Strength and Vibration of Marine Structures

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Lecture - 34
Method of Plastic Analysis

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Take it.

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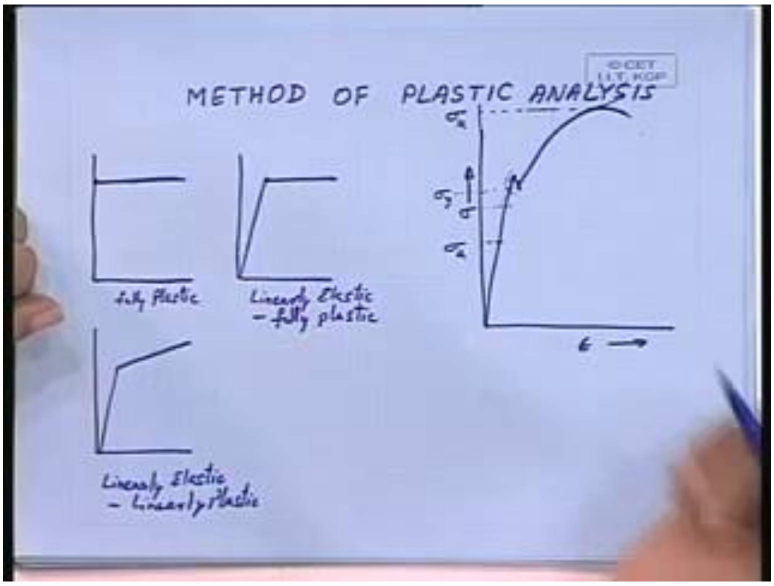

Usefully we try to analyze our structures based on a stress strain diagram like this or stress displacement diagram, and this is a typical curve. Nature of the curve is typical for Steel in which we say that the, as you start applying the load, the stress starts from zero and more or less it goes linearly, up to a point known as the yield point stress. Then there is a small jerk here and then again it goes in non-linear way up, and then it follows a downward trend, basically the ((Refer Time: 01:59)).

This is a typical nature of Steel. In other material you will not find this, this part, and for Steel we say this is the lower yield point and this is the sorry this is the upper yield point and this is the lower yield point. And where it is supposed to reach the maximum is known as ultimate strength. In fact, beyond that the curve goes in this direction, but just before breaking, the cross section reduces and we compare the strain on the basis of the original area, whereas the area here starts shrinking. And therefore, the stress should increase, but because we are taking original area, it comes down.

Most of our structure which we which we design, we try to get a stress value which is in the linear part so that when you load it and then try to unload it, it follows the same path, and that is what we say that the material follows Hooke’s law - stress is proportional to the strain which was given by Hooke.

And we limit our working stress to something known as allowable stress by taking a factor of safety on to the material stress, where we take sigma y to be the maximum, and therefore, sigma y by some allowance which we say is the factor of safety gives you what is sigma A. Now suppose, instead of sigma y and divide it by factor of safety, if I take sigma ultimate and divide it by factor of safety, then also I may get a stress, which I can say allowable stress, still within the elastic limit. And one can question that what is wrong in working with that definition of factor of safety or with that allowable stress because that gives you a larger limit and you can strain the structure or stress the structure to its fullest capacity basically.

In fact, the trend now is that many of the components specially in ships where the stresses are supposed to reach a higher stress value only in some accidental way or once, or from the probability point of union case of emergency like flooding and some sort of a crash; otherwise, in usual lifetime it is not expected to withstand that load. In that case, one can think that if we take a larger value or base of a design on the basis of ultimate strength of the material, then the structure will be still efficient and will be of less weight. So, this stress strain diagram, one can conceive it in a simplified manner in the following way.

One can consider it to be fully plastic; that means there is no elasticity. You keep stressing the thing there is no deformation. And when the material becomes plastic, it goes like this; that means this part of the curve I have taken it back and this non-linear part I have made it flat. So, that is one assumption; so, fully plastic. Otherwise you follow this nature. It is elastic as you get it and once you come to this yield point stress, we assume it to be fully elastic. So, we say linearly elastic and fully plastic. And otherwise we can consider that all this is simplified to this point here and from here to here instead of going to there, you simply join it by a straight line. In that case, we get a line or the curve which gets modified to like this; linearly elastic and linearly plastic.

In fact, this type of material one uses in concrete. You assume that the concrete will not deform till it comes to some sort of a breaking limit, and we want to keep some as reserve and we will not consider it to be linearly elastic, linearly plastic. So, we are left with only one choice that we assume that the material is linearly elastic and fully plastic. So, we will concentrate on this and on this basis will try to see what happens.

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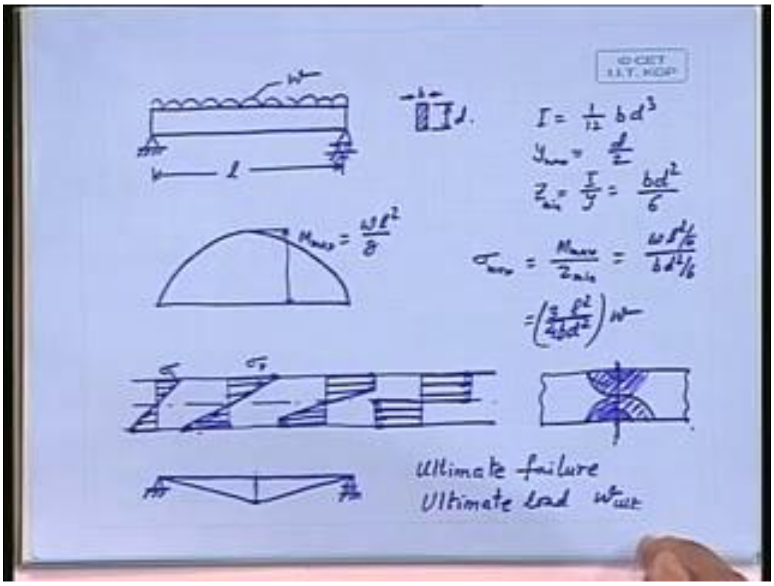

So, if you consider that the material is linearly elastic fully plastic, then let us try to examine a very simple beam and say that how it works in that condition. Say, I take a beam, a simply supported beam, and that beam is also a very simple. That means the cross section I am assuming is just a rectangle and I also assume some uniformly distributed load. w per unit length and we can calculate what is I. I is equal to 1 by 12 bd cube and y max is equal to d by 2. Therefore, Z which is I by y is equal to this divided by this gives you Bd square by 6.

Then, we can also draw what is the bending moment curve and the bending moment will be maximum at the center; I suppose this is the center and this value is M max which is equal to wl square by 8. So, we can write that sigma max, which will be at this particular section here, will be M max by Z minimum, should write Z minimum, which is equal to wl square by 8 divided by bd square by 6 or 3 by 4 l square by d square bd square. So, sigma max is a function of, a linear function of w here; the rate of loading . So, let us see at this stage what happens with the stress.

So, let me draw the stress here. Let us say that this is sigma and we have taken sigma is equal to say sigma allowable. Now, I want to load it further; that means, I can still load it here and this sigma will keep on increasing.

So, a stage comes when according to this diagram, say I have taken some value sigma a here. So, let me say this is sigma a. Now I keep on increasing the load and I go to a stress level which is sigma ultimate or sigma y.

In this case, what happens to this part of the beam? Let me draw the beam here also. There is the section under consideration. So, at this stage, the stress reaches at the extreme fiber to a value sigma y and below that all over the stress is less than sigma y and the entire section is still within the elastic limit.

Now, I continue increasing the load. I further increase and therefore, this stress will further increase. But according to my this diagram which I have assumed for the material, it cannot increase beyond sigma y. So, it remains at sigma y here, but the yield point stress will penetrate down below. That means it was extreme fiber which was plastic. Now, it has penetrated down to some level and all this has become plastic region, having the same stress value of sigma y, whereas, this core is still elastic, and I can still imagine that this particular core can further take the load.

And if I further increase the load here, then what happens? This penetration is full and the stress pattern or the stress variation becomes like this; that means, this has gone right up to here and the entire section has become plastic now; that means, at this section not even a single layer or single fiber is in the elastic region. So, once this becomes totally plastic, then it cannot take further load and under the transverse load given here, like this, it will simply try to collapse in this fashion. So, if I draw a line here, line representation of this particular diagram, then at this point, it will simply go down under the effect of all this because these region are still elastic acting like a rod and this section has become fully plastic. And you have to give usual supports which are hinges and this will also act like a hinge, and if you are supporting it with 3 hinges, this is likely to collapse.

Student: But what if there any difference in the compressive strain failure to the tensile strain failure?

We are assuming it isotropic material. In Steel we considered the same thing; otherwise we will not get the same value here. So, that what happens? It will become fully, you see, actually if you try to have 2 hinges here, supported and loaded, it will simply go down like that. So, if it goes down like that, then we say that this structure has now ultimately failed. So, we term it also ultimate failure. And corresponding to this ultimate failure, whatever load was here, we say that is the ultimate load. And in this particular case, it is the rate of intensity of this loading, but we use a suffix ult here, w ultimate. Right now, with this type of load distribution sorry stress distribution, now what is the moment here?

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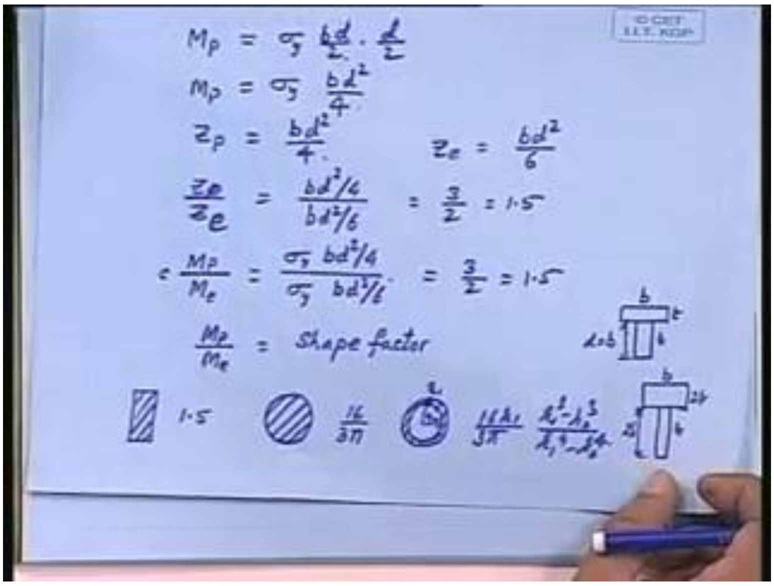

So, let me write in a very simple manner. Mp is equal to what? Plastic moment; this has become the complete plastic. So, what is the moment and the strain just before collapse? This is sigma y all through; this is also sigma y all through. So, sigma y into d by 2 on this side into b; b into d is the total area; half the area is under tension and half that area is under compression. So, the force is sigma y b into d by 2 and this resultant force is acting at the distance d by 4 from here. The other one is also acting at d by four. So, the couple lever is d by 2 because this force is here and this force is here; distance d by 2 apart. So, this is equal to sigma y and I say bd square by 4.

So, what is Z plastic? Z plastic works out to be (Refer Slide Time: 19:38) and we had already calculated what is Z elastic here. If we call it Z elastic, when Z elastic is ((Refer Time: 19:52)). So, this we are getting from the geometry of the structure and the stress, corresponding stress distribution. Now, if you take the ratio of this, it works out to be sorry Z plastic by Z elastic. Or if we say what is the ratio between plastic moment by elastic moment, then plastic moment is equal to this. And what is elastic moment maximum when the first fiber becomes yield point and the entire section is fully elastic? So, that will be, from here you can get, is it not? I by y is equal to stress is equal to M by Z or M elastic is equal to Z into sigma y. So, that Z is elastic Z here.

So, if you take the ratio of plastic moment through the moment when the extreme fibers becomes just yield point stress and the entire cross section is elastic, this ratio is same as the ratio of the corresponding section moduli defined as in the plastic condition and the elastic condition. This is known as, let me write here once again known as, the shape factor because the entire thing is based on the shape here. It was rectangular. So, we got it like that. So, for a rectangle, shape factor is 1.5; if you take a solid circle, the shape factor is 16 by 3 pi; if you take a hollow pipe as a beam with R 1 and R 2 two radii, this is R 1 R 2 and outer one is R 1, then this works out to be 16 R 1 by pi 3 pi R 1 cube minus R 2 cube divided by R 1 to the power 4, and other shapes also one can find out.

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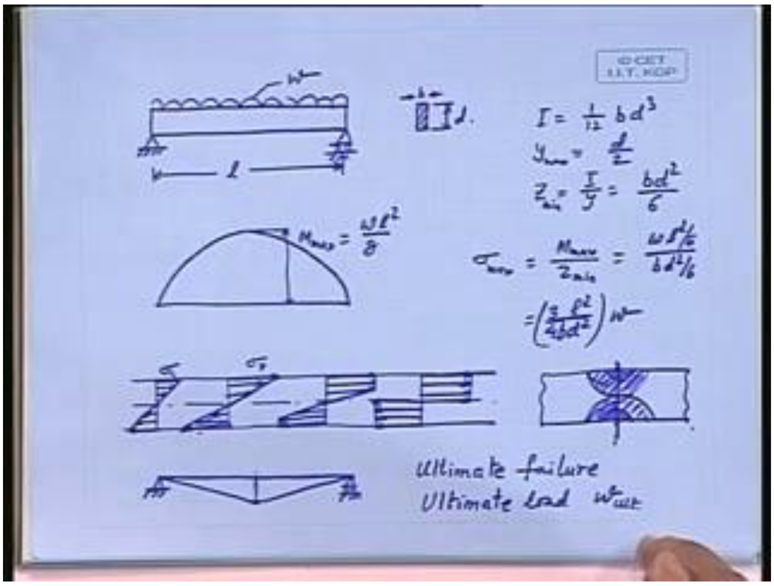

Now, for the section to be in balance, what we can notice here, that the total tensile force is equal to the total compressive force. And in this particular case what will happen is that because the entire stress is of the same value, it will simply divide the section into two equal areas. Unlike in elastic case where the variation is triangular, then we say that the flanges take more forces than this because they are away and so on and so forth because the flanges will be stressed more than the webs and so on and so forth. So, if you give a bigger flange, it will be better. So, there, it is it is not the cross sectional area which is divided into equal parts, it is the force which is getting divided into equal parts.

Here also, it is the force which is getting divided into equal part, but because the stress variation is or the stress variation is not there, it is the same uniform stress we are considering; that ultimately divides the cross sectional area into two equivalent areas. . So, if we try that and say what is the shape factor, then the value will be in a different fashion. So, the section modulus of the cross section in the plastic condition has to be calculated on the basis of how you are dividing the two areas, the cross sectional area into two parts.

You can try on your own a T section where you can say that, this you can take as b, you can take this as t and this also you take as thickness as t, and this depth d is equal to b and find out what is the shape factor of this cross section. Another section you can take in a similar fashion and find out what is this. This is b; this is 2t; this is 2b; this is t. Web is t thickness and twice the width.

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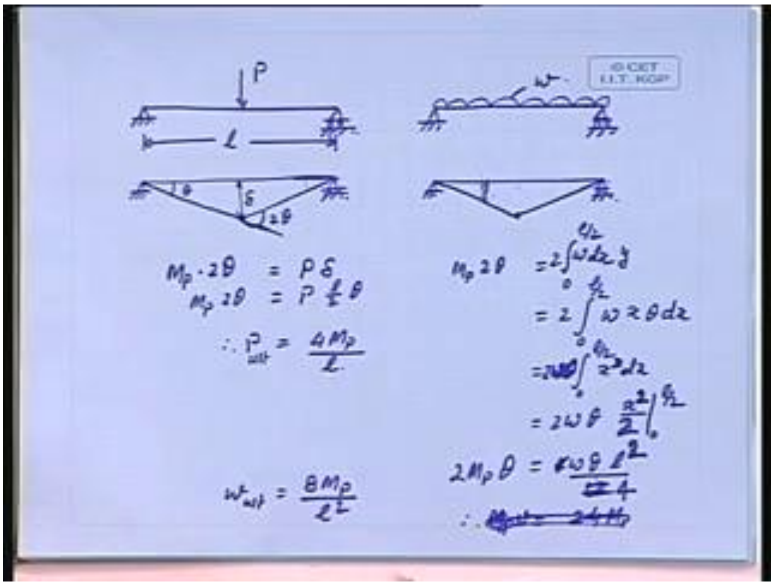

Now, let us try to solve some problems and see that how we try to say it. Say you are having a beam here; just the first one in a simplified manner. We will see that. You take a centrally loaded load P. Consider this to be l and we have to find out the ultimate value of P in terms of MP. So, this will have a mechanism which we have seen that under this load the maximum bending moment will occur and the bending moment diagram we know; it will be a triangle.

So, let us assume that the first hinge is formed here and first plastic hinge is going to lead it to the failure here. This is already a support. This is already a pin support. And under the load as soon as the hinge is formed, it will simply collapse. So, let us say that under the load, this displacement is delta and this angle is theta. So, this is also being theta from equilateral triangle, isosceles triangle, and this will become 2 theta here. So, what is the internal work done here? Internal work done is this plastic moment is turning the thing by 2 theta. So, you have MP into 2 theta and what is the external work? External work is this load P is moving down up to delta.

So, they must be the internal work done must be equal to the external work and that is equal to P into delta. If you convert delta into, after all we are considering small angle; so, delta can be written in terms of theta. So, what is theta?

Student: Delta by l, delta by l into theta.

Delta is equal to.

Student: L l.

L by 2 into theta Yes that is l theta. L is this. L by 2 theta, yes. L by 2 theta. So, l by 2 theta and therefore, your P works out to be both sides theta can be canceled out, and P is equal to ((Refer Time: 31:01)) and this we will say as ultimate. So, the value of P ultimate is 4 MP by l.

Now, if suppose we want to analyze our example, here we say that the load is like this; this will also fail in the same mechanism because now the bending moment curve is a parabolic curve and the maximum value is at the center and the failure mechanism will be same. So, it fails in the same fashion. So, on this side, we still have MP into 2 theta. But on this side, what will you have? This intensity of loading is there. So, this is moving by say, you consider a small load here at dx; so, w into dx. If you consider this 0, this is x; this is dx; so, w into dx, and if we consider this to be at displacement y, this is to be integrated from 0 to l by 2, and the same thing will hold good here; so, multiply it by 2. Now, what is this y? y is equal to X theta.

So, this will be (Refer Slide Time: 33:40) am I right? So, now

Student: But sir why cannot you find just the deflection at the center like what you have done here.

Here the load is distributed. So, every component of this load is going to do the work. It is not at the center.

Student: ((Refer Time: 34:11))

Here it is a distributed load. So, each small component will have this displacement.

Student: Directly whole load can be from the center it is acting?

No, no. It is not equivalent to that.

Student: But uniform load sir

Uniform load

Student: A triangular change in area of parabola, how can you find out? Here it is easy [FL]

If it were like that, then you should have got the same bending moment using the same analogy. Why you are getting a parabolic distribution and why are you getting wl square by 8? wl is equal to total load P; then it should be Pl by 4. Why you are getting Pl by 8?

So, now let us see what happens here. x square dx and w theta we can take out. So, I have not made any provision. w theta, constant, I have taken it out and now 2 w theta and this gives you a x cube by 3; that makes it l cube. So, l cube by 8; 8 means 24. Therefore, MP is equal to 24.

Student: 24.

Sorry not MP 24. w is equal to w is equal to 24 MP by

Student: Sir, how that becomes x square sir?

Yeah, yeah. That is the mistake. It should be x dx. So, it should be x square by 2 and x square by 2 means it is l square 4 2s are 8 and So it is 4. 4 2s are 8. So, w ultimate is equal to 8 MP now. Or if you want to convert it, So, w ultimate into l is equal to P ultimate total load. As you are saying just for comparison sake, then it becomes 8 MP by l; just twice this value.

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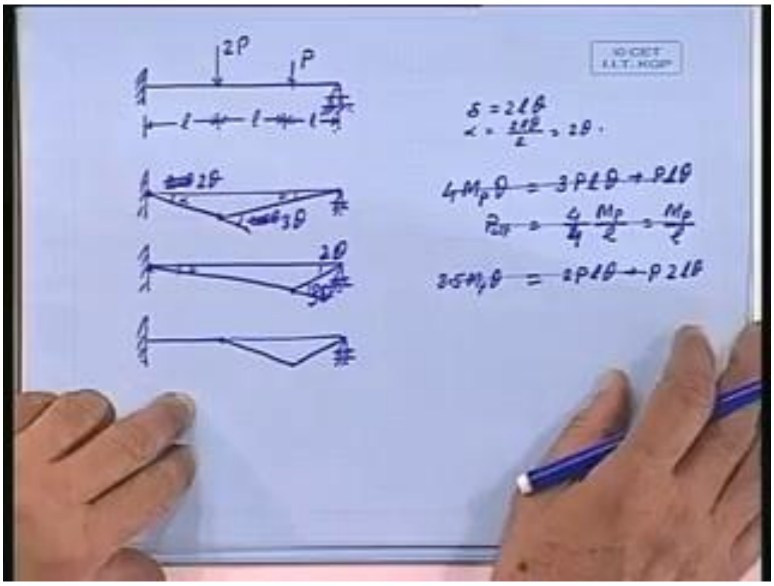

Now, let us take another example. Say we are taking a cantilever sort of a beam, blocked, and I have 2 loads here, and let me make this span in this fashion so that I can have a simple calculation; l, l, l, and this. Now, we have seen that 3 hinges are required for the collapse mechanism and the way it collapses is known as the mechanism. Now, in this case, we can start from here. There is a physical hinge provided here. We also know that, under the load the maximum bending moment can occur. So, this is a physical support where the fixed end moment will be there. So, this can turn to be a hinge. Under the load excessive bending moment will be there; so, this can also turn to be a hinge; this can also turn to be a hinge. This is the physical hinge. Now, we have got this combination here and let us see that what are the combinations we can have.

Let me first assume that this support becomes a hinge and this load also becomes a hinge, and this is a support; so, this is a hinge. So, I can have one mechanism like this. One support under load and physical hinge; the second one I can consider that this is becoming a hinge and the second load is becoming a hinge, and this support is there. Third I can say that let these two remain intact. Under the 2 loads, I can have 2 hinges. So, under the 2 loads if I make hinges here, so, I can find out 3 possible mechanisms under which the collapse of this beam can take place.

Student: ((Refer Time: 40:13))

One is under this load, another under this load, and this remaining intact. So, 2 plastic hinges, one physical hinge; 2 plastic hinges, one physical hinge; 2 plastic hinges and one physical hinge.

Let us say that these are the 3 possibilities for the failure mechanism. Which one will be the failure mechanism ultimately? As you keep on increasing the load, the one under which the actual failure will take place is when the load is minimum. As you keep on increasing, a failure mechanism will occur first. What is that? That is the least value of these because if we increase, may be then the second mechanism will take place. But before the second mechanism takes place, it has already failed in the first, and therefore, that is the mechanism under which it is going to fail. So, that is how you keep on gradually increasing the load. And then the mechanism under which the structure will fail first, that is the mechanism you have to accept; not that because all these 3 mechanisms will give you some numerical value.

So, let us calculate the failure load under each of these mechanisms, and the P ultimate which is the minimum under the 3 is the mechanism for the actual failure. So, what is the load here or what is the value? So, this if you consider this to be say theta here, what is this value?

Student: Theta by 2.

That will be 3 theta by 2. This is 2 and this is 1. 1.5 theta and how much is this? This plus this is equal to this angle. You check up. I suppose I am doing it correctly.

Student: Sir, this third diagram I am little getting confused; this third diagram.

Say it like this. This is clamped. This is a physical hinge. This is the collapse mechanism. A hinge is formed here, a hinge is formed here, and this is the physical hinge available.

Student: Why will we not take this hinge?

Which hinge? This first one, that you are calculating know, this is fixed here know.

Student: After that sir it is here.

No, no. For failure you require 3, 3 hinges to be formed.

Student: So, each one we are considering?

We are considering one at a time here. Is this okay?.

Student: Yes.

So, if this is okay, then what is the work done here? 1.5 plus 2.; So, 4 MP theta. And how much is this? This is 2P into delta, and delta is equal to 1.5 theta into l. So, this becomes 3. So, P ultimate is equal to (Refer Slide Time: 44:18). Next let us take, here let us take this to be theta; then this will be 1.5 theta and this will be 2.5 theta. So, if this is theta, this is 2.5 theta, then the work done is 3.5 MP theta.

And how much is the work done by the load? Oh my God! I have done a mistake here. Plus plus plus; this is also going down; this will go down to half the level here. If this goes down to delta l, this will go down delta l, delta by 2. Now, if I consider this to be delta, then this displacement is delta by 2. So, delta by 2 will give you this. This angle I have considered as theta.

Student: Theta we can write theta l.

We can write theta l for that. So, it is Pl theta and this will be added up to 4 by 4. In this case, this is the load; this point is coming down how much? 2 P into l theta; this is theta; this is 3.5; this is 1.5. So, 2 Pl theta and plus this point is coming down to l theta.

Student: Theta value.

That is what I am having a doubt here whether this angle is correct or wrong. Theta is equal to let me just calculate here. No. This is 2 theta; 2 theta, theta, and this is 3 theta. Once again, theta, 2 theta, this is 3 theta. So, all this calculation is going wrong here. Let me do this calculation once again.

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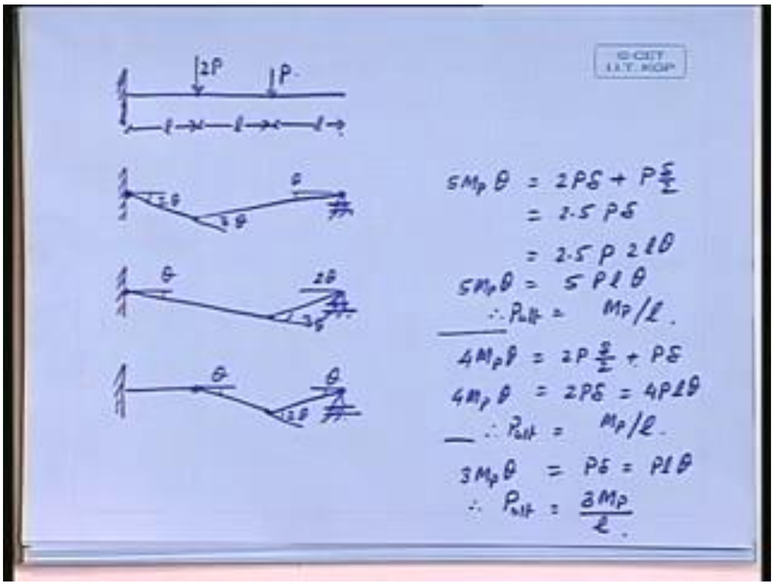

Mechanism one.

Student: Sir, on what basis we keep these 2 theta sir?

If this is delta, then what is this angle?

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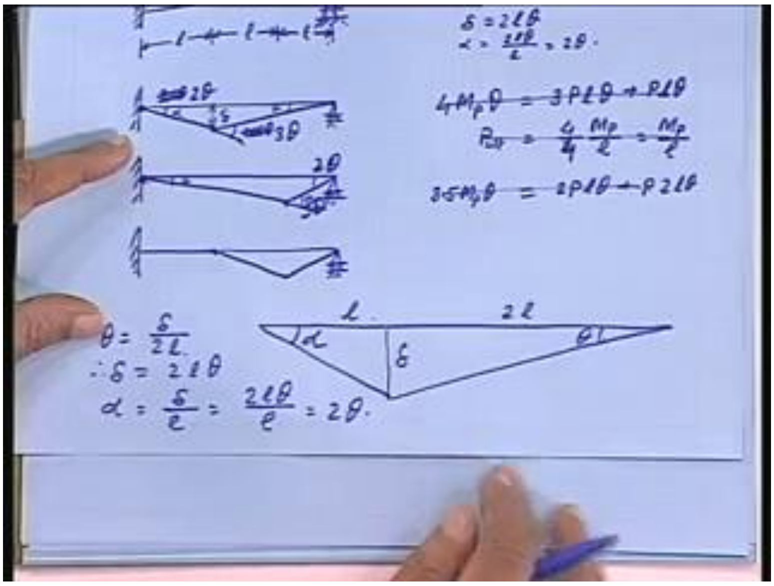

Say, this angle is alpha; this angle is theta. Let me say this is delta and this is 2 l and this is l. So, what is theta? Theta is equal to delta by 2l. Therefore, we can say delta is equal to 2 l theta. And what is alpha? ((Refer Time: 50:15)).

So, this is the relationship. So, 3 plus 2, 5 MP theta is equal to 2 delta 2P delta plus P into delta by 2 which is equal to 2.5 P delta, and delta is equal to l 2 theta or 2 l theta. Therefore, P ultimate is equal to MP by l. In this case what happens? In this case also it is theta plus 3 theta; so, 4 MP theta. This is case 1. Case 2 is 4 MP theta is equal to this load is coming down by half delta. So, 2 P and delta by 2 and plus this load is coming back, coming down. So, it is 2 is 1 1 and 1 2 and this delta you can write it as ((Refer Time: 52:51)).

So, this is a funny situation you are going to get and therefore, P ultimate here is equal to last one. Last one, this, this load is not moving at all; only this load is moving, but the work done by this hinge and this hinge is 3 MP theta; 1 MP here, 2 theta here, and this load is moving down. So, it is P into delta and which is equal to Pl theta.

So, now you see, this has got MP by l, this has got MP by l, and this mechanism has got 3 MP by l. So, the last mechanism is out of question. It can be either this mechanism or this mechanism because both have got the same ultimate load. And it is only the imperfection in the structure or slight imbalance in the load, you say it is 2 P or this is P, or there is a small difference in the time of loading the quantity, it can take either of this mechanism for failure.

So, this is how the beam problems are solved and finding out what is the MP from section modulus, one can try to design. And based on this, one finds that as we can see from the rectangular section that your shape factor is 1.5. In fact, you are getting some 50 percent additional strength there.

For other sections also, the T section which I have given you, you will find that some 15 percent or 30 percent you will get extra there. So, depending on the shape you get an enhancement. And if you base your design on the basis of this elastic theory, though we have assumed that it is fully elastic and fully plastic, that means, in the plastic region we have assumed that once it comes to the yield point stress, it stays there, whereas in true picture, even if you modify it, it is linearly elastic and linearly plastic. It is basically non-linearly plastic, but even if you consider it to be linearly plastic, then also though you are underestimating, but still you are having some additional strength there.

And this way, if you try to design some structures, then you are really trying to save some load, some material, and make a lighter vessel. So, this what I wanted to cover here.

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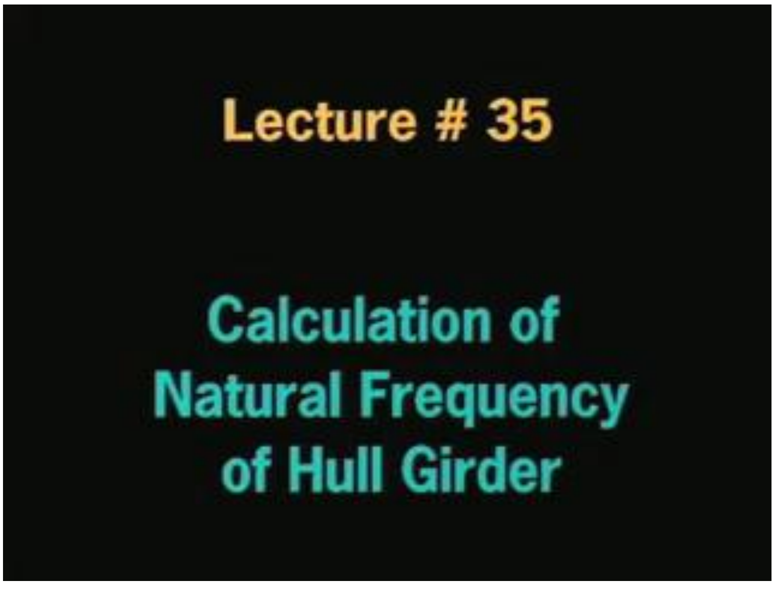

Now, it is okay. So, let us start this coefficient of coefficient for added mass, vertical vibration C v.

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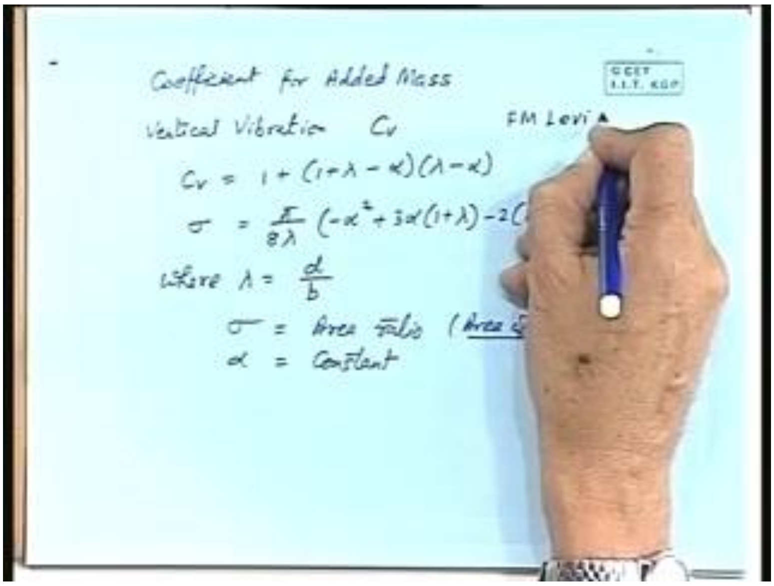

So, this is the coefficient and Levis has given this expression Cv is equal to 1 plus 1 plus lambda minus alpha into lambda minus alpha. I do not know how he got all these expressions, and then he says in which sigma is some another coefficient involved 8 lambda minus alpha square plus 3 alpha 1 plus lambda, where lambda is equal to d by b. Sigma is the area ratio and alpha is constant as defined here. Now, this area ratio is area of submerged section divided by 2 bd. This was given by FM Levis or Levis.