Strength and Vibration of Marine Structures
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Lecture - 4
Deflection of Structure Beam – II

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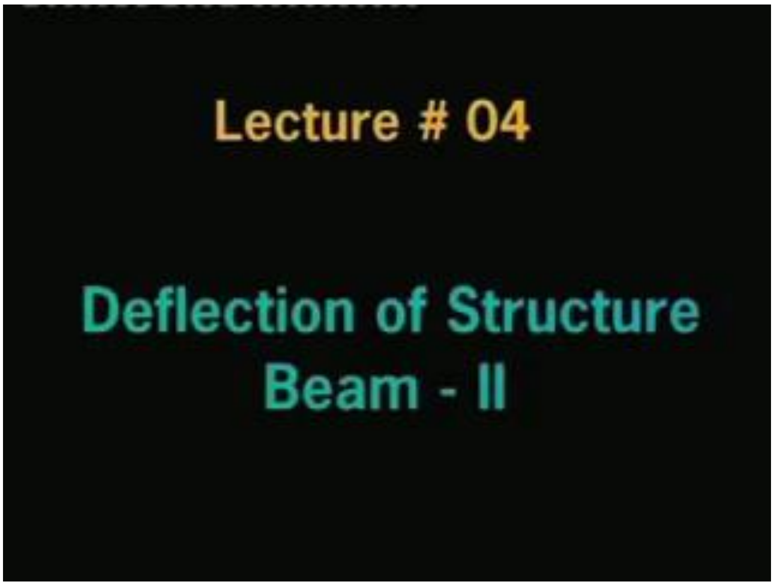

So, we shall continue whatever we are trying to cover in the last class, that is deflection of beam, and we are trying to solve the problem with differential equation technique. We have taken a simple beam problem. It was a cantilever, with some point loaded at the end, with that I have tried to explain you how to find out the final equations, how to find out the constants through boundary conditions, and from there, we try to obtain some standard values of maximum deflection, maximum slope at one of the end.

Now, we will try some other standard cases, and try to get some important relationship. Technique is same; problem will be little different. Now, I can take, again, cantilever problem with a little different type of loader.

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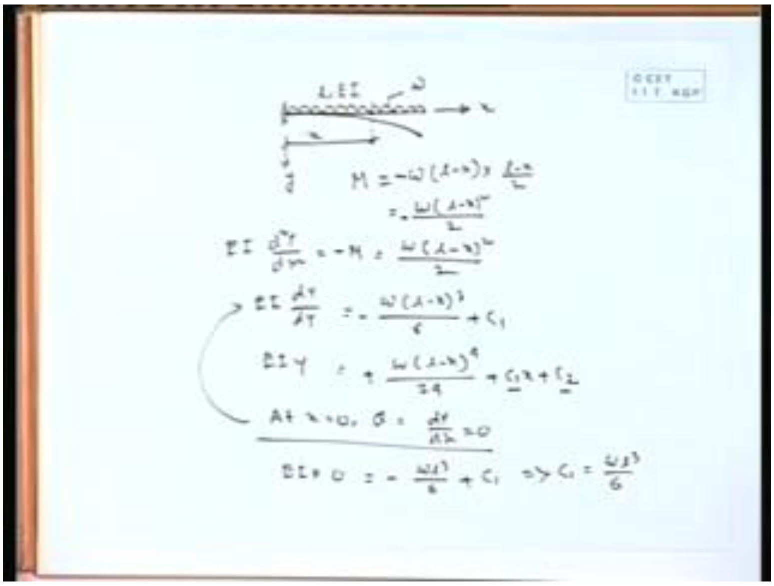

So, structure is same; it has a length l, EI, support identical, it is fixed, that is free, only the loading is different. The idea is to handle a different type of loader. Also, I think, before changing the type of structure; next step we can change the type of structure. So, our first job is we have to find out the expression for the bending moment. Now, at any distance x, so there is one cross section. What will be the expression of bending moment? We can calculate from the right end, because that is a free end it is easier to calculate. So, beyond that section right side, the loading is first of all the length of that will be L minus x. And the loading is say w omega; that is intensity of the distributed load; the load per unit length. So, length is l minus x and w is per unit length - that is the load. So, total load will be W into l minus x and that is any distributed manner.

So, moment will be, it is a uniform uniformly distributed load, means we can assume that load is acting at the centroid of that part again. So, it is it will be acting half distance of lyx. So, we can write omega is the intensity, l minus x is the length, that multiplied by l minus x divided by 2. So, that is one of the very familiar type of expression. If we start from this side, well at omega into x that will be the load into x by 2, or straightaway we will write omega x square by 2; in that case x is not x, it is l minus x. So, it will be omega l minus x square divided by 2.

Now, we have to put some sign here again. At this moment, we have to decide which sign convention you will take positively. If you take sign as positive, it should be negative, because it will give a falling type of moment. So, if we follow the same thing, it will be minus and minus.

I think this part is clear - bending moment expression. Once that expression is ready, we can use that moment-curvature relationship; that directly we will put, the moment expression. So, there it will be EI d 2 y dx square equal to minus M. So, it will be, we can say it is minus M, and we can put here omega l minus x square divided by 2. So, minus, minus; it will be plus. We are sagging; sagging we have taken positive, but here moment is hogging, so we have put minus. And if it sagging as the positive moment, expression will be with a minus M. So, this minus, minus it will be plus.

So, in a similar manner, we will integrate as we did in our previous problem. Iit will be omega l minus x cube divided by 6 minus C 1. And EIY equal to omega l minus x 4. So, 4 into 6 24, and minus and minus it will be plus, C 1 x plus C 2. Absolutely same. Only the right side part is different. Once you have that expression, our job is to determine C 1 and C 2 - these unknowns - through boundary condition. So, it is in a same manner at x is equal to 0, so theta or dy by dx is equal to 0. So, this is the first expression or first condition. So, if we put here, I think. So, it will be EI into 0. It will be minus omega l cube divided by 6 plus C 1 or we can say C 1 equal to omega l cube divided by 6. So, your C 1 part is available.

Next part is the C 2. So, next requirement is to find out C 2. And that we can find out with the second boundary condition. That is deflection at this point - at x is equal to 0, it will be equal to 0.

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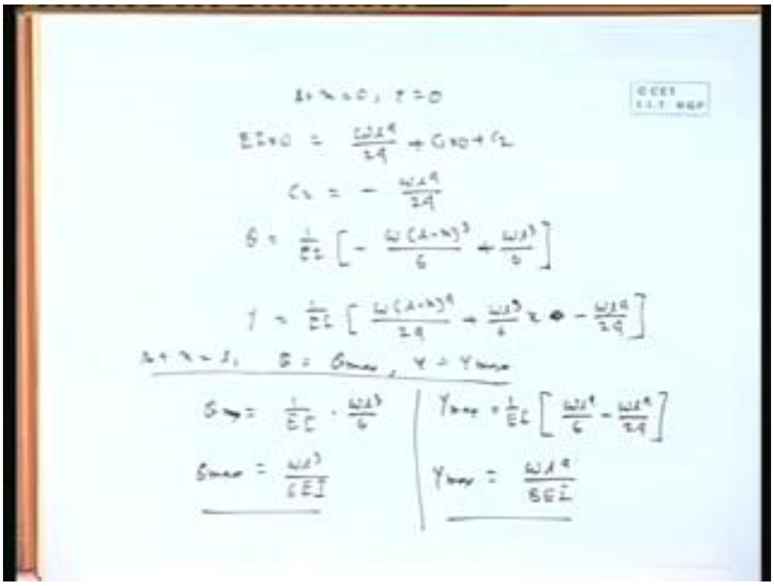

So, substitute that boundary condition - at x equal to 0, y equal to 0. So, we will get EI into 0. That will be omega l 4 24 plus C 1 into 0 plus C 2 or we can get from here C 2 equal to minus omega l 4 24 or these values we can put in the expression of slope and deflection. So it will be 1 by EI minus.

So, C 1 and C 2 we have substituted the values in the expressions we obtained earlier for slope and deflection, and it will be something like this - that is the expression of theta; that is the expression of your deflection. Now, we will get the maximum deflection, maximum slope. So, you here at x equal to l, theta will be theta max, and Y will be Y max. So that part if we find out, so theta x it will be 1 by EI, and here, this term will be 0; it will be omega l cube by 6 or we can say omega l cube divided by 6 EI.

Now, if I find out Y max, 1 by EI; this part - it will be 0; and this part will be omega l to the power 4 divided by 6, omega l to the power 4 divided by 24. So it is omega l to the power 4. So, 1 by 6 minus 1 by 24. So, if you operate, it will be 24; this side it will be 4 minus 1. So 3. So, 3 24 means it will be 8. So, it will be omega l to the power 4 divided by 8 EI. So, this is Y max; this is theta max. So, this is one of the quantity; that is another important quantity. So, same problem with a distributed load; we have tried to repeat that process with a different moment expression, and got the values theta max, Y max.

Now, I will take a problem, where your type of structure will be different. So, beam will not be a cantilever beam, say, we will take a simply supported beam. So, two supports at the two end, and take some load, say, we can take uniformly distributed load, whatever we have taken, at least load will be same like this, but support condition will be different.

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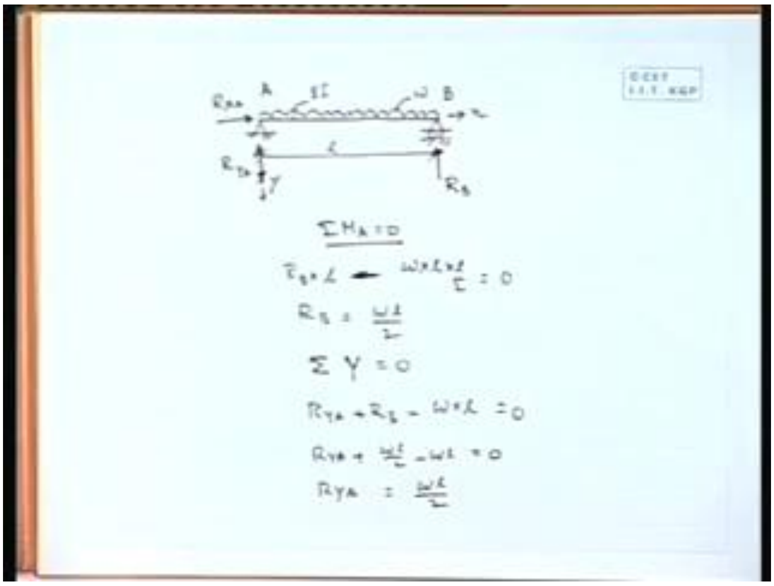

So, this is a distributed load of intensity omega or w; and the member is some EI; the flexible rigidity; this is the length; this is x this is Y. So, our first job is to determine the expression of bending moment. Now, if we start from this end or the other end, both the sides supports are there. So, we have to know the value of those reactions.

Earlier case, support was on the left side and one free end was there. We were trying to proceed from the free end; we need not bother how much will be the reaction at the supports, but here we cannot start from this end or other end; both end are supported. So, first job is we have to find out the reaction. And that will be utilized for finding out the bending moment expression. And that will go to the differential equation.

Now, here there will be one reaction and here. So, this part we can say y; and this part we can say Ry; it is say Rx; let us say A, that A; this is B; say left end if we say A, right end if we say B; it is R along x at A; Ry y at A; this is RB.

So, it is a beam problem and loading here are all in vertical directions. So, RxA automatically it will be 0. Still we have to put, because there might be some inclination in the loading, and that will make the structure unstable. So, we have to keep the provision of some resistance along the horizontal direction.

So, any way this Ry A and R B we have to determine here. So how we will determine it? You can take moment about any one of the end. So it is better to take moment about A, at this both the forces will be eliminated. So, first step we can take summation of moment about A equal to 0. Now, the reaction - RB into l. So, that is one force into the length; that will be balanced by the moment generated by the externally applied load - omega. So, that is will be equal to… Because this is the this way moment and load will be giving in a different way.

So, we can put minus equal to 0 or we can directly put on the other side. So, we can put here minus, and the omega is the load per unit length into total length l, and its resultant will be acting at the centre of the beam, because it is throughout uniform. So, into your l by 2, and total moment will be equal to 0. So, RB you will get from here it will be omega l by 2, because one l will cancel. So, omega l by 2.

Now, the other equation you can apply. It will be say summation of vertical force or the all the forces along Y is equal to 0. Somewhere, we will say summation of B equal to 0; somewhere, Y equal to 0; sometimes, YI; it is a matter of convention. So, it will be RYA upward, RB upward minus omega into l, it will be downward, whole quantity will be equal to 0. Now, RB already you have obtained. So, RYE plus omega l by 2 minus omega l equal to 0. So, RYA it will be omega l by 2. It will be omega l, and it will be omega l by 2; this is minus plus, right side if you take it will be omega l by 2.

So, we have tried to use equation of statics, tried to write so many equations, ultimately we got both the reactions are omega l by 2. From our basic idea also you know it will be omega l by 2. So, it is, you can say, verification of that or from this experience later on we can do one thing. There are some problems where physically we can find out, we need not go for that calculation. So, straightaway we will write it is omega l by 2, omega l by 2, because it is quite symmetrical structure; structure fully loaded. uniform. So, half way, half way it will be distributed. So, omega l, omega l by 2, omega l by 2. Now, once these reactions are available we can write the expression at any point or rather any section.

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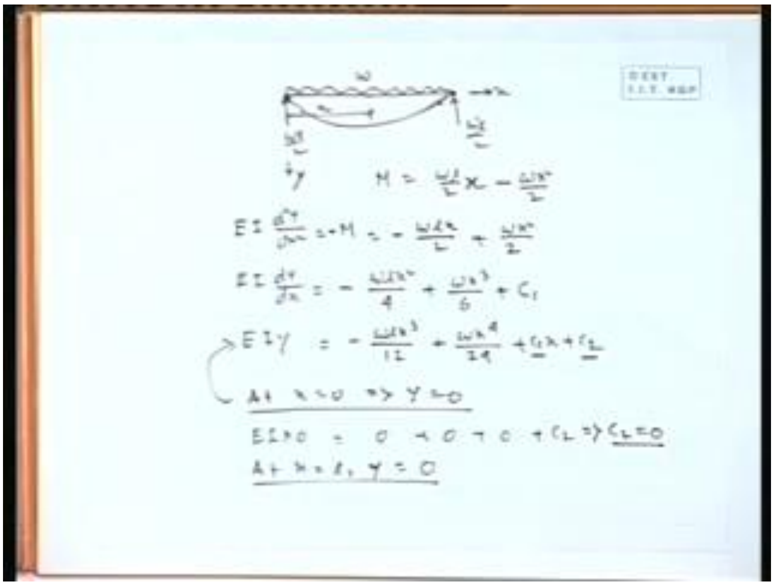

So, we can write the beam in this manner; here it will be omega l by 2; there it will be l by 2; omega l by 2; and this part is omega; and here it is x; and that side y. So, moment will be, so at any station, so at any distance x is omega l by 2, that reactive force multiplied by x, that will give a moment like this – clockwise, and this load it will give a anti clockwise moment. So that will be omega x square by 2, because up to here the length is x omega into x into x by 2, omega x square by 2. So, this is x and per unit length it is omega. So, omega into x will be the load - total load - here that will be acting at the middle of x. So, it will be x by 2. So, omega x into x by 2, omega x square by 2.

And this moment will be a sagging moment, because this is reaction is giving a moment that will generate tensile stress at the bottom. So, here there is a reaction. This reaction will give a tensile type and this load it will give… tensile here means hogging type. So, it is we put minus. So according to sign convention we have written the bending moment expression.

So, next step it will be our moment-curvature relationship. So, it will be EI d 2 by dx square will be the minus moment, because sagging is positive. So, it will be minus omega lx by 2 plus omega x square divided by 2. So, once that relation is available, we have to integrate twice. So, it will be EI dy by dx minus omega lx square divided by 4 plus omega x cube divided by 6 plus some constant will come again. So, second level integration it will come minus omega lx cube 3 into 4 12, plus omega x to the power of 4, 4 into 624 C 1 x plus C 2.

So, next job is to find out the value of C 1 and C 2 with the help of boundary conditions. In previous problem, we got at x equal to 0, slope, deflection - both quantities become 0, but here it is a simply supported problem. So, it will deform like this. So, slope is not 0 here; slope is not 0 here; only deflection is 0; and deflection is 0; these two information we have. So, our boundary condition is at x equal to 0, y equal to 0 - this is one of the boundary condition. So that part we can write here - EI y is equal to this will be 0; this will be 0; this will be 0; and C 2; and that will lead to C 2 equal to 0.

Because everywhere it is in 2 x, x cube, x 4, x. So, x is equal to 0, and entire part will be equal to 0; and y equal to 0, C 2 will be equal 2 - this is one of the boundary condition. Now, the other boundary condition at x equal to l, y is equal to 0. Now that boundary condition if we apply, we will have a relatively large expression.

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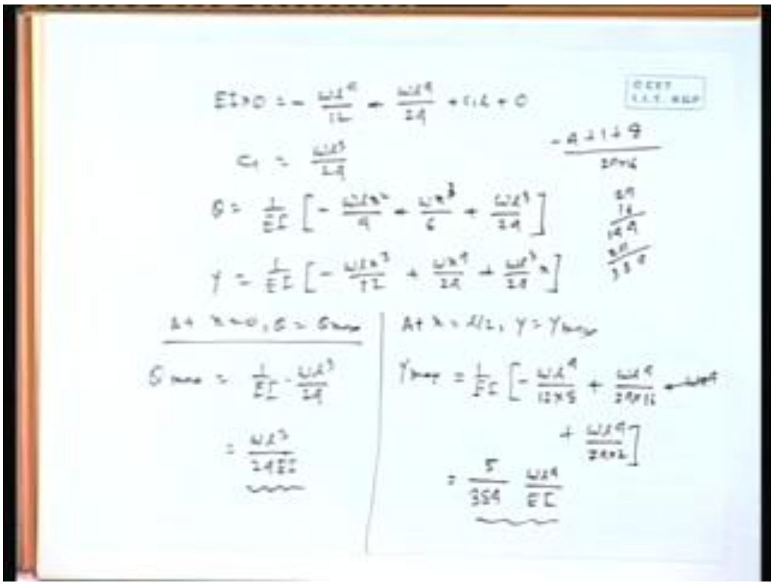

So, that expression will be, your EI deflection is 0, it will be minus omega l 4 divided by 12 plus omega l 4 divided by 24 plus C 1 l, C 2 it becomes already 0, for C2 is 0, C 1 l omega l to the power 4 by 24 minus omega l 2 the power by 12 that will be equal to 0. So, C 1 we can find out from here. So, C 1 will be, this part is 0, this part is 0, 1 l it will cancel, it will be omega l cube, omega l cube divided by 24, because it is 12, it is 24, half of that. And this is minus it will go the plus side. So, C 1 is omega l to the power 24. And

C 2 is 0. And these two quantities we can put in the expression of our slope and deflection. We will get the equation for the deflection slope, of the deflected line.

So, we can write here theta is equal to, because from here already we have the expression, it is 1 by EI, and this part is minus omega l x square by 4 plus omega x, x 3 divided by 6 plus, C 1 is omega l cube divided by 24. So, C 1 omega l cube divided by 24. And other part we have just put there. And y equal 1 by EI, omega lx cube divided by 12, omega x to the power 4 divided by 24, and this part will be omega l cube 24 x C 2 is 0. So, these are the equations. Definitely, you can rearrange in some other form, but I want to find some important parameters.

So, what will be the maximum slope? What will be the maximum deflection? Now, here the maximum slope we will get, where will get it? It will be at the two ends. Slope will be maximum at the two ends. So, gradually, so it is a simply supported beam problem. So, here it will be maximum, and here it will be 0 slope; and again it will go the maximum, it will be minus side; in a minus side maximum value will be this. It is symmetrical one. So, slope this side, slope this side - it will be identical; only one will be plus and another will be minus. And deflection maximum we will get at the center. So, deflection maximum means slope 0. So, that way also we can get it.

So, say at x equal to 0, theta will be your theta max. So, theta max will be here. We can put x equal to 0, in that equation. So, it will be 1 by EI. And these two terms, it will be 0; it will be omega LQ by 24 or sometimes we write omega LQ divided by 24 EI.

Now, if I write the maximum deflection at x equal to l by 2, your Y will be equal to Y max. So, it will be at the center of the beam. So, your Y max. So, here we have to put 1 by EI minus omega l. So, we have to put l by 2 here. So it will be l to the power 4. So it is l by 2, 2 to the power 3 it will be 8. So, 8 will come at the denominator; it will be LQ and one l is there; it will be l 4 plus omega l 4 divided by 24 is there and l by 2 4 times. So, it will be 16 here. And here omega l to the power 4 plus omega l to the power 4, it will be 24 below and 1 2 you will get, because here l by 2 half it will be 16, it will be 8.

Now, all are omega l 4 and it will be EI. Only some factors are there; it is 12 into 8, 24 into 16, 24 by 2. So, this calculation if we carry out here again, so it will be something like this: it will be omega l to the power 4 divided by EI, and this factor we have to find out; this factor we can calculate here in as a rough calculation.

So, denominator is maximum 24 into 16. So, 24 12 means, here 2, here 2, it will be 2 into 2 4, minus plus 1. And here it is 2, 2 into 8 24, 24, it will be 8. So, it will be 8 plus 1 9, 9 minus 4 it will be 5, and 24 into 16, 24 into 16 if you calculate. So, 4 into 6 24 2. 384. I think it will be 384. So, 24 4. So, it will be 384. So, this is one of the standard formal thing, but I want to check whether it is exactly coming out. Because this is one of the important term; this is one of the important term. So, 5 by 3 into omega l to the power of 4 by EI and that is omega LQ by 24 EI.

So, this is one very, very standard case - a simply supported beam with uniformly distributed load, maximum deflection, maximum slope. So, in the next case, I will take a point load on a simply supported type of problem, because cantilever we have started with a point load, then distributed load. Then we have kept the loading same, support was different; now support is again simple support at the both end, but loading is point loader. Now let us see what are the things we will get.

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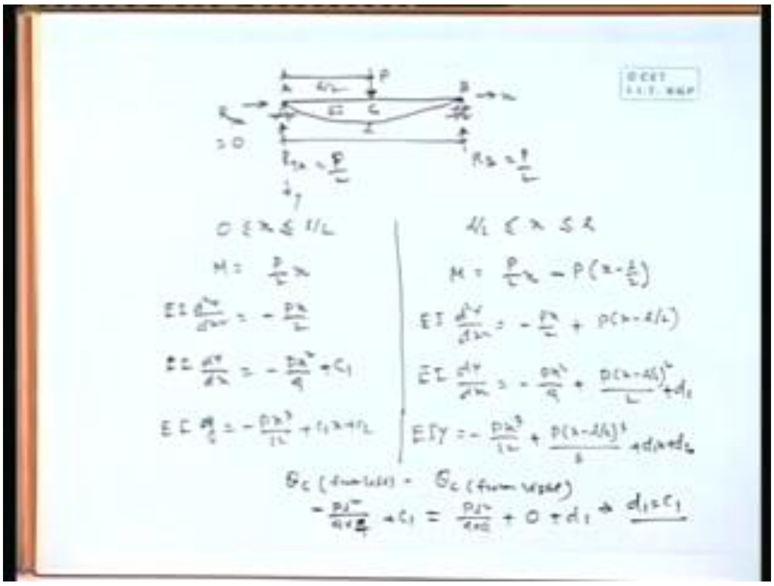

So, the length is l; left end a it is in support. So, Rxa Rya; this is roller Rb; xy is like that; P is acting at the mid span. So, it is at the center of the support. Now, the first job here is to determine the support reactions, because both the end are supported; that we have found in the earlier problem. So, one option is you can take moment at support a, then you can take summation of all the forces. But you need not do it, because from the structure itself or the way load is put there, directly you can calculate. So, what will be the reaction here? PY 4. It will be straight way PY by 2; half, half load. So, Rb we can write straightaway P by 2, and Ra it is P by 2, and Rxa equal to 0. There is no horizontal force. So, the reactions we obtain from the look of the structure.

Next part is we have to get the expression of the bending moment, and that we will substitute in our moment curvature relationship. Here there is one problem; say this a, it b, the center point is c, and total length is l; and here to here, it is say ly 2. Now your moment expression is not valid for the entire span. So, E to C, there will be one expression for bending moment; if you cross C moment expression will be different. Cantilever problem, point load case was much easier compared to the distributed load, but here situation is just reversed.

So, if we divide into two regions, say x, it is l by 2 0, this is say one part; another part l by 2 l; here your moment is P divided by 2 into x; that is, because P into the distance; P into distance that is giving a sagging type of moment, positive moment. So, this P by 2 into x. Now, if we cross point C, your moment will be it is P by 2 into x minus something will be there, that will be P into x minus l by 2. So, additional force will involve additional term in the expression.

So, if you have some other loads, say, if you put another load. So, we will have one segment, two segments, three segments, or from here there will be a distributed load. So, once a different type of load will come, it will go on inviting one, one additional term, so expression will no longer valid there. So, it will be, with any additional term means it will be different. So, for that region we have to apply your moment curvature-relationship with this moment, and for the other region, you have to put this moment in the moment-curvature relationship. So, simultaneously for different segment you have to run it.

Now, let us put the expression and see what is happening. First of all your EI d 2 y dx square equal to minus M. So, it will be minus Px by 2. And this side it will be minus EI d 2 y dx square equal to minus Px by 2 plus Px minus l by 2. So, this side is more or less same thing, only this part is additional term. Now, if we integrate, so it will be EI Px square by 4 plus C 1, and this side will be EI dy by dx equal to minus Px square by 4 plus Px minus ly 2 square divided by 2 plus say d 1.

Again if we integrate, so it will be EI into y minus Px cube divided by 12 C 1 x plus C 2 and here it will be Px cube 12 plus P. 6. So, it is d 1 x plus d 2. Can you read d 2? Or it is going beyond limit. Now, it is more or less similar type of expression. Here it is up to that; here this plus some quantity. And this region we are getting C 1 x C 2 and here we are getting d 1 x d 2.

There might be another segment; it should be, say, C, D, E, F, G, some sets of constants. So, how we will get all these constants? Now, here the deformed shape will be something like this. And if you take the load point, under the load, the deflection if you calculate from the left span, and the deflection if you calculate from the right span, they are suppose to be equal; it cannot be different deflection, because it is a continuous structure. Similarly, the slope, if you calculate from the left equation - means equation for the left part - and the slope if you calculate from the equation for the right part, it should be identical, because slope should be identical; there should be not be any ((kink)) here.

So, there might be a number of span, but in between all those interface point, where there is any new load, your expression will change, your continuity of slope and deflection, we can use; those will give the additional information like boundary conditions.

Now, if we utilize those say at x equal to l by 2, your theta from this side and theta from that side should be equal. So, we can say theta at C from left, it should be equal to theta at C from right. So, if you calculate from the left side equation, if you calculate theta from the right side equation, at C it should be equal. So, that is the equation.

So, if you write that expression, it will be, say here, it will be minus pl square divided by 4 into 4 into 4. So, it will l by 2, means 2 2 4 plus C 1, that should be equal to. Here, it will be again pl square divided by 4 into 4 plus… This quantity will be how much? This quantity, incidentally it will be equal to 0, because x equal to l by 2 l by 2 l by 2 0 plus d 1. But that should be divided by your EI, because this is slope into EI; both the side it will be divided by EI. So, you can put it, again cancel it. So, basically this quantity will be equal to this quantity. Now this pl square by 4 into 4 and pl square 4 by 4, it is identical. And it is C 1; this is 0 D 1. So, from here we are getting your D 1 equal to C 1.

So, our equation is different, and we are getting different sets of unknowns, but the slope continuity at that point where we are changing our equation, that will give the D 1 is equal to C 1. So, first constant in this equation will be identical to that equation on the second equation.

In a similar manner, if we just equate the y, so at x equal to l by 2, so this quantity and this quantity if we equate, we will get D 2 will be equal to C 2. So, you can put and see here, I think.

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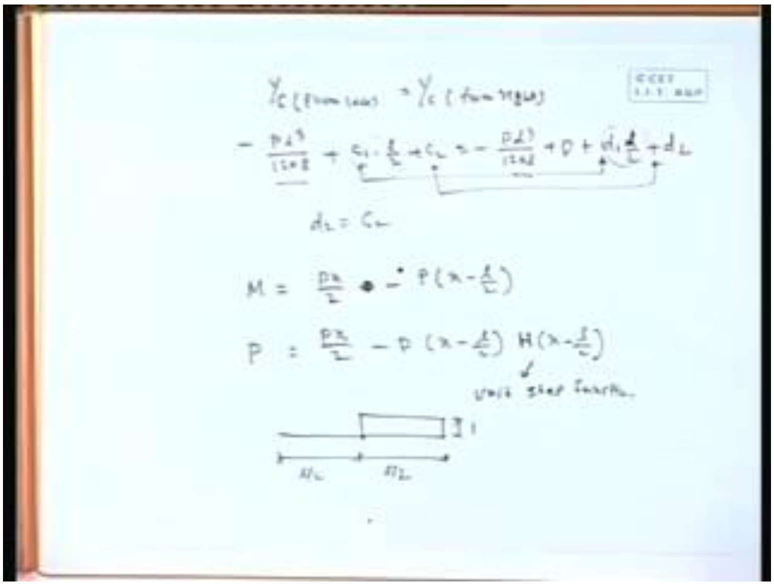

Y at c, it is from left YC from right. So, here we have the equation it will be minus pl cube divided by 12 into 8 plus C 1 l by 2 plus C 2; it will be equal to minus pl cube 12 into 8. Incidentally, that term will be equal to 0; the additional term what you will get at that point plus D 1 already we have obtained it is equal to your C 1 it is l by 2 plus D 2.

Now, this C 1 and D 1 already we have established. So, this term should be equal to this term; and this term is equal to this term; it is 0. So, these two terms should be equal to 0. So, we are getting D 2 equal to C 2.

The main reason is I shall show you again this page - previous page - the additional term x minus l by 2 - this term - will be after l by 2, but at l by 2 exactly the value is 0. This moment, this load will give some moment on the right side, but it will start from 0 value. So, when we are putting the boundary condition, that part is becoming 0 whatever additional term we are getting it. So, ultimately it is becoming identical constant within the equation. So, there might be say two segments like this type of problem; there might be three segments or four segments; only first segment will have some expression; second segment will have expression plus some additional; term third one is the expression for the previous zone plus some additional term. So, same thing will be there plus some one-one additional component plus some constants C 1 plus C 1 x plus C 2 D 1 x plus D 2 like that it will continue, but from the continuity condition all the constants are identical.

So, it is this C 1, D 1, E 1 it will be all identical; similarly, C 2, D 2, E 2, F 2 it will be all identical. Now, from there we can think in that manner, we can take the expression of the last span. So, last span it will retain all the quantity of the previous one with some additional, additional component. And the constant whatever we will get, because it will be identical part all the thing, so it can be rewritten in this form; say, this particular beam problem, we can write, say, M we can write in this manner. So, for the left span, we got it was M equal to Px by 2, and the right span we got it was minus Px minus l by 2. So, it is minus Pxl minus 2 and Px by 2.

Which one?

Student: Extreme reaction.

Extreme reaction it will not come in the second part.

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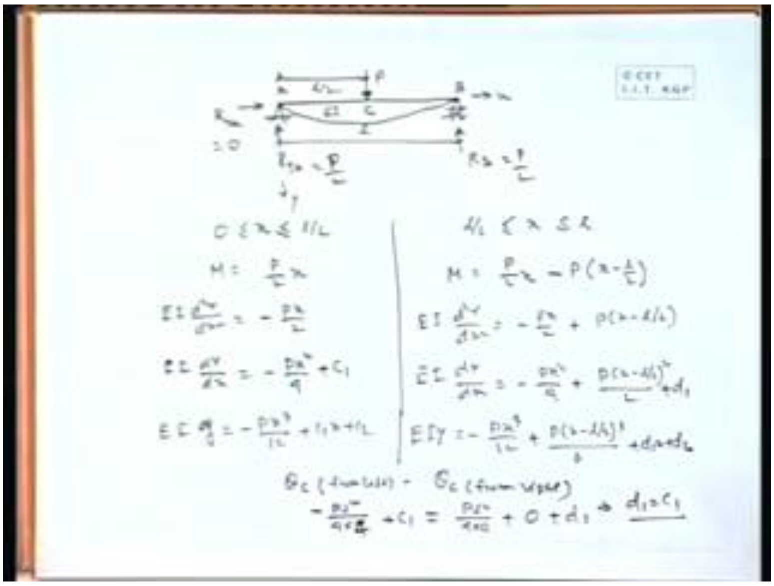

Say here; here to here M equal to P by 2 into x; and here to here it is, say, this force into x that part is there, minus P into x minus l by 2, because here if it is x, so l by 2 minus that is the distance P l minus x P; that part it will give a hogging moment, and this part will give some sagging moment.

Yes, because these two force will give some moment. That moment will be balanced by Rbh. So, if you come from Rb, whatever moment you will get, if you come from this side you will get the same moment. So, either you have to take from this side or that side, I think; both the thing you cannot take that, then it will be cancelling it. So, the main part is, this is the expression for the left part; this is the expression for the right part, where right part is involving one additional component.

Now, this expression, whatever we have written, so we can say this is for left part; and if we take the full component - this plus something, it will be the right part. Now, this equation can be written for the entire structure in that manner. So, whatever we are talking in language, it can be written in some equation form.

So, same expression we are telling when it is left span it is only this one, and right part, it is some additional part. Now, this H l minus x - this is called unit step function, and its value is something like this. Here it is 1 unit; it is l by 2; it is l by 2. So, up to it is 0, then it will be 1. The reason is very simple. The function is 0, when it is less than l by 2. So, when it is beyond the load, it is 0; when it is beyond the load, means it is before the load. When it is after the load the value is 1. So, here also when it is before the load, only this term; when it is after that load, this plus this quantity. So, that physical logic we are trying to put in a mathematical form with some function called unit step function.

So, it will give a general expression for bending moment for the entire span, because both the section here constant, will be identical. I think we can stop here at this class; next class we can continue on.

Preview of the Next Lecture

Most specifically it is deflection of beam the method we used it is basically integration of differential equation we try to handle some cantilever type of problem, then switch over to a simply supported case with uniformly distributed load point loaded case we have taken very standard type of case load is acting at the centre of the beam up to the certain extend we solve it. So, remaining part we can continue. So, last class we got more or less something like this.

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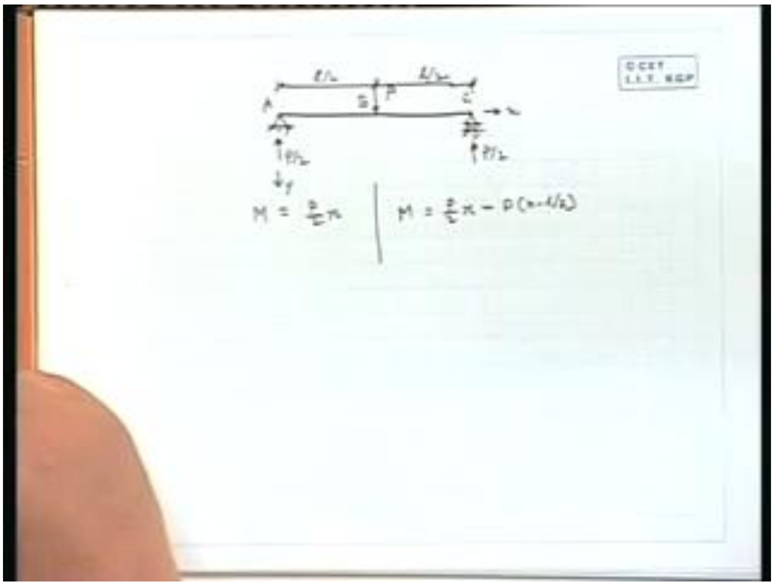

So, we took a beam like this this side some support here also,me support and centre we put P reaction we got P by 2 reaction P by 2 if we say this side is x that side this is y.

Now, the main difficulty what we were discussing in the last class it has 2 regions if we say it is A and if it is B and if it is C. So, A to B are there is 1 segment or a part where your moment has some expression and the second part B to C it has a different expression for the bending moment expression. So, these two parts more or less we have tried to express in this manner say M that we have written P by 2 into x for the left part for the right part it was P by 2 into x minus P xl by 2 right.

So, that we have written in the last class because here only 1 load will generate some moment. So, P by 2 into that distance. So, if x is beyond that limit here the total length is l and we have divided into 2 part l by 2 l by 2, I think. So, that distance we have written earlier it was say l by 2 l by 2 now once we cross l by 2 the centre load P will contribute something in bending moment. So, that was P into l minus we got x minus l by 2. Now, after integration we got 2 sets of constant here and here left side say C 1 C 2 right side d 1 d 2.