small element
- $\delta w$  = Elastic compression of small element
- $F$  = Impact force applied at the pile top
- $P$  = Compression force on the small element
- $\delta P$  = Initial force on the element
- $f_s$  = Frictional resistance of soil
- $Q_{tip}$  = End bearing resistance of soil at the tip

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So we just derive the equation today, so you can see here a picture showing a pile given by impact at the top by a hammer impact force is  $F$  and then take a elemental part of the pile which is  $\delta y$  thickness and trying to just drive by impact force of  $F$ , that  $F$  is produced by the impact which is having a certain weight, certain fall height and energy is transmitted we do not know what is that force, we need to determine that force at the time of impact and how it actually propagates through the pile itself.

And  $w$  is the displacement,  $w$  at the bottom will contain soil deformation plus the pile deformation. The pile top will be  $w$  not just only the amount of going down which does not include the pile compression either elastic or plastic. So these are some of the notations you just understand. So during this process what happens is the small elemental piece of material which is shown in this green colour has got velocity for sure because during the time of impact it is going to have a displacement and if the displacement changes because there is a resistance coming from the soil during the time of propagation of stress wave then it will have a velocity and if the velocity changes because of the varying nature of resistance you will have actual ratio.

So what we are going to look at is that dynamic equilibrium of this piece of small element in the pile during the process of impact and after the impact what happens when the stress waves return back if the soil is having not much of energy absorption characteristics and come to an equilibrium equation and relate it that is what we are trying to drive so various notations are given here.

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**Pile Driveability analysis**

Consider a small element of thickness  $\delta y$ , located at a depth of  $y$  from the pile top, the force on the segment can be calculated using Hook's Law.

Strain in the element due to elastic compression ( $\epsilon$ ) =  $\frac{\delta w}{\delta y}$

Stress in the element ( $\sigma$ ) =  $E_p \frac{\delta w}{\delta y}$

Force acting on the element ( $P$ ) =  $\sigma A_p$

$$P = E_p A_p \frac{\delta w}{\delta y}$$

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So if you look at the strain in that small piece of metal is basically the ratio of your displacement change in displacement with the change in the thickness delta w by delta y and you can calculate the stress by multiplying the strain multiplied by modulus of elasticity from your Hook's Law. And then force on the element is simply by stress multiplied by your area of the cross section which is annular area of the pile itself.

So you can drive a simple equation for the force in that element which is E times, A times the strain which is delta w by delta y, delta y is your thickness. This is basically the you know the force arising from the static component which is basically your Hook's Law.

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**Pile Driveability analysis**

The above equation can be written as (after taking limit of  $\delta = 0$ )

$$P = E_p A_p \frac{\partial w}{\partial y} \quad (1)$$

Due to impact force applied at the top of the pile, the stress wave travels along the pile and causes the element accelerate downwards. The velocity and acceleration the element can be found as

$$v = \frac{\partial w}{\partial t} \quad \text{and} \quad a = \frac{\partial^2 w}{\partial t^2} \quad (2)$$

where  $w$  is the instantaneous displacement of the element. The acceleration of the element causes the inertial force developed on the element equal to

$$\delta P = \rho_p A_p \delta y \frac{\partial^2 w}{\partial t^2} - f_i \delta y \quad (3)$$

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**Pile Driveability analysis**

Dividing by  $\delta y$  and taking limit  $\delta y = 0$ , yields


$$\frac{\partial P}{\partial y} = \rho_p A_p \frac{\partial^2 w}{\partial t^2} - R \quad (4)$$

From equation (1),

$$\frac{\partial P}{\partial y} = E_p A_p \frac{\partial^2 w}{\partial y^2} \quad (5)$$


Substitution on (4) in (5), yields,

$$\rho_p A_p \frac{\partial^2 w}{\partial t^2} - R = E_p A_p \frac{\partial^2 w}{\partial y^2} \quad (6)$$



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Similar thing can be done using your dynamic, so basic the displacement divided with respect to time if you monitor with respect to the time component  $\Delta w$  by  $\Delta t$  will give you velocity and first derivative of that will give you the acceleration and then similarly you can take the acceleration multiplied by thickness multiplied by area will be your volume multiplied your density of the pile material which is  $\rho P$  will give you mass time acceleration will be your force.

And you take this first equation to differentiate with respect to  $t$  and  $(\frac{\partial}{\partial t})^2$  you can rearrange this  $\Delta y$  down here I have written in next page  $\Delta P$  by  $\Delta y$  will be just rearrange the terms you will get a equation of this kind and from that first equation we had a equation for  $P$  equal to something just differentiate with respect to  $y$  and equate them because both of them are same from one from dynamic and another one from static, you will get a equation of this kind you know basically and you can rearrange and reorganize.

Very similar equation you might have seen in your wave theory very similar  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}$ , I think you might have been going through that course and method of separation of variable for both direction  $x$  and  $y$ . We are going to do exactly the same only thing is here you got one of the direction which is your direction of the pile, the other one is time.

So this is both the coupled equation between time and space but only one dimension because the pile is uniform or you can have a cross sections change when you use a numerical method to divide the pile into several segments. And  $R$ , I just forgot to tell you here this is the inertial force generated due to its mass of accelerating component of that small element minus the

resistance offered by the soil, if the soil resistance is uniform throughout nicely then you can take that  $f$  s multiplied by  $\delta y$ , of course you can have your  $\pi d$  multiplied if you have a circumference.

So this is the resistance offered by the soil that means every time the pile is trying to accelerate and the resistance is provided by the soil in terms of skin friction and that if you integrate you will get a value just notation changed and that is what is appearing here. Now if there is no resistance that means it will just keep going down. So that is the equation that we were looking at.

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**Pile Driveability analysis**

Divide by  $E_p A_p$ , we get



$$\frac{\rho_p}{E_p} \frac{\partial^2 w}{\partial t^2} - \frac{R}{E_p A_p} = \frac{\partial^2 w}{\partial y^2} \quad (7)$$

Where  $c = \sqrt{E_p / \rho_p}$ , defined as velocity of stress wave and  $R$  is the soil frictional resistance and hence the above equation can be written as

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2} - \frac{R}{E_p A_p} \quad (8)$$

The above equation is the one dimensional wave equation and can be solved using method of separation of variables or numerical methods

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**Pile Driveability analysis**

Dividing by  $\delta y$  and taking limit  $\delta y = 0$ , yields

$$\frac{\partial P}{\partial y} = \rho_p A_p \frac{\partial^2 w}{\partial t^2} - R \quad (4)$$



From equation (1),

$$\frac{\partial P}{\partial y} = E_p A_p \frac{\partial^2 w}{\partial y^2} \quad (5)$$

Substitution on (4) in (5), yields,

$$\rho_p A_p \frac{\partial^2 w}{\partial t^2} - R = E_p A_p \frac{\partial^2 w}{\partial y^2} \quad (6)$$

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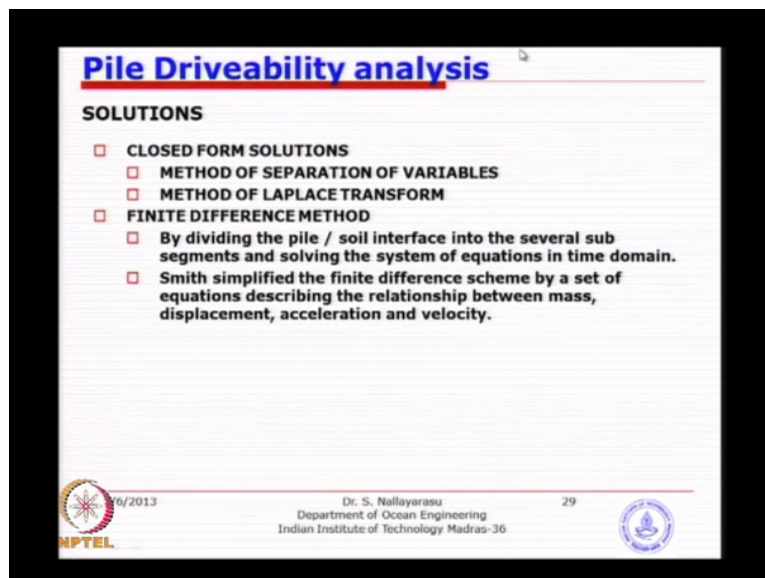
You can rearrange the whole thing something like this by just substitution of the stress wave which is  $E$  by  $\rho$ ,  $E$  is the modulus of elasticity of your steel material,  $\rho$  is the density, just

rearranging the same thing we are not doing anything just rearranging substitution. You will get an equation of this kind and basically this is called the wave equation or stress wave equation very similar to the surface wave equation, only you will not see that this term is slightly different.

And we need to find the solution that is because what we require is just after one impact how the displacement pattern along the length of the pile and during the time until the stress wave goes down to the pile onto the soil and reflects back. So we need a time history of pile movement, pile acceleration, pile velocity for every impact and that is the idea behind and you can actually solve using method of separation of variables if you have a simplified conditions.

For example soil is uniform just only single layer soil no end bearing and pile material is very much uniform you could solve very similar to the wave theory but in reality we do not have such conditions piles are going to have different wall thickness, soil is going to have varying characteristics, you will have skin friction, you will have end bearing. So that is why we normally do not try to go for the simplified solution, we go for a numerical solution for practical applications.

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**Pile Driveability analysis**

**SOLUTIONS**

- **CLOSED FORM SOLUTIONS**
  - **METHOD OF SEPARATION OF VARIABLES**
  - **METHOD OF LAPLACE TRANSFORM**
- **FINITE DIFFERENCE METHOD**
  - **By dividing the pile / soil interface into the several sub segments and solving the system of equations in time domain.**
  - **Smith simplified the finite difference scheme by a set of equations describing the relationship between mass, displacement, acceleration and velocity.**

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So variety of methods have been in use for solving this equation. So I was talking about this one very commonly used for this type of second order differential equation very common, most of solutions for dynamic equation we could find. Method of Laplace transform I think

you might have studied in your mathematics course, also can be used slightly easy to obtain solutions but not very common.

And then for historic region 1960's and 70's this equation has been solved by finite difference method very successfully because it is only one dimension with time marching scheme. So if you know the results of the previous time step they are going to the next time step easier and if you know the results of the previous segment you divide the pile into several sub segments, if it is a 100 meters pile divide them into 100 sub segments.

So previous segment what is the force transfer to the next segment, next segment, next segment and then you can solve for it. So there is forward space scheme and forward time scheme and how you solve it is a interesting technique, it has been established several decades back is a very simple time margin scheme which is being used even now, any software you might offered Newton Raphson scheme, Newton Raphson method I think in mathematics you might have studied.

So we will be looking at that solution which is implemented in several software and the pioneer in doing this work is one professor Smith he has done this one when he was in university and then he sold the program to one company. Now that company has actually owning that software and overall there are not too many software for pile driving analysis, I think only one or maximum one or two softwares which is being used in the industry we will have a look at it when we try to go to that stage.

But otherwise most of the time we use the numerical scheme because of the complexity of the system that we have a hammer, we have a bearing pile segments with soil different and we also need accurate solution because the complexity for offshore projects is slightly more compare to onshore. So we do not want to use dynamic pile driving formula, neither we use the closed form solution because it makes too many assumptions that may not fit into our systems.