Strength and Vibration of Marine Structures
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Lecture - 5
Deflection of Structure Beam – III

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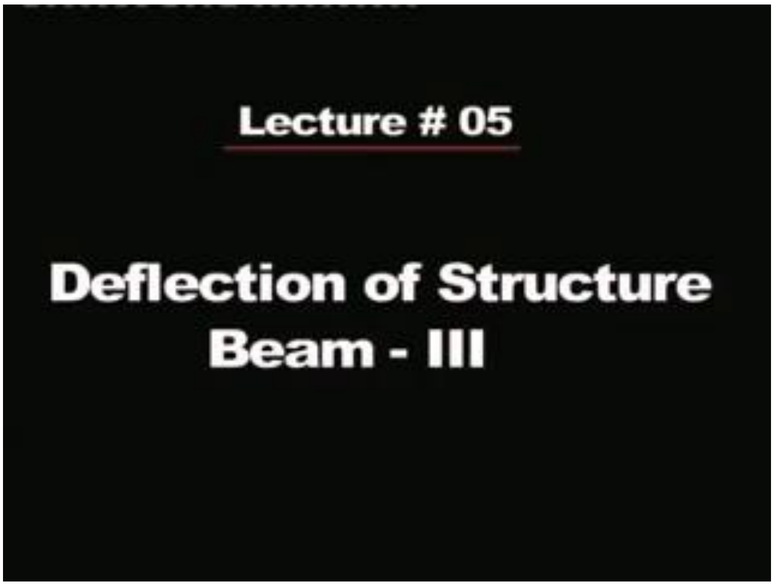

So, last day we were talking about deformation of the structure or more specifically it is deflection of beam. The method we used it is basically integration of differential equation. We tried to handle some cantilever type of problem and then switched over to a simply supported case with uniformly distributed load. Point loaded case we have taken, very standard type of caseload is acting at the center of the beam up to certain extent we solve it. So, remaining part we can continue. So, last class we got more or less something like this.

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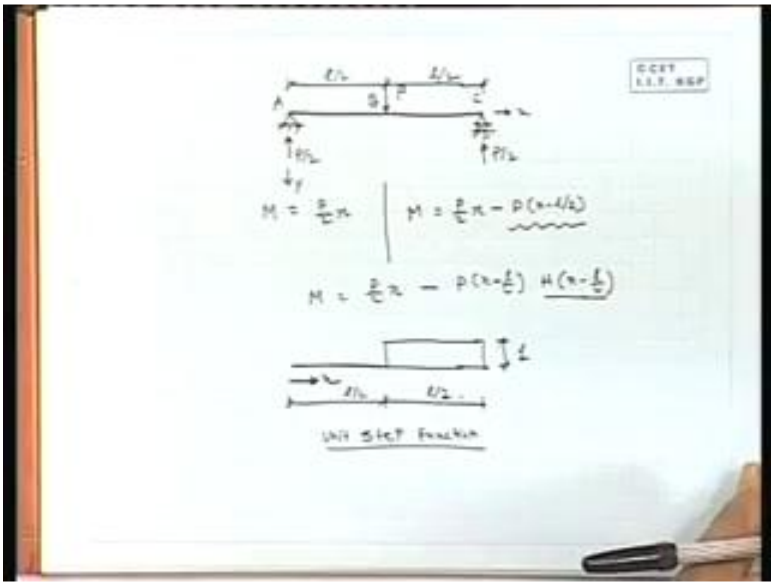

So, we took a beam like this; this side some support; here also some support and center we put P. Reaction we got P by 2; reaction P by 2; if we say this side is x, that side this is y. Now, the main difficulty, what we were discussing in the last class, it has two regions, if we say it is A, and if it is B, and if it is C. So, A to B there is one segment or a part where your moment as some expression and the second part B to C it has a different expression for the bending moment expression. So, these two parts more or less we have tried to express in this manner, say, M that we have written P by 2 into x for the left part; for the right part it was P by 2 into x minus Pxl by 2. So, that we have written in the last class, because here only one load will generate some moment. So, P by 2 into the distance. So, if x is beyond that limit, here the total length is l, and we have divided in two parts l by 2, l by 2. So, the distance we have written earlier; it was say l by 2, l by 2. Now, once we cross l by 2, the centre load P will contribute something in bending moment. So, that was P into l minus x minus l by 2.

Now, after integration, we got two sets of constant here and here left side C 1 C 2; right side D 1 D 2. And we try to establish the slope and deflection continuity. From there we found C 1 is nothing but D 1 and C 2 is nothing, but D 2. So, from that information, we have tried to combine both the expressions. So, we can say, if we integrate that, we will get two integration constant, here also two integration constant, and both integration concept are nothing but identical values, because that common point, from both the side if we compare, the deflection slope, we have checked it; it will be equal. Now, from there we have tried to write a common expression for M equal to P by 2 into x minus Px minus l by 2 multiplied by a function xl by 2.

Now, this function we have defined as unit step function, and that step function, if we just draw below the beam, it will be something like this. Say, it is the load point, this side we have mentioned it; it will be 1; that side it will be equal to 0. So here, the x is starting from here, and total length we are telling it is l by 2 l by 2.

Now, the property of H is such, when within the bracket, the quantity x minus l by 2, if that is a negative quantity and it will yield a value 0. And if it is a positive quantity, it will yield a value 1. So, it is a function; we have defined it is a unit step function. Now, idea here is, if you see the equations here, here P by 2 into x, here also P by 2 into x. So, this part is common; only there will be additional part on the right span. So, this part will be the additional part. So, this is the additional component. This part should not be here when we will be treating the left part, but that part will be there when we cross the concentrate force at the center.

Now, we are talking all in language: it will be there; it will not be there; where it will be there; where it will not be there. So, that part if we try to put it in a mathematical manner, so in this function H what we are defining as unit step function, so basically that understanding in a physical sense, it is trying to express in a mathematical format, because if it is before allowing the mid span, before reaching the appearance of the load, your internal value within the bracket it will be negative, and whole quantity will be 0, automatically this component will not be there.

When it will cross P, this component will be greater than 0 - a positive quantity - automatically this will appear into a equation. So, it is a matter of automatic adjustment. You are talking about Macaulay’s method. Basically, it is same thing and there you put some form; it is basically in a same manner; different places it is representing in a different way. So, basic idea is different equations are applicable at the different zones, but you will find some common part; once you will cross a limit, you will find some additional term. So, that additional term will be there in some part; in some other part it will not be there. So, that will be controlled by this function; this is one of the ways of representation.

Now, this is a simple case; it is P; there might be another load here again. So, there should be one segment, two segment, three segments. So, there should be first is P by 2 into x; second part P by 2 into x minus P x minus l by 2; and there should be a third segment, so this plus something. So, it should H something, H something, like this. So, a number of unit step function may be there; only x minus l by 2 might be x minus say 3 l by 4 or something, where the load will be appearing on the span of the beam.

Now, the whole problem we can treat as a beam, where the moment expression is this expression, valid for the entire span. If that is the common expression valid for the entire span, we can write the moment-curvature relationship in this manner.

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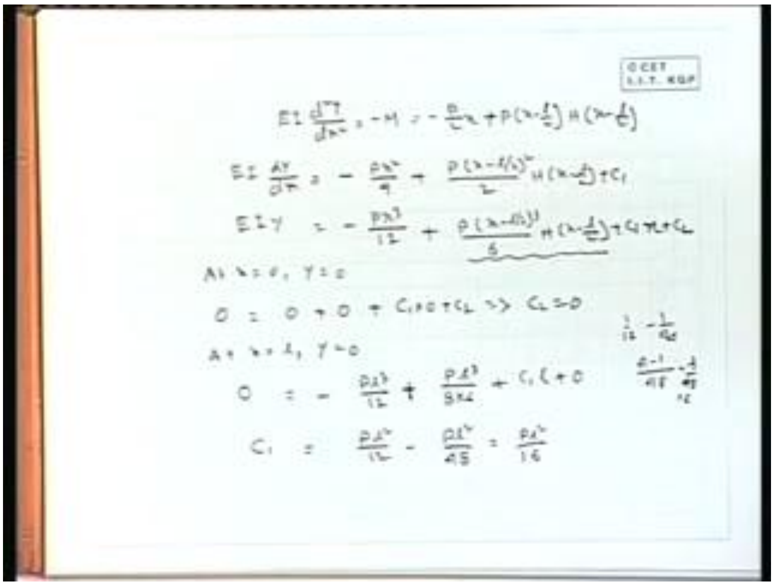

Your EI d 2 y dx square that is equal to minus M or just now we have written as minus P by 2 x minus minus plus P x minus l by 2 and Hl by 2. Now, if we integrate. So, in the first step we will get say EI dy dx; that will be minus P x square by 4 plus Pl by 2 square divided by 2 H x divided by 2 plus C 1. So, again if you take EI y Px cube divided by 12. So, it will be 6; it will be C 1 x plus C 2.

So, it is nothing but writing two expressions simultaneously, we are writing in a combined fashion, because this constant C 1 and D 1 are identical, C 2 will be equal to D 2. Only this unit step function will control this part. We are interested automatically this part will be there or this part will be dropped out. Now, that is a common expression.

Next step, normally, what we do we substitute the boundary condition and try to find out C 1 C 2.

So, let us try here and find out the value of C 1 and C 2. So, we know at x equal to 0, y equal to 0. So, if I put here, so EI into y equal to 0 and this part depending on x. So, this part will be also 0; and this part, if we are in the left side, that x equal to 0 means the unit step function within the bracket, the quantity will be minus l by 2. So, automatically we might be getting some value here, but that should be multiplied with 0 means whole quantity will be 0. So, that will be also 0 and here it will be C 1, 0, C 2. So, from here we will get C 2 equal to 0. If you take the write end that x equal to l, your y is again 0.

Now, here it will be 0 equal to minus Pl cube divided by 12 plus, in that situation this term will be activated, because the unit step function value will be 1. So, that is the important point. You should remember that this part will not be there in the earlier case, but when it is greater than l by 2 it will be there. So, if I put x equal to l, it will be l by 2 to the power 3. So, it will be Pl cube. So, denominator it will come 8. So, 8 already 6 is there plus your C 1 l, C 2 automatically we obtain - it is 0. So, from here C 1 you can find out. So, C 1 will be your Pl square by 12 minus Pl square, because this l part will cancel this will be 48. So, what will be the value? So, it will be 1 by 12 minus 1 by 48. 48 if you pick up, so it will be 4 minus 1. So, 3 divide by 48. So, it will be 3 1 16. It will be Pl square divided by 16.

Now, once we have the value C 1 is equal to Pl square by 16 and C 2 is equal to 0, we have the expression of y we have the expression for dy by dx we can substitute and get the important values, say maximum deflection at the center of the beam, maximum slope at the supports. So, from here we can calculate all those quantities.

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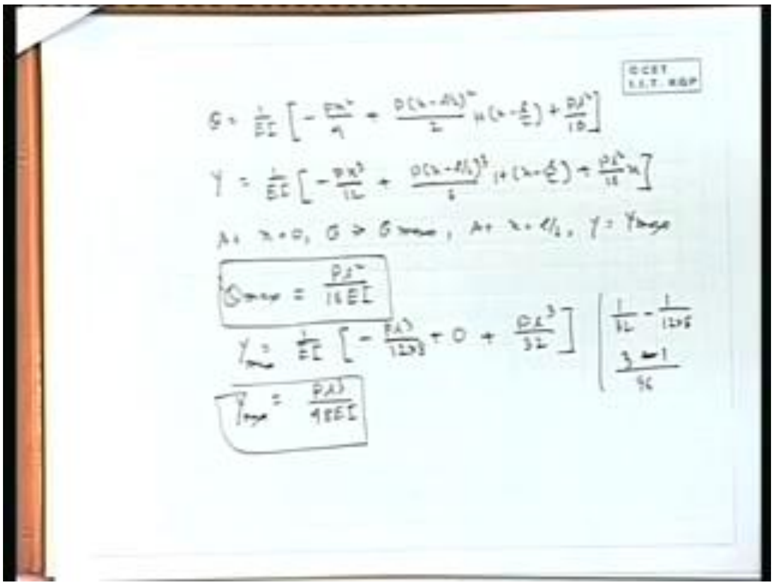

So, we can write the slopes theta 1 by EI, that should be minus Px square divided by 4 plus P square divided by 2 and C 1 will be there; it will be Pl square divided by 16; y will be 1 by EI Px cube divided by 12 Px minus l by 2 is 6. So, whatever expression we have derived, just the value of C 1 we have substituted 1 by EI part I have brought it here. So, it appeared as 1 by EI. So, we will get at x equal to 0, theta will be your theta max. And at x equal to l by 2, your y will be y max. So, theta max it will be just we have substitute the value of x equal to 0. So, this part will be 0, this part will not be 0, but the step function will be 0. So, in a automatic process there is a tendency we calculate something, but there is no point of calculating because the step function will be 0 here. So, both the quantity will be 0 and ultimately the Pl square by 16 will be remaining that will be divided by, so, that will be the value.

Similarly, the maximum deflection, it will not be all 0; we will get some value. So, y will be 1 by EI Pl cube divided by 12. And this term will be 0, because x will be l by 2 minus l by 2. So, this term will be 0, and there is a ambiguity here, whether unit step function will be 0 or 1, because that is the transition. So, if it is just in the right side, it is 1; if it is just in the left, it is 0; and at that point what will be the value? There might be a deviate here again. So, that deviate can be solved by this term, because this term will be automatically 0, because l by 2 minus l by 2. So, whole quantity will be 0. So, you need not think about that confusion what will be the value of H here, I think.

So, automatically that part will be 0, and the remaining part it will be Pl, if I put and if I make l cube. So, here it will be half, it will be 32. So, a little simplification we have to meant. So, it is 1 by 32 and it is 1 by 12. So, it will be how much? Pl cube by 62. Here if we substitute, it is not pl cube divided by 12, this half term will be there, it will be 2 into 2 into 2, 8 terms will be there; otherwise, this term will be larger and we are getting a negative value; it should not be. So, it should be 1 by 12 into 8 plus 1 by 32. So, here I can calculate a little bit; it will 1 by 32 minus 1 by 12 into 8.

96. How much? It is 2 by 96.

That is 1 by 40.

1 by 40 it is the correct answer.

No, you calculate and check it from there again.

Say 96 is the common, into how many times will 32?

That is it will be 3 times.

3 times 1 by 24.

It will be 1. So, automatically it will be 2. So, it will be automatically 48.

Now that 48 the y max value.

Now, this part is one of the common, very standard form. If Pl cube 48 EI, theta max is equal to Pl cube by 16 EI. So, by that time we have to tried to find out some standard expressions, say, beam fully loaded, what whatever maximum deflection we are getting, whatever maximum slope we are getting, a beam with a central load, this value you are getting, cantilever some form we got.

Apart from that, there are some other values, but in this context, this unit step function, which is required for a new load appearing on the span, we are going to use, so if I take a very general type of case I want to show how we will apply this unit step function to represent a number of load.

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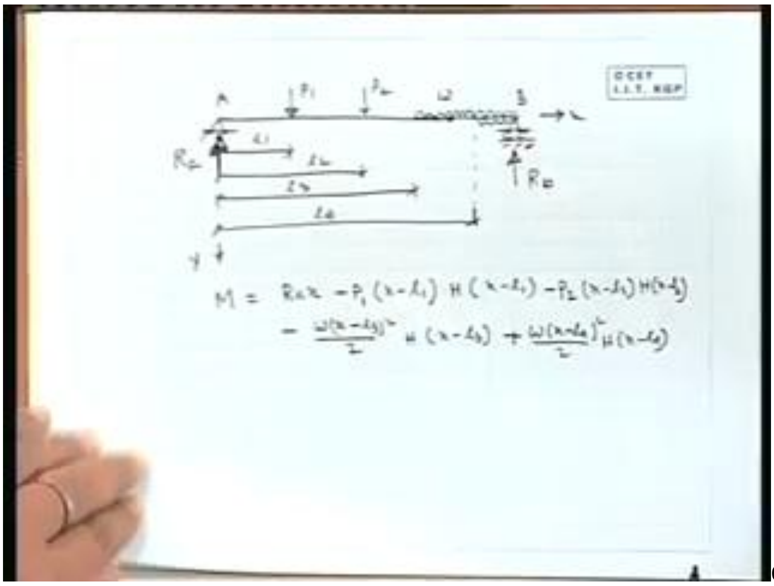
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Say there might some support; there might be some support here. Now, there might be a load here, say, P 1; there might be another load P 2; there might be a distributed load like this, say it is W that intensity of the distributed load per meter length or per unit length; say, I am writing here it is l 1, and this part is say l 2, here it is l 3; say it is extended up to this, it is l 4. You can add some other load as a concentrated load or some form distributed load. I am trying to show you a very typical case. You can increase the number.

Now what will happen? Here we will get some reaction. So, some reactive force; here also we will get some reactive force. Say if we say this is support A, it is support B, that reactive force; I can use a different color, I do not know how much it will be prominent here. So, this is say Ra and this Rb. This side is x; that side is y. Now our objective is to get a common expression of bending moment. In that case, we will use the unit step function to combine different segments.

Now, if it is Ra 0 to l 1, your bending moment should be your Ra into x, or let me write the expression of M. M will be Ra into x. So, far we are within l 1; x is 0 to l 1; this will be valid, if you cross that, you will have say P 1 x minus l 1, but that part will be after l 1. So, here you have to put some step function. So, this part will be 0, when x will be less than l 1; when it will be more than that, so that part will be 1 or you will get some contribution from here.

Similarly, if I take the second load; it is basically P 1 not P. So, it will be P 2 x minus l 2 Hx minus l 2. So, P 2 x minus l 2. So, there is another unit step function; that is x minus l 2. So, if it is less than l 2, this part will be 0, and this part will not be there, but if you cross that it will be there. Now, this part is also coming, if you cross your l 3. So, it will be minus omega x minus l 3 square divided by 2 x minus l 3. So, if I am here, we cross this l 3. So, if we reach here, I think effect of that load will come, and intensity of the load is this one. So, if I am here, so here to here it is x. So, x minus l 3 will be only this small part. So, omega into xl minus 3 will be the load into half of the distance. So, square divided by 2, that will be the effect. And that part is valid when you are here, I think, not before this. So, another step function it will be Hx minus l 3.

Now, here there is a problem. R 1, P 1, P 2 at if you cross R 1 anywhere, it is R 1 into x; if you cross P 1, it is P 1 into x minus l 1; or beyond this, it is P 2 x minus l 2. But if you cross here, up to this it is, but if you come here this expression is not valid. If you are here, because it is x minus l minus 3. So, if your x here, l minus 3 is this one. So, your loading is not there. The load is already over here. So, this part will give a message that it is a extended up to the end, but it is not applicable. In that case, the procedure is, if there is a distributor load starting from some part, if you write that type of form, so we have to follow that equation and add this load, and that load is not in the system . So, we have to make it minus here.

So, upper one it can be tackle by the equation whatever we have written, and the lower part is the correcting force, for that we have to add another term, and that will be in a reverse manner, it will be minus into plus. So, it will be omega and it will be x minus l 4 divided by 2 Hx minus l 4.

So, this is one of the very tricky type of idea. You should remember when there is distributor load, starting from any point, if it is valid up to the end of that member, it is fine, but if it is terminating at some point, then there is a problem, because the way we will take the equation, it will give the message - load is extended up to the end of the member. So, in that case, we have to really extend the force and we have to put some correcting force there. So, that correcting term will come here and this term will take care with this black and ring - both the load.

So, with that type of understanding, you can handle a problem with any number of point load and any form of uniformly distributed load; not only uniformly distributed load, if the variation is little different only that equation we have to take and you cannot terminate. So, you have to extend some of the portion and you have to put some correcting force.

So, finally, you will get a common expression for bending moment, which will be valid for the entire span with a number of unit step functions, and they will start from here, here, here, here, different levels; before that it will be 0, after that it will be invoked, means it will give a value of 1, means it is now 1; it is just behave as a switching function. So, it is now on; it is now off; something like this.

Now, once you get M, you can write M is equal to minus 82 by dx square; integrate twice, you will get C 1 x C 2, and at the two end you can substitute the boundary condition; you get C 1 and C 2. Finally, you will get the expression of y or you can say dy dx. So, any point if you substitute x equal to 0 or l l by 2 l 1, l 2, l 3 you will get the values of y, that is deflection D 2 by dx square slope. So, you can find out all those quantities for very unknown type of problem.

Sir, there we reached to a conclusion. We knew that at this point slope is 0. We have taken the deflection in that y equal to 0; in that at the end we have taking.

Because we starting with the second-degree equation. So, moment minus EI d 2 dx square; we will get two constants. So, two boundary conditions are unnecessary. And the boundary conditions will be in the form of deflection and slope. So, if it a cantilever problem, in one point we are getting the condition deflection as well as slope are 0 at the fixed end; but simply supported case, it is slope is a unknown quantity here; slope is also unknown quantity here. So, slope we cannot utilize. We can only take the help of deflection of A and B, means at the two supported points.

Now, this is more or less dealing with deflection of beam with the help of differential equation technique. We can take some numerical example in our tutorial classes; there we can check different situations. So, my next target here is move on to a little different method, which is called moment area method.

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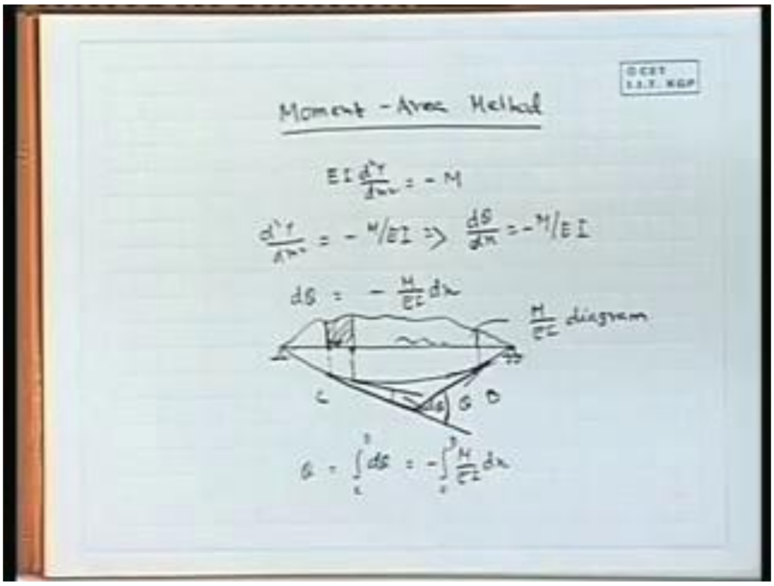

So, this method is called moment area method. So, it is another method like your differential equation technique; very simple and elegant method, typically applicable for a small size problem. Now, the idea is coming again from your moment-curvature relationship. We have the understanding that EI d 2 by dx square equal to your minus M, minus or plus whatever M is there. So, this we can write as d 2 by dx square equal to M by EI or that equation we can write in this form d theta by dx minus M by EI, because theta is basically dy by dx. So, if we put theta equal to dy by dx we are supposed to get d 2 y by dx square. Now this is one equation. So, d theta will be your M by EI dx.

Now, let us draw a beam with some deformation. It may have some load, some support. So, it will undergo some deformation here. So, from our moment-curvature relationship, we have written d theta equal to minus M by EI dx. Now if I take two adjacent points; so, there is a point here; if we draw the tangent on the elastic curve at that point, and if I take from some adjacent point here, if we draw tangent here, the angle between the two lines is the d theta, because this line with x, it is basically theta, and here this line with x, it is d theta plus some increment of theta. So this value is nothing but d theta. And what is the d theta? d theta is M by EI into dx. So, what is dx? dx is… basically this is dx.

So, this is a point say at x; this is point, it is x plus dx; and here at x it is theta; here x plus dx it is theta plus d theta. So, difference in their slope is d theta, and that is equal dx M by EI. Now what that means is some loading are there you find out the reactions; from the reactions, you can draw the bending moment diagram of the beam. If you can draw the bending of moment diagram of the beam, that you can divided by EI of the beam; you will get some diagram; it is the M by EI diagram. Now, here say a bending moment is something, and that we have divided by EI. So, that if we say M by EI diagram. So, M by EI into dx is basically this part.

Say some bending moment diagram divided by EI means we are basically representing the same diagram with some scaling factor. This scaling is 1 by EI. So, that diagram is M by EI diagram, say, M by… it is not M by EI, say it is say y - function of x. So, M by EI is a variable quantity; it is a function x into dx. So, function of x dx; this small incremental; this is function of x into dx; and dx is very, very small; means within that range we can assume this is more or less uniform value. Now, if I take some other point, little bit away, it is not adjacent point, it is little bit away. So, this tangent will move like this; there e is a point here. So, say from here if I draw a tangent. So, that was the slope; that is the lope; the difference between the slope is basically your theta. This was d theta.

Now, if you can go on taking many points. So, d theta, d theta, d theta, if you go on accumulate, you will get difference between the two tangent line theta. So, theta will be, basically, our integration of d theta, and integration will be found here to here. So, if it is point C, and if it is point B. So it will be point C to point B and this is nothing but your integral M by EI dx. So, we obtain integral M by EI dx, means from C to D, if we calculate the area of the curve, M by EI, ranging from here to here. So, the total area from C to D if we integrate, that is shown here, because this is M by EI and dx, and we are integrating from C to dx. And that integration will give, if we draw slope here, and slope here, the difference between their angle is theta.

So, if you have a deflected curve, if you take two points, if you draw a slope here, if you draw a slope, both the slope will meet, and this slope is change. So, the direction is changing. So, this amount of change is basically here to there; whatever area under M by EI diagram. So, this is one of the component of your moment area theorem, in terms of calculation of slope.

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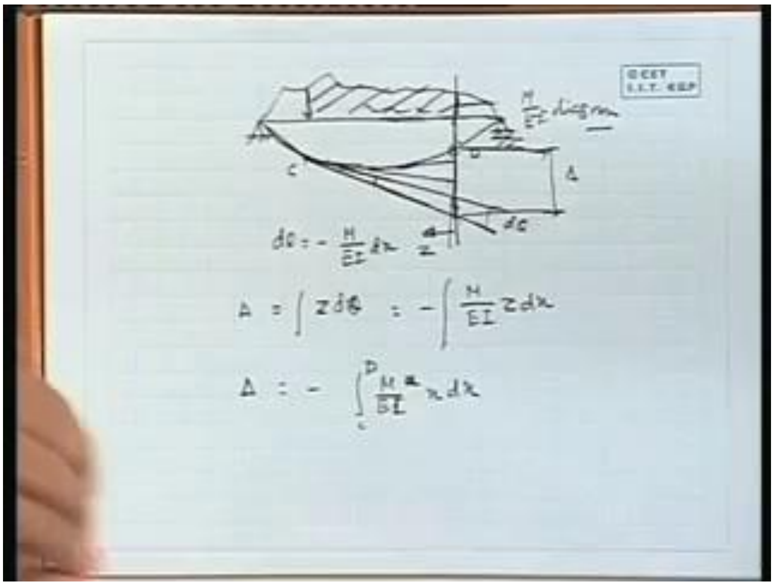

So, next step is the deflection part. So, if we draw the beam again, and try to investigate what is happening with the deflection, it will be as I told, assume any loading. Now, this point we have drawn some tangent. So, that we have defined as C. Now, we took some adjacent point; there we have drawn another slope. Now, there is point D. So, C to D we want to see what is happening here.

Now this angle is D theta. Now if I draw a line perpendicular from D, perpendicular to the initial line if you drop the line so this value will be how much? This value will be D theta into the distance; say this angle is your D theta. And d theta into here to here this distance if you multiply, so you will get this the small distance. Now if we take another point, draw it, there will be another D theta. So, here to here you have to multiply. If you take another tangent, so it is D theta multiplied by this one. So, like that if you go on taking small, small value, so here to here - this part - it will be how much? It will be your D theta, already we have written has D theta you have written minus M by EI dx.

Now, I am trying to tell from here, D theta, D theta into, say, this is some distance, say, z or something, and that part if you go on accumulating, integrate, so if that is delta, you should get this delta. And this is nothing but your minus M by EI into your z into your dx. Now, here this z and x, we will take in a similar fashion, because you have to integrate dx, there is no limit actually. So, this part will be your M x M by EI; you can say it is xdx, say here, we are trying to tell it is z. So, D theta into z.

Now if you take the next one it will be different z. So, you have to go on accumulating; definitely, you will get this value, this value, this value, this value; finally, we will get the total value, so it is… and D theta from earlier expression we have M by EI into dx.

Now, what is D theta? D theta M by EI into dx. So, this dx part is this one. Now dx we are we are measuring x from here. And dx is this one. So, if we shift our origin, say, dx we can take as dz, no problem, because from here it is z, and this part is dz, so dz into…So, entire quantity we can write dz dz or xdx in any form we can write . So, it will more or less same thing. So, here what are we getting? Delta equal to M by EI is the say any diagram here; this is M by EI diagram; that is x into dx. So, we say this is the first moment of area of this diagram, between your C to D, but this x or z whatever we are talking about, you have to take from here, not from here.

So, ultimately, this area you have to consider, and you have to take the first moment of this area about this line, because z we have taken this way, x we have took in that manner; in order to unify x and z, we have taken the expression as x, we could write it is z also; rather it is better to write z, and z we are taking from here. So, it is… this is the ordinate; it is xdx; this is the typical expression of taking this area - moment of that area - about this line.

So, if you take any two points, and if we calculate the first moment of area under that curve from C to D, about d, so we will get this value. This value is what? From the deflected curve D, if we come down perpendicular to the original line, it will meet the tangent drawn at C. So, whatever distance you will get, that can be calculated from here. So, this the second step of our moment area theorem.

So, it has two steps: first was related to slope calculation; the beam will be given; loading will be given; reaction we can calculate; we can draw the bending moment diagram; that bending moment diagram can be divided by EI. If EI is constant throughout, it will be uniform scaling; it may be a beam having a different type of EI, so it will be just divided by EI at the corresponding range. So, you will get M by EI diagram.

Now, any two points if you consider and draw two slopes, that difference between their slope, one will have theta 1 and another will have theta 2. So, theta 1 minus theta 2 will be the actual difference; that will be area of M by EI diagram, from your C to D, or point 1 to point 2. And if you take first moment of area about D or point 2 whatever you can say, it will give the deflection measure from D towards the tangent point drawn at A, but your measurement should be perpendicular to the initial line.

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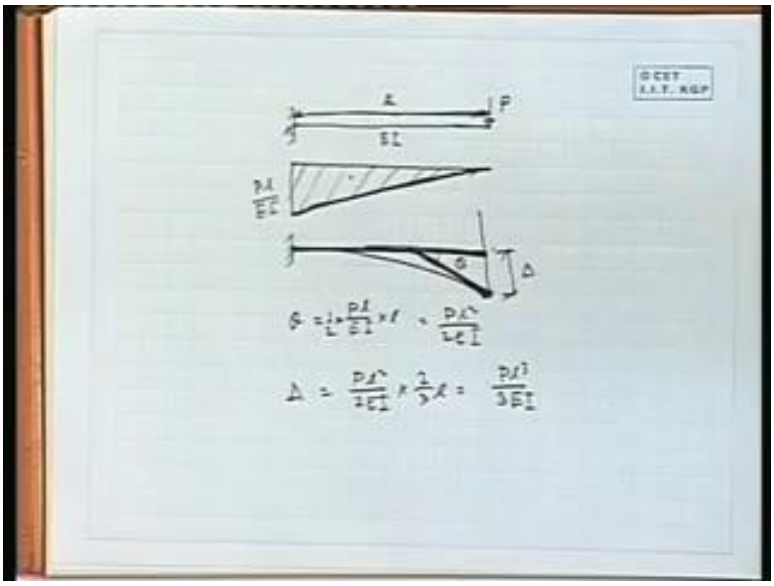

Now, if I take some example, cantilever is a very good example in that case, because, say, this is a cantilever problem; and this problem you have solved by you differential equation technique; some of the information it is already there. So, we can compare our result at this level using moment area theorem. Say, this is the beam, and there is a load P. This case you have handle at the beginning. Say total length is L, and this is EI, and length is L.

So, our first job is we have to draw the bending moment diagram. It is a cantilever problem. We need not calculate the reactions, because we have one free end; from that end we can start. So, P into x that will be the expression of the bending moment diagram or if we draw the bending moment diagram it will be something like this.

So, this value will be B into L. This is what your bending moment diagram. And if you divide by EI, it will be M by EI diagram. Now, here I will draw the deflated shape of the beam. So, this is a initial line; after deformation it will be like this. Now at this point, if I draw one tangent. So, this is one of the point, say, starting point. This is the initial line plus if you draw the tangent also, it will be the same line, because it is a fixed end; it will not undergo any slope; it will be 0 slope and 0 deflection. So, this is the initial line is the tangent to that point, and here tangent is this one.

So, difference between their slope of the two tangent is theta. And according to your first part of the moment-area theorem, difference between the slope is area under the curve. So, theta will be area of this curve. So, what is the area? This is L; this is Pl by 2; and it will be half of that. So, it will be Pl by EI half of that into L. Now we can say Pl square divided by 2 EI. And that value we obtain earlier also. And here one of the advantage - one of the support is fixed. So, as it is fixed, one of the tangent line is the initial line; that is a great benefit here, I think. Earlier we have taken a simply supported type of the beam. The first slope, last slope, it is an not the original line. Here the advantage is left end, the tangent is the initial line. So, any point if you draw the tangent here, that will be the effective slope. So, straight away we can calculate.

Similarly, if you want to find the deflection delta. So, try to remember the second part. Second part means, so from here - this is the deflected line - from here if we move perpendicular to the initial line, so we will meet a tangent drawn at other point. So, that is the deflection, means the area of curve, we have to take first moment of area about this, because we are moving about this point. So, delta will be the area under the curve we have already written - it is Pl square by 2 EI - that is the area, and centroid will be 2 by 3 l. So2 by 3 l. So, 2 by 3 l. So, it will be Pl cube divided by 6 EI. That is also we have derived. Earlier we have written some equation and obtained C 1, C 2, calculated. So, it is much more easier at least that type of problem. Now, with that this class I am trying to finish. Next class we will continue with some other type of problems.

Preview of Next Lecture

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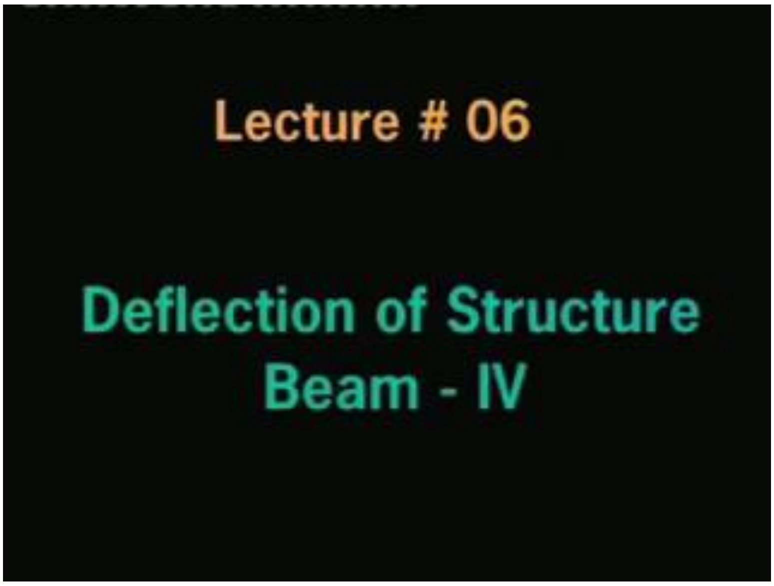

So, we are talking about moment-area theorem, and two cases - simple beam problem, cantilever beam - with some deep load and maximum deflection slope, we have calculated and tried to compare our values with those obtained by your direct integration of the differential equation. And you must have that feeling, that it is much more easier compared to the earlier approach. So, it is not a very general statement, for that problem, it might be easier, I think. So, it depends problem to problem, you have to choose the method in a appropriate manner. Sometimes the direct method may be much more easier or convenient to get the solution. Now, I will take another problem of a cantilever type of beam.

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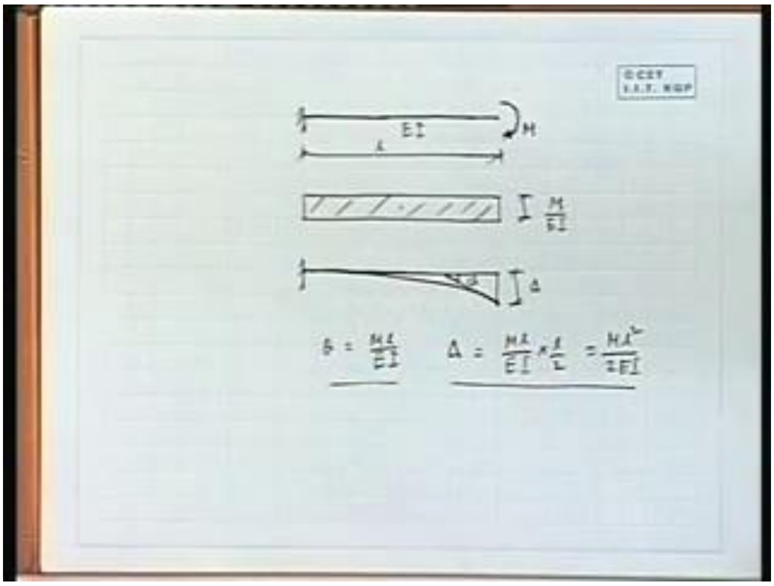

Because the values are quite important here. So, length is l, and again it is EI here. There is a moment acting at the end, say, this moment is a M. Now for that beam, you can draw the bending moment diagram. So, it is M. Throughout the beam, bending moment will be M or if we divide by EI, we will get the M by EI diagram on the beam. Now, if I draw the deflected shape of the cantilever. So, here, there is one advantage - this point is fixed. So, tangent at that point is the initial line. So, that we have already mentioned. So, at the free end, if we draw a tangent, so both that tangent will have a difference in slope, that will be theta. And incidentally, that value will be the actual slope of the point at the free end.

So, it is what? It is basically area of that curve. So, it is l, it is M by EI. So, it will be simply Ml by EI. And delta is, from this point, if we move in a direction perpendicular to the original line, it is meeting that tangent drawn at that point here. So, that is delta. So, we are moving along this line. So, about this point we have to take the first moment of area of this M by EI diagram. So, it will be ranging from here to here, moment about that.

So, area already we have written Ml by EI. And its centroid will be at the midpoint. So, that will be l by 2. So, it will be Ml square by 2 EI. Now these values are quite significant values. Like the case of a cantilever subjected to a load and the free end, we have written Pl square divided by 2 l or Plq divided by 3 EI. Similarly, if there is a moment, it will be Ml by EI or Ml square by 2 EI for deflection and slope.