

Strength and Vibration of Marine Structures
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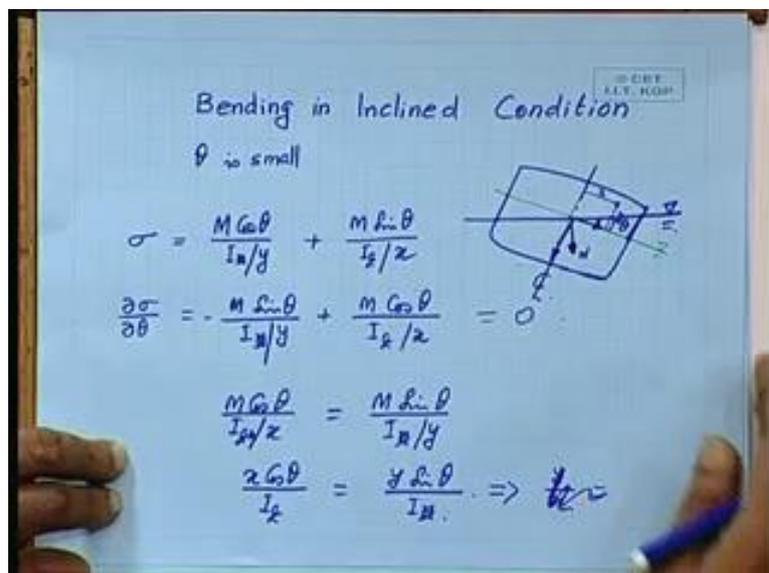
Lecture - 21
Bending in Inclined Condition

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Yes, that is being monitored just like halter in the heart, lap tops. Okay now this is the expression for the stress here.

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Now, this is stress under the bending condition in inclined condition also. Now, what we will try to say is from the definition of sigma, we say that the neutral axis is the place where there is no stress. Now, under the heeled condition, this is the stress. We have expression for the stress we are getting. We would like to know that at a particular heeled angle, what the position of the neutral axis is. So, for that we should find out where the maximum or the minimum stress occurs. So, let us try to find out $\frac{d\sigma}{d\theta}$ and this gives me $M \sin \theta \cos \theta$ will give me a negative sign, and this I will equate to 0.

So, let me write down $M \cos \theta$. I go in a very lengthy manner of writing the thing, so that ultimately we do not make any mistake and goes off. Let me write $x \cos \theta$ by ICL is equal to $y \sin \theta$ I midship. Now, x and y , I will consider here, right. So, from here let me write down y by x y by x is equal to or x by y . Should I write x by y ? I am sorry. I have done a mistake here. Let me rewrite the whole thing.

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Bending in Inclined Condition

$$\sigma = \frac{M \cos \theta}{I_R / y} + \frac{M \sin \theta}{I_L / x}$$

$$\frac{y M \cos \theta}{I_R} = - \frac{x M \sin \theta}{I_L}$$

$$\frac{y}{x} = - \frac{I_R}{I_L} \frac{\sin \theta}{\cos \theta} = - \frac{I_R}{I_L} \tan \theta$$

$$\tan \phi = - \frac{I_R}{I_L} \tan \theta$$

for $I_R = I_L$ $\tan \phi = - \tan \theta$

Inclined condition, I repeat writing here once again. Now, that part will come later on to define what the neutral axis in the inclined condition is. Sigma has to be 0 at that particular point. So, keeping sigma is equal to 0. I get $M \cos \theta$ into y by I midship is equal to minus of x into $M \sin \theta$ divided by ICL and goes off from both the sides. Then, if I consider a point somewhere here which is x away, and let me draw the neutral axis. Also, this is y , this is x and then I should have y by x here.

So, y by x gives me I midship by ICL and $\sin \theta$ by $\cos \theta$ which is equal with a negative sign I midship by $ICL \tan \theta$. That means if the vessel is inclined in this direction, the neutral axis moves in the opposite direction. So, that what this tell y by x is nothing, but y by x if I say this is $\tan \phi$, I can call it. So, if ϕ is the rotation of the neutral axis in the inclined condition with respect to the original neutral axis, then that will be given by this and it moves in the opposite direction of θ . That is what this expression tells me.

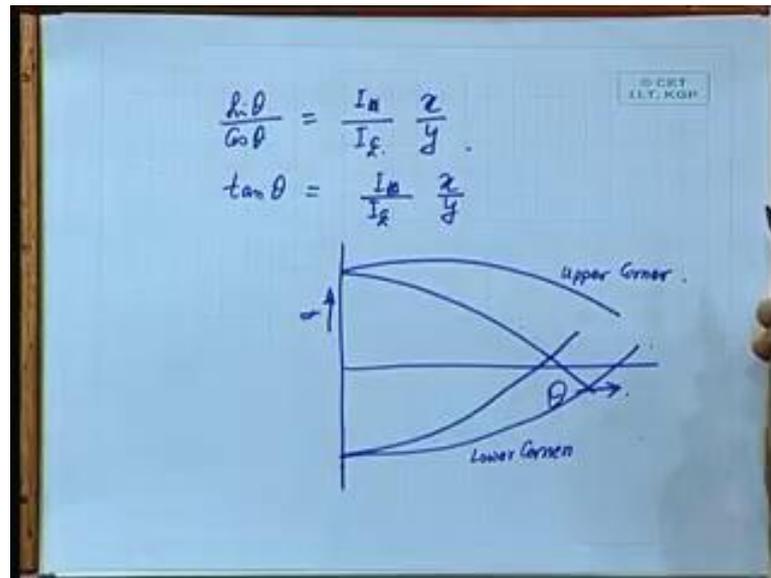
Now, as a special case, let us see I already said that ICL is more or less four times the I ϕ , but suppose let us consider a case where the ship is just like a square box, and we say that ICL is equal to I ϕ . Then, what happens for I midship equal to ICL is that you will find that $\tan \phi$ is equal to minus $\tan \theta$. That means if the vessel has rotated like this, the axis applied condition was INA has rotated like this. It gets back to that is you know.

So, with respect to that again it becomes horizontal. I suppose you are getting it. Tough, say this is a square. If it is floating in this condition, the neutral axis is horizontal. Now, I try to rotate it and it floats in this condition. Now, if it is floating in this condition, then according to this formula $\tan \phi$ is equal to minus of $\tan \theta$. That means this angle is θ . Now, in this condition again I will say that this is the neutral axis because this part is identically equal to the lower part. So, that means, with respect to this my ϕ has rotated by minus θ and then it becomes horizontal.

So, for a square case, horizontal line is again the neutral axis that for this expression tells me that if I midship is equal to ICL which is only valid for a square, then $\tan \phi$ also to be minus $\tan \theta$. That means if I rotate by θ , the neutral axis gets rotated by minus θ , but then a ship is not the case where ICL is much larger than I midship and therefore, it will be a different angle altogether, but it will be away from that.

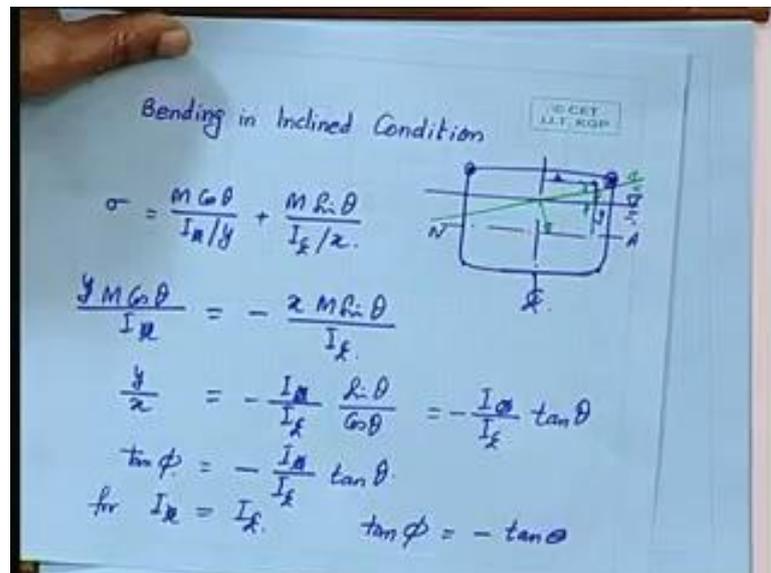
Now, once we know that this is what happens, the second stage will be at what θ angle, the maximum stress will take place and for that we have to differentiate σ expression with respect to θ , equate it to 0 and we get the maximum value or the minimum value. So, after doing that we will get here the condition, where we can write x by y or y by x . Let me write x by y or we can write it in $\tan \theta$ mode.

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Sin theta by cos theta from this expression, what we get here is sin theta by cos theta is equal to I midship. You just check if I am making any mistake [FL]. Correct me. Sin theta cos theta. So, I midship by ICL x by y. So, that means, tan theta is equal to now for a ship let us put this diagram here or this diagram.

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For a ship if we choose a particular point, let us take this point here. This quantity is constant for a given ship I midship ICL because the ship structure is already made. So, this is constant. Now, what we can vary is x and y position. Now, from here we can see

that x if we take it to the extreme right here for a given θ , then the stress increases, either increases or decreases. It is this θ is from maxima or minima. So, if you take x to one extreme, it increases. You take it to the other extreme, it decreases. So, if I say that increase means tensile stress, so plus value. So, you keep on going to the plus direction or if you go to the minus direction, it will decrease.

So, that means, the point is driving meet to a corner here. It is again driving meet to a corner here and again driving meet to a corner here and yeah, it is again a function of y also, right. So, in the upright condition, the stress was at the crown here at the keel here. The maximum stress was either at the deck or at the keel and deck also, not anywhere, but at the farthest point which was the crown of the deck because of the camber. So, it was more or less at the central line of the ship on the deck and the keel, but now the point is shifting from the central line to the extreme ends. So, either it will be at this corner, or this corner or this corner or this corner. That is what I was trying to tell you that once again if we write the same or we go back to the same and you say that your x is positive, y is positive, θ is positive, and then σ is the maximum.

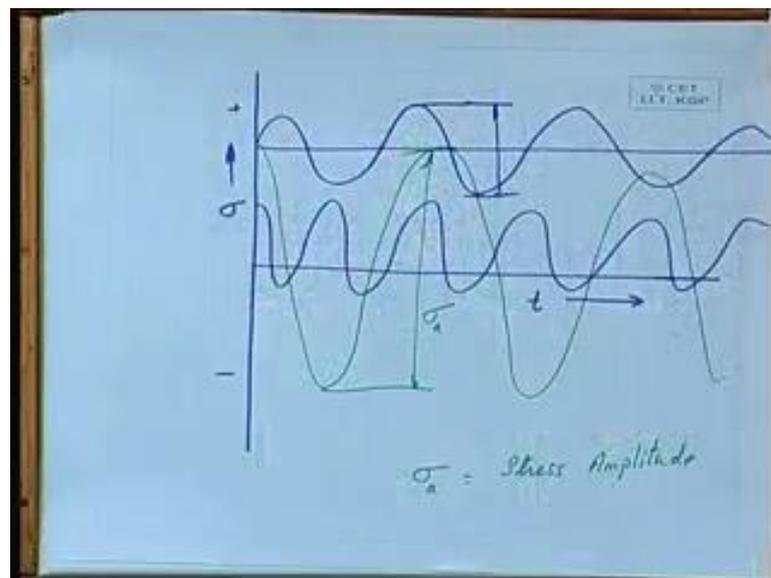
If you say x is negative, y is negative, and then σ is the minimum. Minimum means it goes to the other extreme. So, in one case if it is tensile force, tensile stress, then in the other case, it is the compressive stress and other cases will be either x is positive or y is negative or y is positive, x is negative. Then, the situation is something like this. It is floating in this condition, so either this condition or this condition. These two are two extreme tensile and compression. These are the two other points where it is less than that. So, when the ship is rolling, the maximum stress point is changing from the center line to the two edges and as the ship rolls, it will move like that from center to the deck corner, and then from deck corner to the center cross over to the other point so on and so forth. So, that is what I wanted to explain here.

Sir because it also means that distance of y_b is more than y_k , it definitely means that the corners have more stress than the bridge plates. Yes true and if you want to plot it, it will be something like this. These are the upper corner, these are the lower corner and these are the other two points. So, this is your σ , either positive or negative and with θ , it moves like that. What about the inner curves? Inner curves are the other two points. Now, what I am to trying to say is that this is the upper corner, this is the lower corner and these are the other two points.

In upright condition, it will be same. These two points will have the same value, so that what it show, but as the theta increases, this corner will keep on increasing and this value will keep on decreasing and the other corner in the same fashion, right. Now, this gives rise to another point that when the waves are going across the thing from tension to compression, at the same point the stresses are varying. Here also we see that the value keeps on changing from this end to that end. So, that means, every point on the deck and the keel is subjected to some sort of a varying stress value in a cyclic manner because the waves pass the vessel is also a cyclic process, and rolling of the vessel is also a continuous cyclic process.

Under both the conditions, the stress at a given point will keep on changing in a cyclic manner. We cannot say that it is always going from extreme positive to extreme negative. If it happens, it is a different thing altogether, but it may not even cross the 0 stress line. It may fluctuate in the same nature.

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Let me put the time here. As time passes this, I am putting sigma. This is plus axis, this is minus axis. Now, what we are saying in a particular point is the stress may go like this. It is quite likely that the stress may, so these are the two basic situations. This can be in positive side or it can be negative side also. I am not saying that, but it can be like this and it can be like that. What is basically happening is if we consider this to be our mean stress value, it is fluctuating about the mean. If this happens to be mean, mean happens to

be 0. Then, it is fluctuating about this, but this fluctuation whatever is this value here will definitely give rise to what is known as the fatigue and the structure stress amplitude.

Now, this part of the fatigue analysis has been studied by the metallurgists or material scientists only at the lab, because the true data of this to get, it will take a long time and therefore, in the experimentation you try to accelerate the whole thing. You take a specimen and put it under a given stress condition and a particular frequency which will take it to positive and negative. So, that is basically this much of positive, this much of negative and with a particular speed, it will take it and then you find out that after how many cycles the structure fails.

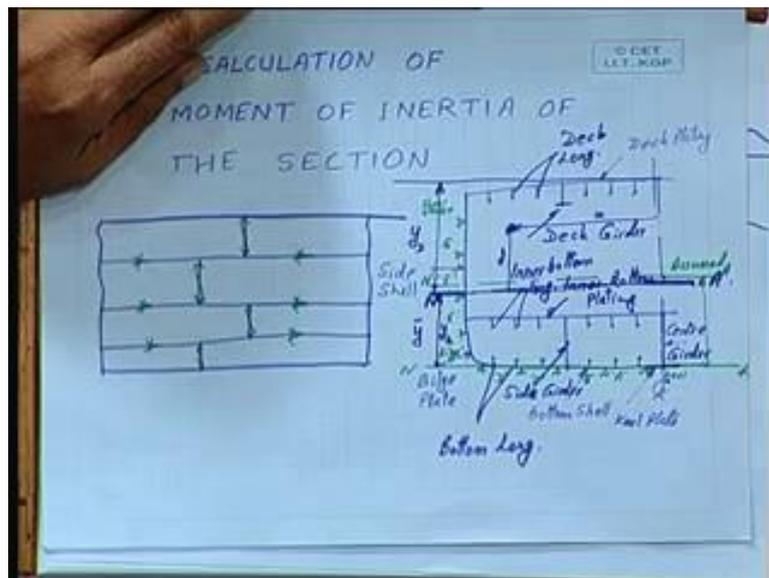
So, normally these types of experiments can be conducted, where your mean value is 0. What you can adjust is this sigma amplitude and depending on that, you can adjust the speed, and find out at what number of cycles the structure fails, but in our condition, we will find that in certain structural items are subjected to this type of cyclic loading. It is always in tensile condition, but over that tensile condition, it is fluctuating. Some may go to some negative side. May be that I need to draw here also. Not that much, but this much. So, the mean value is still the positive value, but it is likely that it may go to the negative side. How to handle these cases? This, the experimental people have not given us.

So, what we do is basically even to handle this position, we are trying to use the same experimental data which is available and try to find out. On that basis you try to find out what is the life cycle of a particular component. So, once we know some sort of stress, you know the sea conditions etcetera, you may be in a position to find out that what are the likelihood of the stresses, but again the sea. So, random that it does not follow a given pattern here, and specially the case which we were just discussing that the cause of. Now, we have a lot of stress margin, lot of factor of safety for our structural items for unknown case, unknown conditions where such cases can be taken care of, but it happens that sometimes it crosses the yield point stress limit.

Now, once it crosses the yield point stress limit, the picture is totally different say in a bulk carry away, we have been saying that the lower knee has got a pitted hole and we have got some sort of a stress concentration there. Now, due to that stress concentration when the cyclic loading is coming, suddenly it crosses the yield point stress limit and

once it crosses the yield, it is in the plastic range. The behavior is totally different. So, such cases are to be handled in a different fashion altogether. We cannot cover it under this. I do not know we still have some time, but I do not think that many things more to cover here. So, under this condition what we try to highlight is basically that when the ship is trying to roll the maximum stress point do change, and it goes to those corners and the corner connections becomes one reliable. That was one of the reasons.

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If you see the olden ships, those who have seen it, I happen to see old ships. The strake which is here is known as the shear strake used to have double rows of rivets. Because of this high stress region here, they were likely to be strained a lot and rivets used to get loosened. So, you to have double row of rivets here, double row of rivets in this region and again here and here. So, they were some special cases which I do not know whether those people had realized all these things or not, but experience was something which used to tell them that take special care of all those regions, where between the light load and the full load condition, you can have any condition there.

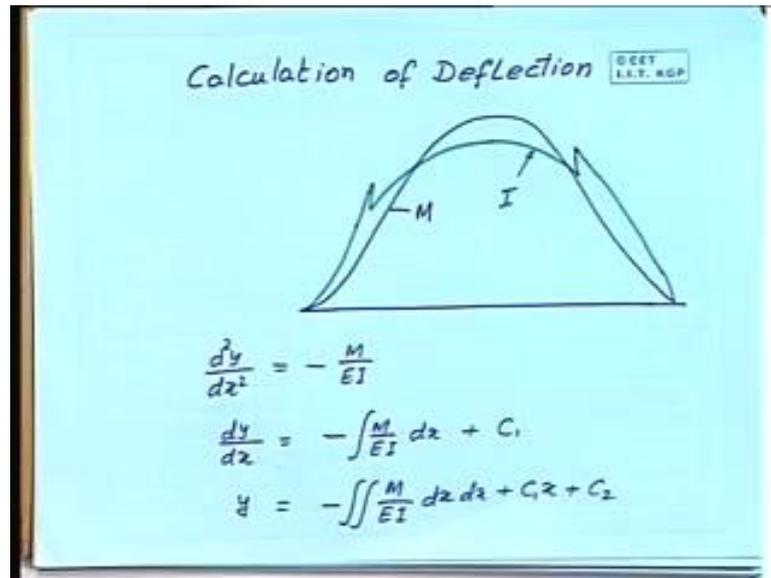
So, above that is always exposed to the air. Below the light load condition is always submerged in the water. So, therefore, the top part is not in contact with the water. The bottom part is not contact in oxygen, but in between it is always in contact with water and oxygen. Therefore, the oxidation is very high, the corrosion is very high and therefore, thus painting scheme was different and corrosion along was different in that

region. Of course with proper aligning elements and better quality paints, we have tried to reduce all those margins, but still, yes tumble from basically you will find in tankers only. You will find in tankers. Yeah, actually wherever you have a curved part, it gives you more strength than the flat part, especially the corner joints.

Tumble home is different actually. It is the rounded gunwale what he said. Yeah, the rounded tumblehome is basically you take it inside. Yeah that is given to tugs. I think tugs will have some tumblehome because their duty is to go sideways push and you know, no it is from operational point of view. Operation tugs are provided with tumblehome because it has to go alongside the vessel, and that time if it is rolling, then that part will try to hit the thing and it cannot. So, let it roll, but still it is in contact with the vessel. That is the basic part. Otherwise, other ships do not have tumblehome rounded gunwale. Yes, all the tankers are provided with that basically. It is a well proven fact that curved plates or shells in that sense has got more strength than the plate. Yeah, no corrugated in which sense? There the case is different. What you are trying to do is basically that one single plane you are trying to shift it. More materials is also being used.

So, basically when you are trying to compress it, then the moment of inertia long back plane is increased, but what you are talking about the corrugated sheets, there it is a circular cylindrical spot. So, there also the strength will be much more. Corrugated particles, the advantage is basically that you do not have the stiffness there. Stiffness, yes sir, but you will have stingers. You do not have the stiffness and if you do not have the stiffness, then other situated problems are not there. You only require the plate welding, but not the stiffener welding. So, that operation is there and two sides are more or less flat for storing the cargo. I think you tell him that I will stop now.

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Now, last time we have seen one thing that the bending moment curve was something like this, and depending on the shape of the hull, we may find that the moment of inertia distribution along the length will not be a very smooth curve, but it will have some smoothness and then depending on some abrupt change. That means, as this I am trying to depict that suppose there is a poop, then suddenly when the poop deck is over, then it comes to the main deck. Therefore, there will be a drop in the main sectional area, and then when it again goes up. And then let me continue and say that it ends up like this. So, this may be we can say that this is I variation along the length whereas, this is nothing, but the bending moment curve.

Now, the differential equation which we will be using is d^2y/dx^2 is equal to minus M/EI depending on our sign convention, and we have seen that if we are taking a sagging bending moment as positive concave up. And if we consider that bending moment to be positive bending moment, then the curvature works out to be in negative and that is how curvature is nothing, but this d^2y/dx^2 . So, M/EI with a negative sign gives us the curvature. So, this is the moment curvature equation and therefore, when we try to integrate it twice, the first integration gives me dy/dx minus M/EI integral. This is dx plus a constant of integration C_1 and the second integration will give me y , where C_1 and C_2 are two constants of integrations and they have to be evaluated by considering the end conditions.

Now, what we find that we have to integrate M by EI curve. For all practical purposes, we consider that the ship is made of one single material and therefore, E is constant all through. We may use two different steels. For example, mild steel and high tensile steel, but if you see the mechanical properties or the elastic properties, you will find that more or less E is constant. It is the tensile strength, which varies in two materials, but E is more or less constant. So, even in such condition, we may assume that E is practically constant. Now, if that is the case, then this particular expression which we have just written to y is equal to minus double integration 1 by E, I take it out.

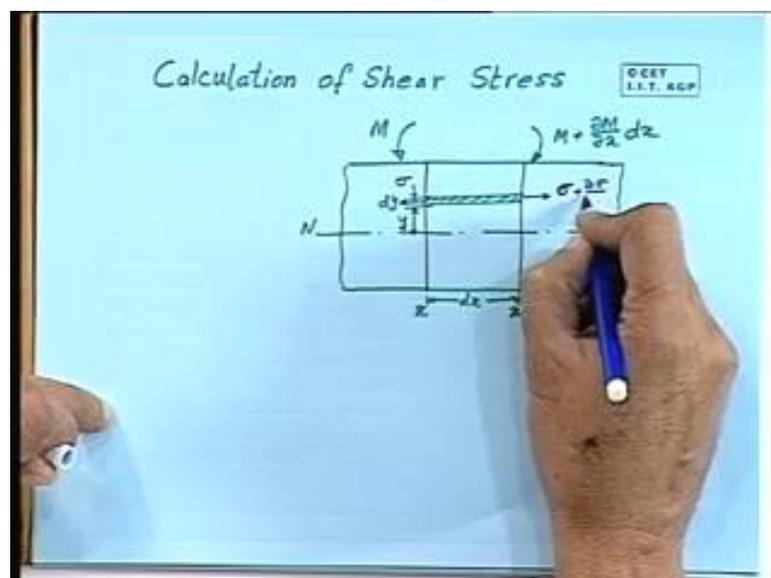
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$$y = -\frac{1}{EI} \int \int \frac{M}{I} dx dx + C_1 x + C_2$$

This becomes M by I dx dx plus C 1 x plus C 2. So, what is required is we have to integrate M by I curve. Now, if we see this variation of M, M is this curve and this is I curve and therefore, I have to find out what is M by I curve. Now, for ships basically for a naval at section or which element under that section is strained or stressed heavily that idea we do not get. Now, if you go back to the riveted ships, you will find that towards the middle streak, there always use to be two rows of rivets. In the shear streak also there used to be one double row of rivets and so on and so forth, but today the things are all welded and welding is something like gluing and tearing. That is all that technology has become so nice and in fact, at the welded joint you can have the efficiency more than 100 percent.

In fact, seldom you will find that the joint fails at the weld line, rather it is slightly away from the weld point. That means the welded line or at the welded joint, the efficiency is more than 100 percent. There can be two reasons. One reason is very simple that at that particular section or at the joint because of the metal deposit, the cross-sectional area increases or if you say that there is no increase in the cross-sectional area, then because of the heat affected zone in the neighborhood, the mechanical properties have deteriorated. So, what we are interested in is how the stress or how the shear stress where across the depth of the cross-section. So, that will be very interesting. So, let us say that we try to depict a cross-section here.

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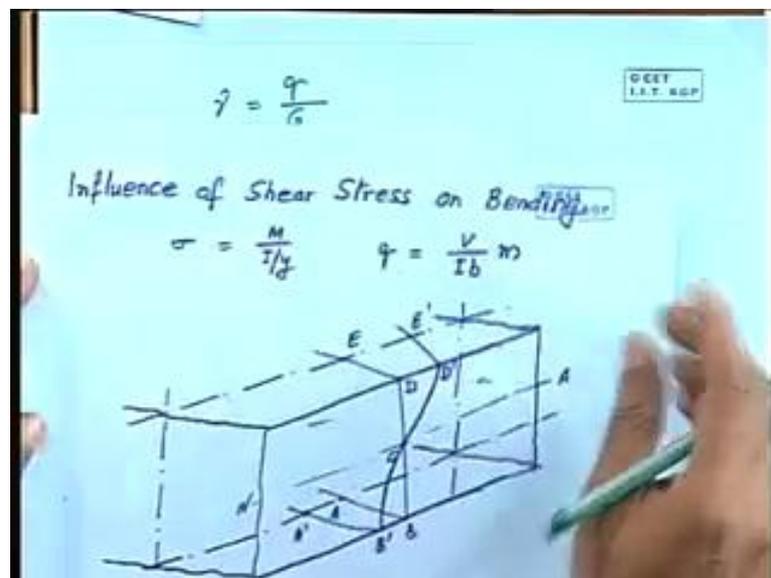
Some length we take representing the ship. We say that this is the neutral axis and we consider two sections. One somewhere here and another somewhere here which we say one is at x 0 is somewhere here 0 to x , and another one is xx plus dx . That means this distance which we are considering is dx apart. We say that the bending moment applied here is M , obviously at a distance dx . This will be M plus $\frac{dM}{dx} dx$. At a small we consider a small layer of thickness dy at a distance y from the neutral axis, and try to find out what the stress is here. So, in this horizontal layer, we assume that the stress at this end is σ and stress at this end is σ plus $\frac{d\sigma}{dx} dx$.

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$$\begin{aligned}\gamma &= \frac{q}{G} \\ \gamma &= \frac{\partial \epsilon}{\partial y} = \frac{q}{G} \\ \epsilon &= \frac{\partial \epsilon}{\partial z} = \frac{\partial}{\partial z} \left(\frac{q}{G} \right) \\ &= \frac{\partial}{\partial z} \left(\int \frac{q}{G} dy \right) \\ \text{Long. Stress Induced } \sigma &= E \frac{\partial \epsilon}{\partial z} \\ &= E \frac{\partial}{\partial z} \left(\int \frac{q}{G} dy \right) \\ &= E \frac{\partial}{\partial z} \left(\int \frac{ym}{IbG} dy \right)\end{aligned}$$

There will be longitudinal stress induced due to the distortion of an amount. Let us first write the longitudinal stress and then here longitudinal stress induced is equal to E into longitudinal strength E epsilon that is de by dx.

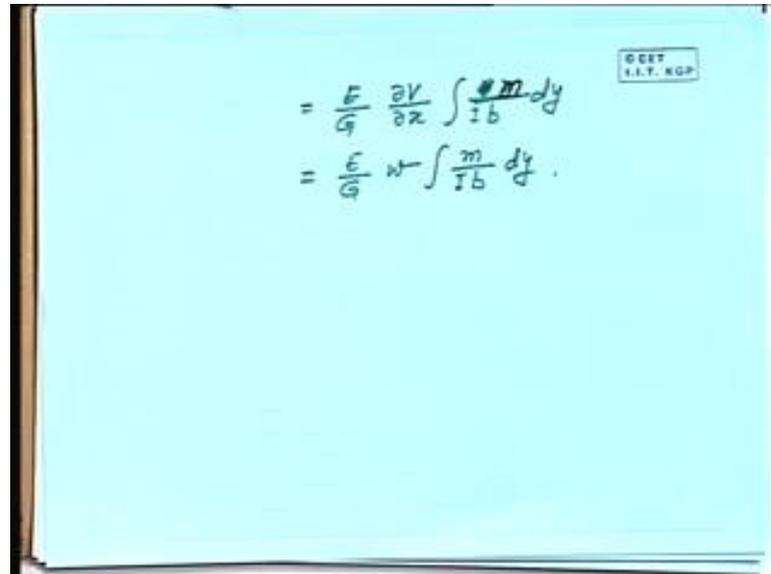
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Now, here I will try to substitute this value and this stress value q by. Now, once again what will do here q is already equal to V by Ib into m. So, you put V by Ib into m. There will be G here and then comes dy. So, now after putting substituting this value, what is a function of x here, and what is a function of y here? So, what we can do here? Now, this

shear force is a function of x. These are all function of y m is a function of y. So, what we will do here?

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$$\begin{aligned} &= \frac{E}{G} \frac{\partial V}{\partial z} \int \frac{m}{I_b} dy \\ &= \frac{E}{G} w \int \frac{m}{I_b} dy \end{aligned}$$

We try to put this equal to E and G, we take it out. This is constant. So, we take out and then we say that dv by dx integral V by Ib m into dy. Sorry, V has gone. It is only m by Ib. V has come out and what is dv by dx rate of loading. So, rate of loading normally is used as w. This is the induced longitudinal stress and it can be shown.