Strength and Vibration of Marine Structures
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Lecture - 26
Ship Vibration IV

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So, now we continue further.

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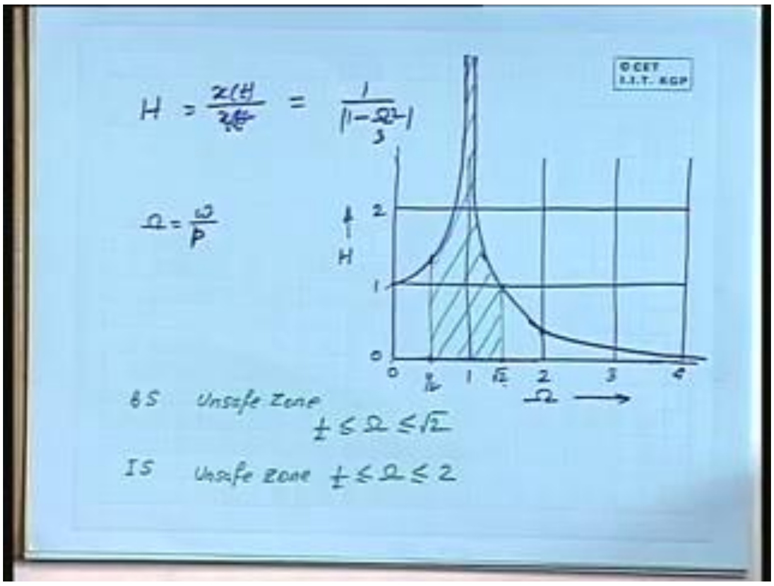

So, this is now let us try to see this expression H is equal to 1 by 1 minus omega square. If the frequency ratio omega is equal to, let me rewrite exciting frequency by natural frequency. If this omega exciting frequency is 0, then H is equal to unity. That is this point here. If omega is equal to 1, then this blows up to infinity somewhere here. If I take capital omega is equal to half that is somewhere here, then what is the value of H here? H works out to be 4 by 3.

So, 4 by 3 is 1.33 is somewhere here, right. So, with these three points, I try and you see that this blows up exponentially like this, all right. Now, when I say that omega is equal to say root 2. So, this becomes 1 minus omega square 1 minus 2 is equal to 1 and 1 by 1 is equal to 1. So, if root 2 is somewhere here, so this works out to be this point here. If you take it as 2, then it is 1 by 3 2 2 4, so 3. I am taking the modulus, so 1 by 3. So, 1 by 3 is somewhere here. So, that means the plot goes like this.

So, this is an indication of the displacement pattern. So, as omega starts from 0 and increases, say let us now take a real situation that you have a plate or a beam on which you are having a motor or some rotating machine which has got some eccentric frequency. You switch it on. It will start from rest 0 rpm and it will attain a highest rpm of say 3000 rpm, 6000 whatever is the designed rpm. So, from 0 to 7000 or 6000, it is attaining. So, it moves from here, this omega I am taking about 0 to say 7000.

Now, this P natural frequency of the object or the structure on which this is mounted may be say 3 hertz. Now, 3 hertz is around how much? It is 180 rpm or say 30 hertz. So, 1800 rpm. So, you are starting from 0 going up to say 70 to 100 rpm. So, 0 to 4, it is going frequency ratio. The motor is attaining a speed of say 70 to 100. You are starting at 0 and slowly it attains that. So, it will go from here to here, right.

Now, when it goes from here to here, it is passing through all these points. So, at some place, the thing is building up and then in fact it will not build up to infinity. You get that increase in the frequency and then decrease. The other day I was just talking to you that when the bus is in stationery condition, what happens that suddenly the thing starts rattling and that now the same thing happens because the driver wants to accelerate also at the same time.

So, he is trying to increase the rpm of the engine and then something suddenly happens. If he keeps it in the idling condition may not be any rattling or something, but he cannot sit just like that. You know he is waiting and then he keeps on accelerating, decelerating, accelerating and decelerating. So, at that case you will find that suddenly the thing goes up and then comes down. So, what is actually happening is he is trying to move the rpm from here to here. In the process, this zone is getting crossed and therefore, the displacement is jumping to this high value and then coming back and that is what that rattling sound comes at times.

Now, I give you another example that all of you have used the main engine, and on the telegraph, the wheel house, you find that there is a red zone. You have 0 stop. Then, half speed or half ahead or whatever it is, then full ahead and then down below you have the astern, but from stop to full ahead in between, there is a red zone sector painted red. That red sector is denoting some rpm belonging to this zone, and the meaning is that you do not have to operate the engine at that. You have to cross over immediately. So, that is this red zone.

Now, according to this diagram, we see that it blows up to infinity because this expression gives me that, but every system has got its own inherent damping property and therefore, it will not go to infinity, but it will be limited to a particular value. That we will see only once we take up with damping. So, let us now try to see the case with damping.

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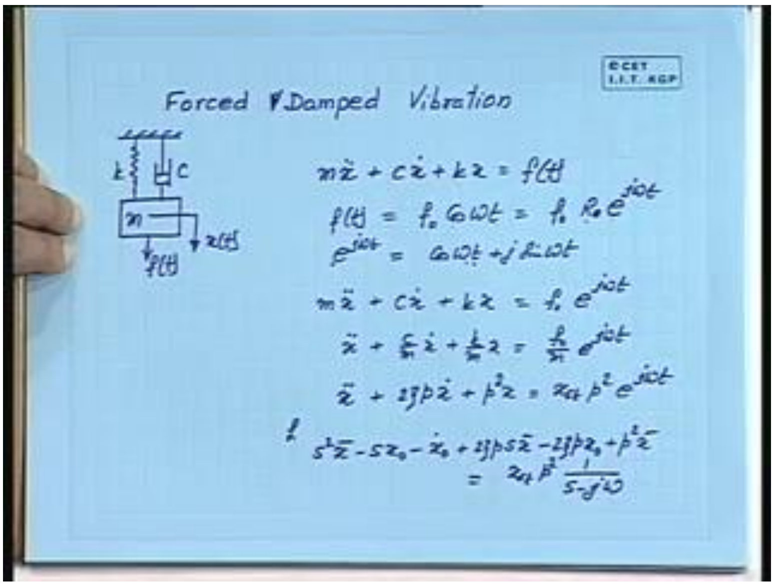

Let me say forced vibration, forced damped vibration. I will consider the same simple diagram. Only thing now I will put a dashpot here. I am considering the free body diagram of this which we have considered earlier. We arrived at the governing differential equation as mx double dot cx dot plus kx is equal f of t. Once again we will assume that f of t, the exciting force is given by f 0 cos omega t, but this time we will consider this to be f 0, real part of e to the power j omega t.

Now, what is e to the power j omega t is basically when we talk about real part, real part refers to this. This is the imaginary part of it. So, we are taking cos omega t. So, this cos omega t is the real part of this particular function and therefore, we are writing Re e to the power j omega t, but now onwards I will not write this Re because it is implied that I am only taking about, I am basically concerned with the real part of it. So, writing this function with this here, I have mx double dot cx dot plus kx is equal to f 0 e to the power j omega t. As usual dividing all through with m, we have x double dot c by mx dot k by m x f 0 by m e to the power j omega t making the usual substitution which we have defined earlier. We write this expression as x double dot plus 2 zeta px dot plus p square x is equal to xst P square.

Now, taking the Laplace transform of this, I get s square x bar minus sx 0 minus x 0 dot 2 zeta p sx bar minus 2 zeta p x 0 p square. X bar is on the left hand side. Right hand side I will get xst p square and this will give you s minus j omega. This is the transformation of this part.

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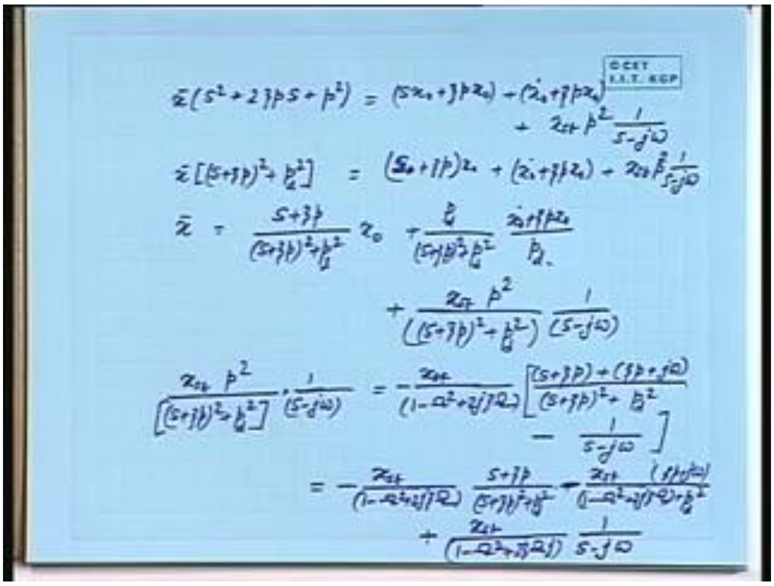

Now, writing the transformed response here, s square plus 2 zeta p s plus p square and as usual writing this in a splitted form sx 0 plus zeta px 0. This is one part and this I am splitting into 2 plus x 0 dot zeta px 0 and xst p square 1 by s minus j omega. Now, here we had done this manipulation or mathematical readjustment, whatever you call is s plus zeta p whole square plus pd square with usual definition, and this is x 0 plus zeta p. Sorry s response expression is this. So, dividing by this and multiplying and dividing by p d here, we get this expression here.

Now, by taking the inverse of this, we should have requisite answer or the solution. This part is no problem. The problem is going to be created by this part and one has to do the partial fraction of this. So, let me just take the third part here, and see what happens with this. Now, this will give you, I will straight away write the solution part of it. This has to be done that partial fraction and the solution will work out to be slightly lengthy here, and that lengthy solution is something like this.

This plus this divided by this part minus this by this, and here you get minus xst 1 minus omega square plus 2 j zeta omega and this can be further splitted. This is again minus, minus and minus will become plus. Is it coming on the screen? Yes sir. 2 zeta omega j also let me write here. So, this is what happens. You are basically getting from this single term here. There are three terms that is what I just wanted to show you that three terms you are getting here. Now, you take the inverse of this.

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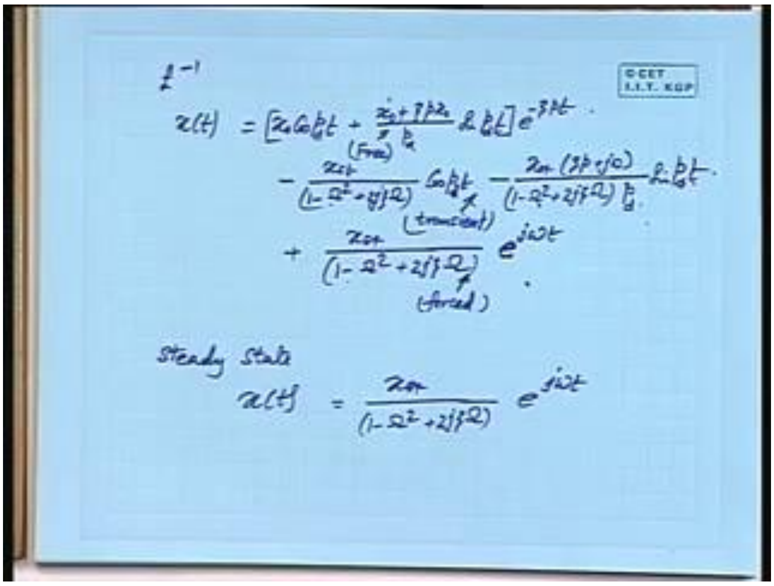

What you get is x of t. The first term which is this here gives you x 0 cos pdt. This term will give you x 0 dot plus zeta px 0 pd sin pdt. Now, this is getting splitted into three terms. What are these three terms here? 1 minus omega square plus 2 j zeta omega and what is this. Once again cos pdt. Then, what is this term? Zeta pj omega and this pd by this part will give you, I have done a small mistake here which I will try to upgrade here.

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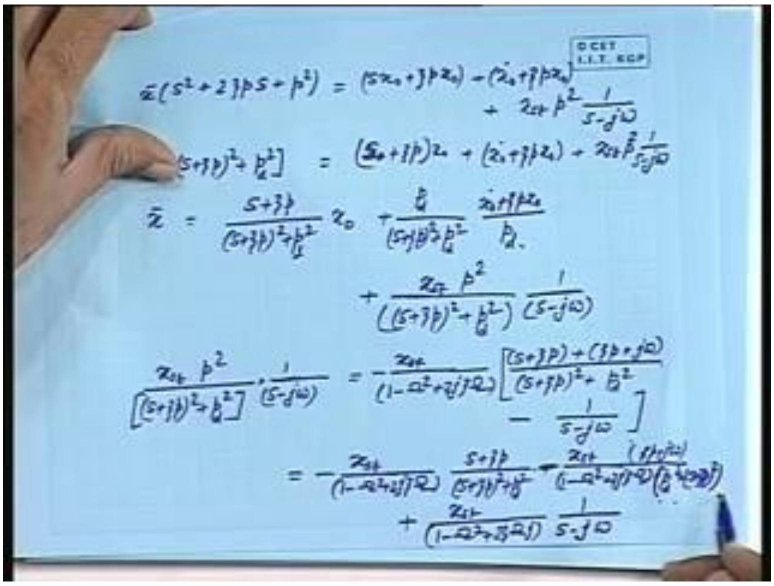

This part is missed from here. Okay. So, pd by this part will give you sin pdt and this divided by pd will come here, right. So, these two terms taken care of last term is xst 1 minus omega square plus 2 j zeta omega and inverse of this is nothing, but e to the power j omega. So, this is the full solution of this.

Let us see here what happens. This is x 0 x 0 dot x 0 pd. This is same as if we were not having any forcing function. This was the, sorry there is another term here which we are missing here is e to the power minus jpt zeta pt. This was the response of damped free vibration. This part is that. This part here, if you see what are we getting. I suppose I have not done any further mistake here. So, this part is dependent on the natural frequency.

Pd is the natural frequency of the system. There is no forcing function is coming here, but only frequency ratio is coming somewhere here. That is the only part. Otherwise, the response which varies is based on natural frequency. In this part, the response is governed mainly by the forcing frequency here. So, this part we call is the free vibration part. This we say the transitional vibration because it is changing from natural to force and this is the forced vibration part. So, this has got three components, natural, transitional and forced.

So, this is we say as free vibration. This part is free vibration, this is transient and this is the forced part. Now, our experience says that in such a case as time elapses, this will die down. In fact, this part we have seen it dies down. There is an exponential decay function and even this part will also die down with the same analogy. So, only this part will continue as time increases and therefore, we can say in the steady state vibration by this expression here j omega t t, I am missing here. Is this clear, this part?

We do not have to bother about all those big expressions, but that is part of this game, but only thing what we are trying to say is they are not the steady state vibration. It is governed by this expression, right. Now, this expression as such creates a lot of problem because j term is involved here and here, and we must get rid of it and we do again a little bit of mathematical operation.

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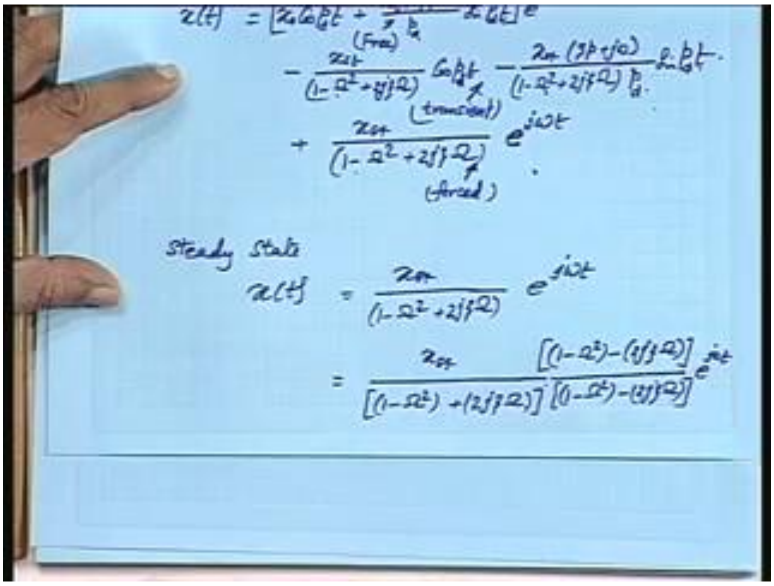

What we write is, this put it into parts like this and I multiply by and divide by the same thing, multiply and divide.

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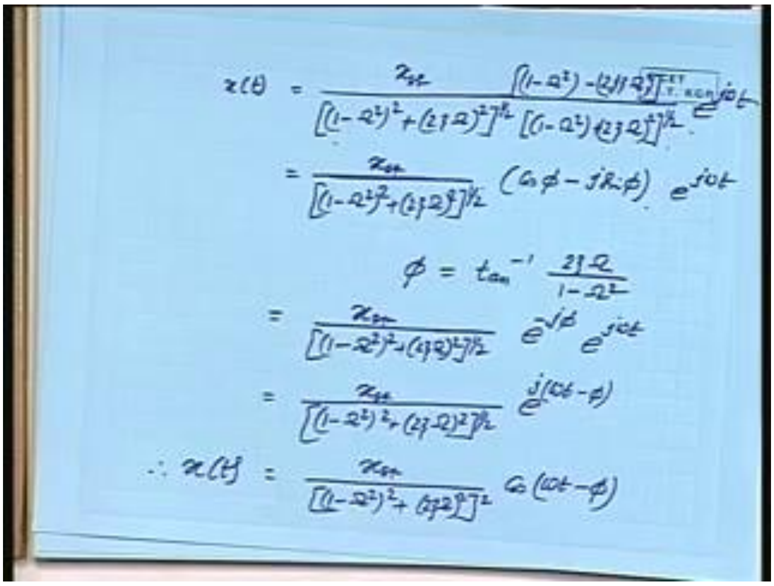

Now, what happens is this. This becomes whole square of this part and minus whole square of this. So, whole square of this means j square is coming into picture and j square is equal to minus 1. So, I will make it plus here and xst is here and this part I will leave it as it is for time being, right. Now, if you try to split it up into two parts like half you take here and you take another half here, right then 1 minus omega square by this part will give you cos phi t, where phi will be defined and minus 2 j omega will give you minus j sin phi t.

So, I write this as this is square root and this part I will write as cos phi cos phi minus j sin phi, where phi is given as phi is equal to tan inverse 2 zeta omega phi 1 minus omega square. So, this part gives you that and I have as usual e to the power j omega t, right. Now, cos phi minus j sin phi is nothing, but e to the power minus j phi into e to the power j omega t, and this zeta omega square root over e to the power omega t minus phi j right.

So, in this way I have been in a position to remove this j part from here. So, this part is now free of j. Only e to the power j omega t minus phi is there. What I am bothered because I started with a forcing function f 0 e to the power j omega t. The real part of it and I am coming to a solution which also has something here, and e to the power j omega t minus phi here and therefore, when I started with the real, I will take the real part and therefore, x of t is nothing, but xst divided by if I take the real part of it, then the response is given by. So, this is the expression first steady state response. Now, if you try to compare this response, let me try to put this response along with it.

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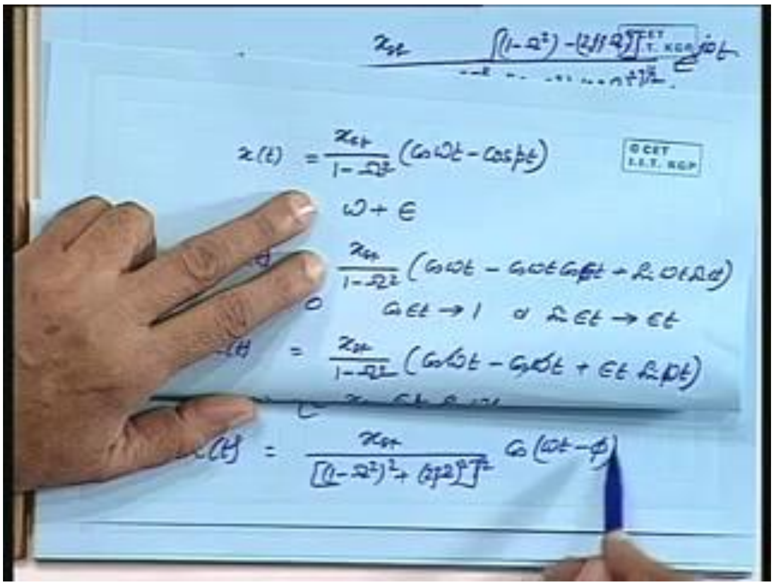

This is the response for free undamped and this is the response for free damped. Sorry, this is the response for forced undamped and this is for forced damped vibration. So, what is you are getting here is 1 minus omega square. What you are getting here is 1 minus omega square whole square plus 2 zeta omega square, square root of this. You are getting cos omega t term here. You are still getting cos omega t minus phi here. There was a natural frequency, which would have died down. So, we would not have bothered about it. So, there is a phase lag coming here. So, this is an additional term. This is the term you are getting. Now, when you try to say this term and try to plot, what we had or let me first try to say what is the amplification factor is x of t by xst.

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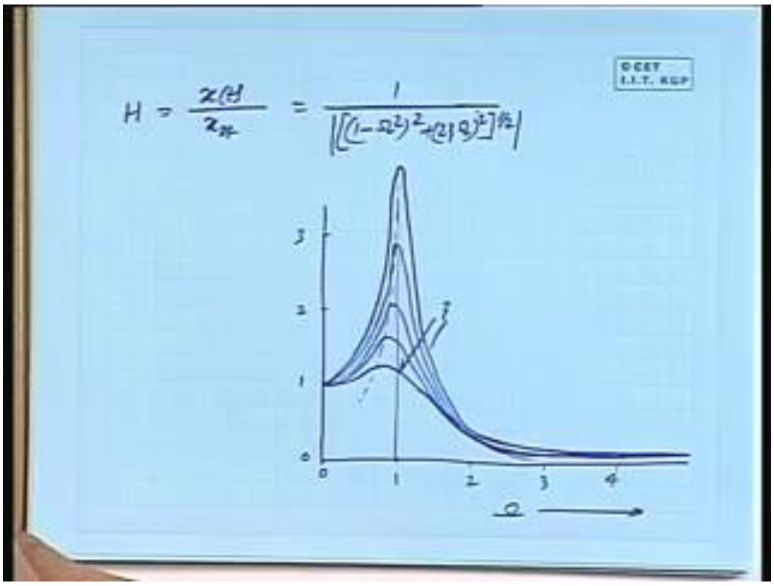

Now, this term when it becomes unity for some value of y omega t and phi, this will be unity and under that condition I take this ratio xs xt by xst which will be given by, let me take the modulus of this once again and this is the definition of H here amplification factor. Now, try to see this part here when omega is equal to 0, then you have H is equal to 1. When omega is equal to 1, this part is becoming 0, but this is not becoming 0. This is 2 zeta. So, if zeta is if the structure is likely damped zeta, may be a small value and we say that for example, zeta is of the order of say 0.01. So, 2 zeta will give me a value of 0.02 and 1 by 0.02 will give me a finite value of say around 50.

So, what is happening is that the amplification will be 50 times, but it is a finite value. In the other case, we found it is blowing up to infinity undefined value, but here it is a defined value as because zeta is there. So, if suppose you are having a structure with say 0.1 as the damping ratio, you are getting 1 by 0.2 and 1 by 0.2 is 5 times the displacement. So, it is not blowing up to that value, but it is blowing up to 5 times, 10 times, 20 times, may be or something of that sort depending on what is the damping ratio in the structure you are having.

So, what we normally see it is not that. That is why it just vibrates and then it comes down. Is that clear? Now, before going to this, let me try to go back to the same diagram.

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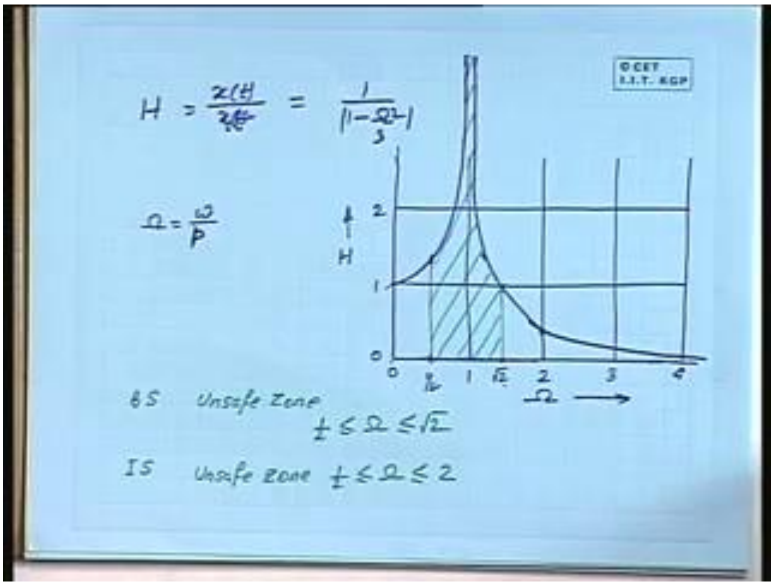

Here three values I had taken. One for this at omega is equal to half, one here I took at root 2. Now, this zone from half to root 2 according to the British standard, the unsafe zone is defined as half less than equal to omega less than equal to root 2, and according to IS standard, Indian standard unsafe zone was between half is up to this.

Now, I do not find any justification that this is unsafe, this is quite, but only thing that we do not want to simply accept what British’s have said. So, we try to make it, but even if you try to see this particular thing, even root 2. Why it should be root 2? If you can accept a value of 1.33 here, amplification you should be in a position to accept the same value here, but the only reason one can see that when you are trying to put some sort of, put forward some design code, you try to give a value, which one can remember memorize or some such thing.

Here it is 1 by 2, here it is root 2. It is the same 2. Only thing is how you handle the two. So, in this case, it is root 2 and here it is half, but the digit is only 2. Indian standard also did the same thing. They simply removed this root part and say this is half. This is inverse of that, that half 1 by half, half and 1 by half. So, it becomes 2, but actually speaking that when the ratio is coming down, it becomes safer and that part you will find that at a high frequency, a calm situation prevails. At low rpm or low frequency, low excitation, there is every chance that it will blow up, but at a higher excitation, a quieter situation will be available.

Now, with this also one can try and draw all these curves. What you will find how it was going one-third here now. Now, if you increase the value of zeta it is, now the peak will be shifting like this. So, one can say that in this direction, value of zeta is increasing. So, the peak value is shifting.

Now, what is the resonance frequency here? How do we find out what is the resonance frequency? It is p is equal to omega or something else. So, to get the resonance frequency, what is to be done is one has to find out at when we say resonance frequency at that time, we expect that the amplification is the maximum with respect to that frequency. That amplification is maximum.

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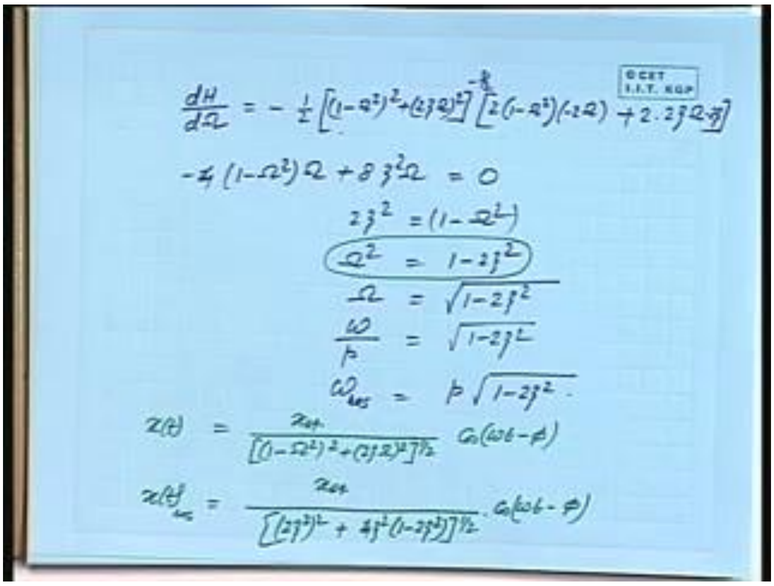

So, that means, we have to find out at what point this is equal to 0 dH by d omega. So, now that expression if you want to find out what is dH by d omega, then that works out to be like this, where you can see from this expression which I have written here. So, what can be done is the whole thing can be taken up with a negative sign. So, when you take the relation, so you will get 1 minus omega square whole square plus 2 zeta omega square minus half minus 1, that is minus 3 by 2 and then let me write here and then omega this d of this which is 2 into 1 minus omega square and you will get minus 2 omega from here, and then plus you will get 2 into 2 zeta omega 2 into 2 zeta omega into 2 zeta. So, this has to be equated to 0.

So, let us take this part because this part is not equal to 0. So, that we now forget about it. So, this part let us take. So, this part means 2 into 2 into minus 2. So, minus 4 1 minus omega square, then omega and plus 2 2 4 8 8 zeta square omega is equal to 0. That means, 2 and 4, I have 2 zeta square is equal to 1 minus omega square. This is negative. So, if I take it on the other side and remove the omegas and 4, so 2 zeta square from here and 1 minus omega square into omega. Am I right? Zeta no omega also gets canceled out. So, this is the value I am getting here and therefore, the frequency ratio omega square if I shift here, and this goes there 1 minus 2 zeta square or omega is equal to 1 minus 2 zeta square root over.

Now, omega is equal to this by this which is equal to 1 minus 2 zeta square and therefore, omega resonance is nothing, but p times. So, this becomes the resonance frequency. Now, this is the value of omega square, right. I will use this color now. Now, xst x of t x of t is given by xst 1 minus omega square whole square plus 2 zeta omega square under root, and then this is the response for the damped forced vibration.

Now, if this is going to the resonance frequency, then this is the ratio and this is the ratio square. So, ratio square if I substitute from here, I get the response for resonance here as 1 minus omega square. I have already calculated. So, this is 2 zeta square. Now, 2 zeta omega square omega square is here. So, omega square is this part. So, this is 4 zeta square and omega square is equal to this is this. Let me see what is this.

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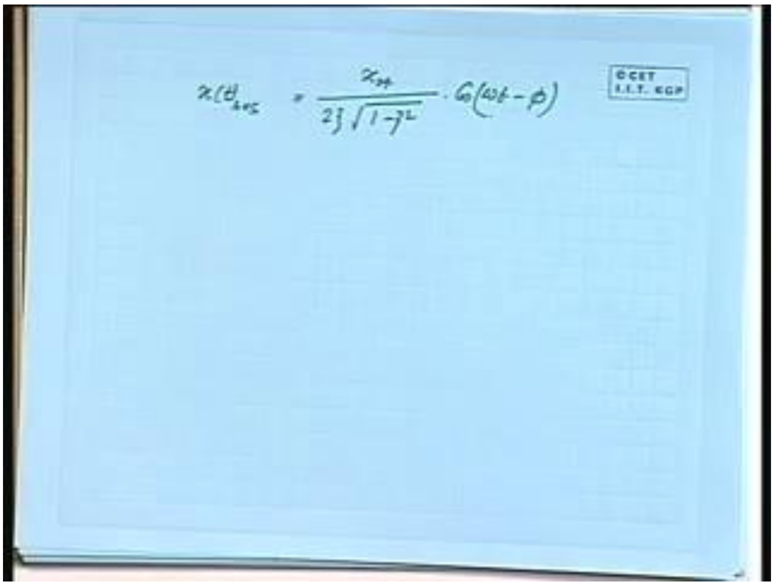

So, if you try to solve this part, you will get x of t at resonance will be given by. So, this is what we will get the displacement at resonance frequency, definitely it is a finite value and there is a phase line. So, this part is not unity at resonance, this is not unity. You will get some sort of a fraction here and then xst divided by this quantity, right. So, this is one part of it. He has already shown me the time. So, let me wait for some more time.

Now, this explains actually that why the structure at the resonance does do not go to infinity, and that is why we are thankful that something reserve is here. So, when we try to design the structure, fortunately for ship structures we have the stiffened plates basically and the frequency depends on how the mass is distributed. We need not increase the weight though the frequency is equal to k by m root over k by m. You can play about with mass; you can play about with stiffness. So, increasing mass is no problem. You just add stiffness here or there you can increase the mass, but at the same time, you can play about with the stiffness.

So, if you are at the design stage, if you try to find out what is the natural frequency of the structure, you take few combinations of it say you change the stiffness spacing or you change the section profile or you see some sort of a combination and find out. Say for example, this is the place where I would like to put a machine. In fact, these days it has become important that on the deck house, in the bridge deck, you would like to put your PC. Now, PC specification you know when you try to buy it, the manufacturer will tell you that how much of G it can take, what are the displacements it can sustain and therefore, it becomes important where to locate the thing.

You have the wheel house and in wheel house, you have to locate it somewhere. Where do you want to locate it? So, when you do the final analysis of the structure entirely, then you see that how the responses are coming onto that particular structure. Find out a nodal point, where the disturbance is the least and then you say that I would like to put the PC here and then you see with the design group whether it is possible to house it there or not. If not possible, you say that from functional utility, my PC must be here only. Put it there and then you play about with the structural arrangement there and do the analysis and see what the response you are getting now. This can be played about.

The same thing happens with say engine room layout. Certain machineries are there. You cannot say that my main machinery will be here. No, I have to offset this because it has to align along with the shaft of the propeller. So, how can you offset it? You have to put it there. Now, in that position if you are going to get a sever vibration, then do something with the engine seating change. The girder scantlings may be the scantlings by changing by 2 millimeter here or by increasing the flange of that, you can change the vibrational property or the response totally. So, that can be played about and that is what we are seeing here.

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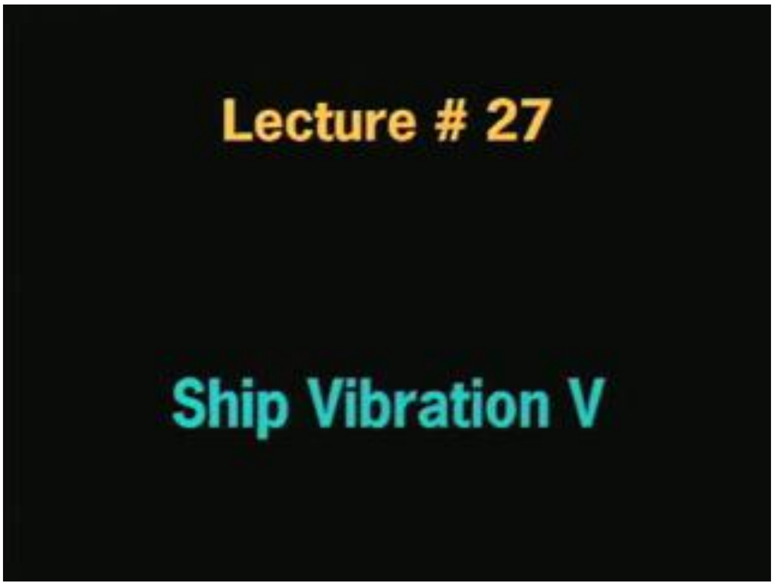

So, now, we continue further. So, this is the response of that structure when it is in a resonant case.

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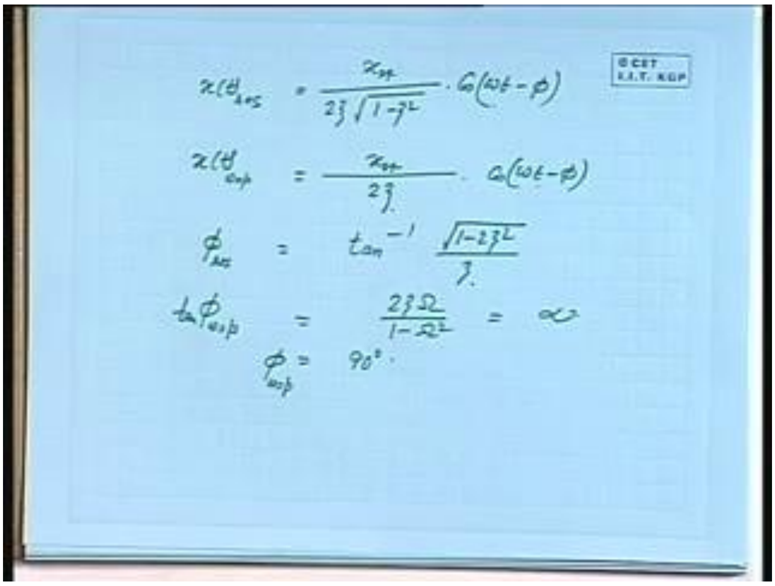

Let us try to find out the response at omega is equal to p. What happens when omega is equal to p? Capital omega is equal to 1 and then if you substitute that value here, what you will get is 2 zeta square of 2 zeta, right. At that time let me say phi at resonance will be tan inverse. It will be 1 minus 2 zeta square root over by zeta. This is the value of phi and phi at omega is equal to p will be 2 zeta omega by 1 minus omega square which tends to infinity. That means, phi is equal to 90 degree or phi by 2 tan phi is becoming, sorry tan phi. Tan phi is equal to this. Therefore, phi at omega is equal to p is 90 degree.

So, this is what we get here. Anything here you would like to ask me because this part I think I will not go any further, beyond this impact etcetera are there which I will not try to take it up.

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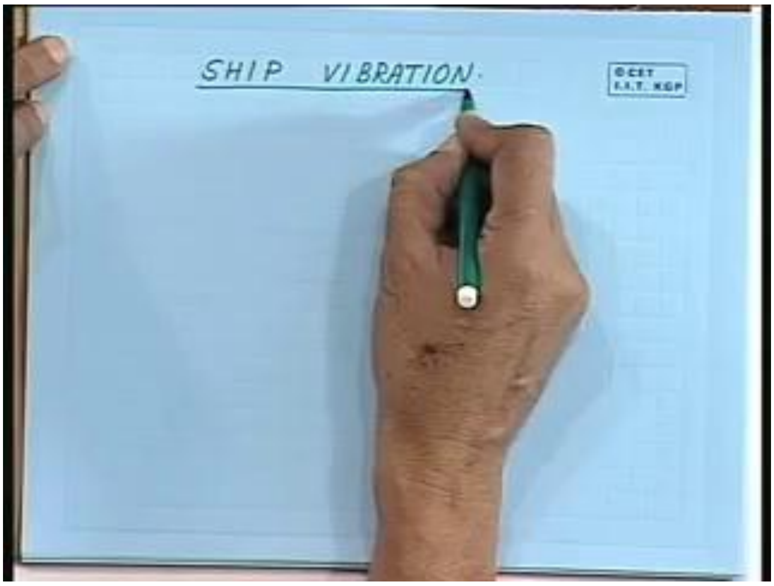

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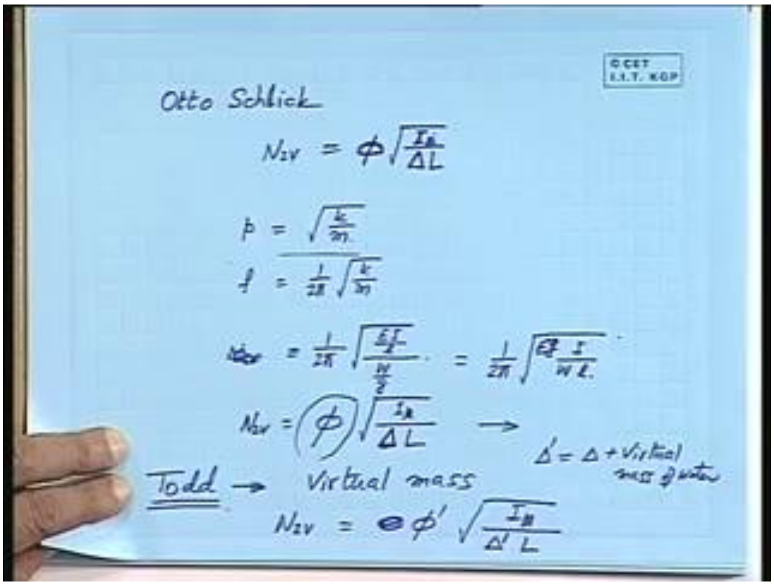

Now, I will try to do out something on ship vibration plus virtual mass of water. A lot of work continued on this virtual mass.

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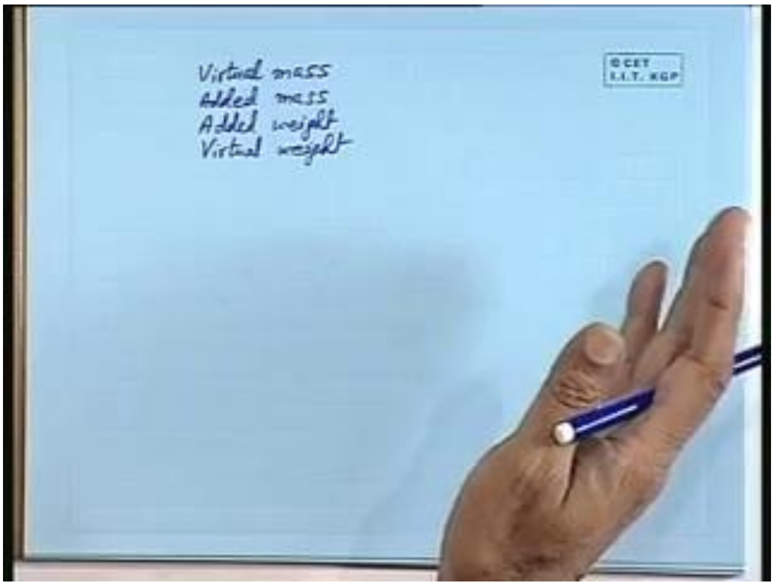

It is also known as added mass is also known as added weight also known as virtual weight. Virtual because it is not the mass of the vessel. Something is there. Now, one can try to interpret it in many ways. You say that when the vessel is vibrating up and down, something is taking along with it that also vibrates up and down. Some people will dispute about this. They say that as soon as the thing is going up, the liquid will come down. So, how can you say that the mass is going up and down along with it? It is not, yes. So, there is a dispute in that definition.

Now, mass and weight are interchangeably used because when we are taking weight, weight is mass into acceleration and therefore, in engineering terms we always try to mix around like this, like kg force and kg mass. What is when we say kg? Sometimes we also get confused that we are talking about the mass or the force. When you go to the market, you say that you give me 4 kg of potatoes. There it is a force, but sometimes when you do the calculation, kg is in mass. Sometimes it is with g, sometimes without g. So, these are interchangeably used and one has to be very careful with the unit and the usage.

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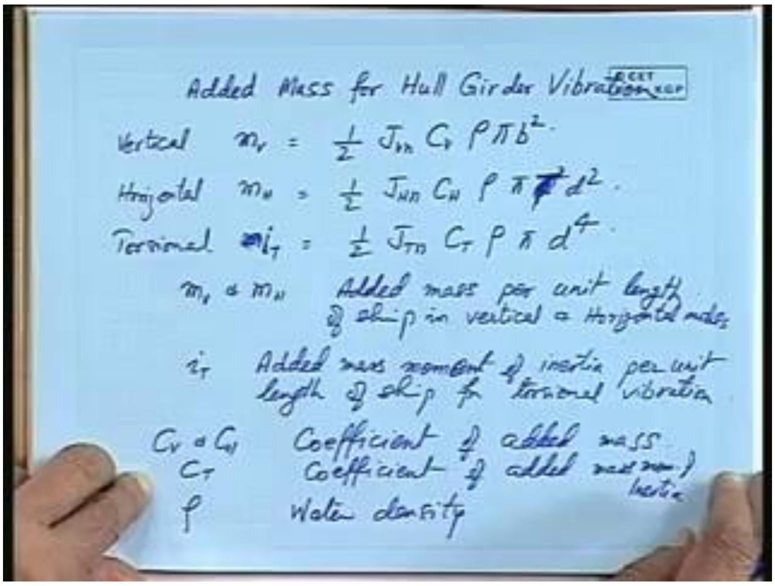

Vibration CV and CH coefficient of added mass. CT is coefficient of mass moment of inertia rho is water density.

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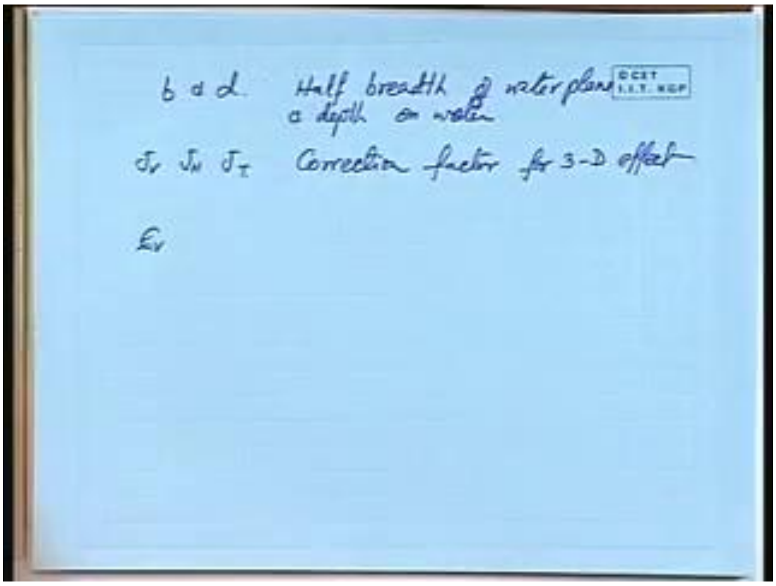

B and d half breadth of water plane and depth on water. Then, what JV JH JT correction factor for 3D effect. That is all and he says that this can be of course, this I will do it in the next class. Little more is there. So, we will do it in the next class. So, these are the moment of inertia horizontal and vertical mode of vibration which is being calculated on this basis.

So, what we try to find out is the fundamental modes and we also like to find out the higher modes. It is important. We will see that in the next class what are going to do with all these frequencies, what we are going to do with them once it is ready.