

Course on Foundation for Offshore Structures
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Module 1
Lecture 14
Pile Foundation 5

So we will continue with the lateral bearing capacity of the pile foundations.

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Pile Foundations

BASIS OF BROMS THEORY

CLAYEY SOIL

- Lateral soil reaction assumed uniform along the depth
- Value of soil reaction assumed as $9C_u D$
- Top $1.5D$ depth of soil is taken as ineffective

SANDY SOIL

- Lateral soil reaction proportional to the depth and is increasing with depth linearly
- Value of soil reaction taken as $3K_v P_{ov} D$

SHORT OR RIGID PILE

- Soil failure leads to pile rotation or pile displacement depending on long or short pile

LONG OR SLENDER PILE

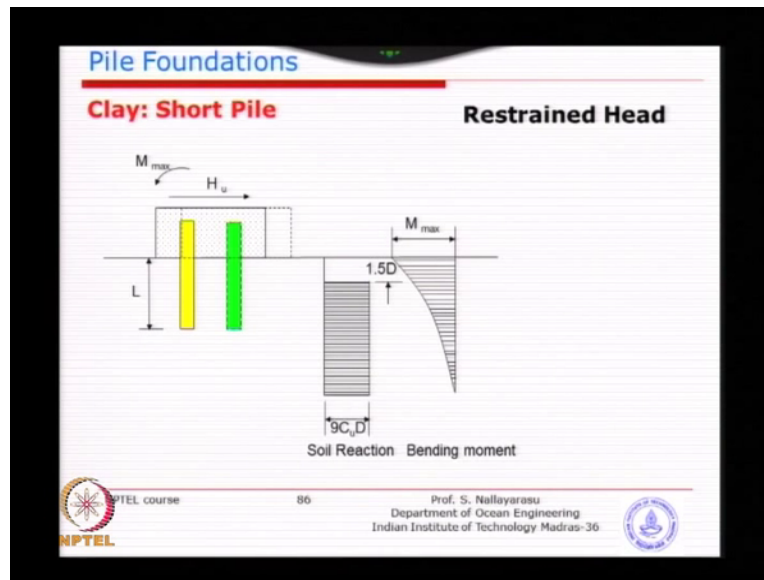
- Soil failure at top leads to pile failure by yielding or bending. Ultimate pile moment of resistance governs the capacity

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Today we will just continue on the few cases of Broms' theory I think we have already seen few of them. The basic idea of this Broms' theory we can apply to clay, soil as well as sandy soil and you can see here very clearly for clay soil it is uniform after certain initiation with lower capacity but for sandy soil it is linearly proportional to the overburden pressure so it is basically similar idea like what we have for vertical capacity.

And also can be classified into short pile and long pile depending on the relative flexibility of pile with the soil.

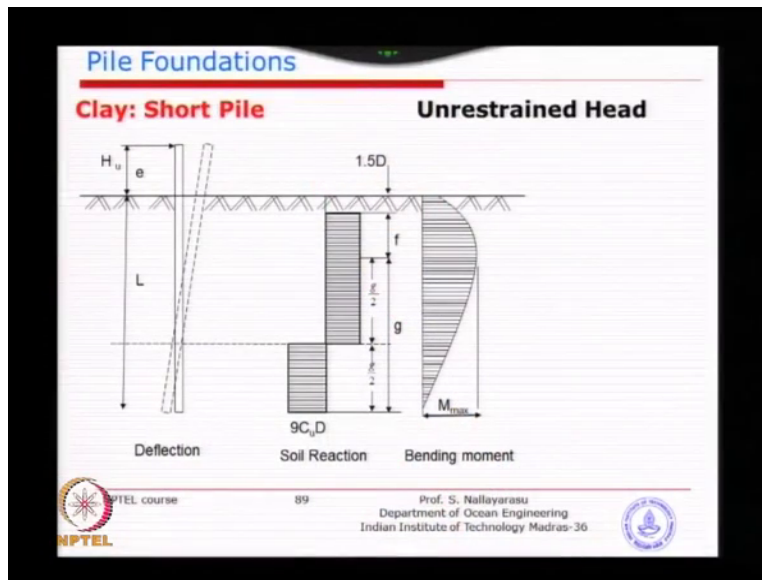
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We have already reviewed through some of the cases like short pile with a pile cap that means the few piles connected together the variation of lateral resistance of the soil is almost uniform except that starting we have a loose soil where does not give you that much of resistance. So he has assumed 1 and a half diameter and restrained head, so just like a translational characteristics which is what will happen if you try if you have a nail in certain ground and try to apply a large load and if you have the pile is almost rigid not must flexibility like a concrete short pile you will see something like this.

So several cases of limiting configurations he has investigated and come up with some solutions basically based on elastic theory. Instead of assuming completely rigid he has taken the elastic values of if it is a concrete pile you can use EI or steel pile so you just incorporate but as long as your EI value of the pile is very large compare to the soil EI value so that is where the difference.

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In the last one we were seeing is this the short pile and unrestrained head this is the case mostly you will encounter in offshore or costal applications where there is not much a big structure at the top you may have a structure but is not going to as rigid as onshore structures where we have a very big building with a pile cap restraining the pile foundation moment.

So most of the cases something like this the value of eccentricity which we call (e) (2:25) is not actually eccentricity is the height of the structure above the seabed in our case. So if it is longer and longer it becomes quite flexible so then it will become not short pile it become a slender pile. In some of the cases like coastal waters 5 meter water depth, 10 meter water depth you may actually have a short pile category.

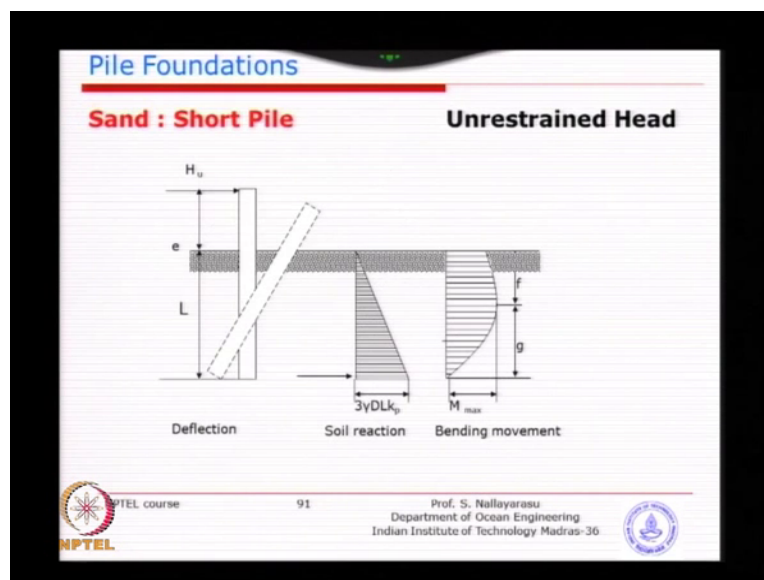
The distribution is almost similar to the Brinch Hansen theory you can see here the forward reaction and backward reaction the pile is trying to rotate almost similar the idea is similar and the bending moment is going to happen somewhere maximum just below the seabed and that is the point of interest that we are looking at. And the variation as you can see here because it is a clay soil is a uniform distribution of resistance forward and backward 9 times the C_u multiplied by the projected width of the pile which is the diameter so it is just giving you the resistance against the lateral moment.

If it is a long pile, the only big difference is after certain distance the pile becomes almost not influenced by the load itself as you can see you drive a very long nail what will happen is the pile

is influenced by the load applied at the top to a certain depth because that is where all the loads are taken and below which it becomes just not much influence that means the displacements or the reaction from the soil is almost diminishing which will be the case when we are looking at offshore pile like a jacket structures you will see that mostly first 20, 30 meter of the pile will be influencing pile will be influenced by the loads and the reaction from the soil after that it becomes almost then we can as why we require that we require because of the vertical capacity because we need a lot more vertical capacity than the horizontal load.

So what we need is take the pile down to a bearing stratum or distribute the frictional stresses or frictional loads to the deeper soil. So that is where you will find the length is required for vertical capacity but of course the fixity will not be happening very deep maybe within first 30 meters or 40 meters. So that is why you will see most of the time the bending moment for the piles will be maximum at certain depth below the seabed and then after that it will start reducing you will see many times this can go into secondary distribution.

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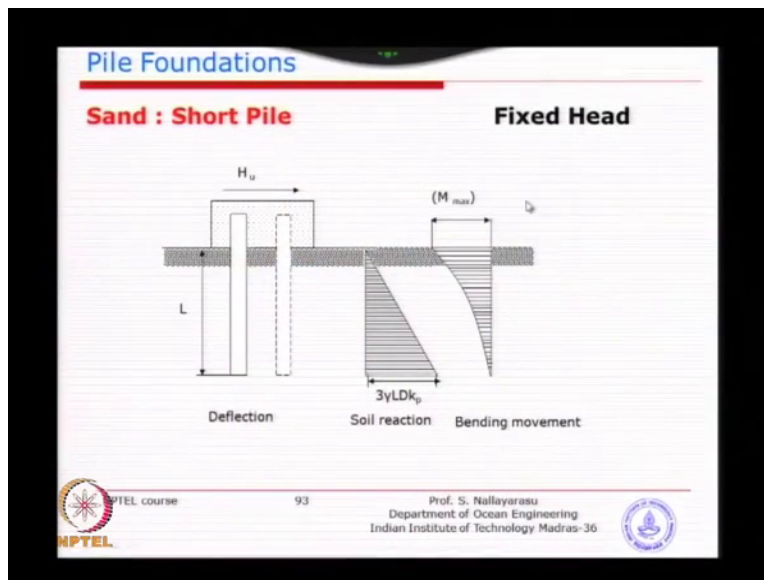


And then again we will just go into the sandy material that what we saw was is a clay then we can go into sandy material the only change is the resistance is linearly proportional to the overburden pressure so you will see a triangular distribution and that is the measure difference between the clay soil and the sandy material so reminding is behavior will be short pile or long pile and with and without the pile fixity at the top, so you will see 4 cases similar idea only you

will change the distribution of resistance unrestrained head basically that means just very similar to a cantilever.

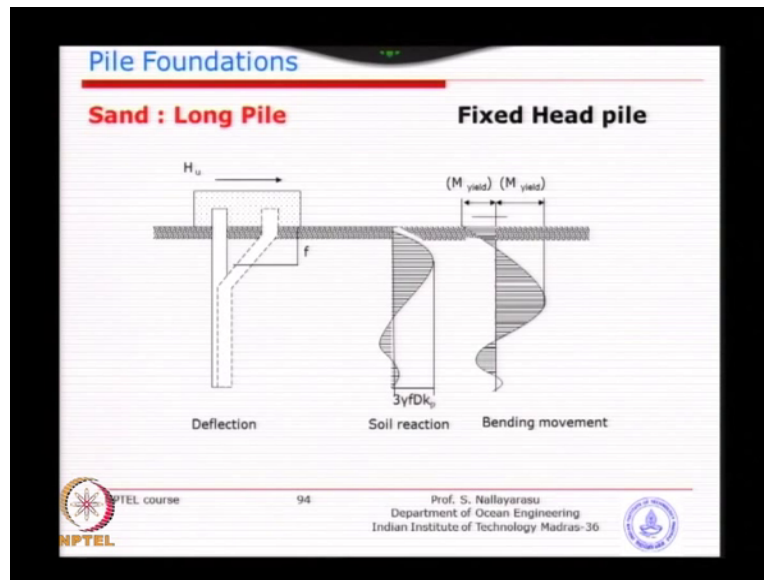
So what you saw in the case of clay it was just no resistance and then we have a kind of rectangular distribution whereas here is slightly complex you can see here because of the nature of type of soil and that is what has assumed based on his testing. Similarly a short pile with fixed head almost very similar to clay soil except the distribution is triangular it is just moving horizontally and the bending moment is maximum it almost behaves like a reverse cantilever because the fixity is on the pile head and the soil is trying to push the pile and get the bending moment developed at the maximum at the top.

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Of course you will see the my bending moment will be 0 at the other end is just a reverse cantilever.

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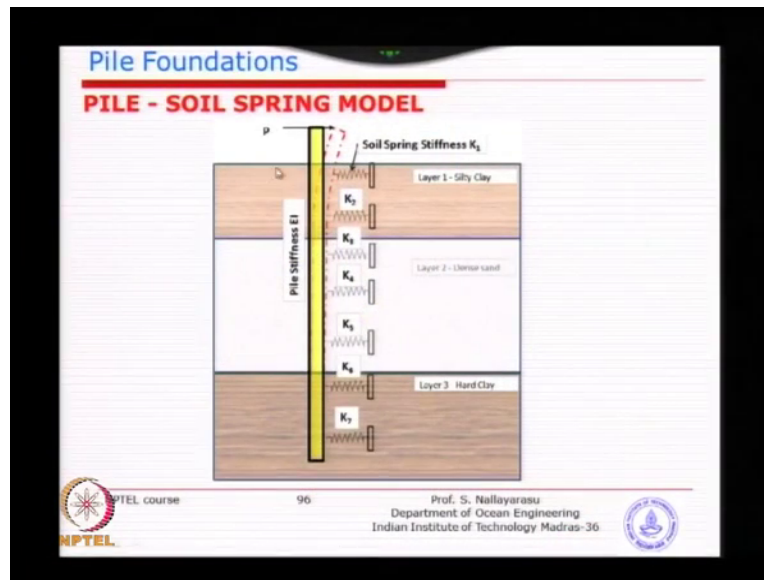


Similarly you have a long pile but still fixed headed almost you will see double locations of bending moment, bending moment maybe maximum here and because of the long pile you also have the bending moment developing at the some depth below the so all this is various limiting cases. So how do we get the general solution is we have to solve the beam on elastic foundation, so we will just look at some of these cases.

So first let me introduce the pile soil spring model so these are the cases proposed by Broms' as well as Brinch Hansen as a limiting cases which or having several restrictions you know except one of them like Brinch Hansen can actual model multilayer soil different type of piles whereas all these other cases are very much limiting cases which may not be applicable to offshore applications. So when it comes to realistic long pile with various layered soil with the different characteristics how do we model is one of the biggest question always will come to offshore engineers especially the jacket type of foundations.

So we need to look for some numerical methods instead of close form solutions because is quite difficult to model in such manner. So that is why the spring model is one concept very similar to beam and elastic foundation if you turn this upside down you can see here the beam having uniform bending stiffness of course in reality you have actually varying at different segments you will have different wall thickness different diameter if require.

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And of course modulus of elasticity will be uniform but you can have a varying thickness as required resisted by the soil springs especially when you apply horizontal loads very similar to what we were looking at the vertical springs and the end bearing spring very similar only thing is on the horizontal direction you could see here when the pile is too flexible it might experience larger displacement and actually push the top soil quite a bit and as you go down the force on the soil springs will be reduced as the soil springs at the top take more and more loads because the pile is more flexible there the inducing displacement is more.

So if the inducing displacement is more is getting more compression because of the pile relative flexibility with the soil.

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Pile Foundations

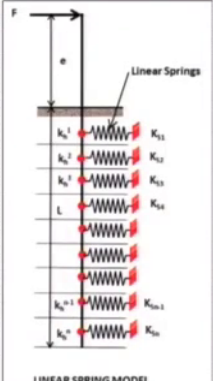
LINEAR SPRING STIFFNESS METHOD

Modulus of subgrade reaction is determined using the following equation (Vesic - 1961)


$$k_n = \frac{0.65}{D} \sqrt{\frac{E_s D^4}{E_p I_p}} \frac{E_s}{1 - \mu_s^2}$$

Where

- K_{ni} Modulus of subgrade reaction
- E_s Modulus of soil
- D Diameter of pile
- E_p Modulus Elasticity pile material
- I_p Moment of inertia of pile
- μ_s Poisson's ratio




LINEAR SPRING MODEL



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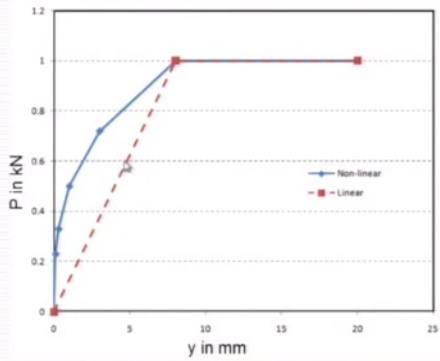
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Pile Foundations

LINEAR AND NONLINEAR LOAD-DEFLECTION CURVES




Deflection (y) in mm	Load (P) in kN (Linear)	Load (P) in kN (Non-linear)
0	0.0	0.0
2.5	0.25	0.5
5	0.5	0.7
7.5	0.75	0.9
10	1.0	1.0
20	1.0	1.0



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Now this is what we want to study and do a numerical solution how we can make a solution to this type of problem many many numerical methods are available but of course the simplest one is new mark method which he proposed a concept basically a linear if you go back to the previous picture I think we have discussed few days before the spring linear versus nonlinear we discussed the advantages disadvantages I think If you look at this for a given displacement you know if you look at 5 mm for linear you can take capacity of 0.6 kilo newton whereas for nonlinear if you actually look at projection from here you can go up to 90 percent or 85 percent.

So what we are assuming is conservative approach taking a lesser capacity than what actually the soil going to offer.

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Pile Foundations

Based on Newmark's distribution the spring stiffness is given as follows

First spring
$$K_{s1} = \frac{Dl}{24} [(7k_n^h) + (6k_s^{n-1}) - (k_n^{n-2})]$$

Intermediate spring
$$K_{s-m} = \frac{Dl}{12} [(k_n^{n-1}) + (6k_s^n) + (k_n^{n-1})]$$

Bottom spring
$$K_{sm} = \frac{Dl}{24} [(7k_n^h) + (6k_s^{n-1}) - (k_n^{n-2})]$$

where

- D Diameter of the pile
- l Spacing between two adjacent springs
- k_n^h Modulus of subgrade reaction of n^{th} spring

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So that is the approach taken by New Mark and basically he proposed empirical method to calculate the resistance which is basically the lateral capacity and distribute the springs in a manner similar to what the assumptions made by Brinch Hansen and Broms' what we he is assuming the last most spring or the last few springs will be weaker and that is what we saw in our assumptions made by Broms' I think first 1 and a half diameter of the depth the soil will offer limited resistance or maybe no resistance which is realistic true because when the pile is pushing the soil the soil will start squeezing away or the bottom of the pile it may not experience enough resistance because the pile is trying to rotate.

So that is what the assumption he was trying to assume and that means the intermediate spring is having more strength than the starting and ending spring you can see the denominator basically this 24 is decision by 24 we will look at one of this case basically if you look at the middle spring D is the diameter of the spring l is the segment so what they are doing is diving the whole pile into several sub segments say for example total length of the pile embedded into the soil is 100 meters you divide them into 1 meter segment each so l will be 1 meter D.

And the stiffness of the or the spring of value of the previous segment and the next segment is available so you do a weight average. So you can see here the middle springs will be or the

springs in the middle throughout the pile length will be stronger than the pile the springs at the top and the bottom which is a reasonable assumption. Basically even today we do this method of New Mark distribution for many of the applications in fact many of the computer programs written is based on this type of approach which simplifies our.

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Pile Foundations

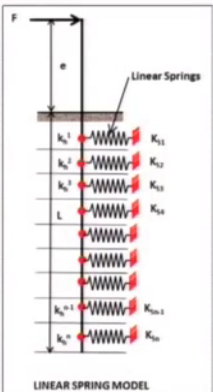
LINEAR SPRING STIFFNESS METHOD


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Where

- K_h Modulus of subgrade reaction
- E_s Modulus of soil
- D Diameter of pile
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- I_p Moment of inertia of pile
- μ_s Poisson's ratio






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So what is the value of k_h is this something that we need to find out k_h is nothing but modulus of subgrade reaction now this we need to introduce ourselves what is that is resistance against the pile moment horizontal direction and we need to just look at how it is related to modulus of elasticity. So you can see here most of the time obtaining this reaction value is so difficult so Vesic has proposed empirical method relating the modulus of elasticity which is available from either (12:16) test or you do a specialized horizontal or vertical load test to obtain this E_s value and using that E_s value he has related the pile flexibility into a factor so that the amount of soil displacement depends on how much the pile is going to deflect.

So that is the idea behind of course this is not an derived equation it is a empirical method after doing experiments. So he has proposed the resistance against the pile moment will be offered by the soil depending in the type of the pile and its flexibility or rigidity together. So he proposed this method to calculate so that means if you have multilayered soil for each of the layer you have value of E_s you have the value of Poisson's ratio of the soil and you will have the pile

stiffness E_p , E_p is the modulus of elasticity of the pile I_p is the modulus of moment of inertia of the pile cross section and you have the soil and diameter of the pile is available.

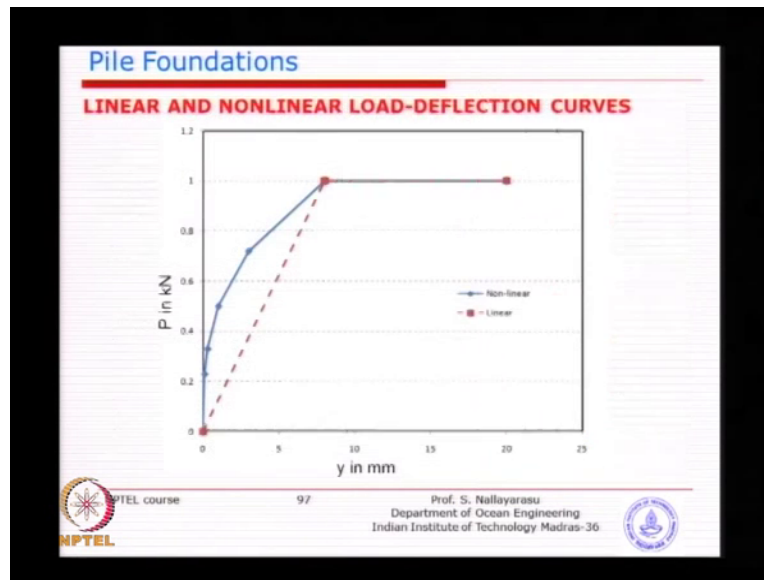
So everything will be available for each of the layer you can calculate the K_h value what you are trying to do here is distribute the or adjust the stiffness of the soil in relation to the what is the soil above and what is the soil below and as you go across when you come to the near end or near the seabed you will see that it is a reduced stiffness because you will lose one of the upper layer because it is no more and you will see that it is very similar assumption to what the other people have done.

And if you know this value of K_1 to whatever the end now the problem has been translated into a simple structural analysis because you have a structural pile which is embedded into the soil and you have converted the soil into a spring which is very similar to the mechanical spring and you can do using any computer analysis you can solve for displacements of the total system because in order for you to do a hand calculation for example if you are asked to do a hand calculation of this pile soil spring model you need to do a sequential transfer of horizontal forces from the pile and you need to find out what is the displacement of this pile at this point (14:28) in order for you to find out what will be the amount of load taken by the first spring then go to the next spring and go to the next spring.

So it will be an iterative solution until an equilibrium is found for each load step, for example if you have 1000 kilo newton here you just divide 1000 kilo newton into say 100 steps and apply the first load find out what is the displacement of the pile all along the several segments as you know very well the displacement here will be more than here more the displacement here the more load will be taken by that soil but if the soil is weaker there it will displace more. So you need to do an iteration until such time the spring and the pile will be in equilibrium condition.

So that means the spring displacement and the pile displacements should be matching until then you do a iterative and then go the next load step. So it is basically that is what most of the computer program now a days even now like any program you take they use this iterative solution for finding out a equilibrium of soil and equilibrium of pile to come to a convergence solution.

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So only assumption here instead of using a nonlinear spring Vesic has given a quick solution that the iteration becomes easier because every time you are going to do a linearized if you go back to this picture you will go through this line rather than this line.

So you do not need to do iteration too many times because is anywhere linear so that is conversions will be faster. Whereas nonlinear you may have to divide the load steps into several small segments otherwise you may not converge the solution may diverge. So the linear versus nonlinear the argument of solution technique depending on type of problem you can decide most of the soil problem linear will be conservative of course imagine if your displacement is somewhere here do not matter whether it is a linear solution or nonlinear solution.

For example, if you have a very soft clay material does not matter you adopt whichever the method your displacement is so large that the soil is already under the plastic stage so you do not need to worry about which type of soil many times that is what will happen for a (())(16:44) clay especially at the top few meters. So one of the method which is adapted in numerical solution is this soil spring stiffness methods.

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Pile Foundations

SUBGRADE REACTION THEORY

This is based on Winkler soil model, the pressure (p) and the lateral soil deflection (y) is related through the **modulus of subgrade reaction** (k_h)

Lateral Pressure $p = k_h y$ where k_h has the units of kN/m^3

Soil reaction / unit length of pile $w = Ky$ where K has the units of kN/m^2

$K = k_h D$

And K is called "**Subgrade Modulus reaction**"

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So let us introduce I have just introduced this term called modulus of subgrade reaction so how do we actually get this understanding of this and we will just quickly go into a subgrade reaction theory which the definition of modulus of subgrade reaction which is called K_h is nothing but the resistance of the soil against the horizontal displacement. So you can see here the pressure induced on the soil is equal to K_h which is the soil stiffness multiplied by y and K_h has the value of or the units of kilo newton per cubic meter.

In other way if you look at the reaction itself this is the pressure this is a reaction you have k times y and k will be taken as K_h times D . So if you multiply by the projected width of the pile multiplied by K_h then it will take the units of your pressure or stress which is called subgrade modulus reaction. So this is basically a reaction whereas this is the slope of the curve the modulus of the subgrade reaction so that is sometimes you have to be little bit careful the names will look almost similar and I think most of the text books they use universally this type of symbol but if some other text books use a different symbol you have to be little bit careful.

So you have to be cautious use cautious in using the units of this modulus of subgrade reaction or subgrade modulus reaction that is why I wanted to give you a clear picture what the units we get. Depending on the application or depending upon which equation you are using some of them will be using k value in fact if you look at some of the older text books they will use the capital

K value which is just taking the units of pressure or stress which is you know something like this or if you are taking the lateral pressure which is Kh times y.

So you have to little bit be careful there but most of the time for offshore applications we will be using this small kh value which is the modulus that means the variation and that will be applicable to steel pipe piles. In many cases for concrete piles they do use this capital K value because that is how the empirical equations have been setup so you have to be cautious in doing that.

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Pile Foundations

MODULUS OF SUBGRADE REACTION THEORY

Governing equation

$$E_p I_p \frac{d^4 y}{dz^4} = -pD$$
$$E_p I_p \frac{d^4 y}{dz^4} = -k_h y D$$

The diagram shows a vertical pile of length L fixed at the bottom. A lateral force F is applied at the top, causing a deflection y. The z-axis is vertical, and the y-axis is horizontal.

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Now the general solution to what we were looking at you might have already studied in your linear elastic mechanics you know the beam theory simple elastic beam theory which is basically a fourth order differential equation.

The only difference here you know you have a lateral load which is supported as the soil spring you know you have a load as well as continuously supported whereas in case of beams you will have a boundary condition at the ends and free elsewhere with a intermediate load, so that is what we are going to just look at that. So if you look at this equation I think as long as if it is simple you can find a close form solution. If it is not simple then you can find a numerical solution something similar to what we have seen here if it is a simple means one layer of soil single cross section single material load is simplified then you can find a close form solution

which is very easy which is what many people have developed and I think some of them are matching with Broms' theory.

Some of them which just I have got a 2 or 3 limiting cases for this this we can just look at it so that when you apply the principle to you know multilayered soil you can easily understand.

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Pile Foundations

Closed form solution for Constant k_h

Lateral deflection
$$y = \frac{2F\beta}{k_h D} \left[\frac{\sinh \beta L \cos \beta z \cosh \beta(L-z) - \sin \beta L \cosh \beta z \cos \beta(L-z)}{\sinh^2 \beta L - \sin^2 \beta L} \right]$$

Rotation
$$M = -\left(\frac{F}{\beta}\right) \left[\frac{\sinh \beta L \sin \beta z \sinh \beta(L-z) - \sin \beta L \sinh \beta z \sin \beta(L-z)}{\sinh^2 \beta L - \sin^2 \beta L} \right]$$

where
 z - vertical coordinate
 D - Diameter of pile
 L - length of embedment
 F - Applied horizontal force

$$\beta = \left(\frac{k_h D}{4E_p I_p} \right)^{1/4}$$

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So one of the close form solution for constant K_h , so that means the modulus of sub grade reaction is uniform and basically the lateral deflection and the rotation moment is as the pile head is given by this complex equation basically using the derivation of this K_h value means the stiffness of the soil is same and the it is related to a parameter called beta we call it pile rigidity or pile relative rigidity and it is related to diameter and K_h is the representative of the soil you know the modulus and related to the E and I of the pile itself.

So you can see here the equation of a simple beam on elastic foundation is this and you have a simple assumption of one single layered soil single pile material cross section and you could derive the deflection which is what is more important for us because using this displacement you can go back to the soil spring find out what is the capacity and then do an iteration. So this parameter what we call it beta will give us the story that whether the pile is behaving as short pile or a long pile that is what we are going to see just in.

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Pile Foundations

Limiting solutions for Constant k_h

$$\beta = \left(\frac{k_h D}{4E_p I_p} \right)^{1/4}$$

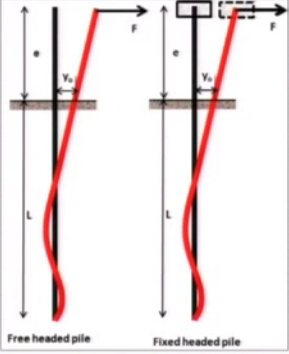
Long Pile ($\beta L > 2.5$) - Free Head

Lateral deflection $y_0 = \frac{2F\beta(e\beta + 1)}{k_h D}$

Rotation $\theta_0 = \frac{2F\beta^2(2e\beta + 1)}{k_h D}$

Long Pile ($\beta L > 1.5$) - Fixed Head

Lateral deflection $y_0 = \frac{F\beta}{k_h D}$



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And limiting case solution for constant K_h beta is defined as this from here you can see just the same equation I have just rewritten here long pile when this must be beta I think is a mathematical sometime it changes during computer to computer. Any why beta times L greater than 2.5 its free head then basically it is called long pile. So when you have computed the beta value or a beta value here multiplied by the length of the pile embedded into the soil is greater than 2 and a half then if it a free head then it is called long pile.

If it is a fixed head if it is beta L is greater than 1 and a half then it is long pile. So that is the idea behind why we need the values of beta is to find out the relative flexibility or rigidity of the pile so that you can classify them then you can apply corresponding limitations. So that is the idea of this beta value is given here, similarly we can go into a different type of soil because here we have got a case where K_h is constant. Now if K_h is varying for example a sandy material it is not going to be uniform then even in single layered soil as you can see pressure distribution in sandy material is a triangular distribution because is increasing with respect to the overburden pressure.

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Pile Foundations

Limiting solutions for Constant k_h

$$\beta = \left(\frac{k_h D}{4E_p I_p} \right)^{1/4}$$

Short Pile ($bL < 1.5$) - Free Head

Lateral deflection $y_0 = \frac{4F(1+1.5e/L)}{k_h DL}$

Rotation $\theta_0 = \frac{6F(1+2e/L)}{k_h DL^2}$

Short Pile ($bL < 0.5$) - Fixed Head

Lateral deflection $y_0 = \frac{F}{k_h DL}$

Free headed pile Fixed headed pile

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Pile Foundations

Limiting solutions for Constant k_h

$$\beta = \left(\frac{k_h D}{4E_p I_p} \right)^{1/4}$$

Long Pile ($bL > 2.5$) - Free Head

Lateral deflection $y_0 = \frac{2F\beta(e\beta+1)}{k_h D}$

Rotation $\theta_0 = \frac{2F\beta^2(2e\beta+1)}{k_h D}$

Long Pile ($bL > 1.5$) - Fixed Head

Lateral deflection $y_0 = \frac{F\beta}{k_h D}$

Free headed pile Fixed headed pile

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And same case when it is free head so in this case we saw free head and fixed head long pile and for short pile if it is less than 1 and half or less than half. So these four cases short pile and long pile free head and fixed head we can classify if you want to do a empirical solution because once you find out which type of category of classification you can go back to the simplified solutions given by either Brinch Hansen or Broms' theory and then take a coefficient for bending moment and design it quickly.

So that is the idea behind some of this cases derived by past literature. And the next case is the limiting solution for linearly varying K_h , that means the value of K_h is not going to be uniform

throughout unlike clay type of soil most of the clay type of soil they have a constant K_h value. In cases of dense clay or sand you may actually have values of K_h varying and K_h varying is related to a value that means if you have a K_h value keep on increasing the slope of that K_h will give you the value of λ that is the idea.

So that λ is related to the flexibility parameter called lambda instead of beta in here we had a beta value, so instead you relate lambda with λ and λ is the increasing or the slope of the K_h value that means a change and the similar derivations could be arrived but slightly complicated and you can classify as a long pile, again that lambda has changed to λ . So lambda times L is greater than 4 for free head and fixed head for most of the cases you have long pile classification.

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Pile Foundations

Limiting solutions for Linearly Varying K_h $\lambda = \left(\frac{\eta_h}{E_p I_p} \right)^{1/2}$

Short Pile ($\lambda L < 2$) - Free Head

Lateral deflection $y_0 = \frac{18F(1+1.33e/L)}{\eta_h L^2}$

Rotation $\theta_0 = \frac{24F(1+1.5e/L)}{\eta_h L^3}$

Short Pile ($\lambda L < 2$) - Fixed Head

Lateral deflection $y_0 = \frac{2F}{\eta_h L^2}$

The slide also contains two diagrams: 'Free headed pile' and 'Fixed headed pile'. Each diagram shows a vertical pile of length L subjected to a horizontal force F at a height e from the top. The deflection curve is shown in red. The fixed head diagram shows the pile is rigidly connected to the ground at the top.

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And for short pile if lambda L is less than 2 you have (())(25:08) for both cases of fixed and free it is called short pile. So you see here we have got 8 cases and 4 cases for K_h value is constant 4 cases for K_h value is varying. So we represent the varying K_h value in terms of λ and this is where you will find you know if you look at API, API will give you the values of λ for varying modulus for most of the sandy material of course they used at different symbol, so we will just need to be little bit careful.

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Pile Foundations

VARIATION OF MODULUS OF SUBGRADE REACTION (k_h)

Variation of k_h along the length of pile essential to predict the displacement of the long pile. Several distributions has been used in the past and the following is general expression developed by Palmer and Thompson (1948)

$$k_h = k_L \left(\frac{z}{L} \right)^n$$

where k_L is the value of k_h at the pile tip ($z=L$)
and n is the empirical value set equal to or greater than zero

For clay type of soil n is generally assumed to be zero and for sand type soil, n is taken as 1 and usually the equation written in the following form

$$k_h = \eta_h \left(\frac{z}{D} \right)$$

Where η_h is the coefficient of subgrade reaction and has the units of kN/m^3

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So in general how do we actually approach this K_h value so originally proposed by Thompson in as early as 1940's K_h value is taking a nonlinear (\quad) (25:53) something like this and linear means is n to the power 1 that means is just or it can be constant or it can be nonlinear. So you can see here most of the clay type of soil n is generally assumed to be 0, so that means is constant value so and for sand type of material n is taken as 1 which will be having linear distribution with respect to z is the depth below seabed and l is the pile embedment length.

So this was originally the idea behind how the K_h value is varying and you know you see here the last one what we were seeing K_h related to ita h is basically the change in value of K_h along the depth, so we call it slope of the K_h curve and ita h also will be taking the same units of K_h so you have to be little careful with K_h and ita h is same only the capital K what we saw was subgrade modulus reaction will be taking units of pressure because sometimes in some of the references you will find only k value will be given.

So once you know the k value you can find out K_h or ita h depending on which application you are using in.

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Pile Foundations

MODULUS OF SUBGRADE REACTION

Modulus of subgrade reaction can be estimated by any one of the following methods

- ❑ **Full scale pile load tests**
Full scale pile load test will give the correct load displacement characteristics of the soil but it is time consuming and expensive
- ❑ **Plate loading tests**
Horizontal plate load tests can be carried out to determine the load deflection characteristics and this can be extrapolated for lateral direction
- ❑ **Empirical correlations**
Various empirical formulae proposed by researchers can be used. This is possible only when other properties are known

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So modulus of subgrade reaction how do we find out there are several methods of obtaining it one of the simplest method is if you know the modulus of elasticity of the soil the you can relate using empirical methods in the past if you look at the literature many of them used that idea I think in first few classes I have given a big table where you have the values of modulus of elasticity relating to either C_u value or with spt value once you know the E_s value then you can use the E_s value to find out the k value or ita h value we will see some of the some more empirical formulas.

Or you can do a full scale pile load test which is something not very easily done because of its expensive and time consuming or you can do a small scale plate loading test this is very common in most of the onshore applications the first thing we do is the simplest plate load we have a square plate of 150 mm by 150 mm or 300 by 300 the maximum size is 600 by 600 so you go to the level of the foundation put this plate and load it in sequence of say whatever the maximum load divided by several time several load steps and note down the vertical displacement.

And using that you can arrive at the elastic modulus from there you can find out the modulus of subgrade reaction as early as 1940's Terzaghi he also has proposed because those days doing a horizontal load test was not at all a feasible idea he was using a vertical plate load test and using the data how you can derive a horizontal capacity. So he proposed one simple empirical formula

from there onwards I think many people have proposed many many different ideas I would say different relationships we will see some of them in due course.

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Pile Foundations

TERZAGHI METHOD OF EXTRAPOLATION

Terzaghi proposed the following relationship to extrapolate the value of modulus of subgrade reaction from horizontal plate load tests.

$$k_h = \left(\frac{1}{1.5D} \right) (k_{s1})$$

Where k_{s1} is the modulus from the horizontal plate load test (1ft x 1ft size) and D is the diameter of pile in ft

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So the first one proposed by Terzaghi he was something like this, so this k_{s1} is the modulus from the horizontal plate load test, horizontal plate load test means the plate is horizontal load test is vertical those days there was no actual horizontal test. Nowadays we have almost every project we do a horizontal test that means load test itself the load is applied horizontally and measure the horizontal displacement of the pile along the length one of the biggest challenge is do the instrumentation along the depth because before the pile is erected you need to have the strain cages fixed to the pile so that when you are actually doing horizontal load the pile bends and the soil compresses and the relative displacement can be measured which is very very troublesome and difficult.

So this formula is empirical and you have to be little bit careful we use of this formula is because this is particular to I think feet and cubes units. So you have to be little bit careful in applying this formula so you have to convert your units to this unit and then convert backwards.

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Pile Foundations

EMPIRICAL FORMULAE

Following formulae can be used to estimate

$$k_h = \left(\frac{0.65}{D} \right)^{1.2} \sqrt{\frac{E_s D^4}{E_p J_p}} \left(\frac{E_s}{1 - \nu_s^2} \right) \quad \text{Vesic (1961)}$$
$$k_h = \left(80 - 320 \left(\frac{C_u}{D} \right) \right) \quad \text{Skempton (1951)}$$
$$k_h = \left(67 \left(\frac{C_u}{D} \right) \right) \quad \text{Davisson (1970)}$$
$$\eta_h = \frac{A\gamma}{1.35} \text{ (Tons/ft}^2\text{)} \quad \text{Terzaghi (1951)}$$

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The other empirical formula what was given we saw this one earlier on remember in spring methods so he has used modulus of value something like this which we saw earlier on and all other different formulas also some of them are this is definitely unit depending this is also this not necessarily unit depending you could see the original paper but most of then you have to be careful with the units any empirical formula you take for soil mechanics the conversion was not so easy so we have to use that with the corresponding units.

So you can see all of them give the values of K_h only the last one you have it a h value γ value for that could find from the original paper I have not taken here.

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Pile Foundations

**MODULUS OF SUBGRADE REACTION (k_1)
(OVER CONSOLIDATED CLAY)**

Consistency	Firm to Stiff	Stiff to very stiff	Hard
Undrained shear strength (c_u) kN/m ²	50-100	100-200	>200
Range of k_1 MN/m ³	15-30	30-60	>60

$$k_h = \frac{\alpha K_0}{D}$$

$$\alpha = 0.52 \sqrt{\frac{K_0 E_p I_p}{B^4}}$$

$$k_0 = 0.305 k_1 \quad k_0 = 1.67 E_{50}$$

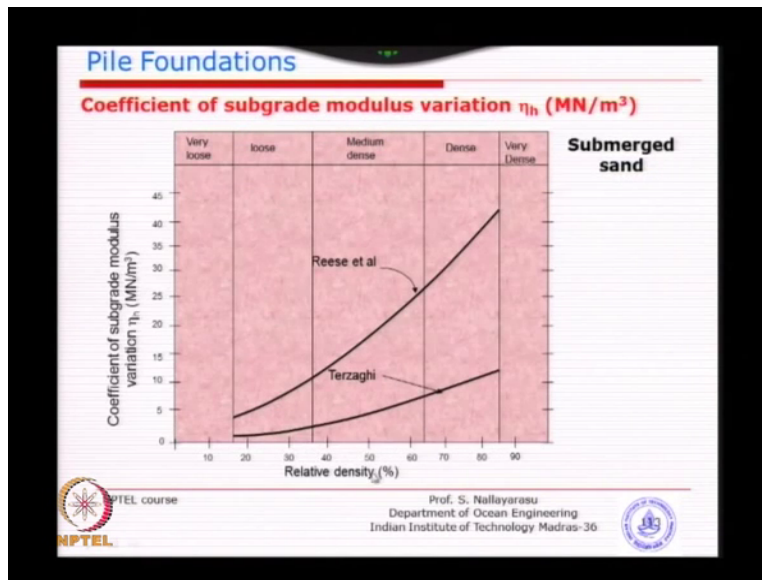
E_{50} – Secant modulus obtained from plate load test at 50% of the pressure.

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Similarly you have the values of k_1 which is for normally consolidated clays slightly complex formula instead of directly giving the k value or K_h value from triaxial test if you have the 50 percent the modulus when you are doing a triaxial test at that tangent at the 50 percent deviated stress you can take the modulus value which is called E_{50} , in fact we will be using this one very often in lateral loading test when you are doing a triaxial test for p_y curve we will take the modulus at the 50 percent deviator pressure and that will be the representative of the soil failure and we will taking that using that value you can find out the k not and k_1 then finally you go and find out K_h , so is related.

And a typical values of k_1 is given in the literature you can see here almost for each range of C_u value you have the value of k_1 and just from references when we are doing actually problem you should have the value of ranges so that you do not take arbitrary values is quite important to understand the ranges that is normally there even though you have a empirical formulas when you substitute all of them if it gives out of range there must be some thinking before using the values B in this case B is the diameter and D is the depth, so you have to little bit cautious in using these formulas.

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If you look at the API, API has given such a graph relating relative density with respect to coefficient of subgrade modulus variation which is its η_h value especially for sandy material and you can see at the top loose or very loose to very dense type of soil and original proposal by Terzaghi and this is taken from one of the publications by Reese we will see later the work done by Reese is very much you know close to Terzaghi but he has done much more than what Terzaghi proposed in the initial stages and most of the pile foundations especially with respect to lateral loads Reese has published almost more than 100 papers.

And some of the papers as early as 1977 we will see we are still using it for today's design because after that not much has been done because he has done a lot more experimental studies on land of course marine applications he has done only one test because doing lateral load test on marine condition is so difficult. So even today 1977 paper is being referred in most of the you know practical applications. So that is why API has adapted taking his work and just comparing it with what the value is given by Terzaghi you can see here the values of Terzaghi are quite low because those days the number of tests and the available information was too little.

And that is why you can see here almost 3 to 4 times higher than what I was giving. So this modulus we call it coefficient of subgrade modulus variation because it is actually the change in k 's value which is basically denoted as η_h . So if you know η_h if you want K_h value you can actually find out depending on which depth you are looking at and predominantly this is the

value that you require for sand, if it is a clay material any why you may not use this because the relationship between the horizontal capacity and the type of soil for clay is anyway uniform and depends only on the C_u value so you may not actually require for clay type of material.

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Pile Foundations

Coefficient of subgrade modulus variation (η_h)

Soft Organic silt : 150 kN/m³

Soft normally consolidated clay : 350-700 kN/m³

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Typical value for consolidated clay and organic silt is something like this typical values is remember so that when you are not given you should not unduly worried and you should be able to assume certain values.

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Pile Foundations

PILE FLEXIBILITY

Piles can be classified as Short or Rigid and Long or Slender based on relative pile stiffness with the soil.

Constant k_h (Hard clay)	Linearly varying k_h (Sand or soft clay)
$R = \left(\frac{E_p I_p}{k_h D} \right)^{1/4}$	$k_h = \eta_h \left(\frac{z}{D} \right)$
$L < 2.0R$ Short or Rigid	$T = \left(\frac{E_p I_p}{\eta_h} \right)^{1/5}$
$L > 3.5R$ Long or Slender	$L > 4.0T$ Long or Slender
L - Length of pile	$L < 2.0T$ rigid
$E_p I_p$ - Pile Stiffness	

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So I think this is something that we were looking at earlier on we were having a parameter called beta and lambda, beta for constant K_h value, lambda for varying K_h value a similar idea you could see here slightly modified this is done by one professor in geo mechanics in Australia called Tomlinson and he has done lot of work and at the end of the day many of the coastal applications or land based pile foundations people still design using this method of classification instead of either beta or lambda I just wanted to compare that you know you can see here K_h value constant for most of the clay type of soil or sometimes you have soft clay or sandy material you have K_h value vary.

So he proposed parameter called R instead of beta, so remember we were having beta K_h divided by EI K_h times diameter divided by EI so he has just reversed it he call it pile flexibility instead of pile rigidity and he uses this number to define how do you classify whether the pile is short or rigid just and l less than 2 times or for short and rigid piles and l greater than 3, 3 and a half to 4 times for long or slender piles. So you can now see here the first step is to find out the relative flexibility and compare what the pile length is, suppose if you have 20 meter pile and you find out R value you can just simply find out whether the pile is going to behave as a and many times this is quite useful.

And when you have a sandy type of material or a soft type of clay you could use this formula instead of R you are going to find out T and then. So if you are interested in solving the problem by means of simplified solutions this will be very useful, something like this you could easily design it if it is not too much complexity in the soil layers.

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Pile Foundations

PILE BENDING AND BUCKLING

A partly embedded pile may require to carry axial and bending loads and need to check against the buckling using Euler buckling capacity as stated below

$$P_{cr} = \frac{\pi^2 E_p I_p}{4(e + Z_f)^2}$$

For constant modulus (clayey soils)

$$Z_f = 1.4R \quad (\text{if } L/R > 4)$$

For varying modulus (Sandy soils)

$$Z_f = 1.8T \quad (\text{if } L/T > 4)$$

File embedded in to soil Equivalent fixed base pile

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So the most important thing is ultimately what we are looking at is when the pile is getting fixed whether it is short pile or rigid pile if you are able to translate the problem into a simple structural problem, for example in this case what we are interested is trying to find out do we have enough length of the pile embedded into the soil so that the pile does not come out. For example if you have a very limited length of the pile when you apply large horizontal load the pile will rotate and come out.

So that is why the depth of fixity is one of the important parameter whether you get the pile is fixed below ground that means is not going to rotate, so most of the applications when you have a horizontal load is large enough to uproot the pile you need to make sure the pile is emended that means it want to convert the problem from a short pile to a long pile.

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Pile Foundations

PILE FLEXIBILITY

Piles can be classified as Short or Rigid and Long or Slender based on relative pile stiffness with the soil.

<p>Constant k_h (Hard clay)</p> $R = \left(\frac{E_p I_p}{k_h D} \right)^{1/4}$ <p>$L < 2.0R$ Short or Rigid</p> <p>$L > 3.5R$ Long or Slender</p> <p>L - Length of pile</p> <p>$E_p I_p$ - Pile Stiffness</p>	<p>Linearly varying k_h (Sand or soft clay)</p> $k_h = \eta_h \left(\frac{z}{D} \right)$ $T = \left(\frac{E_p I_p}{\eta_h} \right)^{1/5}$ <p>$L > 4.0T$ Long or Slender</p> <p>$L < 2.0T$ rigid</p>
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That means we need to find out the length of embedment so in this case the reason why we are actually finding out this R value is not just to classify the pile itself we want to make sure that the R value if once you find you put the pile foundation depth in such a way that automatically the length becomes longer enough to convert the pile into a longer pile that means you will get the fixity so that the pile will not get uprooted but pile will start bending that means you will be able to design the pile for that bending moment so that is the idea behind.

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Pile Foundations

PILE BENDING AND BUCKLING

A partly embedded pile may require to carry axial and bending loads and need to check against the buckling using Euler buckling capacity as stated below

$$P_{cr} = \frac{\pi^2 E_p I_p}{4(e + Z_f)^2}$$

For constant modulus (clayey soils)

$$Z_f = 1.4R \quad (\text{if } L/R > 4)$$

For varying modulus (Sandy soils)

$$Z_f = 1.8T \quad (\text{if } L/T > 4)$$

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So in this particular case once you have the R value is found or the T value is found you make sure that the length is longer enough and find out where the depth of fixity. So you can see here the depth of fixity is 1.8 times T or 1.4 times R this is the assumptions or in fact reasonable assumptions made at that time. Now if you fix the pile length is equal to less than 1.8 times T what will happen, the pile will definitely just come out. So you need to make sure that you fix your pile length should be longer than Zr otherwise there is no meaning because the pile will be coming out.

So that is the idea behind the initial stages of design many times we use this screening type of design you find out R value and just quickly fix up your depth of fixity and then use your analysis.

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Pile Foundations

Limiting solutions for Linearly Varying k_h $\lambda = \left(\frac{\eta_h}{E_p I_p} \right)^{1/5}$

Long Pile ($IL > 4$) - Free Head

$$y_o = \frac{2.4F}{k_h^{1/5} (E_p I_p)^{2/5}} + \frac{1.6Fe}{k_h^{1/5} (E_p I_p)^{1/5}}$$

$$\theta_o = \frac{1.6F}{k_h^{1/5} (E_p I_p)^{2/5}} + \frac{1.74Fe}{k_h^{1/5} (E_p I_p)^{1/5}}$$

Long Pile ($IL > 4$) - Fixed Head

Lateral deflection $y_o = \frac{0.93F}{\eta_h^{1/5} (E_p I_p)^{2/5}}$

The slide also contains two diagrams showing deflection curves for a 'Free headed pile' and a 'Fixed headed pile' under a lateral force F. The free headed pile shows a larger deflection at the top compared to the fixed headed pile. The diagrams include labels for length L, deflection y_o, and rotation theta_o.

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For example after you find out Zr value pile is fixed here once the pile is fixed it becomes a structural problem rather than a soil mechanics problem because you already have converted the relative stiffness of the soil to the pile and the depth of fixity can be easily found and after that you make the pile length in such a way that it is definitely longer than what is required so that the pile will behave nobody wants the pile to be in a shorter classification because once you start the potential of coming out is more.

So for this type of soil L by R is greater than 4 then you can take the depth of fixity as 1.4 times R value and this is very very useful in fact even today when you do a numerical solution either

using linear spring or nonlinear spring many times we find this approximation is almost true if you do a computer analysis today and later you go and find out plot the bending moment distribution along the depth you will find maximum bending moment will be occurring somewhere nearby in this type of range and you know that is the kind of approximation those days people have used even without doing any computer analysis I think many many structures have been designed using this.

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Pile Foundations

PILE DEFLECTION

Free headed pile

Lateral deflection $y = \frac{F(e + Z_f)^3}{3E_p I_p}$

Fixed headed pile

Lateral deflection $y = \frac{F(e + Z_f)^3}{12E_p I_p}$

$E_p I_p$ - Pile Stiffness

Free headed pile Fixed headed pile

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Pile deflection I think is a similar derivation which we were using earlier on you know solving the beam and elastic foundations the deflection at the pile head is quite important because when you are looking at the depth of fixity that is there you are worried about you can substitute this and find out whether your assumptions are correct.

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Pile Foundations

ESTIMATION OF SOIL MODULUS (E_s)

Clay

$$E_s = (300 \pm 100)C_u \quad \text{Vesic (1961)}$$

Sand

$$E_s = 1.6N \quad \text{MPa} \quad \text{Kishida and Nakai}$$

Where N is the SPT Value

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Earlier also I have given you some you know empirical relationship this will be very useful directly you can see here C_u versus E_s as early as 60's he has proposed this empirical formula and then N related to though I have given the bigger table for our most of the classifications as well as our examples we will use this simplified formula which is 1.6 times N value which is easy to remember, so what some of the examples I will give you using this method.