Strength and Vibration of Marine Structures
Prof. A. H. Sheikh and Prof. S. K. Satsongi
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 23
Ship Vibration I

(Refer Slide Time: 02:01)

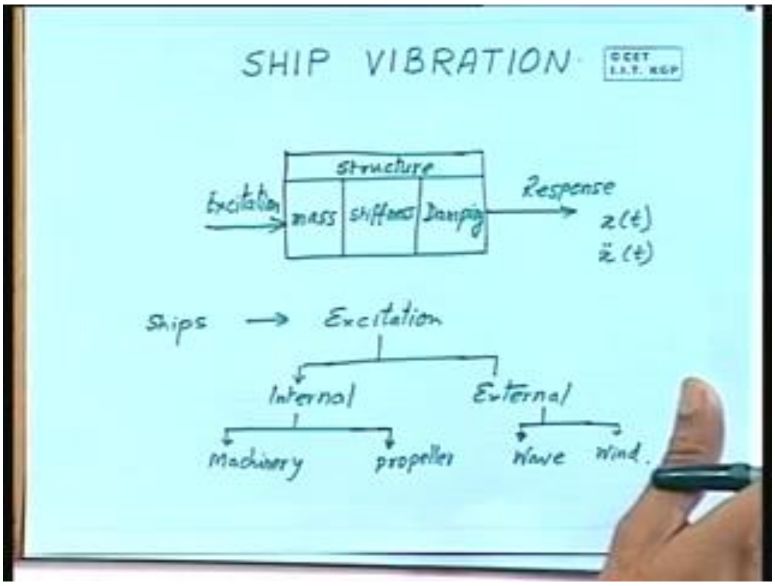
So, now if you take a small element like this of the stiffness, say I say this is a beam element, then this is replaced by two nodes at two ends, and the properties of this structure whatever is used for the bending analysis or for that type structural analysis will be considered at this and this point only. So, if this is the cross-sectional area, so we say at this point that the beam has got this much of cross-sectional area. Whatever is the moment of inertia, we will say that this beam has got this moment of inertia at this point.

If you want a horizontal bending also, then you say that the beam has got central line moment of inertia at this point is so much. If you want to do a torsional analysis, then also you say that this is torsional rigidity at this point and these two points are so many centimeters or so many meters away from each other. Whatever force you apply here, that equivalent force you find out at this moment of point. So, if heat is loaded uniformly, then this will have half the load coming directly here and the effect of this as a bending moment here. So, that load will be broken up into the components as direct load bending moment may be shear force, may be a twist or whatever it is.

So, you have a method to calculate that and apply it at this node, and like that computer can handle thousands and thousands of these points and once the capability of calculating effort has increased enormously, so one can break it down to this level and start it, but again when you come to the ships breaking down to this level, again poses a problem. And therefore, what we say that we make a course mesh and in the course mesh, many such small items are added up together. Therefore, we do a course mesh analysis and then from there we take this panel out and then apply the forces. What we calculate at these points are the displacement and the forces and the moments.

Now, those things become the applied load for this structure, and then we do the final analysis of this and for that all those methods, the stiffness method especially is very much useful which he has already talked about if I am not mistaken . So, we will not go for that. Now, the other most important thing I was thinking that if we are not going to do anything there, then what we can start is what we call vibration. Should we start now vibration of ship, ship vibration?

(Refer Slide Time: 05:34)



Ship vibration is important because ship is always moving in a very hostile environment, and under that at hostile condition if something really goes wrong, you do not have anybody around you to help you out and therefore, nobody wants a situation to come even as an emergency case that you require that much of help at the open sea. Therefore, one has to be very careful about the vibration analysis. Now, any failure which takes place due to vibration is always a catastrophic failure because one does not know how this failure has progressed. Actually the progress is very slow and very silent. Progress you will not notice it till actually the disaster takes place and therefore, one has to be very careful regarding this aspect, and even if you try to notice it, then you do not get much time between the noticing of the failure and the actual collapse. Therefore, we always say that these type of failure, some mostly catastrophic failures and therefore, a lot of health monitoring needs to be done.

We always talk about failure as far as the vibration is concerned, but just let me tell you that every coin has got two faces. So, when we talk about vibration as such mechanical vibration, then this has got the plus part of it and the minus part of it. The minus part of it that this leads to sometimes catastrophic failures, total disaster and so on so forth plus point that we use vibration for many useful work mechanical or entertainment. When you talk about entertainment, all other instruments which we or world over people have designed based on the fundamental of resonant vibration you talk about sitar, you talk about table, you talk about guitar, you talk about saxophone, anything you talk about, flute, everything depends on how to create resonance at a particular given frequency. That is the beauty of the artist who generates it, and in which order he generates that and the way he generates it. That resonance frequency or the resonance sound he converts it to a melodious music.

The person who cannot generate it in that fashion will create noise. So, the same instrument depending on how you generate those resonance frequency, one is in a position to convert it to a music and another will convert it to a noise, and as far as the noise is concerned, it is totally unacceptable to the human being and may be that the PSB will come into picture whereas, the music he can make meant money because he is trying to entertain. So, many people this is the music part of it. If you come to the construction part of it that wherever concrete is compacted, we use a vibrator there and unless you use a vibrator, you cannot pack it and therefore, the strength of the structure will not come.

If you want to open it up even today in the department, you have seen that they are trying to remove the top layer from the roof top. They are using a vibrator to chip it off. In ship building to generate the v groups for the welding, first they do the tag welding of the two plates and then they used a chisel with the vibrator to create the grove for a multi-pass welding. So, the vibration is used for useful purposes there. It has also got many harmful things and therefore, we are more bothered about the harmful rather than the useful things because anything which is good, we do not try to appreciate it, but anything bad, everybody will come forward to criticize it. So, let us try to see what ship vibration is and how we try to understand it.

Now, once we know that it is bad aspect, then how it is generated and how it can be controlled. So, one thing is sure that ship is made of an elastic material. Any material which poses elasticity has got an intrinsic property that it will vibrate and when it will vibrate is when it is being disturbed. This is an elastic material. If we just leave it as it and do not disturb it, it does not vibrate, but if I try to give some force and try to leave it, give quite tuff here, so it vibrates a little and then stops. So, what happens that once it is disturbed, then only it vibrates and how it vibrates, will depend on its own properties, so that we can try to put it down in a block structure like this or a block diagram.

We say this is a structure and how this structure is defined? Structure is defined by its mass. It has mass and it is made of elastic material and therefore, it has got some stiffness. Being elastic it can observe some energy and being elastic, it has got to give away that energy and what we say is the damping property. So, these are three properties of a particular structure which will define what the structure is apart from its own geometry and other thing, but how it will behave provided your input, the excitation. You excite it and what will happen to it, it will give its response.

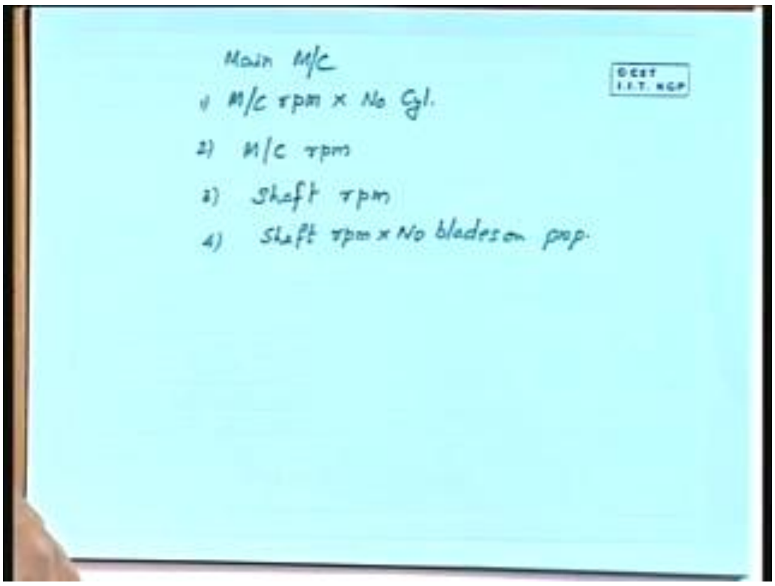
Now, you take any structure and see whether it posses all these properties or not. If it poses this property, then given the excitation, it will respond. How we measure the responses? The usual way is you find out the time dependent displacement or time dependent force. That means time dependent acceleration. Now, when we try to put ship in this block diagram, then we find that ship has a finite, this thing has got some mass. It has got some stiffness because we have already seen the longitudinal strength, and we said that it has got p and I and stiffness at any cross-section is a function of EI and L. And then damping is moving in a sea way its own elasticity plus it tries to displace the water moving up and down and so on. So, it has got the damping of property.

How does it get excited? It has got internal excitation and also external excitation. So, in case of ships, we have excitations which are internal and external. What is internal machinery? Machinery includes not only main machinery, but auxiliary machinery, pumps, gensets, anything you talk about anchor chain and I will put propeller separately, propeller or thruster etcetera and in external, basically it is your wave forces and wind force. Wind is also there. We can say without wind there is no wave, and wind effect is comparatively much lesser than the wave effect. Anyway let us to complete the diagram. Let us put wind and wave.

So, this is basically what we can see here. Now, when we talk about the machinery, let us first see the main machinery. We normally have a diesel engine, internal combustion engine basically or external combustion engine. When we talk about say diesel engine, then you have a diesel engine which has got number of cylinders connected to a crank shaft and through the fly wheel, you take out the power. It may be high speed, it may be slow speed, or it may be medium speed. Now, if it is slow speed, it is directly connected to the propeller shaft. If it is medium speed or high speed, then it is connected to the propeller shaft through the gear box. So, one shaft is coming from the main engine and another one is coming out from the gear box.

So, the power transmission is you have number of cylinders which converts the chemical power to the mechanical power, and this mechanical power is transmitted through a series of shafts and gears or flanges to the propeller. So, the propeller to the water and from the water thrust back to the hull.

(Refer Slide Time: 17:46)



So, in the machinery, you have machine rpm into number of cylinder is one sort of an excitation force. One particular frequency is this, second one is machine rpm. Third one I will say is the shaft rpm. It may so happen that the shaft rpm is same as the machine rpm, it may not. Fourth one is shaft rpm multiplied by number of blades on propeller. So, main machinery generates these frequencies in a similar fashion. The gen set will also generate similar frequencies that the prime movers, this thing number of cylinder multiplied by main rpm. Then the rpm, then again it may have a gear box and then you have the generator, this thing generator shaft rpm.

Then, other machinery which you talk about is your pumps are there. You have a series of them for each purpose. You have fuel pump, you have lube oil pump, you have sea water pump, you have fresh water pump and you have this, yeah you have this sewage pump. So, you have compressors for air conditioning, you may have something else also. Then you have anchor windlass, you have crane operating drums and so on and so forth. So, large numbers of them are there and each one of them operating at different rpm’s and different things. So, internally you have a large number of excitation. Some of them may be very huge in forces which will be transmitted and some of them may not be in a position to transmit that much of force. So, that defect may be localized, but again it will try to vibrate even that local area.

Now, when we come to the external, this thing wave hitting the vessel is one of the largest forces. Now, when it is the machinery forces, then precisely we know at what rpm or at what force the structure is getting excited, but it comes to the external sources and then we actually do not know. Actually here we should write another thing is the blast. You may have underground, underwater blasting and all sorts of things. Enough bomb blast and enemy is firing at you or there may be a collision sort of a thing. So, that impact load, a blast load wave load bend load. So, these are all external forces.

Now, as far as all the external forces are concerned, precisely we are not in a position to determine the type of force the frequency and the quantum. You design a ship and only if the war breaks, then it may be subjected to some sort of a bomb blast, blast load. Otherwise, if it is operating always in peace water, there is no blast. Yes that is the wave effect. Wave effect motion is giving rise to the wave. So, that is the motion effect or wave effect. Whatever you call it slamming force you have say. You are going through somewhere, and suddenly your enemies seize you and try to put a bomb and the bomb blast somewhere close by, or may be that you are going through and there is an underwater submarine is or something which has blasted off. So, you do not know, but you get the impact of that.

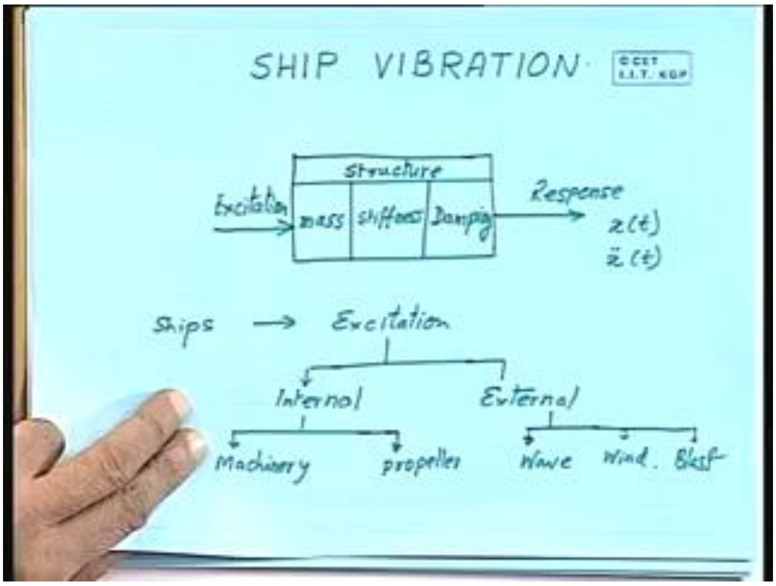
Now, once again when we talk about all these forces, we are talking about the response of a ship which is a huge size structure and we can say it is a continuous structure. Now, let us see in Cartesian coordinate what happens when we take a point, then we say along x, y and z directions, it has got freedom to move. One is laterally, another is rotationally. So, there are three laterals and three rotational. Total degree of freedom is 6. So, any point in a structure has got 6 degrees of freedom to be considered. One particular point and if we consider a beam. We normally consider the beam to be the simplest of the lot of all the structure because all the properties are considered only on a particular line.

The whole structure is replaced by a line. Line does not have any cross-sectional area, but line is generated through a series of infinite points and therefore, the line also contains infinite number of points, and each point contains 6 degrees of freedom. Therefore, even in a beam which is a continuous material has got a large degree of freedom or infinite degree of freedom. Fortunately, we try to analyze such structures by finite element methods, where we say that this beam is considered by replacing with only few points on it. If you join the two points, consecutive points that gives me an element of finite size, finite length and so on and so forth.

So, at this particular line which represents the beam is now replaced by this series of points, where we assume that all the geometrical and other properties are concentrated and each point contents 6 degrees of freedom here. So, we are having say 5 points here again, total 30 degrees of freedom. So, then we try to say that how we try to reduce the degrees of freedom? Out of this 6, when we talk about a ship’s vertical vibration, then we are not considering the horizontal vibration or the longitudinal vibration. So, the two displacements we eliminate. We do not consider the torsional vibration in any of this axis system.

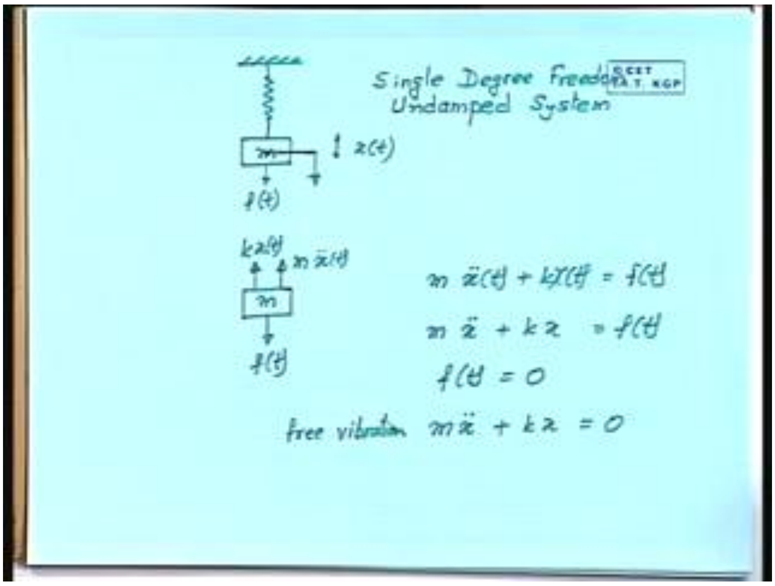
So, from these 3 laterals, we choose only one and from these 3 rotations, we choose nil and then the total degrees of freedom reduces to one at 0.1. So, if you now consider these points, then we are left with say 5 and dealing with 5. Again we find problem. So, what we do that we first consider, what happens if we consider a single degree freedom system? Then try to generate it to 2 degree freedom system and then you make it a multi-degree freedom system. So, what we will try to do is, we start generating our knowledge from a single degree and progress slowly to the multiple degree freedom system.

(Refer Slide Time: 26:33)



Now, once again when we go back to this diagram, we take a structure which is represented by one single point and the displacement will be only one single displacement. That is how we are going to get one degree of freedom. So, there will be some excitation to this one single point or single mass structure, and we get one single response here.

(Refer Slide Time: 27:05)



Now, the easiest way of mathematical modeling is you consider a mass m concentrated at some point here, and the stiffness of the elastic property is depicted by this spring connected to it. So, you consider a spring to that, you consider a bomb, a mass and that represents a single degree freedom system. I put my coordinate system from this mass center downward positive, but the moment is up and down is to and fro motion x of t, and I say that at this point, I apply a force or a forcing function which is time dependent f of t. So, I have not considered damping here and therefore, we say it is a single degree freedom system, freedom un-damped system.

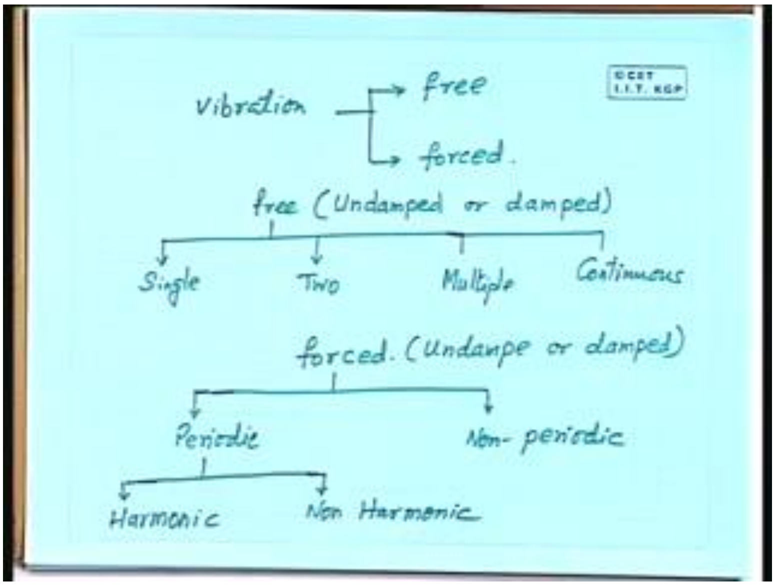
Now, I take the free body diagram of this. If you take this mass out at this situation, there is force acting on it and because of this force, some force is being generated in the spring which we will try to pull it up and that is given by kx and there is a inertia force developed which is given by. So, if we now consider under this set of forces, this mass is in equilibrium and then the equation what we get is. Now, this x we are considering is a time dependent force and every time within a bracket carrying the t with acceleration, and displacement is annoying. Therefore, we drop it and we write it simply as mx double dot plus kx is equal to the f of t.

Now, if we assume that f of t is equal to 0, then the equation boils down to mx double dot plus kx is equal to 0 and this we say is a free vibration case. So, this is the governing differential equation which governs the motion for a single degree freedom and damped free vibration system. There is no force acting. We have developed the equation by assuming the force here, but what we will suggest here is that what I was trying to think once again if you can give me the ripple. I think that is much better. No, that also is not much because the force required is much more. Stiffness is very high.

Now, here in this particular case I can excite it in two ways. One is I give a bending here. I consider this to be a cantilever. I give a bend and I leave it. So, as soon as I leave it, it vibrates and then comes to this stand stiffness. Another is I apply an impact force here and then it vibrates. You can see that it is vibrating and then coming to a steady position. So, what I do at time t is equal to 0, I am applying the force, but after that there is no force. Time t is equal to t 0. I have the force and then I am not giving any force. When we say a forcing function is being applied, then you keep doing like this, but when I am disturbing it and leaving it at t is equal to 0, I am disturbing it and then I am leaving it alone. That is a free vibration here. After that how it behaves. Thank you.

That is why after writing this expression, I say that f of t is equal to 0 and then I say this is the free vibration governing equation. Now, let us see that which are the conditions we will be trying to? I already told you that we will start with the single degree freedom system. We will go to that the two degree freedom system and then we will generalize to multi-degree freedom and that multi-degree if you tend to infinity should lead to continue system, and all the structures which we have basically all continuous. So, we should be in a position to see that how we are going to analyze those structures. So, now the structure can be or the vibrating structures can be categorized under two conditions.

(Refer Slide Time: 33:58)



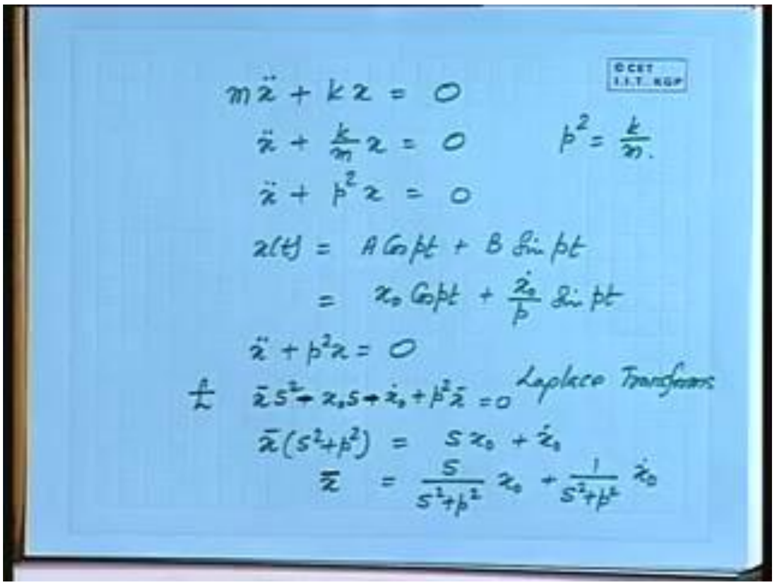
When we talk about vibration, we can categorize under two headings. One is free vibration, another is forced vibration. When we talk about the free vibration, then we have single degree freedom system, two degree freedom system, multiple, continuous and this free can be again undamped or damped. So, you can have one set of free vibration, undamped single degree, undamped two degree, undamped multiple, undamped continuous. You can have another series of damped single degree, damped two degree, damped multiple, damped continuous, right and then forced. When you consider the forced condition, it can be again undamped and damped, but the forces can be one. We say a periodic force and another we say is non-periodic. Under periodic forces, one part can be harmonic and another is non-harmonic.

Now, let us see that how many conditions are there. We have 1, 2, 3, 4, 4 into 2, 8 here. Here, we have 1, 2, 3, 3 into 2, 6 and again 1, 2 multiple continuous 6 into 4 is 24. So, 24 plus 8 minimum 32 conditions we have to find. So, for minimum 32 types of vibrations, there will be many more. When we talk about say multiple degree freedom system, we can have a beam vibration, or we can have a plate vibration, or we can have a shell vibration, or we can have stiffen plate vibration and so and so forth. Now, I have not tried myself nor I do think that many people have tried, but many of these conditions we have to write similar equations, and for simple cases, we do have these simple equations and those governing equation have to be solved to get these solutions.

Now, even if we say that there are say 10 types of equations, which are to be solved, it becomes say troublesome job for us to remember which type of equation will have what solution, and where to find the solution because one of these will be a practical problem at hand. Any one of them we do not know you may say that as he just pointed out that ship is panting, so environmental. So, you are under this category here a non-periodic force. The structure is damped and this non-periodic force you have to generate through the motion of it is coupled along with the motion of the vessel and then you find out that solution. So, you set up a differential equation for this solution of that in a continuous system with damping force than damped property and non-periodic force. It really becomes say troublesome job and once you have written down the equation, then what will be the solution, what is the mathematical solution.

So, what we find that if so many problems are there at hand, and at any time we have and one is suppose to solve all these problems, it is not that you get the problem and you search somebody will solve it for you. You will try to put your own hands to get the solution and get some value to engineering accuracy, right. So, in that case how are we going to handle these cases? You open any book in mechanical vibration; you will find that there is differential equation. There is the solution. Similarly, when a different case is there, you will say this is the differential equation and this is the solution, but are we going to do that or not. So, what we will try to do is, we take up this equation and see that how it can be solved.

(Refer Slide Time: 40:18)

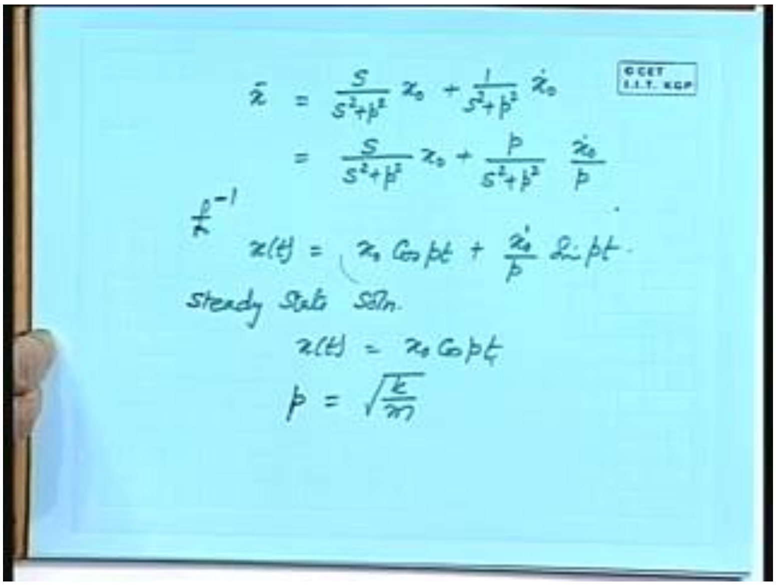


Now, what we will do? We will divide all through by m and rewrite this equation, and we put substitute this value for k by m. So, if you substitute this, we get the solution here. Now, solution of this if you open the book, you will find that x of t is equal to and they will say that A and B constants which are to be evaluated by putting the initial conditions. After you put the initial conditions, may be that you will get the solution like this. That means, a is equal to x 0 and b is equal to x 0 dot by P x 0 is nothing, but initial displacement at time t equal to 0, and x 0 dot is the initial velocity at time t is equal to 0 of that object.

Now, I have written this from my memory that may be you can say that I have mugged up this equation, I have mugged up the solution and I have also mugged up the values of the constants and I have reproduced it, but can I do it for those 24 cases. I do not think that I can do it. Let the simplest case and therefore, I could reproduce it. Now, what is the way in which you can do it? I suppose that we should start with this equation itself and try to solve it. Now, when so many variations are available, the best way of solution is using the Laplace transformation and for this I will refer you that you take any book on Laplace transformations, and try to see what it gives you. Basically I am not very much interested in what are those, but standard cases are to be taken from that table.

So, you will find that there will be one page or two page materials which if you have along with you; you can solve all the problems. So, there will be one operator which is defined as S and the notation for Laplace transformation is, so this Laplace you can say that you get x bar S square plus sx 0 plus x 0 dot plus p square x bar is equal to 0. So, the transformed equation will be something like this and then I will take the x bar here. Sorry, I think this will be minus here and the remaining part I will take it. These are minus here on the other side. Then algebraically I will try to solve it because this is the transformed response and I put it in this fashion, I will reproduce it to start with the new page.

(Refer Slide Time: 45:10)



So, x bar is equal to s by s square plus p square x 0 plus 1 by s square plus p square x 0 dot. This I will do a little manipulation. I multiply by p and then I divide it by p and then I will take the inverse of this. This is the notation and once I take the inverse, I get the solution. So, x bar will give me x of t and this parameter when you invert it, this gives me cos pt. So, I get x 0 plus cos pt, and this is the transformed form of sin pt, so I get x 0 p. So, you see what I have done? I have taken this differential equation or the governing differential equation, I have transformed using the Laplace transformation and I get this algebraic equation.

So, in the transformed response which is x bar, I am only doing an algebraic manipulation here and after doing this manipulation, I bring to this shape here and then I take the inverse which gives me the solution and if you compare this solution with what I had written here, it is the same solution I get. I do not say what initial condition is. In fact, these are the initial conditions I am getting. X 0 and x 0 dots are nothing, but the initial conditions. So, instead of solving for this A and B, I am directly getting the complete solution here. So, this is one way handy. Of course, you may argue here for this simple case that you could have got easily here, and all these also can be obtained from the text books, but I have already said that there are number of cases and each case will not be handled in the text books. Therefore, you will have to refer back to some sort of a mathematical book.

So, instead of that when you are referring it to the mathematics book, why not you try to approach the solution from a unified way and the best way will be used for Laplace transform. You have the transformation tables with you and the inverse of that. If you have that much of material, then you find out your differential equation. Just apply this to this algebraic manipulation, get the inverse of that from the table and you get the solution. Now, once we have got the solution, now what are we going to get from here? So, let us try to study this. This has got two initial conditions. One is x 0, another is x 0 dot and both the initial conditions independently showed you with the refill.

In one condition, I say that you pull it down and then leave it. So, in that case, x 0 was provided to that as an initial condition, but not x 0 dot. You know velocity was given, but I had given an initial displacement at time t is equal to 0 and then I left it. So, if that was the case, then this was the steady state response. In the second case, I try to hit it to give an initial velocity, but the initial displacement was 0. That time this was the solution. You may have a situation where you have given both these initial conditions. That is also there which I could not show you, but these two parts I have shown you and simplest of the lot was you just pull it, and then leave it.

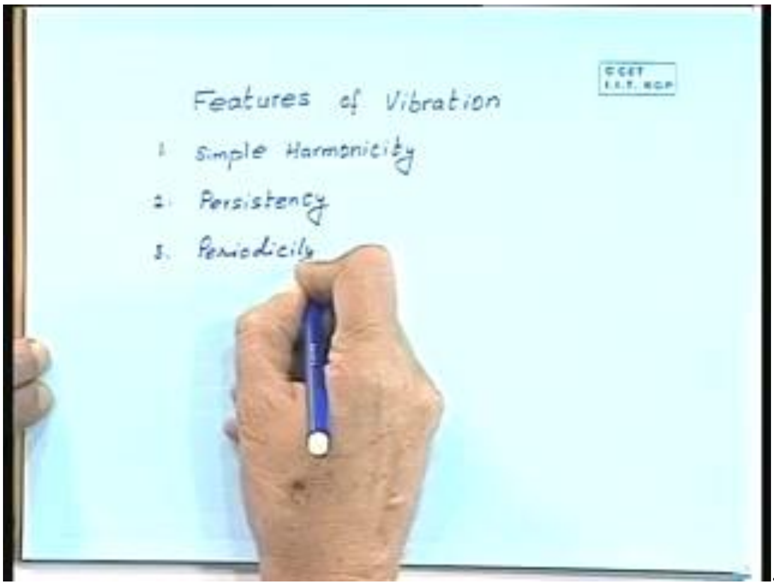
So, we can also say that steady state solution if we want to write, we can say xt is equal to x 0 cos pt. Even if you take this, there is no problem. Sin p and cos p, just there is a shift here and both of them behave in the same fashion. So, what does this equation tells you? This equation tells me that once the structure vibrates, it will keep on vibrating for an infinite time period. There is no stoppage. You start giving this t value anything, it will keep on vibrating. So, once it has been disturbed, it will be in that mode throughout, but actually speaking it does not happen. What we have seen that it stops after few cycles and that few cycles after which it stops is because of the intense damping property that material has got.

So, the thing decays and then it stops, but in the idealized condition, the equation tells me that it will continue indefinitely. That means persistency is there. It persists. There is some fixed amplitude x 0 and with that x 0, the whole thing will start vibrating. What is p? P is the cyclic frequency and p is given by root over k by m. We have assumed here that p square is equal to k by m. What we get in this solution is p is the circular frequency. So, p is given by k by m. Any other property can you think about from here? Sin curve you shift it, pi by 2, it will match with the cos curve. The same thing, you know it is the same thing whether it is sin curve or cos curve, it only shifts of pi by 2 pi pi radiant. So, this has got a circular frequency given by k by m.

Now, from this what we can find out that frequency is governed by the stiffness and mass. Now, when we take a structure and we find that particular frequency is not suitable for me, we said that excitation can be all those. Now, those structural items or that machinery, etcetera for the ship are all bought out items. So, you do not have any control in that, but in the structure which is being designed by you have certain control by manipulating k and m, and you know that if you increase k, the frequency will increase. That means, if you make the structural stiffer, then the frequency will increase. If we make it heavier, then the frequency will come down, but it so happens that when you make it heavier, the strength also increases.

So, it depends on how you dispose the material, how you place the material, so that you can make use of this property. This is what normally we try to do. So, this gives us that type of a clue that the structural frequency will vary on the basis of k and m. Now, we are basically not interested in increasing the structural weight. So, we will not touch the material. What we will do? We will try to change the stiffness. How the stiffness can be changed? Stiffness can be changed because we are using in ship structure stiffened plates mostly. So, the disposition of the weight you change, you play around with the size of the stiffener, the spacing of the stiffener, the positioning of the stiffener etcetera will change the vibration property of the structure. It is I think with this I have introduced what is vibration. Expression for single degree freedom system of vibration we try to see the response which is your x of t is equal to x 0 cos pt plus cos 0 dot by p sin pt.

(Refer Slide Time: 55:23)



Now, what are the features, we will see from here. Now, those features we can list it in this fashion. Number one is simple harmonic motion that is to write it simple. Harmonicity is depending on what are the initial conditions, whether you are having initial displacement present or initial velocity present. One of them is usually 0, either x 0 is 0 or x 0 dot is 0. So, the response will be given by say x 0 cos pt or x 0 dot by p sin pt. Now, this cos term or sin term, they are both simple harmonic motions and therefore, we first of all see, that this particular motion is simple harmonicity. It possesses simple harmonicity.

Second is persistency. Now, from this expression, we do not find that if suppose we say that x of t is equal to x 0 cos pt as t keeps on increasing, the value will fluctuate from plus x 0 to minus x 0 and it will continue for t is equal to infinity. There is nothing which we try to decay this function and once the vibration starts, it will continue forever that what the equations says, but actually speaking we do not see this. There is a simplicity in this because we have assumed there is no damping in the system, but every physical system will have some inherent damping. And due to that what we actually see is that once the system is disturbed, it does oscillate, but the amplitude decays and within a finite time period, it comes back to the stationary condition. That is because of the inherent damping feature of the system, but mathematically we have ignored for the time being.

Therefore, what we are saying from the expression that once it is disturbed, it will continue because the energy has been put and there is no means by which the energy can be dissipated or taken out from the system. So, it will keep on going. Then we also find periodicity.