Strength and Vibration of Marine Structures
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Lecture - 22
Calculation of Deflection / Shear Stress

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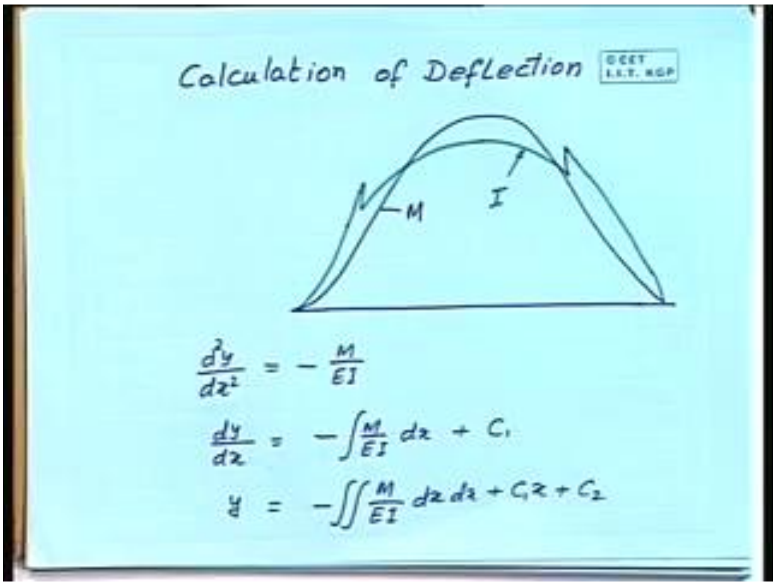

Calculation of deflection to start with now last time we have seen one thing that the bending moment curve was something like this, and depending on the shape of the hull, we may find that the moment of inertia distribution along the length will not be a very smooth curve, but it will have some smoothness, and then depending on some abrupt change. That means, as this I am trying to depict that. Suppose there is a poop, then suddenly when the poop deck is over, then it comes to the main deck and therefore, there will be a drop in the main sectional area. Then, when it again goes up and then let me continue and say that it ends up like this.

So, this may be we can say that this is I variation along the length whereas, this is nothing, but the bending moment curve. Now, the differential equation which we will be using is d 2 y by dx square is equal to minus M by EI depending on our sign convention, and we have seen that if we are taking a sagging bending moment as positive concave up, if we consider that bending moment to be positive bending moment, then the curvature works out to be negative and that is how curvature is nothing, but this d 2 y by dx square. So, M by EI with a negative sign gives us the curvature. So, this is the moment curvature equation and therefore, when we try to integrate it twice, the first integration gives me dy by dx minus M by EI integral. This is dx plus a constant of integration C 1 and the second integration will give me y.

Now, where C 1 and C 2 are two constants of integrations and they have to be evaluated by considering the end conditions, what we find that we have to integrate M by EI curve. For all practical purposes, we consider that the ship is made of one single material and therefore, E is constant all through. We may use two different steels. For example, mild steel and high tensile steel, but if you see the mechanical properties or the elastic properties, you will find that more or less E is constant. It is the tensile strength which varies in two materials, but E is more or less constant. So, even in such a condition, we may assume that E is practically constant. Now, if that is the case, then this particular expression which we have just written will boil down to y is equal to minus double integration 1 by E. I take it out.

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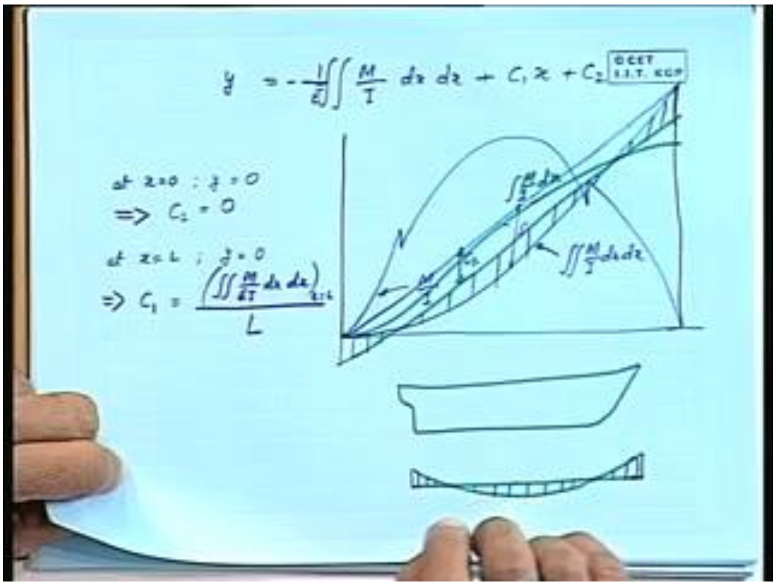

This becomes M by I dx dx plus C 1 x plus C 2. So, what is required is we have to integrate M by I curve. Now, if we see this variation of M, M is this curve and this is I curve and therefore, I have to find out what is M by I curve. Now, for ships, basically for naval architectural purposes, neither M can be expressed by mathematical expression nor I can be expressed as mathematical expressions over the entire range. So, what we have to do is, we have to consider various sections at convenient points and then evaluate what is M and what is I and take the ratio and construct a curve exactly like this.

So, you evaluate what is M at that particular point, what is I at that particular point and find out the ratio M by I and then plot the variation. So, let us try to do that and once we do this, we get a sort of M by I variation, something like this. I am taking a general curve here not too much of variation. So, say this is your M by I curve. So, once we have the M by I curve, we can now try to integrate and once we try our integration scheme, we find that this integration, this is the first one which is M by I dx. Now, this integration we do in the numerical fashion. That means, you find out up to certain areas and keep on adding it and this curve you will get.

Now, what you will find here is that the tangent is increasing till a particular point, where the rate of additional area is increasing and then suddenly the rate of increase is reduced and therefore, you will find that the tangent starts going down like this. So, there is a point of inflection somewhere here, where the maximum ordinate is there. Now, integrating this, once again we get what is your double integration curve M by I dx dx. Now, for convenience sake we try to join these two lines, these two ends of the curve. I thought that not touching the point because both are drawn in a different scale. It is not touching. Actually I am trying to join this and point to this and point with a straight line, and try to measure the displacement with respect to this in this direction at any point.

Now, we go back to this expression here. Now, in this expression we said that C 1 and C 2 have to be evaluated. Now, if we construct the displacement curve in this fashion, then we can define our boundary condition and the boundary condition says that at x is equal to 0. That is this point, the displacement is 0. This leads to that you put x is equal to 0 here, then C 2 works out to be 0 and then when you come to this particular point, then we have the condition at x is equal to L. Once again y is equal to 0 and this leads to after putting the value, this is 0. This is already 0 here. So, what we get is C 1 into L is equal to this integration. So, from there C 1 is equal to minus I will put EI here itself at x is equal to L divided by L.

Now, this is equal to say that C 1 is a coefficient which is nothing, but the mean value of this displacement curve. Double integration at x is equal to L divided L that practically tells me that C 1 is nothing, but the mean value of this displacement curve. So, if I try to place a reference line which is C 1 away from this particular straight line which I have drawn, say let me draw a parallel line. So, this I say is C 1 away, and then this should be my new base line for calculating the displacement. So, that means, you can measure the displacement in this fashion. Now, let us come to the solution of this. We say this is the ship which we say is replaced by a line and the displacement of this which is free at both the end is something like this.

Now, if you compare this displacement curve with the curve here, they are more or less of the same nature and therefore, we can say that by evaluating the values in this fashion and trying to shift the base line or the datum line for the measurement of the displacement, we are trying to get what is the displacement. Now, many a times this is a debatable thing that whether this is going to be neutral, axis is going to be at the mean level. No, it is not. It only gives you a representation. That is not the exact value and therefore, you will find that the displacements and seldom given in absolute manner rather it is given as a fraction of maximum displacement. So, wherever the maximum displacement is there, remaining displacements are given in terms of fraction of that maximum.

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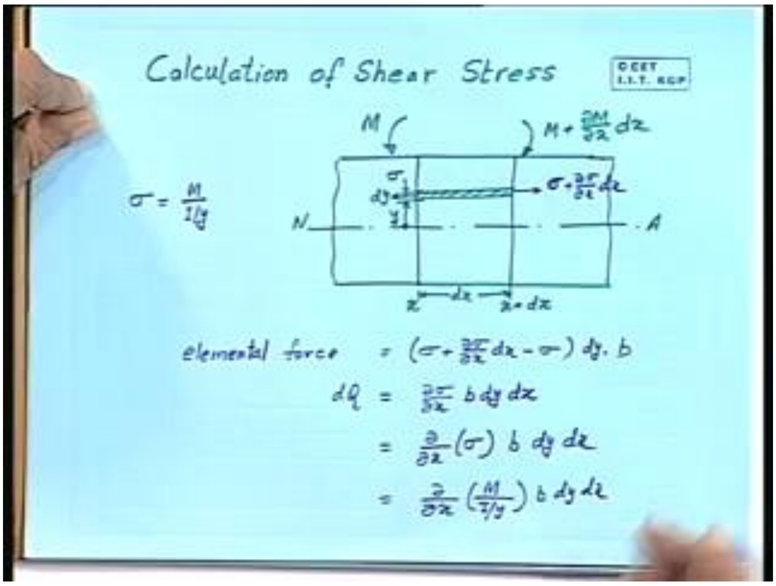

Next small topic we go for the calculation of shear stress. Now, we have already drawn five curves that is the load curve, the shear force curve, the bending moment curve, slope curve and the deflection curve. Apart from that we have already drawn what the weight curve is and the buoyancy curve. After doing all these, in between we have drawn what is shear force curve. So, that means, at any cross section of the ship, we are in a position to know what shear force is. Once we have that shear force along the length that divided by the cross-sectional area which undertakes the shear, we get what is the average shear stress at that particular section.

Now, shear stress does not give us that how the stress is varying across the section. We only get a measure of average stress. Now, that average stress is from one point of view. That means, simply have some idea, but which section or which element under that section is strained or stressed heavily that idea we do not get. Now, if you go back to the riveted ships, you will find that towards the middle streak, there is always use to be two rows of rivets. In the shear streak also, there use to be one double row of rivets and so on so forth, but today the things are all welded and welding is something like gluing and tearing. That is all.

The technology has become so nice and in fact, at the welded joint you can have the efficiency more than 100 percent. In fact, seldom you will find that the joint fails at the weld line, rather it is slightly away from the weld point. That means the welded line or at the welded joint, the efficiency is more than 100 percent. There can be two reasons. Now, one reason is very simple that at that particular section or at the joint because of the metal deposit, the cross-sectional area increases or if you say that there is no increase in the cross-sectional area, then because of the heat effected zone in the neighborhood, the mechanical properties have deteriorated we can say. So, what we are interested in is how the stress, how the shear stress is across the depth of the cross-section. So, that will be very interesting.

So, let us say that we try to depict a cross-section here. Some length we take representing the ship, we say that this is the neutral axis and we consider two sections. One somewhere here, another somewhere here which we say 1 is at x 0 is somewhere here 0 to x and another one is xx plus dx. That means this distance which we are considering is dx apart. We say that the bending moment applied here is M, obviously at a distance dx this will be M plus del M by del x into dx at a small. We consider a small layer of thickness dy at a distance y from the neutral axis and try to find out what is the stress here. So, in this horizontal layer, we assume that the stress at this end is sigma and stress at this end is sigma plus del sigma by del x into dx. So, this is nothing, but the bending stress which is due to the bending moment applied.

So, the next force in this particular element, elemental force is equal to sigma plus del sigma by del x into dx minus sigma multiplied by dy and the width across this which is let us assume is b. Now, this force we can write as dQ and these two sigmas will cancel out and therefore, this is del sigma by del x b dy dx b is the width of this across or transfers width. Now, we can rewrite this in this fashion here where sigma is the bending stress. Now, we also know that what sigma is. Let me write here sigma is equal to M by I by y. Now, substituting this value here, we get the expression in this fashion or the elemental force.

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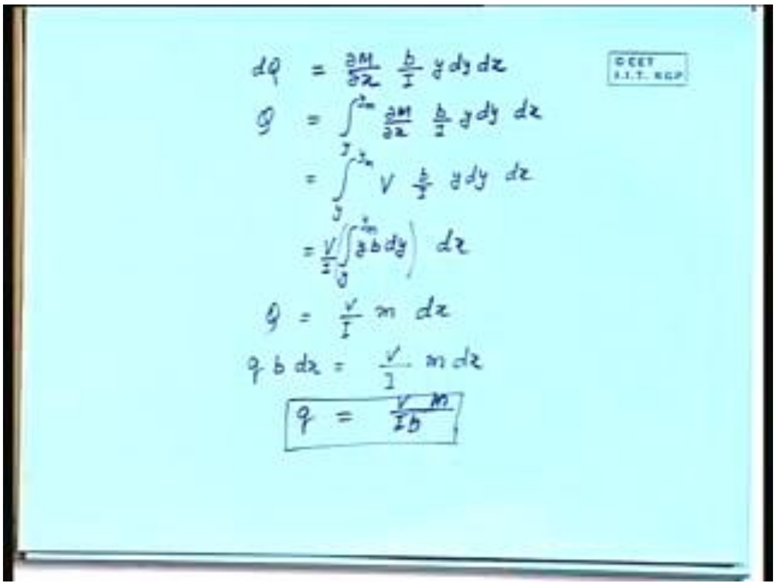

Now, this elemental force I is not a function of x at that particular point, but what we are trying to say is the variation along the depth we are concentrating at a particular cross section. So, we are keeping the x fixed. What we are interested is along the depth this is our y here, right. So, at that cross section I is constant y is there. So, we remove I and y from there, and this differential will operate on M. So, that will become b by I y goes up. So, this is the force in this element here. Now, if I want to find out that what are the other elemental forces above this what is the sum total of this. That means, this has to be integrated from here to here.

Now, that means, I have to integrate it from y to this extreme position which is say y max and therefore, Q is nothing, but integration from y to y M del M by del x b by I y dy dx. Now, what is dM by dx? It is the first differential of moment that is shear force. So, let me write it V and b by I. I keep it as it is y dy dx. Now, what is b into dy? It is the elemental transverse area of that layer dy. If I go along this axis here, if I try to draw it like this, this is b and this is dy. That means this area I am talking about and from y to yM is the entire cross-sectional area here. So, this is the meaning of this b into dy, but what is b into dy into y is the first moment of this area about the neutral axis and that I am integrating right up to the maximum.

So, that means, the first moment of the area of the cross-section from y to yM. So, I put it in this fashion. V I take out, I take out and I put bdy and then dx here and this entire quantity I put as first moment of area. I am giving this notation. Now, what is this Q here? This Q is the horizontal force. If you not try to say it is the horizontal force at this layer between this. Now, what is that if I say that this is the sliding force at that particular layer of length dx width b? So, at this interface the cross sectional area is b into dx. B is the width, this is b and this is dx. So, this is this area b into dx and if the stress intensity there it is this stress. So, it is a horizontal shear. See if I say that q is the stress small q. So, q into b into dx which is equal to V by I m into dx and if I remove this dx from both sides, then I can write this q is equal to V by I m and I put this b here. That is all. So, this becomes the expression for the horizontal shear stress.

Now, from the elementary strength of materials, we know that whenever you take a section, the horizontal shear and the vertical shear always appear in pair, this stare and this share and therefore, this Q is also the vertical share at that particular point. So, now from here one can find out I have to go back to the same diagram V into m by I by b, where the maximum shear stress will occur at the neutral axis. So, at the neutral axis since the shear stress always appears in pair, this is also the vertical shear stress. This I have already told you and now one thing is to be noted here that this expression which we have derived is an approximate method because to start with, we have assumed that there is no shear stress at this section because we based our calculations only on the basis of the bending stress here and then we have tried to prove that there is a shear stress.

So, therefore, if you go for a very regress calculation, then we must go from the theory of elasticity point of view which we have made. Now, let us try to see that how the shear stress varies in a cross-section, which is normally used in ship building. Let us take I section sort of a thing.

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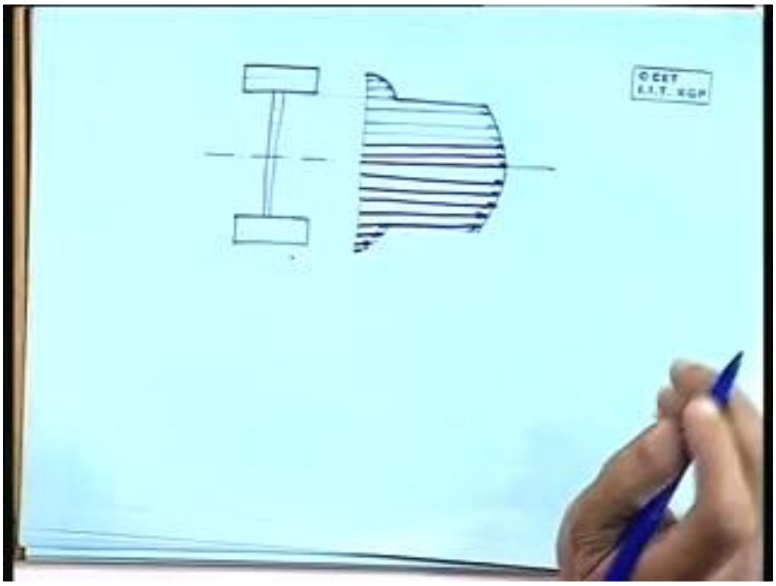

I am considering a symmetrical section. Neutral axis is somewhere here and I will draw the shear stress on this side. Now, try to see this expression. V is constant at this section at any length. Suppose this is the girder and a cross-section, the bending moment is fixed. That means, a shear force is fixed at that particular place I. I am considering a uniform cross-sectional I beam and therefore, the moment of inertia of this is its neutral axis is also constant at that particular section b. B changes from this place. It is this b and as soon as you cross this junction, this becomes b and this continues till you come to this place and then this becomes the (( )).

So, when you try to find out what is the stress here, the stress here will be 0 and it will increase parabolically because what is ydx dy yd d dy integration. Now, dy is dy. So, that integration will give you y square by 2 term y into b into dy. Integral from y to ym will give you y square something. So, y square means it is quadratic ny. So, quadratic ny will give you a parabolic distribution. So, from here to here, this will be a parabolic going like this. Then, suddenly b reduces. So, if b reduces, there will be a jump in the stress because it is in the denominator and we can say that this jumps up here and then again it will go like this and then it jumps down and then it becomes 0 at this side. So, this is the shear stress variation across the depth for I section (( )). So, this is the variation which we get.

Now, area under this curve should be the total shear force. Now, what we find that this part, this is the flange here. Contribution towards the total shear force is only this much of this part. This flange will take up this part and this area which is the major chunk of the area or shear force is being taken by this thin web here. So, we conclude that practically all the shear stress is being borne by the web of the cross-section. The flanges do not participate much here. It is the web which takes the practically more than 90 percent of shear force. Now, in a ship's case which are the sections to be considered as web? It is only the side shells and any other vertical member, may be a little part on the deck, and inner bottom and outer bottom, those stiffener webs, but their contribution because they are far away from the mutual axis will again be less. So, it is the side shells and if you have a longitudinal bulkhead which continues will shear the entire part of shear force there. Now, influence of shear stress on bending. So, this is the second, this is the next topic we will try to take.

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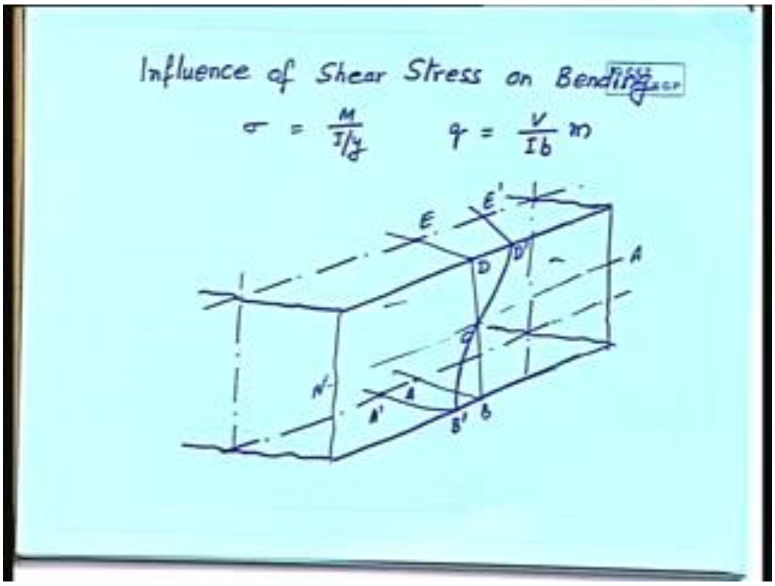

Now, we have seen that in the hull girdle or girdle in general, we have the bending stress, we have the shear stress. Now, how does this affect the bending deflection? Now, when we try to see the ships structure, it has got a very complexion because the shape is designed from minimum resistance point stability aspect safety aspect and what not. We have given a very complex three-dimensional shape to the hull and then that hull we are trying to give a structural envelop by plates and stiffener, so that it retains that shape and keeps the sea above out and the cargo in. That is the basic purpose.

Then, it is hollow in nature. The cross-section is varying from section to section from one end to another end and therefore, we cannot consider it to be a uniform being, but to get the overall behavior of it, we really consider it to be a hull girdle, a girdle of variable cross-section and a hollow girdle. Now, if you take a simple structure which is hollow in nature, say for example, a pipe or a square of tube. Basically a tubular structure, a thin section here also it is a thin shell structure. Then, as it is the structure possess a lot of challenges to the analysts because of the complexity of its nature. On top of that and ships structures, we are making it more complex by giving a complex shape to it and from stability point of view, we are also playing with l by b ratio b by d ratio l by d ratio and so on so forth.

Now, that also creates some sort of a problem. You have certain opening. You try to close those openings, make it if whether title like hatch openings, some man holds even this and that. Whatever you do that when it really goes in a sea way, then all sorts of motions are there and the deformation really takes a very complex shape. Now, in that case we talk about simple part and see that how simple things try to complicate our calculations, and on top of that the ship itself is a very complicated structure. So, what we can do is only try to guess that how the effect comes. Sometimes we only depend on the qualitative effect, sometimes we try to quantify it and in that quantification, we make a lot many assumptions and then we say that it is within the engineering accuracy and so on so forth.

So, let us try to see that what are the effects or what is the effect of shear stress on bending of ship girdle? So, first of all we will try to use an approximate method to study this behavior and this was developed by J. L. Taylor. One second the theory assumes that the hull girdle is subjected to a stress bending stress given by M by I by y, and there is a shearing force which we have just calculated q given by y by Ib into m. All these terms are defined. Now, the effect of this shear stress on the bending stress and deflection is to be computed. Now, let us try to assume a portion of the vessel. So, let us try to draw here a box to depict our ship structure. Let us consider a section here somewhere. Let the neutral axis be somewhere here. Let us name this half section A here, B here, C at this point, D here.

Now, we are considering this section before bending and after bending, we find that it takes a shape like this with addition of the forces. For the bending like that I have taken a box here itself it is looking a little complicated now because it looks little complicated. So, what I will do?

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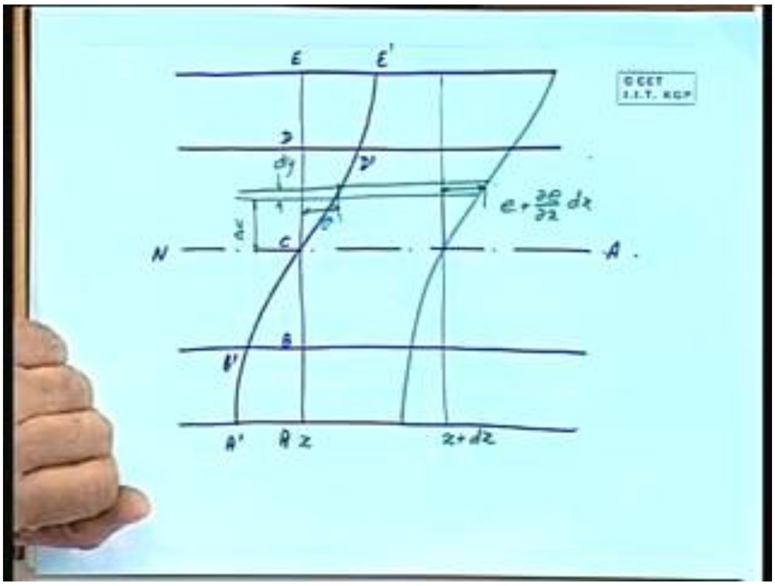

I will try to open it up say this is the neutral axis, and I open up this from central line to central line. That means, in the section which I have considered is and what you are saying there. So, when you try to open up this cross-section, you put it on a plane. This box will look like in this here. Let us consider another section in a deformed condition somewhere here from the neutral axis y away. We consider again a small slice here, and let us assume that this displacement in the longitudinal direction is e, and it is section here. We consider this to be x. This section is at x plus dx.

Now, what I am trying to tell you is that when it undergoes bending that cross-section will definitely twist like this. Now, usually what we assume for standard bending case that the normal at a neutral axis before bending remains normal and straight. Even after bending to the bend surface or the bend line, it will be I. Anyone of you can give me the scale. It will be better if I have a riffle. Somewhere if there riffle can be opened, then I can, yeah you see this I consider as a B. So, this line is given as normal here. So, what I will say that this is the normal here. Now, if I try to bend it say I have applied some bending movement and this as bend which was normal here, when you bend my assumption says that it will be normal to the bend surface. So, I also say that it is straight and normal to the bend surface. So, this one rotates like this. This is straight and went according to this here.

So, if I join this, this is the situation, but actually it does not happen like this. What happens is that this normal gets distorted in this fashion. The assumption which we make is that the normal remains normal before bending and even after bending. Not only normal, it remains straight line also. So, if you consider this to be normal here after bending, this should be the shape, but if I bend it like this and if the bending line goes like this, it may not be normal once again and also it is not straight. So, this is the actual shape. This perpendicular will attain after it has gone bending deformation, but of course in some theory we try to say that we join these points, and we assume you as you can see here.

This is a straight line and we say that instead of taking of this curve, we will try to represent that by this straight line, but it is no more normal to this, but it is still tilted. That is a different thing all together, but actually speaking this will be the shape. So, we go with this shape and see what happens. Now, because of shear force, this had happened and therefore, there will be a shearing strain. At this layer at this position, we say that there is a shear stress. So, if there is a shear stress, then what is shearing strength?

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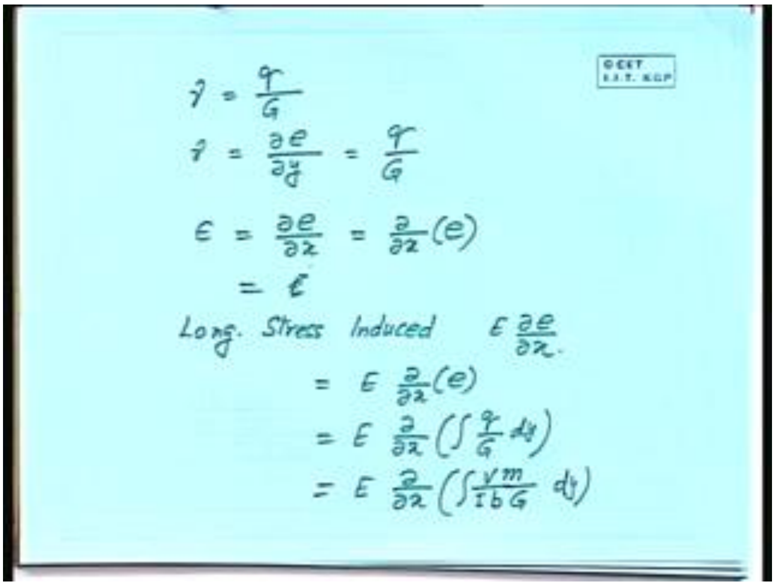

So, we say that gamma is the shearing strength which will be given by the shear stress divided by modulus of rigidity G. Now, what we are saying that because this is what happens here. The normal gets shifted and this is the displacement here, and this is the displacement here. So, if this is going to be the displacement, then this strength should be written as this is the rotation which we are getting along the y axis. So, d epsilon or de by dy should be the shearing strength which now considering an adjacent section here, we find that there is some change here about the longitudinal deformation is nothing, but the longitudinal deformation. So, there is some change here.

So, we can write that epsilon that is the longitudinal strain is given by de by dx. See this is the displacement here. So, net displacement is e plus del e by del x into dx minus e and that is nothing, but that is over a distance dx divided by dx. So, what you will get is, d del e by del x and that is the longitudinal strength, right. Now, this we can write it in this fashion here. So, this we can write it in this fashion and what we can do from here is we substitute the value from this expression here. So, how do we try to substitute the value? From this expression, this is del e by del y is equal to q by G. So, if you integrate this you will get dy here.

So, what we will to do here? Let us put it in this fashion and then we will say neglecting the transfer strength, there will be a longitudinal stress induced due to the distortion of an amount. Let us first write out the longitudinal stress and then this here longitudinal stress induced is equal to E into longitudinal strength E epsilon that is de by dx. Now, here I will try to substitute this value and this stress value q by. Now, once again what will do here is q is already equal to V by Ib into m. So, you put V by Ib into m. There will be G here and then comes dy. So, now after putting substitutes this value, now what is a function of x here and what function of y is here. So, what we can do here now this shear force is a function of x, these are all function of y, m is a function of y. So, what we will do here?

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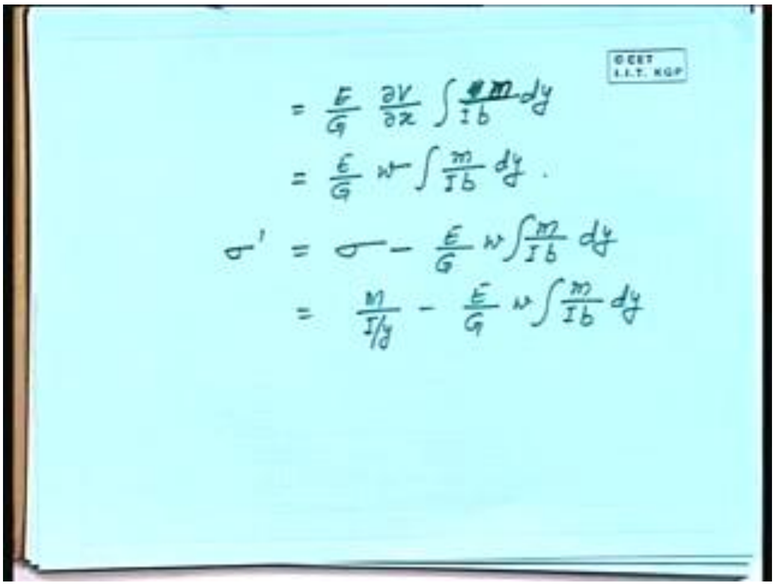

We will try to put this equal to E and G we take it out. This is constant. So, we take out and then we say that dv by dx integral V by Ib m into dy. Sorry, V has gone. That is only m by Ib. V has come out and what is dv by dx is rate of loading. So, rate of loading normally is used as w. This is the induced longitudinal stress, and it can be shown that this will be often opposite nature to the usual bending stress at that particular layer because when you are trying to pull it this way, there is a force tension here. So, tension means this upper slide will go down like that and the lower one, the stress is in this direction. So, if this stress is in this direction, then obviously this stress in the lower fiber is in the opposite direction. So, the net bending stress sigma dash will be usual bending stress minus this quantity.

Now, if we try to put the values here. So, this becomes the expression for the modified stress value. I think I stop here for the time being. Then, this is replaced by two nodes at the two ends, and the properties of this structure whatever is used for the bending analysis or for that type, structural analysis will be considered at this and this point only. So, if this is the cross-sectional area, so we say at this point that the beam has got this much of cross-sectional area. Whatever is the moment of inertia, we will say that this beam has got this moment of inertia at this point.

If you want a horizontal bending also, then you say that the beam has got central line moment of inertia at this point is so much. If you want to do a torsional analysis, then also you say that this is a torsional rigidity at this point, and these are two points so many centimeters or so many meters away from each other. Whatever force you apply here that equivalent force you find out at this moment point. So, if it is loaded uniformly, then this will have half the load coming directly here and effect of this as a bending moment here. So, that load will be broken up into the components as direct load bending moment may be shear force may be a twist or whatever it is.

So, you have a method to calculate that and apply it at this node, and like that computer can handle thousands and thousands of these points. And once the capability of once calculating effort has increased enormously, so one can break it down to this level and start it, but again when you come to the ships breaking down to this level, again poses a problem. And therefore what we say that we make a course mesh and in the course mesh many such small items are added up together. Therefore, we do a course mesh analysis and then from there we take this panel out and then apply the forces. What we calculate at these points are the displacement and the forces and the moments. Now, those things become the applied load.