Strength and Vibration of Marine Structures

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Lecture - 9
Statically Indeterminate Structures – III

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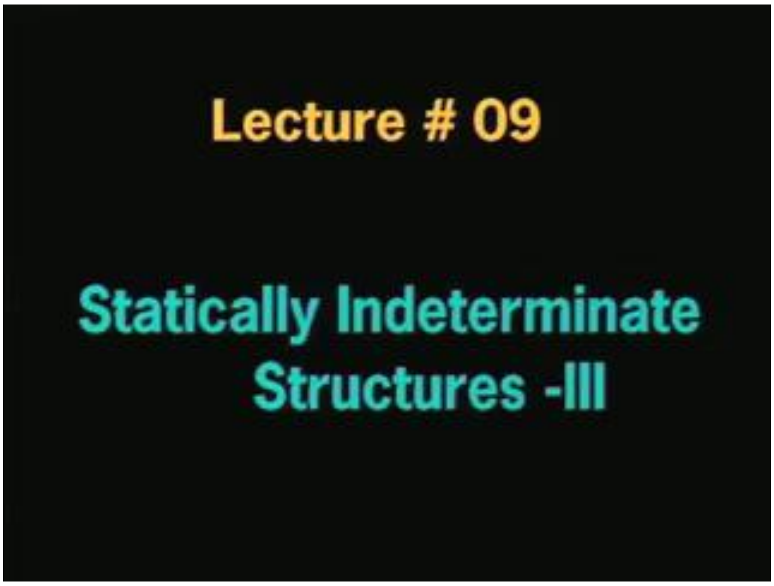

So, we are talking about energy method. Talking about, means just we have started talking on this. The technique is energy method. It is based on basically the concept of strain energy. The strain energy, we are trying to explain with the form of a simple bar problem, under tension.

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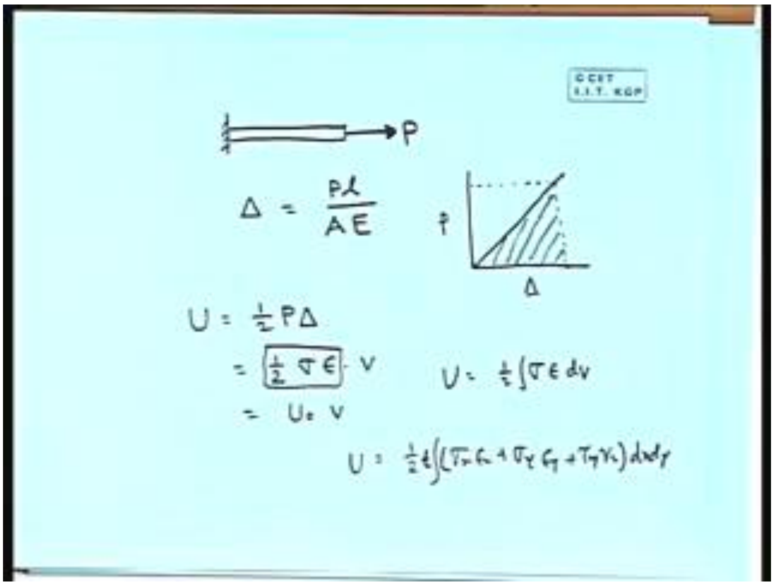

So, we took a bar in that form and applied a load P. Now, this is a case of a very simple stress problem, because entire bar will have identical stress and strength. So, anywhere stress is P by A and strain will be your total elongation divided by length of the member. So, it is a case of uniform strain, uniform stress, whatever you can say.

Now, the elongation, we wrote in the last class it was Pl by AE, and we have drawn a curve like this; your load versus elongation, straight line, because whole thing is a linear system. And for any load, if you start applying load from 0, gradually increase, go up to a load P, that elongation will increase from 0 to delta, and the area under this load deflection curve, we have found that it is basically the work carried out the force P.

Now, it is not straight away P and delta, because P is not constant throughout, because P is starting from 0 to that P and it is delta is starting from 0 to delta. So, total work, so that we have put in the form of half P into delta. So, that work, it will be stored in the form of some energy. We said this strain energy; that strain energy will be equal to this work done half P into delta.

Now, here, load why we are putting in a gradual manner? Because we want to maintain the static condition. If we apply suddenly, load whole thing will be dynamic problem. So, that will be much more difficult case. So, we want to understand with a very simple type of problem. So, load in a static manner if we want to apply, we have to apply gradually. So, always there will be balance between internal-external force system.

Basically, this strain energy, though it is expressed in the form of P into delta, it is rather the measure of some internal quantities; internal quantities means there will be deformation that we have defined in the form of strain and due to the strain some stress will be there. So, whole thing is a measure of the strain and stress inside. And you must have remembered we have written this one as this sigma epsilon into v, because that P we have divided by area, epsilon divided by l. So, it became stress; it became strain; and this l and A, it becomes V, that is the volume, or sometimes we write it is U 0 V. So, U 0 is this part; this sometimes say it is strain energy, density.

So, half stress into strain is the strain energy density multiplied by the volume will be the total energy. Now, this is a case of uniform stress and strain. So, everywhere your sigma and epsilon is constant. For a general case, we have written U equal to your half sigma epsilon dv. Whereas, this was written for a case, for an arbitrary type of problem, as stress strain will vary from point to point. So, this is valid for this problem as stress-strain is uniform. So, if we take, again, where stress-strain is not uniform, it is varying, so this concept is valid, for a small point, for a small element. So, that concept applied, say, V become dV. So, all the element if we combined, so we are supposed to get the expression of a strain energy is equal to U equal to half integration sigma epsilon dV.

Now, it is basically strain energy. So, related to strain or related to stress. So, it is basically internal thing. Though we have tried to explain at the beginning in terms of external work, because external work, internal work in the form of strain energy we have tried to equate, for applying a load in a gradual manner, in a static condition. But in this strain energy concept is valid for dynamic problem also. You apply load, your internal stress-strain it will gradually increase from 0 to a certain value. So, it will not be suddenly generated. So, this half term will be coming there. Only thing, the internal system and external system will be not be in equilibrium; naturally, it will be in a moving condition, in a dynamic condition; otherwise, this concept will be there.

So, for a expression of energy, in a generalized form, we can say U equal to half sigma epsilon dV. Now, here number of stress component is one, number of strain component is one, because it is a one-dimensional problem. If we have a stress analysis problem where number of stress component is more; say, if we take a plate it is under stretching; so it will be a biaxial system of stress. So, it will be a plane stress type of problem. So, there will be three stress components, two normal stress components, one along x another y plus shear stress. Similarly, three strain components will be there. So, in that case, we can define sigma x multiplied by epsilon x plus sigma y multiplied by epsilon y plus your tau xy into gamma xy. So, for a specific case, we can write in this form; for a biaxial system of force, say, it is sigma x, epsilon x, sigma y, epsilon y, tau xy, gamma xy into dv. Say, if we take a thin plate, we can take it is dx dy; we can multiply it by a thickness; t is the thickness of the plate. So, this is a special case of strain energy for a stress problem, where number of stress components are more. So, this idea can be generalized.

Now, at this moment, we are interested with simple problems. So, it is only one component of stress. Now, this expression of strain energy, we want to apply directly to a bending problem, because we are handling a problem having some bending of the beam or bending of the frame member. So, bending part we are mostly dealing at this moment. So, this strain energy if we put in a bending mode, so what will be happening?

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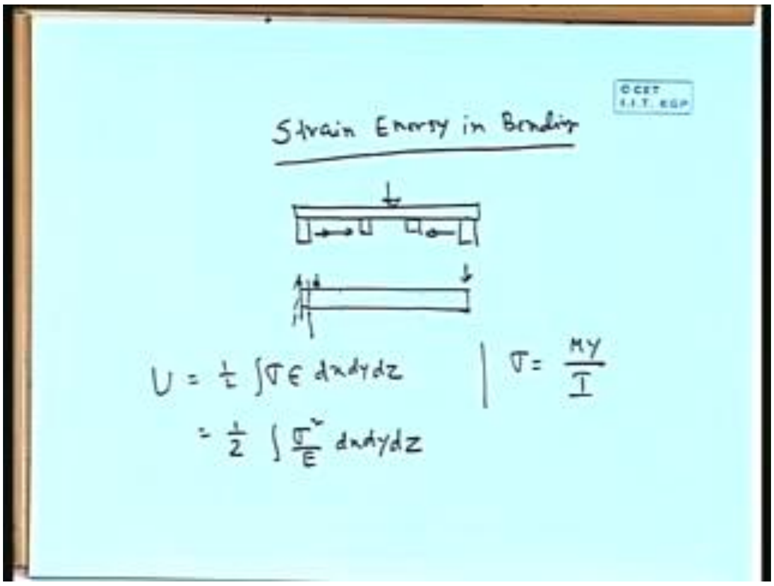

So, if we write strain energy in bending, we know any member, if it is under bending, will get bending stress. And, that bending stress, it will vary from lower fiber to upper fiber, in a gradual manner; it follows a linear manner; a neutral axis, it is basically 0; and it will increase, it will go away from the neutral axis. So, along that depth variation it will vary. And along the width, normally, we assume it is uniform. So, depth wise variation is there; width wise variation is not there; plus along the length it is minimum;

it will vary; naturally it will vary. So, if we think in terms xyz along length it will vary, along depth it will vary, along width it will be constant. So, it is not a uniform state of stress for the bending stress component.

Now, in bending, there is only one stress component; that is a bending stress. Now due to shear or shear force there will be a shear stress, but due to shear stress, we do not consider any shear strain. In elementary bending theory, the effect of shear deformation normally we do not consider; this is one of the assumptions. Because the beam theory what we have the understanding, we assume a plane perpendicular to the neutral axis before bending, remain straight and normal to the deflected neutral plane or neutral line after bending. So, that assumption is based on that idea that we are not considering the effect of shear deformation. Because shear stress, it varies in a parabolic manner. If you take the effect of shear deformation, so it will be not a normal line; it will not be a straight line; and it will not be normal. So, the normal it will be violated plus it will have a working of the whole section.

Normally, what happens, if we take a beam, its length is quite… the value of the length; normally, it is higher compared to the depth and width of the beam. Now, that is mostly common in our structural member. Now in that mode, so bending is predominant. So, bending deformations is predominant; the effect of shear deformation is little less.

Just if we take a case, say, there is a structure like this, there is a support, there is a support, and if we put some load here. So, along member, under load, so bending will be predominant. Now, if we try to shift the supports; so, if you shift the supports; if support is here; support is here. So, this support will be shifted here; this support will be shifted here. Your bending deformation will be reduced. In one case, say, the supports - both the supports - will be tending towards the load. So, there are two supports; there is a load; what will be the mode of failure? It will be just, it is shear mode of failure.

Just we can think there is a cantilever beam, or same this problem, we can think in terms of cantilever. If we put a load here, it will try to bend. Now if you try to bring it here, if you bring it here, what will be happening? So, there will be certain shear of the whole thing, but if you put the load here, it will undergo bending. So, fiber will start failing gradually, form top or bottom, but here once the length will be reduced; so it depends, basically, the depth versus this length. So, once this length is more compared to this, bending will be predominant, if this length is very small compared to this. So, it will be in a shear mode.

We cut a piece of paper by scissors; means two forces we are putting more or less on the same point with the slight shifting. So, if we put two blades, it will never cut, it will be under bending. So, when length is more, your bending aspect will be predominant. When length will be small, it will be in shear mode. This shearing of the rivet; two plates with the rivet, if you try to pull, practically there is no gap, so it will shear. But if you place here, there is a long neck, two plate; so, whole thing will try to bend.

So, it depends what is the size of our structure. So, normally, our structure member will have a large span compared to the depth. So, bending will be predominant, and normally, we take the deformation due to the bending action. So, bending stress will produce bending strain and shear stress will be there; but shear strain will be very small. And if you count, if it is a 1 percent or less than 1 percent, there is no point of making the analysis very complicated, because we have to take the effect of shear, some other expression will be coming in to our equation; we have to handle that; and maximum benefit we will get, say, 1 percent or half percent, if the span is quite large, so we do not consider that.

So, in bending mode, basically, though there are two stresses, but the strain due to the bending part will be predominant, that is why we take the bending strain only. We do not consider the shear strain; shear strain means strain produced by the shear stress. So, two stresses, and two strains. So, one of the strain is basically 0, and energy is stress into strain. So, energy due to the shear force, shear stress, shear strain - this part we can simply neglect. So, it will be a one-dimensional problem; means, it is a problem with only one stress component; that is a bending stress and its corresponding bending strain.

Now, the second aspect is the stress components will vary. As I am mentioned, it will vary along the depth, along the length; only width wise we are keeping it more or less uniform. Now, if we write the energy, so U already we have written; it is sigma epsilon dv, we can write dx dy dz. So, dv is dx dy dz. So, we are putting it in a separate form. And here this sigma, for a bending problem, we know it is MY by I.

So, it is depending on y. So, if x is the length; x is the coordinate along the length of the member, along the axis of the member; y is perpendicular to that along the depth; and z is along the width. So, sigma we can write MY by I. Now, here, if I put the expression of sigma, now what we will get? Now, strain is also depended on stress. So, strain is stress by E. So, this expression can be defined in terms of stress entirely or it can be defined entirely in terms of strain. So, this sigma we can write as E into strain. So, it will be totally strain dependent or this we can express in the form of sigma YE. So, it will be entirely stress dependent, but here we know the expression of stress. So, it is better to write in the form of stress. So, we can write in the second step half it is epsilon square divided by E dx dy dz. So, it will be sigma sigma sigma square divided by E into dx dy dz. Now, here we will put the value of sigma - that is MY by I.

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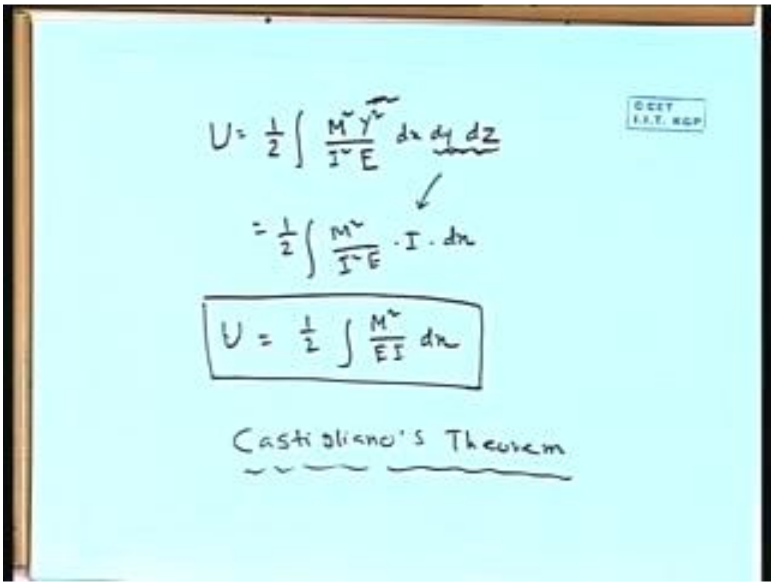

So, if I go to the next page. So, we can write U equal to half integral. So, it is sigma square; means, it will be M square, Y square, I square, denominator E is there. So, dx dy dz. Now, this half, this M square by I square E, here you just observe. IE more or less - this quantity - will be a constant quantity. Here M it will be fixed at a station; for a particular value of x it is constant.

So, if we integrate about y and z. So, yz means along the cross section of the member, because this is x; it is y; it is z. So, along the cross section if we integrate, along the cross section means at a particular station M it will be constant. So, M will vary from from one station to another station. So, here we can integrate. So, if we integrate about dy dz about yz, so here y square will be coming into the equation. So, it is what? dy dz, if we write, it is da. So, integrant y square da is what? Second moment of area. So, we can write this one as I dx. So, this one and this one, if we handle y square dy dz. So, that will lead to your I. Now, this I and this I will cancel. So, it will be your M square by EI dx; rather this is a very common expression for the energy in bending.

So, we have started with U equal to half sigma epsilon dx dy dz. Epsilon we have expressed in the form of sigma ye. And the sigma we have written MY by I. So, this Y square dy dz. So, that part is simply I. So, ultimately it will lead to M square EI dx integral half; it will be the energy.

Now, for bending mode. So, whatever bending moment is there, square of that, divided by EI into dx half, that will be the U. Now, if you try to compare with your basic expression of strain energy, that was half sigma into epsilon into dx; here, sigma, you can think as the analogous term of sigma is bending moment M. And M square is M into M, you can write this M into M by EI, M by EI is what? d 2 y by dx square. So, d 2 y by dx square is what? Curvature. And d 2 y by dx square curvature multiplied by EI is equal to moment. So, we can say moment curvature with a factor EI. So, EI is just like your E; and d 2 y by dx square curvature is strain; and this is stress. So, it is not stress; it is stress resultant. And curvature is not the strain; it is a generalized curvature. So, that thing, in this form we are trying to express.

Once that is there, the energy, after beam under bending, now there is a theorem of energy in terms of calculating the deflection. And that theorem was defined as a energy theorem, and that was first proposed by Castigliano; we will say it is Castigliano’s theorem. So, Castigliano. So, the energy principle is based on the theory proposed by Castigliano. We say it is Castigliano’s theorem. So, what is that theorem? Basically Castigliano has two theorems or whatever he has proposed it has two components. So, one of the components, it will be utilized here. So, if we say U, if we write the Castigliano’s theorem, so U you can express the energy of the system, we can express in the form of the forces, because the energy here, energy is half M square EI dx.

So, if we take a beam problem, M we can find P into X omega X square by 2 something. So, it will be expressed in the form of some force. Now, it can be also expressed in the form of displacement. Now we are interested for the force part. So, U if it is a function, and it is defined in terms of some force, it may be one force; it may be a number of forces.

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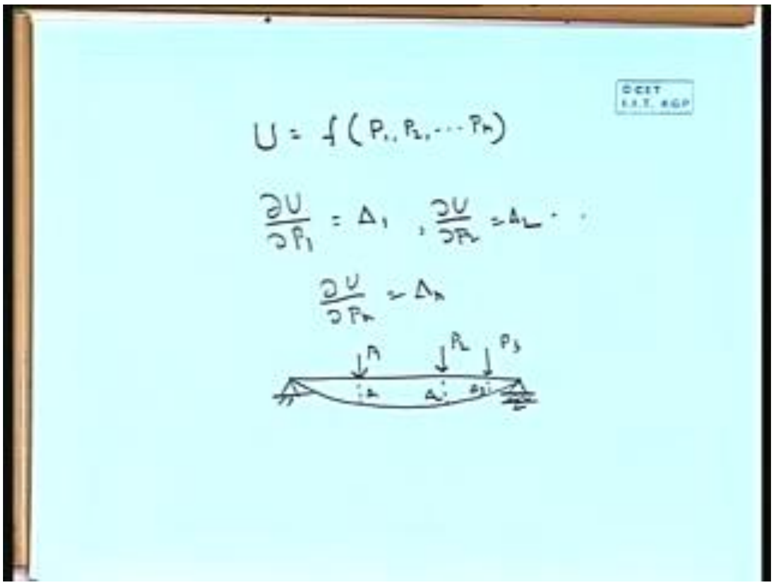

So, there is a beam problem; it is subjected to P 1, P 2, P 3, P 4 like this. So, the bending for the different region, you can express in the form of… because reaction would be in terms of P 1, P 2, P 3, P 4, Pn, and that reaction into that distance, then the force into some distance, automatically the moment expression will be in the form of P 1, P 2, P 3, P 4, Pn. And energy expression is nothing but your M square EI dx dy. So, whole thing will be express in the form these forces. Now one of that expression will be available.

Now, if we take partial derivative about U for P 1, we will get delta 1; or if we just P 2, it will be delta 2; or like this Pn, it will be delta n. Now, if the force is only 1 P. So, energy will be expressed in terms of P. So, partial derivative will be full derivative; it will be dU by dP will be delta. So, what is delta? Delta is the deflection under the load P 1. So, delta 1 is deflection under P 1; delta 2 is deflection under P 2.

I think, if I draw a figure it will be better. Say it is P 1; if it is P 2; if it is P 3; this will be your delta 1; this will be delta 2; this will be delta 3; like this. Say, here I have just drawn any arbitrary beam subjected to P 1, P 2, P 3. So, reactions can be evaluated in the form of P 1, P 2, P 3. So, moment expression you can find out in terms of P 1, P 2, P 3. So, if moment is in terms of P 1, P 2, P 3 ultimately your energy will be a function of P 1, P 2, P 3.

Now, if you take derivative of U in terms of P 1, you will get delta 1; delta 1 is the deflection under the load; and delta 2 is deflection under the load P 2; delta 3 is the deflection under the load P 3; like that you will get. So, if there is only one load P, so if you take that derivative above that P, you will get the deflection under that load.

Now, say there might be a question - at the middle there is no load, and we want to get the deflection here - is the theory valid there? Should I apply this? Definitely, we can apply. Here we have to put a load, say, P at the center. And we will apply this theorem; at the end, we will put Pc is equal to 0. So, we will put PC as a unknown and apply this theorem. Actually Pc, there is a no force, so this value will be 0. At the end, we have to apply. But initially we have to take it; otherwise you cannot proceed. I think those part, if you take some example, it will be much more clearer.

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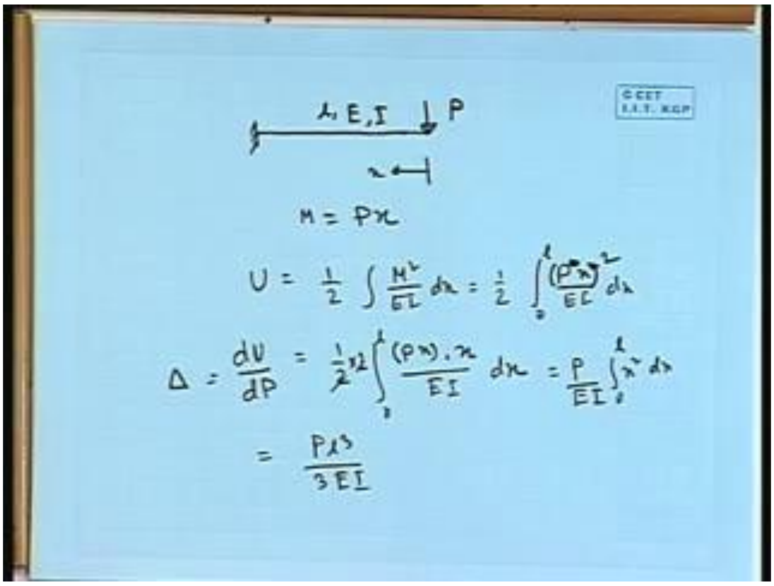

Say you start with the problem, which already we have solved by other method. Now we know the result. Say it is a cantilever beam; it is subjected to load P. It has some length l, some E, some I, something is there. So, length is l, E the material property, I is the second moment of area. Now, due to that load, say, deflection under that load we are interested.

So, first job is we have to find out the expression of the bending moment. Now, you can start x from this way or x from that side. So, it is better to start from this side, because expression will be much more easier, P into X. This side, it will be P into l minus x. So, if we start x from here, so M will be your P into x, and our energy is half EI dx. What is M square EI by dx? It will be half P square x square EI dx, limit will be 0 to l. So, entire span, it is the expression. Now, this advantage we may not get in other cases; if we put a separate load moment expression, it will be different. So, the integration will be from, say, 0 to l by 2, l by 2 to l like this. Now, this integration if we perform. So, it will be... There are two options. We can perform the integration, after that we can take the derivative; other option is we can take the derivative - so, there are two options. Now, it will give the same result.

Say, we put it in that manner, say, half M square part we can write as a M into M, or let us put the derivative first: dU by dP that will be the delta. Now, if you take the derivative, so here what will be happening? Here P square will be there. P square will be 2 P. Or this expression we can little bit change, that will be easier to understand. P square x square we can write as P into x. So, here moment is P into x P into x whole square of that EI dx. Now, delta dU by dP, it will be this square term will be 2 into that Px. So, here this 2 if you bring outside, so it will be Px plus derivative of Px will be in terms of P will be x.

We are taking derivative about P not about x. So, that divided by EI into dx and definitely it will be 0 to l. So, our energy half M square EI dx. So, half here M we just put Px whole square. Now, delta is derivative of U about P. So, this part 2 we have taken out. So, 2 and 2 it will cancel, and this will be the Px, and it will be x. Now, what is this? This Px is basically M and x is derivative of M with respect to P. So, it is derivative of M with respect to P. So, that part we can explain later also. So, that will be the expression.

Now, if that is the expression, so it is Px x, Px square divided by EI into dx. So, it will be P if we take out, EI if we take out, it is basically integral dx 0 to l. So, integral 0 to lx square dx would be your x cube by 3, and limit if you put, it will be l cube by 3; other part 0 it will be cancelled. So, whole thing will be leading to Pl cube 3 EI. And that expression we have to determine by moment-area theorem or differential equation technique; it is a standard expression. So, here we obtain through your energy principle, we have written the expression of M, under U we have put there, and taken derivative.

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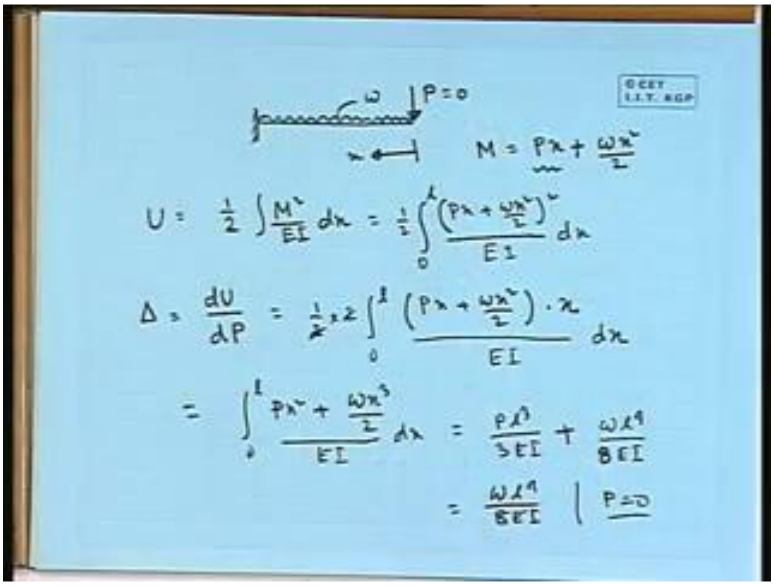

Now, I shall take a different case. What I was trying to tell little bit earlier, say, there is a similar type of cantilever beam, what it is subjected to a distributed load omega, and we are interested to finding out the deflection here; under the free end there is no load. In that case, there is a load; under the load, we have calculated the deflection. Now, I am just talking about the simply supported beam, span, deflection, no load. So, more or less similar situation; fully distributed load; we want to find out and practically there is no physical load there.

Now, this problem, we will just put a force here P and this force is nothing but will be equal to 0. Now, in a similar manner, if we take x axis from this side, we will have a moment expression of M of Px omega x square divided by 2. At any x it is P into x into omega x square by 2; it will be plus; both are in the in the same direction. Now, this P is 0. So, at the beginning, if I put P equal to 0, this part will not be there. Why it is important? That if we carry out that problem, it will give that field. So, our U is equal to half M square EI dx; that part we know. So, we can write, here it is Px. So, just I have substituted the value of M Px plus omega x square by 2 here. Now, next job is, one option is we can perform the integration finally, take the derivative. Other option is we can take the derivative, and then, perform the integration. I think second option is better.

So, delta it should be du by dp. Now this will be half. So, this 2 will come out, and here it will be Px omega x square by 2. So, we are taking derivative of that quantity. So, it was square 2 into that quantity, 2 we are putting it outside, and we have to take the derivative of this quantity with respect to P. So, it will be straightaway x divided by EI dx. So, this term was with square. Now, we are taking derivative of that. So, something square, it will be 2 into that quantity, plus inside quantity we have to take derivative.

So, if that internal quantity if you take derivative of P, so it will be simply x, this part; it will be constant; w may be 10 kilo Newton per meter or something. So, it is not a variable quantity. Now, at this level, now you can go ahead with the integration or at this level we can substitute P is equal to 0, before that we have to keep P, because this part we have taken square. So, it is there after derivative. Now, once the internal part we have taken derivative, we were getting x. So, x it is coming from Px and Px is this one; though this part is not, there P is equal to 0. So, at the beginning if we put 0, if we take derivative, nothing will come. So, artificially we have to put a P and it will carry. So, it will give x here. So, at this level we can substitute 0 or we can integrate, later on we can put 0, anything you can make it.

Now, say let us keep it. So, this will cancel. So, it will be one part will be, we can say, let us put the integral it will be Px square. Another part is omega x cube divided by 2, whole quantity divide by EI into dx; or we can write it will be Pl cube divided by 3 EI plus. So, this x square x square will be x cube divided by 3. If we put the limit, it will be l cube by 3 divided by EI, and here, this omega x cube will be x 4 divided by 4. So, it will be omega l 4 divided by 8 EI.

Now, at this level, we can make P is equal to 0. So, this part will be the deflection. So, what will be the deflection? How much we have derived earlier? We can check it out; no problem. It is ok. It is 8 here. So it is ok. So, I was suspecting there might be some… because we are calculating, we drop some 2, so the whole thing will be a different value.

Now, the actual problem was omega and the deflection under the load it should be wl to the power 4 divided by 8 EI. Now, this will be equal to… because your P is equal to 0. Now, I kept it intentionally, because in previous problem we have determined that deflection is Plq by 3 EI. So, this Plq by 3 EI part is also there.

Say a beam is subjected to this load plus this load omega plus P. So, this part is due to the load - point load; this part is due to uniformly distributed load. Noe, if that part is not there, it will be dropped out; if that part is there… So, this is the value. So, I could put it

0 earlier, I think. So, we need not carry out one of the component. So, unnecessarily I have kept it, just get the feeling that on this part is also there; it will come after.

So, the objective of taking this problem is, say, there is no… we want to get the deflection and we found there is no force there. So, you need not bother; you take a force P; go ahead, you know it will be 0. So, you write the expression in the form of P; you put it there; take the derivative; after taking derivative, at this level you could put it is 0; but if you put 0 at the beginning, you are not supposed to get this one. So, for getting this one, at least at this level you have to keep, after taking the derivative, then only you can make P is equal to 0; before that you cannot make it.

Now, it may be some value; positive, negative, and 0; up to there you have to maintain; after that you have to really substitute the value. If there is a problem of say 10 kilo Newton, so you have to write P, because you have to take the derivative; you cannot take 10, then take the derivative above 10. So, you cannot make del U by del 10. You have to give some variable name. So, it will be P. So, at this level you can substitute the value. So, here integration will be only in terms of x. Here w, P, everything you can put numerical value; 0 or any non 0 value; then integrate; you will get the value of deflection.

Now, if we take the problem of rotation, slope, we are talking about only deflection. Now, there might be a feeling that this method will be valid for finding out the deflection only; it cannot be used for slope; it is also valid for finding out the slope. And we will take a case like this, where there is no direct load under this. So, if we want to get slope, so what should be the corresponding force? There should be a moment and if we take a case where there is no applied moment, say, same problem we can take.

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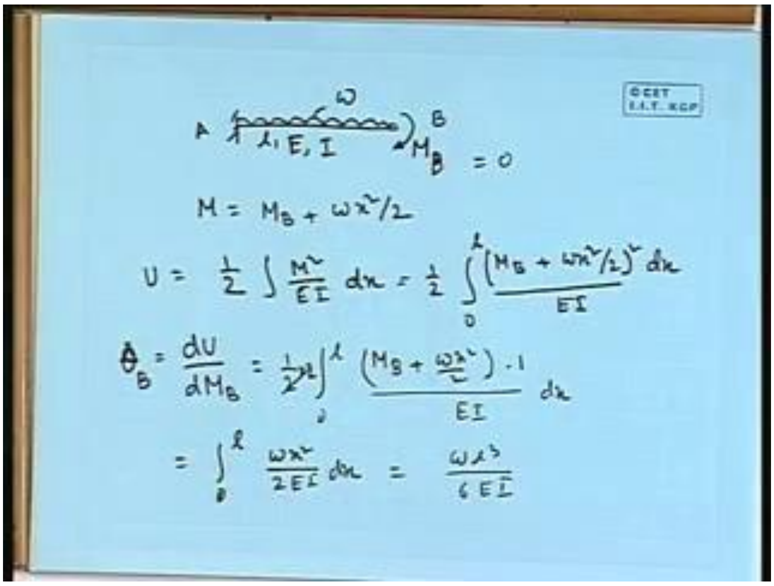

So, it has l, E, I; these are all. Now what will be the rotation at this point - we are interested. Now, here just we will put something, say, MB. Say if it is A, if it is B, I could put it is M, but we will write expression of M as M. In order to have a different notation, say, this end is A this end is B. So, there we put MB, and we know these values will be equal to 0.

But we have to put that and we have to get that expression for the bending movement. So, expression of M, at any station, it will be MB omega. So, your U half. So, just write MB. Now, if we calculate delta, it is basically du by here it will be MB. So, we are interested for finding out the slope at B. Corresponding to slope, we are suppose to give some movement M. So, it is at B MB. So, this delta here is nothing but your, we can write, these are there, write theta B. So, theta B will be dU by d MB. Now, this half 0 to l. So, this part will be MB omega x square by 2 into 1 into EI into dx. And there will be a a 2, that we can put outside. So, this 2 2, we can cancel. So, square 2 into this quantity and if you take derivative about MB; MB is MB; so, it will be 1.

Now, MB we have to carry at least for getting 1; other wise, if that is not there, whole things should be equal to 0. Now, you can keep MB or we can make MB is equal to 0 at this level and perform the integration. So, if MB is 0, this part will not be there. So, remaining part we can take. So, this will be equal to integral 0 to l omega x square 2 EI dx. So, it will be x square, means x cube divided by 3. And if you put the limit omega l cube 3 into 2 6 EI; I think this value also we can check. So, distributed load slope omega l cube by 6 EI.

Now, here after this level, MB we have made 0. So, only one term we have kept and we are getting this value. So, this problem, we are interested not deflection, it is slope, and there should be a moment. First of all, we should know there should be a moment there. So, force is corresponding to displacement and moment is corresponding to slope. So, there should be some corresponding moment and there is no moment. So, we have put arbitrarily one moment MB, though the value is equal to 0. We have processed that and finally, we got the value omega l cube by 6 EI.

Sir, before integrating MB you already considered it as 0.

Yes.

Can we assume?

Not assume; it is actually 0. So, at this level, we can afford for putting the value MB is equal to 0.

Sir, for finding slope we should take a moment and then we can find out at that point?

So, there are two aspects here: number 1, for getting deflection we have to find…we have to think for a force; there should be a force corresponding to that. So, in that direction, we want a displacement; there should be a force in that direction. If we want to get a slope along this; so, there should be a moment along this. If there is a force or there is a moment they are fine; we can go ahead, but that should be in a variable form. And finally, we will actually put the real value of that; it may be 10 Newton or 100 kilo Newton meter. But if it is not there, we have to take the thing in the variable form, but at the end, we have to put that is equal to 0.

So, this level or up to this level, it will be there. In the next level, we can make it 0; this level or at the end also we can make it 0. So, if you do not take that this is 0 at this level, it should be MB into x integration; after putting the limit it should be MB into l; it should be Mb into l. Now you could put MB equal to 0. So, finally, it should be 0. Or if you retain, there should be a case combination of there is a moment, there is a distributed load.

Now, so, with that, at least we came to that level that we have to write the expression of bending moment in terms of force or moment; then, put in that energy expression; then, take the derivative; at that level, we can really put the value of the load and perform the integration; finally, we will get the deflections.

Now, here, this energy theorem, what we are doing, you try to observe here. U equal to M square; we are putting it here. So, after taking derivative, this part will be there into this one. Now, what is this quantity? This quantity is basically M. This one. So, this was M square. So, after taking derivative you are basically getting M and this part is derivative of M with respect to MB, MB or P anything. Except this one, earlier problem, so, it was M square; this is M; and this part is derivative of this with respect to P.

Artificial or real?

Artificial or real, it should be the force or moment where we want to get the deformation or slope. So, whole thing can be redefined in a little different manner and from there the concept of unit load will come. So, that part, we can take care in the next class.

Preview of the Next Lecture

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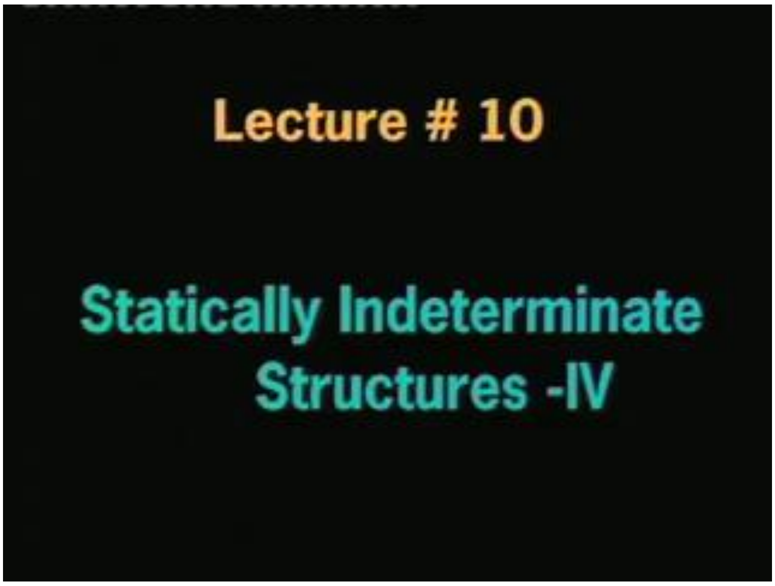

And I was talking there is a method called unit load method. It is basically derivative of the, your unit energy method. So, energy method, if we look in a separate angle, we will get this method, which we are trying to define as unit load method.

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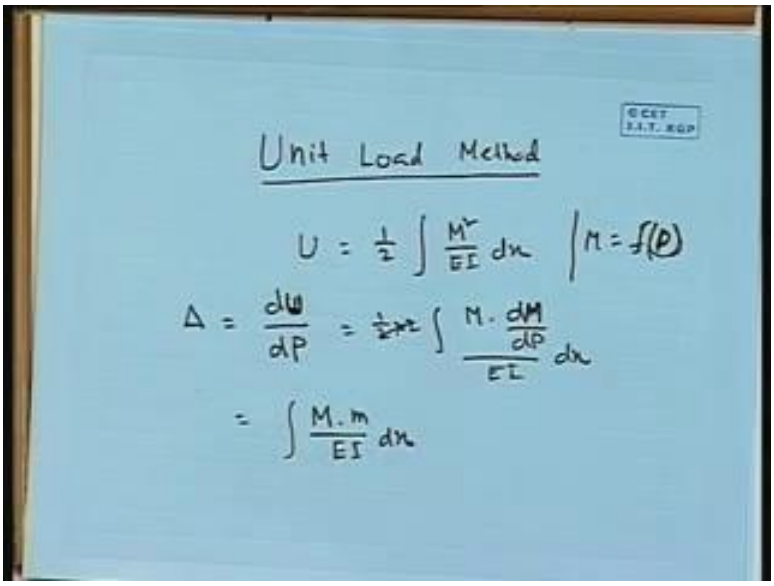

So, it is not a separate method. It is basically energy method, but looking the whole thing is looked in a different angle. Idea is to make the treatment much more systematic or much more mechanical. So, we need not start from the very basic thing. So, energy principle is basically its background, but in a different form we want to apply. Rather I was trying to explain you here.

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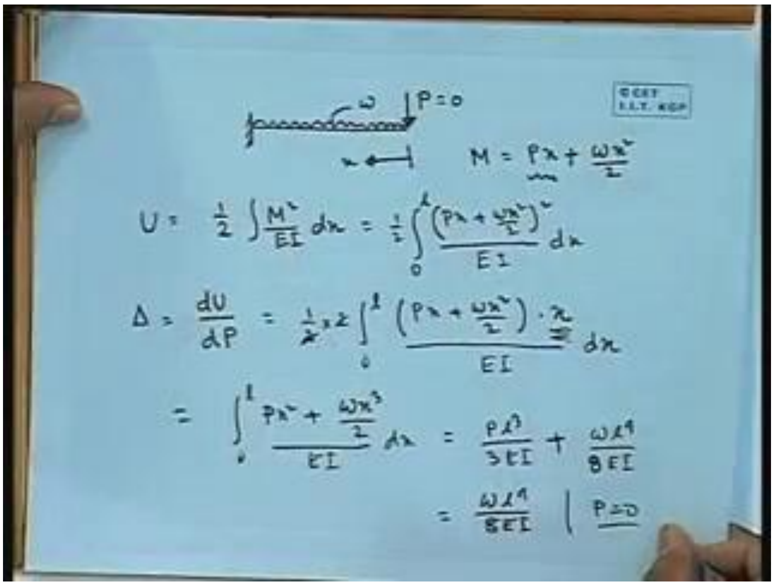

This problem we have solved. M square part we have put there, after taking derivative we are getting this plus another component. So, this part was basically the moment and this part is the derivative of that moment about that P. Now, if we write here, in general, U equal to half integral M square EI dx. So, delta, we are trying to tell this is du, dU by dP; it will be this half 2 M dM dP divided by EI dx. So, U is equal to half M square EI dx, where this M was say function of P . So, P I am writing; it is not the P what we have considered earlier; P may be P MB or any other quantity. So, it is a variable. So, if P is some function of, if M is function of P, so M square by EI dx; this part if we take derivative, so it will be dU by dP. So, here, this M square will be 2 M, 2 it will cancel, it will be M, and this part will be dM by dx EI into dx. Now, this part, we can write as this is capital M, and this derivative part, if we write small m divided by E EI into dx.