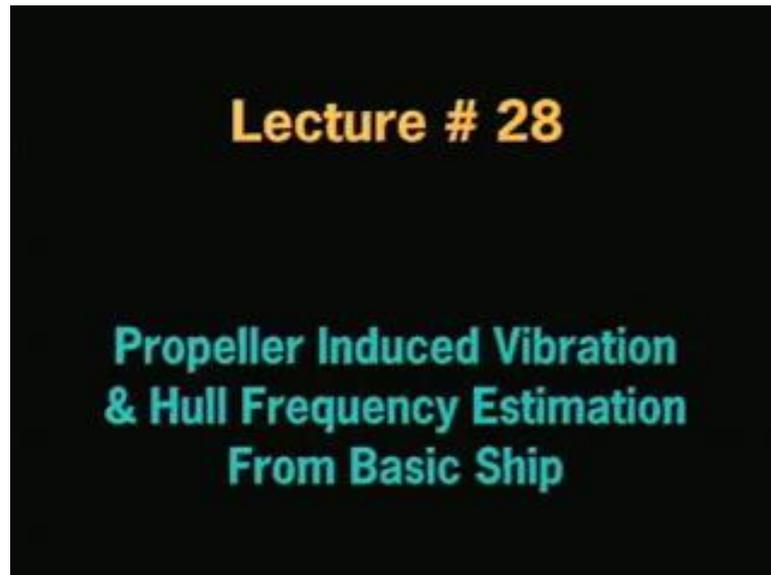


Strength and Vibration of Marine Structures
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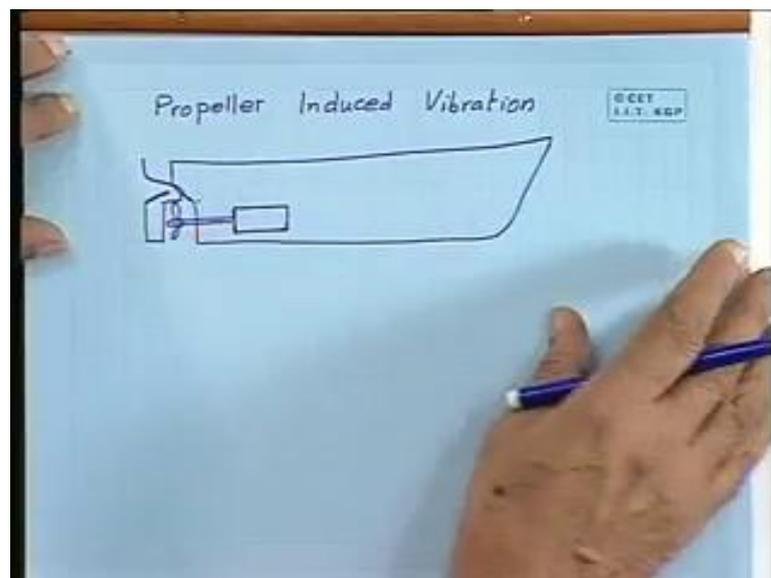
Lecture - 28
Propeller Induced Vibration and Hull Frequency Estimation from Basic Ship

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Let us start now. Should we talk something about propeller induced vibration?

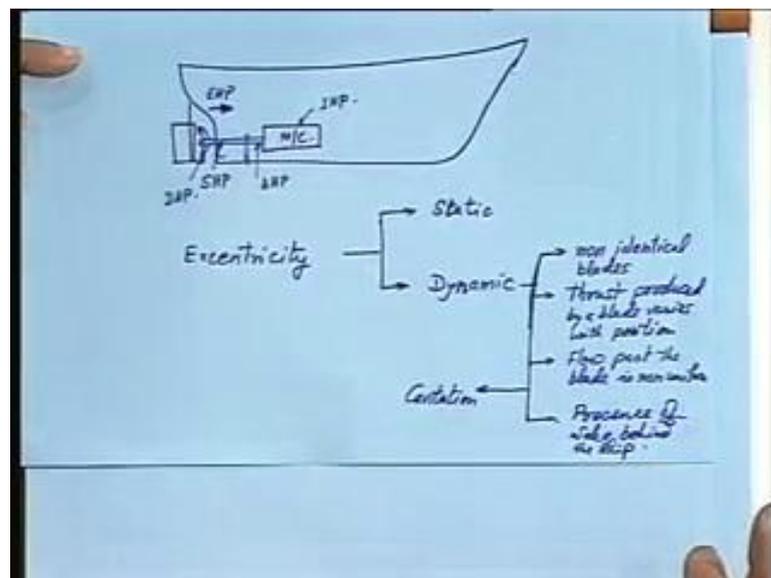
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The main prime over is a propeller in a ship and from maintenance point of view, we always try to select a single propeller and therefore, a lot of power is to be transmitted through one single propeller. Usually we have 4 blades, but there is no restriction on that unless it meets our requirement.

So, we can have a 3 bladed propeller, a 4 bladed propeller or a 5 bladed propeller, but in general, we will find that majority of the single screw shapes are provided with a 4 blade propeller. Therefore, we will find that each propeller blade is transmitting a huge amount of power to the ship or rather when we say the diagram, how the power is transmitted. Let me talk about that also, and I should have drawn as somewhere here I think I will just change the diagram.

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I am not trying to draw the bulkhead, double bottom, etcetera, just machinery is there, thrust block is there, and shaft is there. Now, when you try to measure the power here, it is indicated horsepower. We say then you try the measure the power here, we say BHP British. Not British, but break horsepower and then when you try to measure the power here, it is the shaft horsepower and at this place we say delivered horsepower and then what makes it move is the effective horsepower. So, from here onwards, ultimately when we reach from here to here, there are may be 50 percent losses and we try to move the vessel.

So, a lot of power is being delivered here, and in the process each system will try to induce some sort of an excitation. So, we talked about this because the number of cylinders and the eccentric crankshaft, you can have some sort of an excitation from here, so that excitation may be the number of cylinder into number of rpm because for each revolution of the crankshaft, all the cylinders will be fired one after another if it is a two stroke engine. If it is a four stroke engine, then every two revolution there will be one firing per cylinder. So, in that case, the engine is going to produce a frequency which is rpm multiplied by the number of cylinders or number of cylinders divided by 2 depending on two strokes or four strokes.

Then, depending on the gearing ratio, that crankshaft rpm multiplied by the gear ratio will be another exciting frequency which the shaft is going to transmit and then with that frequency or that rpm, the shaft is coming here and the propeller is mounted here. So, that shaft rpm multiplied by the number of blades on the propeller is going to give you another exciting frequency. So, these are the main frequencies which will be generated here in this system because of the main machinery. Similar things will hold good for other rotating parts like your gensets, your pumps, or any other rotating part.

So, now we will talk about propeller induced vibration. Now, if the propeller is perfectly balanced and this delivered horse power by each blade is also balanced, then there is no eccentricity and we should not have any propeller induced vibration, but what happens is that each propeller is tailor made or custom made to suit the ship that is manufactured in that fashion. It is not mass produced, but it is a tailor made equipment or a item which is to fit into the particular ship, and if you know how the manufacturing process takes place, it basically caste the propeller and then you are try to grind propeller blade to the given profile.

So, here we introduce eccentricity of two types. One is static and another is dynamic. In the static eccentricity, we say that the mass center of the propeller do not pass through the center line of the propeller shaft there. If the mass center of the propeller is slightly away from geometrical central line of the propeller, then we introduced what is known as the static imbalance in the propeller.

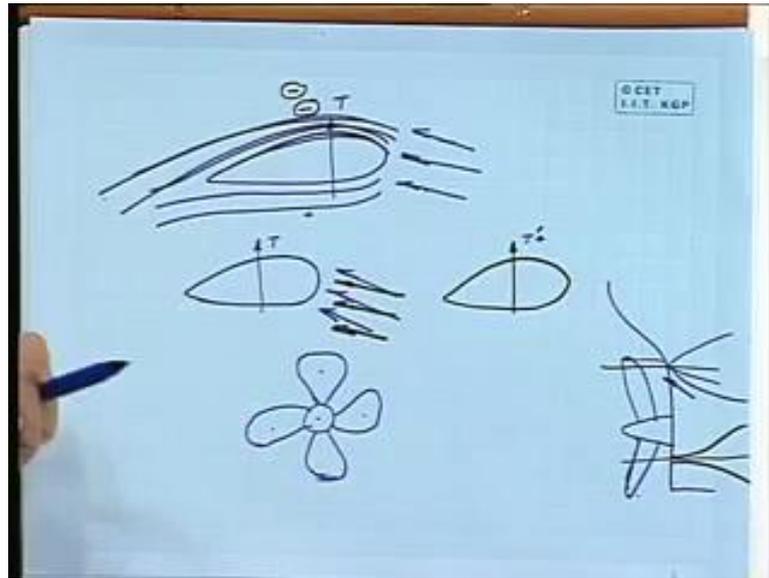
Now, many times this static balancing is done by grinding the propeller blades properly. If some mass is extra here, you try to grind out just like the weights. We try to remove

something and fill it up with some other material, lighter material and try to do that. So, static balancing is normally done, but still there may be some sort of an imbalance remaining there, but dynamic imbalancing is thought to be there due to various reasons. The reasons can be non-identical plates.

See there are three or four blades. Though they are made from the same drawing, same dimensions, but they may not be identical. There may be some difference. Number 2, when it is moving in water, it is developing the thrust and that thrust development property may vary from or the thrust develop by the propeller may vary from position to position. Thrust produced by a blade varies with position, blade position because it is rotating, at sometime it is the top, and sometimes is at the bottom.

Now, there are many reasons for this and we will see that reason and then because the propeller blade is working behind the ship and the shape of the ship in the after end condition is very complex. Therefore the flow pass, the propeller blade is non-uniform. Again one can say that by the flow is non-uniform, there are two things. One is because of the complex shape, the angle of the attack changes. There the entry point or the entry direction of the water to the propeller disk is changing and another thing, because it is moving and viscosity is there, there will be a wake behind the propeller. So, presence of the wake behind the ship and we can also add another part here that sometimes at particular point of blade get overloaded, or the lifting pressure is such, or pressure behind or the negative pressure at the back side of the propeller is such that it falls below the vapor pressure of the fluid or the water which generates some sort of cavity is there. So, it is cavitation of the propeller.

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So, let us see how the propeller blade will we know that the propeller blade is something like this in the airfoil section, and flow pass of blade is at this angle. Therefore, when the flow takes place and goes like this and this side is the negative pressure, this side is the positive pressure and therefore, the resultant works like this and you get the lifting here or thrust in this direction. Now, if the blade profile is slightly changed. I am using a different color to show that. Then, the flow pattern will change here. There will be if this is slightly more here, more negative pressure will be there, so the lifting property changes.

So, suppose one blade is of this section, it is very difficult to draw actually identical blades for me and another one is something like this. Both are looking same, but they are not identical. Then, the lift generated here and lift generated here will be different. So, I have written that thrust generated by this is T , this will be T plus say or T dash, this is the T , this is T dash. T dash can be slightly more or less than this. So, each blade if we consider that the four blades are there and each one of them is generating a different thrust, so the thrust center will be something half of centroidal of it. If these generates T , this generates T , this generates T , this generate T , then all the T together will be considered as generated from the center, but if this is generating T , this is T plus ΔT , this is T plus ΔT and this is also T plus ΔT , then it will be slightly off center.

So, the thrust generating property of each blade under the uniform flow will be such that it is going to give you some sort of eccentricity, right. Therefore, this will try to give you a frequency which is same as the shaft frequency, but as we are talking about the position here, if it is in this position, the flow passed the propeller blade may be like this. If it is in a different position here, may be the flow changes to this direction. Why? It is because the lines here if you draw and if you draw a line here, then this will go like this. So, the water here will be directed in this direction and the water here will be directed in this direction. This is what I am trying to say that this direction is this and this direction is this here.

So, even if the propeller blade is same, depending on the position that the flow pass the propeller blade will change and once the flow passed the propeller blade changes, the lift generated by the propeller or the thrust generated by that particular blade at different position will keep on changing and that will be through with each one of them. So, with the assumption that suppose that all the propeller blades are identically made, each one of them when it comes down or when it makes a complete revolution, its thrust generating property will keep on changing. Does it enter the thrust imparted to the blade when the blade at the top position is much less? We do not know actually whether it is less there. Water flow to the blade is more from the bottom.

No, actually the weight is also there and the flow is really complex. So, it is the combination, say actual situation behind the condition which is a very complex part and people still keep on doing the research to find out how the wake is there and how it effect the propeller performance. Wake is basically back flow because fluid is real flowing. There is a viscosity and there is some tendency that it will follow the ship. So, depending on how the water is churned, there with the propeller action is there, it is churning in a particular direction.

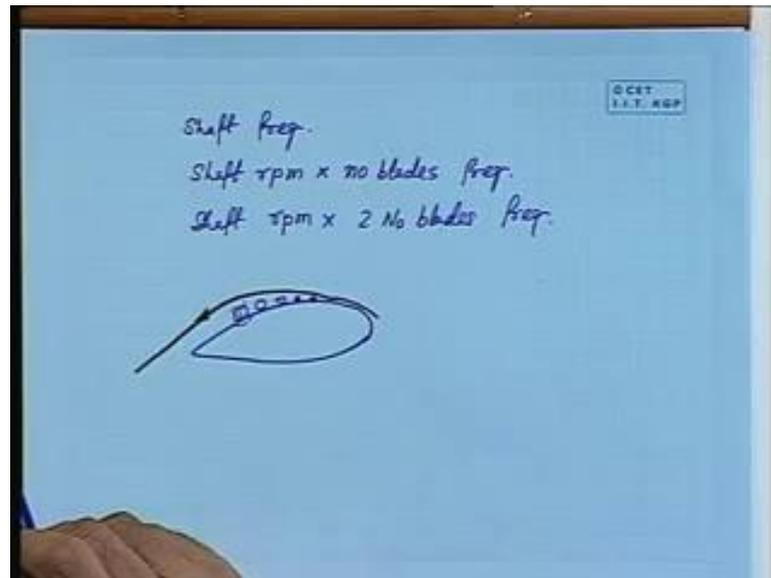
If we are having two propellers, then the churning may be counteracted, but it is the single propeller. It is churning in one direction, the ship is moving in one direction, the water is cross flowing there which is mixed with this churning and therefore, the actual flow pattern behind the ship is too complex. You cannot say at which position the maximum lift will be generated, in which position it may change, say at this instant at this position the maximum thrust is being generated. In the next instant, the ship moves ahead, the flow pattern may change there and it is different position where the propeller

will be generated the maximum thrust. The water flowing to some parts of the blade is that a pastor blade, so automatically. No, but that keeps on getting modified. It will depend upon the girder.

No, suppose the girder angle you have kept it at 0. Still it will change because actually what is the current there, there may be in the sea, there must be some ambient current due to change in water temperature when you are churning the water, hot water from water may come to the top and so on so forth. You do not know. So, what happens is that the thrust generating property of each blade will keep on changing. So, now, in actual case what happens that because they are tailor made, it is non-identical and therefore, because of the non-identical nature if we assume that the flow pass the blade, all the blades are uniform, then also we expect that it will try to excite the hull with the shaft frequency.

Now, the blades are moving and we assume that the blades are non-identical due to some reasons or the other. The flow pass the blade will keep on changing and one will affect the flow on to the other blade. So, this blade is coming here. When it goes away, the other blade is coming here, but it is trilling behind its own effect and that will change the flow pattern to this blade. So, at that position even if the blade is identical, the thrust generating property may still change. So, each one of them will try to generate a frequency of the shaft, but there are four. So, four times the shaft frequency or the blade number of blade times, the shaft frequency will be generated.

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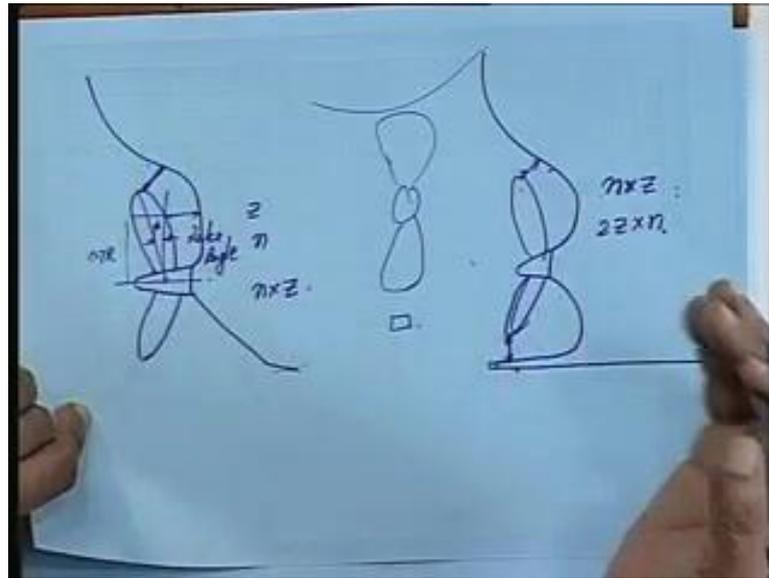
So, because of this reason, you are getting a shaft frequency excitation, you are getting shaft rpm multiplied by number of blades frequency.

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Then let us assume that say I have two types of turns for the ship. One is a clear cut up, and another one having some sort of a sole piece. What we say which supports the radar here and this is the configuration.

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Obviously, this type of arrangement you will find in a small ship, not in large ships. Large ship will have normally this; the smaller ship will have like this. Now, if in this particular case as soon as a particular blade comes closer to the hull, it will try to impart the energy to the hull and excite with that frequency. So, if there are say Z number of blades, Z times the blade will come closer to the hull and shaft rpm is n . So, n into Z times, there will be a frequency generated here. In this case, n times Z will be Z times n will be one frequency, which will be generated by this here.

Now, if there are even numbers of blades, that means, if there are four blades here, rarely we have two blades. If there are four blades and if one comes closer to this part, the next one will go closer to this sole piece here. So, you generate this n times or Z times. The n frequency will be generated by this, but if the number of blades are odd, then the one which comes here, there is no corresponding blade which is coming here, but when it passes, then another blade goes there. So, here you have twice the Z time and frequency.

Suppose, let us take an example that in a tug boat you are having say three bladed propeller. So, if one blade is coming here, the next blade is not coming here. After this blade moves out from here, the next blade goes closer to this and generates some sort of frequency here. When it moves out, the third blade goes pass this. So, in one revolution, three times on this side and three times on this side, so six times it will be generating the

frequency. So, six times the frequency here and if it is even number, then number of blade times the frequency.

So, these two possibilities are there. So, you have shaft rpm, number of blades, frequency, then you have shaft rpm times twice the number of blade frequency. Now, if we want to get the effect less, then there has to be some sort of a minimum distance kept between these tips here and these pieces here you will see that the rules will tell you that at $0.7r$, what should be the minimum clearances given here and what should be the minimum tip clearance given here. Total distance need not be vertical. No, this tip distance and this distance at $0.7r$, this distance and the tip clearance the rules will specify that what should be the minimum permissible, so that it does not.

Of course, the rule will talk about the strength from strength point of view. So, they will say that if you are giving those clearances, then the shock given to after structure will not be much the impact sort of a loading. In fact, that also helps here and even this clearance will also be mentioned because this part is not much here. If you see here the cross section, it will be like this and the blade is moving like that, but here this blade is coming here and this piece is only this much here.

So, the effect here is not much, but the effect here is from here to here. So, these clearances will be declared dictated by the classification society. In the process, what happens is that to maintain the clearance, you cannot have a propeller blade which is straight rather it has to be raked rake angle. Usually some 5 degrees, 6 degrees of rake angle is given. It will be nice because if one can have the propeller to be absolutely straight, so that the stress at the root of the propeller blade is not much because it acts like a cantilever. This is generating a thrust here. Obviously, this is acting like a cantilever.

So, there will be some sort of a bending movement and there will be torsion. So, at the root, a lot of stress is there, but if you try to rake it, then that effect increases and then the torsion is increased and therefore, the root thickness increases. So, from strength point of view, we want like to have any rake, but from vibration point of view to maintain the minimum clearances, you require the rake in the propeller. Then, one has to checkup that you see if the negative pressure goes below the vapor pressure of the water, then you will find that the water starts evaporating and there will be cavities here, and as the water

flows, these bubbles will keep on imploding and suddenly a position will come when the bubble is not in a position to withstand the pressure and it collapses.

So, once it collapses, the water rushes in here and at a particular position, you will find that the pitting takes place. You must have noticed that the propeller blade get splitted and that is because of this reason. If you heavily load the propeller, then also the same thing will happen because the negative pressure here, and the negative side increases. That means it goes below the vapor pressure. In fact, it has been found that basically it is not the water vapor pressure which creates the problem, it is the dissolved gases. When you reduce the pressure there, they try to come out and much before attaining the water pressure here, the vapor pressure here, the bubbles starts forming. So, the dissolved gases, they come in the form of the bubbles. They are also function of the temperature.

So, now as soon as the bubble forms here, you will find that the flow pattern changes say it was suppose to travel like this. Now, because of the presence of this flow pattern changes and those changes, the hydro dynamic lift of the blade and then it collapses. So, there will be some sort of a disturbance of the flow and therefore, some sort of excitation will be generated from the blade here. So, cavitation is one of the forms. Now, in some cases, what happens is when the rpm is too high like in small powered engine the rpm is high and one cannot stop this. In that case we try to design the blade with a cavity here.

So, the cavity itself is a part of the propeller lifting surface, and the flow is such that the cavity is taken into account. So, instead of the bubble cavitation, we try for a sheet cavitation, a continuous cavity and that type of propeller design is known as the fully cavitating propeller. Mostly you will find that the cavitation starts from the tip of the blade. Why from the tip of the blade? It is because the linear velocity there is more. If the linear velocity is more, then the velocity of water with respect to the propeller blade is high there and the lift generated is more. So, it starts with the tip cavitation and therefore, you will find that when you are making the propeller, the propeller blades are flatter towards the tip. So, these are the causes by which propeller tries to induce some sort of vibration, and one has to take into account at the design stage to reduce this effect because once the thing is built, you cannot do it. Therefore, at the design stage itself one has to see.

Now, what should be the propeller output and what diameter? So, that is why you will find that a tug whose basic demand of the thrust is much more because it has to pull or push a particular ship, and bring it to the position and being a very small craft and with a heavy pulling force. Obviously, the thrust generated by the propeller blade is much higher there and therefore, you will find that the tug propellers or tug design is such that Kort nozzle is a different thing all together, but what they do to start with that they try to give a permanent rate to the keel, so that at the draft after is always more than the draft forward, and you can fit if larger propeller in the place. That is number one.

If that also do not satisfy, then to avoid the cavitation you try to put a kort nozzle. Kort is basically a trade name, the engine Buro Kort. So, it is a shrouded propeller. So, you cover it, so that cavitation does not take place and you are giving a jet effect and therefore, the thrust generating property of the propeller increases. Even that helps in (()) also if you are having a swiveling type of nozzle. Any questions so far in this case, then we can answer that. No, actually what it show is that when a high power blade, high propeller is there and if the flow of water is not sufficient, then it will start cavitation.

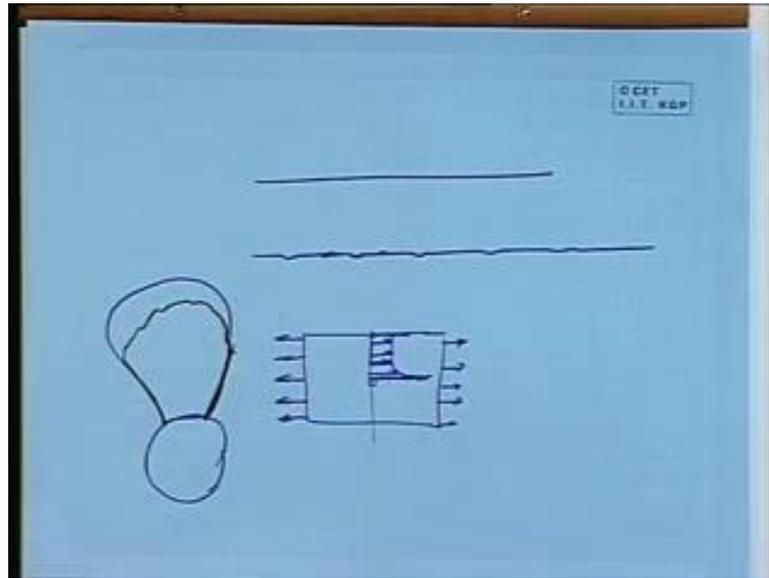
In fact, the high rpm, high loading, there is every tendency that the air will be sucked from the atmosphere to the propeller is not only that it is going to go below the vapor pressure, but it is going to suck air from the atmosphere. So, when you shroud it, you are trying to restrict that air entry. Number one, you are trying to say that the propeller blade is moving in the water and then the design of the nozzle is such that it is the nozzle. So, it tries to give you a nozzle effect also, but the friction increases there when you are carrying, you are increasing the wetted surface area of additional item. You have to overcome that friction also.

Normally it is given for tugs, smaller ships, and bigger ships. You have large space to get the propeller diameter. Yes, it is. It is the problem. So, you require a net for that also and that also increases the resistance and sometimes all this garbage gets stuck to it, restricts the flow to the propeller. That is there because at the entry level, you have to stop it. Very small clearance between the tip of the blade and normally 3 to 5 millimeter. Now, that creates another problem that the tip will get eroded and therefore, you require a special type of material.

Now, if the two materials say propeller blade is made up of manganese and bronze, and if the nozzle is made of mild steel, then coming very close to each other, there will be galvanic action and galvanic action we will try to crude the propeller blade. So, to avoid that and more over, your this effect wherever you are having shallow water and so on so forth and you require the high power output. Now, wherever you have shallow water, obviously the mud particles are the sand particles will be moving along with the water. Even if you have a say 8-10 meters of water depth in river Hooghly and when it is moving, you know that the silt will be moving along with the water. It is always muddy water and that sand particles. However, small it is will be doing some sort of an abrasion there.

So, along with the corrosion, there is erosion also. So, what is done to avoid a galvanic action? They use the stainless steel inside. So, stainless steel claue material is used for the inner side and erosion will take place. Of course, stainless steel is slightly harder. So, erosion is taken place there. There was a case study about a propeller which was short blasted for cleaning. By error and then the blades broke off. The harden particles where embedded in the surface which created the anodic cells. May be anodic cell may be because of the sand blasting the surfaces has become pitted. It must have become pitted. So, stress concentration is another and stress concentration is a very serious thing. Many times you know this corrosion creates stress concentration because the thing is pitted like you know if suppose this is the surface, a smooth surface and after corrosion, it becomes like this and now, there is a small dump here. Now, if the force is given in this direction, it behaves like this.

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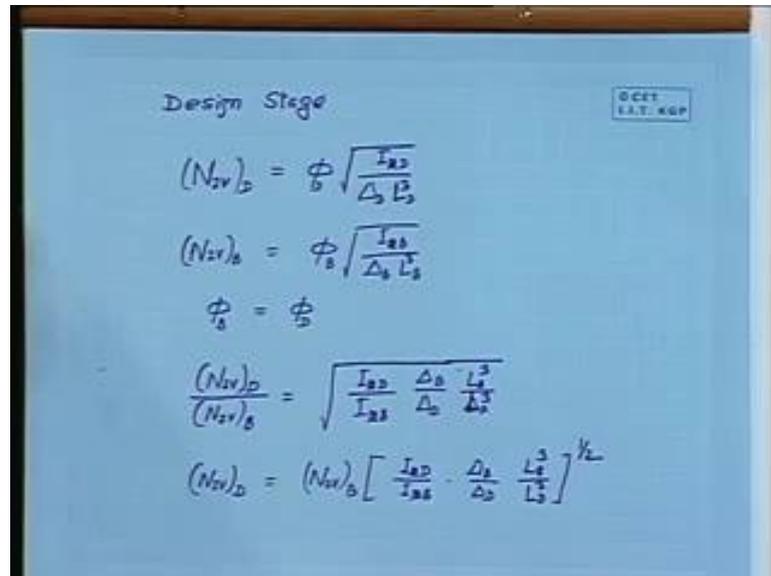


Now, if we take the stress contour here, stress variation you will find that if this is the force here, the stress will be jumping like this. Now, this jump here it will be far larger than this here, and this will easily take it beyond the plastic limit. Many times you will find for such tugs etcetera the blade is at the tip is all twisted here. You will find that it is not having a smooth surface, but it is all bent like that is very thin there and the whole thing gets in fact I am having some photograph where you will find this blade. So, this blade I think for a tug only. I have after a use of about 6 or 8 months, this part of the blade is no more. They simply vanished. So, the final blade contour is something like this. Remaining part is all missing.

It is not going to deliver you the power, number one. Then, this blade has been broken here, other blade and this blade there is no match about it. So, it will give you vibration definitely, but certain vessels you simply do not bother. Noise level let it be there and in fact, smaller vessels not much of problem with the stresses because of the corrosion, because of the minimum thickness required by the classification and also, the minimum plate thickness which is available in the market. All these things will dictate. We will find that the tugs and barges which are build for river use or canal use, do not have less than 8 millimeter plate thickness whereas, if you put 5 millimeter will also do. That will be sufficient, but they do not have less. 8 or 7 is not available, 6 will be too thin and I think 6 is the requirement.

Any further questions?

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Design Stage

$$(N_{sv})_D = \phi_D \sqrt{\frac{I_{2D}}{\Delta_D L_D^3}}$$
$$(N_{sv})_B = \phi_B \sqrt{\frac{I_{2B}}{\Delta_B L_B^3}}$$
$$\phi_D = \phi_B$$
$$\frac{(N_{sv})_D}{(N_{sv})_B} = \sqrt{\frac{I_{2D}}{I_{2B}} \cdot \frac{\Delta_B}{\Delta_D} \cdot \frac{L_B^3}{L_D^3}}$$
$$(N_{sv})_D = (N_{sv})_B \left[\frac{I_{2D}}{I_{2B}} \cdot \frac{\Delta_B}{\Delta_D} \cdot \frac{L_B^3}{L_D^3} \right]^{1/2}$$

Now, one small thing I will try to tell you that at the design stage, we like many times to estimate what is the fundamental frequency and vertical vibration. So, we try to calculate two noded vertical vibration for a particular ship which is under design, and if we try to use the Schlick type of formula, you will find that Schlick gave the equation something like this. Am I right? Just open back and see. So, it is of this order. So, for the design ship, we can say that put a suffix D all over and when we are designing a ship, we have something in front of us. A proved design or a proven design, which has already served of similar time, we try to look at that ship and say that we would like to design something like this.

So, that ship we say is a basic ship. So, on that basic ship, we try to design. Now, for the basic ship we assume that all the calculation and all the particulars are available. So, the same formula if applied to the basic ship will look like this. Now, if the current design is based on the basic ship design, then it will have the features of the basic ship. So, if it has the features, then many of the things will match. It may match, it may not match, but they will be very close to each other and therefore, the first assumption we make is that this coefficient of the basic ship is equal to the coefficient for the design ship. If this is true, then we say that the frequency of the basic ship or the design ship in comparison to the basic ship taking the ratio can be given like this. Now, if we further assume that the

basic ship, the design ship is having all the features of the basic ship, then that means all coefficients must be identical.

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The image shows a blue board with handwritten mathematical derivations. The equations are as follows:

$$\frac{\Delta_B}{\Delta_D} = \frac{\rho L_B B_T C_B}{\rho L_D B_D C_D}$$

$$= \frac{L_B}{L_D} \cdot \frac{B_T}{B_D} \cdot \frac{C_B}{C_D}$$

$$= \alpha^3$$

$$\frac{L_B^3}{L_D^3} = \alpha^3$$

$$I_M \propto B D^3$$

$$\therefore \frac{I_{MD}}{I_{MB}} = \frac{1}{\alpha^3}$$

On the right side of the board, there are three separate equations:

$$\frac{L_B}{L_D} = \alpha$$

$$\frac{B_B}{B_D} = \alpha$$

$$\frac{T_B}{T_D} = \alpha$$

In that case, we can write that delta B by delta D is equal to L length breadth draft, and CB is equal to length. Sorry breadth draft of the design CB row of water. Now, row of water you cannot change for the design vessel and the basic ship, they are identical in the same sea. They are operating and I say all the features are identical. That means the CB's are same. So, this two ratio will work out to be, and if I assume that the linear proportion of the two ships are identical and if we say that LB by LD that is basic ship to design ship is alpha B of the basic ship B of the designed ship is also alpha, and their corresponding draft is also alpha. That means, it is just a scaled up or scaled down ship.

In that case, this will become alpha cube, right and LB cube by LD cube will also. The alpha cube I assume that I midship varies as B into D cube in that case. Therefore, I midship of D by I midship of basic will be 1 by alpha cube. I think I will stop now for the time being. After we get this for an identical type of shape and not the dimension, then plugging these values into these expressions here.

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$$\begin{aligned}
 (N_{sv})_D &= \phi_s \sqrt{\frac{I_{RD}}{\Delta_D L_D^3}} \\
 \phi_D &= \phi_s \\
 \frac{(N_{sv})_D}{(N_{sv})_S} &= \sqrt{\frac{I_{RD}}{I_{RS}} \cdot \frac{\Delta_S}{\Delta_D} \cdot \frac{L_S^3}{L_D^3}} \\
 (N_{sv})_D &= (N_{sv})_S \left[\frac{I_{RD}}{I_{RS}} \cdot \frac{\Delta_S}{\Delta_D} \cdot \frac{L_S^3}{L_D^3} \right]^{1/2} \\
 \therefore \frac{I_{RD}}{I_{RS}} &= \frac{1}{\alpha^4} \\
 (N_{sv})_D &= (N_{sv})_S \left[\frac{1}{\alpha^4} \alpha^3 \alpha^3 \right]^{1/2} \\
 &= \alpha (N_{sv})_S
 \end{aligned}$$

I will just write here for your convenience. So, N_{sv} for the design vessel can be written in terms of the frequency of the basic ship. Now, this works out to be 1 by α to the power 4 . This works out to be α^3 , this work out to be α^3 and then half power. So, 6 by 4 α you see α^2 and taking the square root that of you will get α into. So, if the entire features are identical, then it is very simple to estimate the frequency of the designed vessel on the basis of the basic ship frequency, and the design stage when nothing is available to you except few main dimensions.

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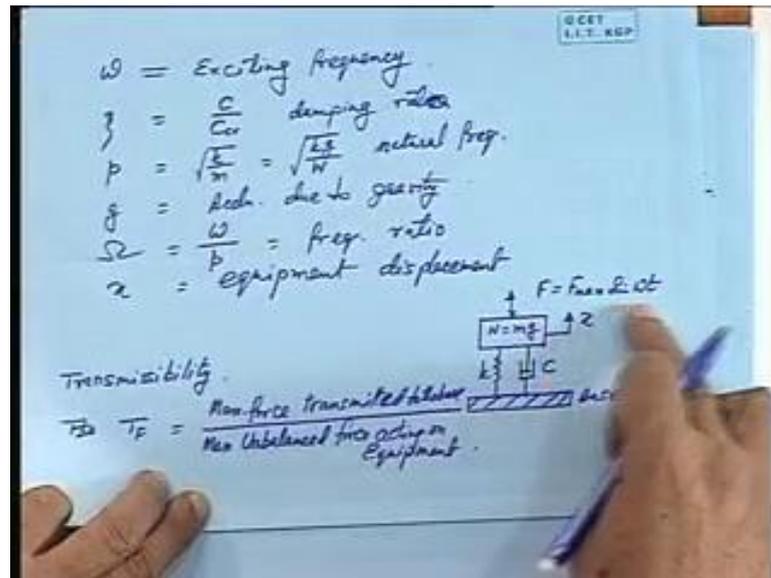
$(1 - T_F) \rightarrow$ Insulation or Isolation

$$T_{DS} = \frac{\text{Max displacement of equipment from static}}{\text{static displacement of equipment under force } F_{max}}$$

Max force transmitted through the spring = $k X$
 max force transmitted

For the ship which is already performed built and served, you have all the detailed calculations. Find displacement transferability of equipment, TDE as maximum displacement of equipment from static condition to the ratio of static displacement of equipment under force $F \max$. See we have taken $F \max \sin \omega t$, so under force $F \max$.

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Now, maximum force transmitted through the spring. Say the maximum force transmitted will be the force transmitted is k into x and under this, you will find that $\sin \omega t$ minus something will come. So, $k x t$ is the force. So, the maximum will be the maximum value of $x t$. So, I will write it as capital X here and maximum force transmitted. You wanted some problems; I have got the problems here on vibration, single degree, freedom system.

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Exciting force.

$$k \cdot z_{st} = \frac{k \cdot z_{st}}{\sqrt{(1-\Omega^2)^2 + (2\beta\Omega)^2}} \sqrt{(c_0 - d)}$$
$$\text{max. } k \cdot z_{st} = k \cdot X \sqrt{(1-\Omega^2)^2 + (2\beta\Omega)^2}$$
$$T_F = \frac{\sqrt{1 + (2\beta\Omega)^2}}{\sqrt{(1-\Omega^2)^2 + (2\beta\Omega)^2}}$$
$$T_{SE} = \frac{X}{\sqrt{(1-\Omega^2)^2 + (2\beta\Omega)^2}}$$
$$= \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\beta\Omega)^2}}$$
$$\text{Max. Isolator Deflection} = \left(\frac{F_{max}}{k} \right) T_{SE}$$

Let me see mostly taken from book. You know there are 20 problems here. If you want to, you can Xerox. Yes sir. Give back this to me and we have from the books, only text books. Now, yeah. (FL). No hurry. You can take your own time. If you want to give me today, you can give me today. If you want to give me tomorrow, day after tomorrow, next working day, it is ok. I think I will stop now.