

So, p by EI there will be something. So, K will be there. So, B will be equal to 0, and this part became 0. And from here we can say A is equal to minus delta because this is 0 A , this is delta. So, in that equation A is equal to minus delta, B is equal to 0. So, we can write y equal to minus delta $\cos Kx$ plus delta, right. So, B equal 0 A equal to minus delta.

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$y = \Delta(1 - \cos Kx)$
 At $x = L$, $y = \Delta$
 $\Delta = \Delta - \Delta \cos KL$
 $\Delta \cos KL = 0$
 $\cos KL = 0$
 $KL = \frac{(2n-1)\pi}{2}$
 $K^2 L^2 = \frac{(2n-1)^2 \pi^2}{4}$
 $\frac{P}{EI} = \frac{(2n-1)^2 \pi^2 EI}{4L^3}$

Or we can write y equal to delta 1 minus $\cos Kx$, right. Now if we put x equal to 0. So, \cos will be equal to 1 $\cos 0 = 1$, so $1 - 1 = 0$ means y equal to 0. If we put x equal to L , we will get the other condition. So, let us get that. Now rather this is the equation of our deflected shape of the cantilever. So, the shape we have drawn. So, this is the equation, right. So, y equal to delta 1 minus. So, it will start from 0, and casually, it will be equal to delta and in between it will follow a \cos curve.

Now delta is any arbitrary value; we can give here 1 millimeter deflection or 5 millimeter deflection; based on that it will take that particular form. So, whole thing multiplied by delta. So, that is basically the shape of the curve. Now here at x equal to L y equal to delta. So, from there we can get some clue about the critical load and all those. So, at x equal to L y equal to delta; so, if we put delta equal to delta minus delta $\cos KL$. So, it is x equal to L and delta delta if you multiply. So, this delta will cancel, right, and ultimately it will be delta $\cos KL$ equal to 0.

Now if delta equal to 0, it will be that straight case. So, it is a trivial case, right; we know when there is no problem, load is beyond the range or load is 0. So, it will be straight; we

have that idea. So, we are basically trying to get something else. So, we have to get $\cos K l$ equal to 0 or $K l$ will be where $\cos \theta$ will be equal to 0, at 0 it is not 0. So, it will be at 90 degree it will be 0, again if we come this side missed 270. So, it will be π by 2 then. So, it will be you can write. So, here it will be 0, here it will be 0.

Again it will be 0, again it will be 0. So, here it will not be 0, right. So, here it is 0 π by 2, here it is 3 π by 2; again it will be 5 π by 2. So, if we put in a n form. So, it will be 1, 3, 5, like this. So, we can write $2n - 1$. So, if n equal to 1 it will be 1; if it is n equal to 2?

Student: It will be 3.

Professor: It will be 3.

So, that we can write, right; so, this part there should be a factor $2n - 1$ π by 2; the minimum value will be n equal to 1, it will be straightaway π by 2. Now if we try to square that equation. So, it will be $K^2 l^2$ equal to $(2n - 1)^2 \pi^2$ by $4l^2$ and K^2 equal to P by $E I 2n - 1$ square π square by 4, okay. Now if you rearrange we will get from here. So, P equal to your $(2n - 1)^2 \pi^2$ here π square E square by $4 l^2$ π square $E I$ π square $E I$; this is π^2 $E I$ by $4 l^2$.

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Handwritten mathematical derivations on a whiteboard:

$$P = (2n-1)^2 \frac{\pi^2 EI}{4l^2}$$

$$= (2n-1)^2 \frac{\pi^2 EI}{(2l)^2}$$

$$P = \frac{\pi^2 EI}{4l^2} \text{ or } \frac{\pi^2 EI}{(2l)^2} \quad n=1 \checkmark$$

$$= 9 \frac{\pi^2 EI}{4l^2} \text{ or } (3)^2 \frac{\pi^2 EI}{(2l)^2} \quad n=2$$

$$y = \Delta \left[1 - \cos \left\{ \frac{(2n-1)\pi x}{2l} \right\} \right]$$

Or if we write in a fresh way it will be P equal to $(2n - 1)^2 \pi^2 EI / 4l^2$; that part if you take here it will be $\pi^2 EI / 4l^2$ or we can write $(2n - 1)^2 \pi^2 EI / 4l^2$. Now

if n is equal to 1. So, p will be $\pi^2 EI / 4l^2$ or we can say $\pi^2 EI / 2l^2$. Now this value will be if it is n equal to 2, it will be nine times, right. So, it will be n equal to 2. So, it will be?

Student: 3 square, 9 sir.

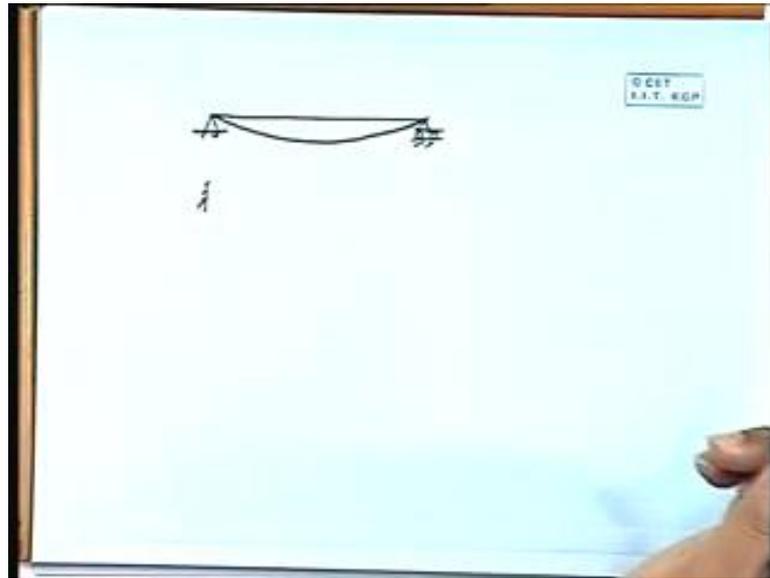
Professor: 3^2 minus 3, 9, right.

So, here $\pi^2 EI / 4l^2$, here there will be 9 or we can write $3^2 \pi^2 EI / 4l^2$. So, in different form we can express n is equal to 2 if n equal to 1, right. And we have the expression y is equal to $\delta (1 - \cos Kx)$. Now K if we put $1 - \cos$, K already we have determined. So, K is or $2n - 1$ divided by $2l$. So, here we got Kl equal to something. So, we can write K equal to $(2n - 1)\pi / 2l$. So, here $2l$ will be there, right, or I think it become very small size; we can write y equal to $\delta (1 - \cos (2n - 1)\pi x / 2l)$, right.

So, that will be the expression for the deflected curve; it is dependent on your δ . So, if we make it 1, you will get this safe; safe will be identical only it will be magnified with the factor δ . And the p value it will be $\pi^2 EI / 4l^2$ multiplied by 1 or 9 or next time it will be I think 3 5, it will be 25. So, like this the different values you will get, but this one will be the minimum value, right. So, if there is no resistance. So, it will try to deflect at n equal to 1 and the value we will get $\pi^2 EI / 4l^2$.

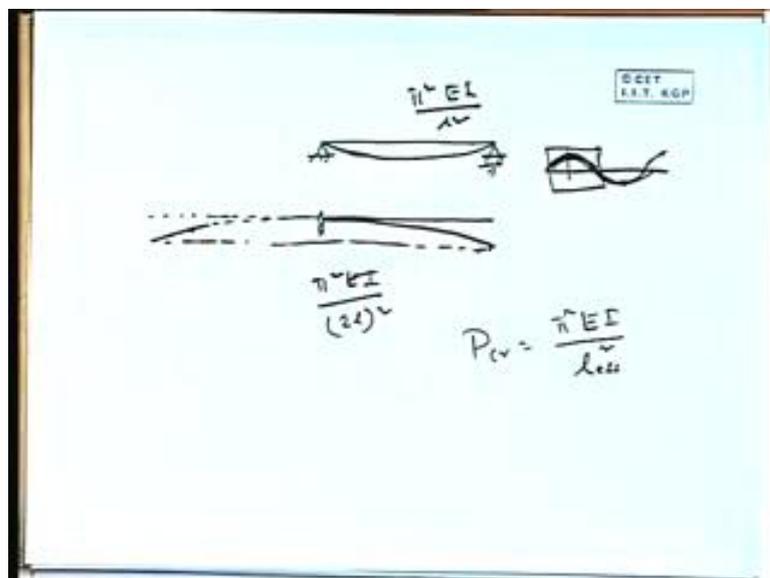
So, earlier we got how much $\pi^2 EI / l^2$. So, $\pi^2 EI / l^2$ is a very standard form; with that if we just divide by 4. So, here also if we change the amount there is a factor multiplied. So, $\pi^2 EI / l^2$ is a very unique case. So, we are talking about a column; both end is hinge hinge to apply. That is the basic case or fundamental case because there the first mode value is $\pi^2 EI / l^2$. Other mode we are getting some factor with this or other boundary conditions, say, cantilever case or fundamental itself just it is one-fourth of that. Now we can physically correlate the two different problem, right.

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Now a simply supported problem that deforms shape is a sinusoidal form fundamental case. Now if we take a cantilever, I think I should draw in a different way; otherwise, it will not may be possible to explain; from here I am drawing it again.

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This curve was a sin curve and that was a cantilever that was just now the equation of what we got. So, y is equal to $\delta (1 - \cos Kx)$, right. So, it is a cos curve. So, it is not direct cos curve; it is with some minus and all those, okay. Now we have the understanding that if we draw a sin curve like this. So, if we start from here it is a cos

curve, right. So, sin curve start from 0, it will be like this. So, if you start from here. So, it will be, right. So, there is a just phase lag, and this value is 0, and this value is 1. So, here you try to see this $1 - \cos$.

So, that deformation pattern is more or less same. So, it is represented in a sin form that is if in a cos form, right, or this part also you can represent; cos we can make it $1 - \cos$, say, $\cos^2 \theta$ equal to $\cos^2 \theta - \sin^2 \theta$. So, $1 - 2 \cos^2$ is there. So, one one we can cancel. So, it can be represented in that form again. So, anyway both are a similar type of curve. Now here if we draw if you put a mirror, we will get a mirror image of that, right.

So, this is the cantilever. If you put a mirror here, we will get a mirror image. Now if you join end to end and if we compare this one with the fundamental case; so, it is like this it will be like that. So, this is a sin curve; this is a cos but I told this; here it is like this, here it is like that starting from higher value a mirror image. Now if you shift the axis, say, if you take this one, this is the base and think in a reverse manner. So, what will be happening? This curve is more or less same thing.

So, if it is l this will be l . So, here how much you got, $\pi^2 EI$ by l^3 . Here $\pi^2 EI$ $2l^3$. So, it is l and l . So, it is $2l$. So, we can write $P_{critical}$ equal to $\pi^2 EI$ by l^3 the effective square. So, that is the general expression of a critical load. So, $\pi^2 EI$ by l^3 effective square. So, l effective is equal to l for this case; in that case l effective is?

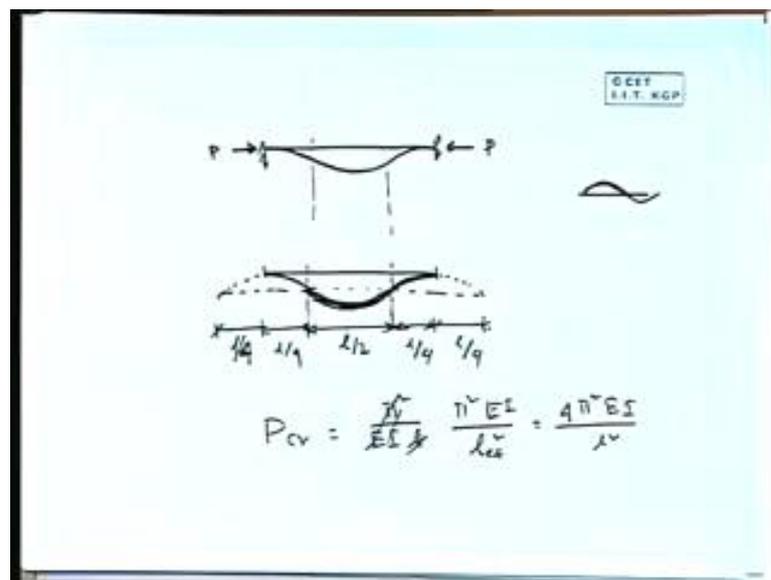
Student: $2l$.

Professor: $2l$, right.

So, if you put here $2l$, you will get the critical load in that case, right. So, what is fundamental case? It will start from 0. So, if you take a sin curve. So, this is a full sin curve. So, if you take a half sin curve that will be found in a fundamental case. Now here if it is cantilever you can take a mirror image, and that part will be more or less like this. So, if it is l . So, if you take $2l$, $2l$ will be identical to that case. So, in that case we say the l effective for a cantilever beam is twice l .

So, what is l ? L is the actual length. So, without dividing that equation if we just compare the mode of buckling, from there we can make some analogy and find out the l effective value and get the value of the critical load. Now I am taking a different case now and without deriving we will find out the critical load from that understanding. So, two cases we have derived; one is a fundamental case both end hinged, another is the cantilever case; next case, say, both the end are fixed, right.

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So, if we take both the end are fixed, say. So, both ends are fixed and it is subjected to p . Now initially there might be a problem in the feeling that if both are fixed if we apply load, how load will be transmitted? It is fixed means the rotation is not allowed; the vertical moment is not allowed, horizontal moment one end it may be resting, other end it should be allowed, okay. So, sometimes you will find a hinge hinge case both the end it is hinge load is there; it should not pass, but here at least I have tried to made one thing is roller I think . So, that filling should not be there, but sometimes in some case you will find it is hinge both.

So, we have to assume that that awareness of moment is there in that case. So, here also it will be there, okay. So, main idea is the moment part is allowed here. So, load will be transmitted. Now if we draw the deform shape fundamental one. So, here slope will be 0 it will be maximum, and it will go like this. So, here it is clamped; there will be no

deflection, no slope. No deflection no slope it will be maximum; symmetric slope will be 0. So, shape will be something like this.

Student: Then why cannot we think that it will go off?

Professor: Okay, it may go in the other way also I think. So, that part I am not interested at this level at least. So, it may bend this way or that way I think.

Student: This or that column which side is positive?

Professor: Now in this figure if we divide into, say, let us draw the deflected shape, say, it will be something like this, okay.

Now here you can just divide how many divisions? This is the central part, say, it is 1 by 2; this will be your 1 by 4; this will be your 1 by 4. Now if you put a mirror here at a distance of another, say, 1 by 4 1 by 4, right. So, this is the original portion. Now that we have divided central part 1 by 2, 1 by 4, 1 by 4; so, beyond the support if we continue this curve in the mirror image form 1 by 4. So, it will be like this.

Now in this part it is just like a fundamental case because here slope is there plus and this part is also the fundamental case. So, it is 1 by 2; this is also 1 by 2; this is also 1 by 2, right. So, there are three different parts each are having a length of 1 by 2, and both the end some slope are there plus it is symmetrical at the middle maximum amplitude. And really if you solve that problem with our differential equation $E I d^2 I d x$ is square equal to the moment equation.

We will find here at 1 by 4 you will get the curvature equal to 0. Normally, if we take a simply supported case both the end bending moment are 0 means $d^2 y d x$ square are zero. So, if you just solve and get the deflected curve at 1 by 4 where y will be there; if you take derivative you will get slope; you will get another derivative and put l equal to 4 you will get here curvature will be equal to 0.

So, if curvature equal to 0 slope is there. So, you can just assume this is a part just like this portion. So, similarly it will be like that. So, here what will be the effective length? It will be 1 by 2, yeah. So, this is representing one fundamental case 1 by 2. So, your p critical it will be $\pi^2 E I$. It is $\pi^2 E I l$ effective square or we can write π

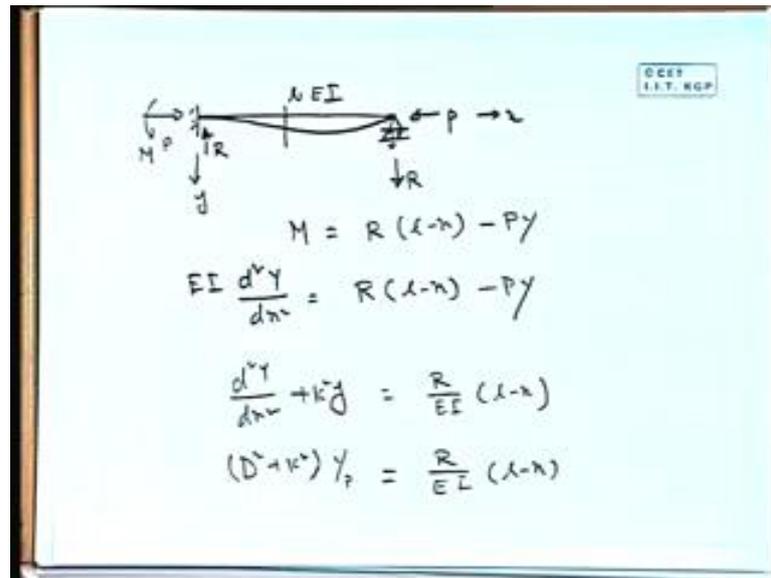
square $E I l^2$; it will be 4 above because it will be $1 \times 2 \times l^2$ by 4, 4 will go up, right. So, we will get four times the basic case $\pi^2 E I$ by l^2 .

Now fundamental case the value was $\pi^2 E I$ by l^2 ; for cantilever it was $\pi^2 E I$ by $4 l^2$. Here 4 into $\pi^2 E I$ by l^2 . Now physically also we can get some feeling that if these two ends are simply supported if we apply load, we are going to get $\pi^2 E I$ by l^2 . Now when it is critical, it will try to buckle; buckle means bending will be initiated. Now if there are some fixed at the end, structure will be much more strong if it is clamped means slope is also restrained.

So, support is much more strong. So, we should expect more strength from the structure definitely your critical load wise if you calculate. So, from simply supported case to clamped case; that critical load value will be you have to apply four times. So, if it is hinge hinge case if we apply 100 and if you get the buckling, here it can carry up to 400. So, four times load it will carry because it is much more strong, and if it is a cantilever case cantilever is very weak that one of the end is absolutely free.

So, strength is one-fourth time. So, at 23 it will fail, alright. So, here it is all square of that effective length. Now I will take a case where this concept cannot be applied because if we make it. So, far both end hinge both end fixed. It is more or less symmetrical type of section; if we take this is fixed this is hinge; what will be happening? It will be unsymmetric, right. So, that is nice deform shape we will not get. So, definitely we have to start from the basic equation.

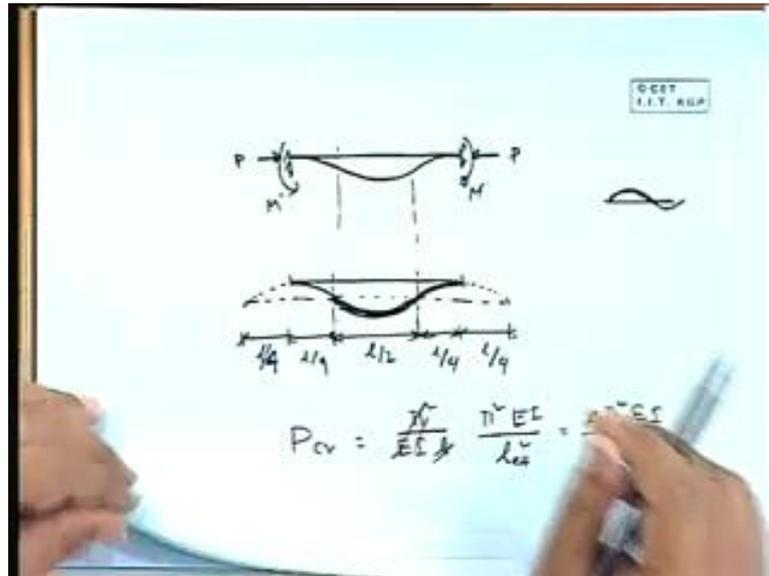
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But there also we will get something; it can be correlated with some other values. It is l and E and I ; l is the length, $E I$ is the flexural rigidity; there is a force p acting here, it is a x y deforms it. Now that problem just now we considered this one or the earlier one. The support reactions are quite complicated, because if there is a p this side there was a p . Now there will be a moment generated here; at this end there will be no moment. Now this moment will be balanced by some equivalent and opposite force.

So, you can say there will be a horizontal p , there will be a moment here M , because it will not try to rotate freely; automatically, some moment will be generated. And if there is a moment, say, p p will cancel, there is no load only moment; that moment will be balanced by this reaction force. So, it is anticlockwise. So, here there will be a reaction; there will be a reaction R and R . So, R into l will be M . So, if we know R we can calculate M ; if we know M R we can calculate but what will be the value that is not known.

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Now in the case of fixed fixed condition. So, some moment was there at this end; some moment was there at that end, say, if it is M it is N. Now here M should be balanced with that. So, there will be no reaction, but here some reaction some moment will be there and it is unknown, okay. Now this problem it can be solved in two ways; number one, say, at any station at x, there will be y; what will be the moment at the station? So, you can come from this side or you can come from that side. So, it is better if we come from this side.

So, if we take any station here. So, this is x that is y. So, your M will be your R into?

Student: L minus x.

Professor: L minus x, and that will be a hogging type of moment.

So, we are taking it is plus minus p into y. Now if we write E I d² by d x square. So, it will be simply R l minus x minus p into y. Now here there is a quantity R that is not known. So, if that is not there, it should be more or less like your earlier form. Now this problem can be handled in two ways; one of the option is you can take fourth-order equation if you take derivative twice. So, R l will cancel in the first option; second option it will be Rx will be R, another derivative R will go there at least. So, fourth order derivative it will be vanishing.

So, we have to integrate four times, and we have to put the boundary condition at the two ends; here deflection 0 slope 0, here deflection 0 curvature 0 or moment 0 here you can see. Now problem will be much more involved because four unknowns four equations; another alternative we can try with the second order keeping this R, right, because it is a buckling problem; it will be in the form of R, just like the cantilever problem there was a delta.

So, that part will be associated with all the equation, and finally, we will get something equal to 0 and $k l$ will be some parameter I think. So, you can make and try, and I am hopeful will we get the result if we retain R in the system. But in other problems, say, if it is a static problem, we cannot solve it, because it will be always in the form of R, but here our attention is little bit different. So, we are not if something multiplied with R. So, let it be there at least.

Now this EI part we can put here and it will be your K square and rearrange the equations. So, it will be your $d^2 y$ by $d x$ square plus k square y . So, this part you are bringing here divided by EI that will be your $R EI l$ minus x . Now for the first part if we consider right hand 0, we will get a complementary part. So, that will be simply a $\cos k x$, $B \sin k x$, and right part the particular integral that part we have to determine.

So, here we can calculate, say, we can calculate in this form. It is d square plus k square; again in the same form y particular $R EI l$ minus x . Now this part we will put on the other side; again we will try to expand, it will discard on all those. So, here maximum order it is x . So, final it will go on cancelling it.

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Handwritten mathematical derivation on a whiteboard:

$$\begin{aligned}
 y_p &= (D^2 + k^2)^{-1} \frac{R}{EI} (l-x) \\
 &= \frac{1}{k^2} (1 + \frac{D^2}{k^2})^{-1} \frac{R}{EI} (l-x) \\
 &= (1 - \frac{D^2}{k^2} \dots) \frac{R}{k^2 EI} (l-x) \\
 &= \frac{R}{P} (l-x) \\
 y &= A \cos kx + B \sin kx + \frac{R}{P} (l-x) \\
 \theta = \frac{dy}{dx} &= -kA \sin kx + Bk \cos kx - \frac{R}{P}
 \end{aligned}$$

So, this y_p we can write $D^2 + k^2$ minus 1 or this part $1 + \frac{D^2}{k^2}$ we can take out $1 + \frac{D^2}{k^2}$ to the power minus 1 $\frac{R}{EI} (l-x)$ or we can write here this part $1 - \frac{D^2}{k^2} \dots$. This part is $\frac{R}{k^2 EI} (l-x)$. Now this is the second derivative; if you go beyond it will be fourth, sixth. So, if you take derivative twice, this part will be zero. So, first derivative x will be 1, second derivative will be 0; other part with first derivative it will vanish. So, 1 into this part will be there, remaining part it will vanish.

So, here k^2 equal to p by EI . So, EI we can cancel it. So, we can write R and here we can write P and here $l-x$, and it will be multiplied by 1; other part if you take we will not get anything or y we can write the complimentary part $A \cos kx + B \sin kx + \frac{R}{P} (l-x)$. Now this part we have to apply boundary condition. So here slope is also 0 one of that condition. So, $\theta = \frac{dy}{dx}$ that part also you find out, it may be required. It will be $-kA \sin kx + Bk \cos kx - \frac{R}{P}$, and this part will be $-\frac{R}{P}$, right. Now we have the problem. So, at $x=0$ $y=0$ plus $\theta=0$; at other end deflection equal to zero. So, these boundary conditions we have to put.

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At $x=0, y=0$

$$0 = A \cdot 1 + B \cdot 0 + \frac{R}{P} \cdot 1$$
$$A = -\frac{R}{P}$$

At $x=0, \theta=0$

$$0 = -K A \cdot 0 + K B \cdot 1 - \frac{R}{P}$$
$$B = \frac{R}{K P}$$
$$y = -\frac{R}{P} \cos Kx + \frac{R}{K P} \sin Kx + \frac{R}{P} (1-x)$$

At $x=1, y=0 \Rightarrow$

So, boundary condition at x equal to 0 y equal to 0. So, it will give 0 equal to A , $\cos 0$ is 1 plus $B \sin 0$ equal to 0 plus your R by P into 1 x is 0. So, you can say A is equal to minus R by P . Now the next boundary is at x equal to 0, θ equal to zero. So, here 0 will be minus $K A$ into 0 plus B , say, $K B$ into 1 minus R by P . So, here we are getting B equal to R by $K P$ and A equal to R by P . So, we are getting both the value $A B$.

So, if we put in the expression. So, y will be A will be if minus R by $P \cos Kx$ plus R by $K P \sin Kx$ plus R by $P (1-x)$, right. So, $A B$ we obtained. So, equal to minus R by P and B equal to R by $K P \cos \sin R$ by $P (1-x)$. Now we can put the last boundary condition that x equal to 1 y equal to 0, right. Now here you see R , R is a common quantity. So, we can take that part is common. So, R at least we can take common. So, we do not have any problem at this level. So, P , okay, it may be there I think so. Now if we put, say, at x equal to 1 y equal to 0; that will lead to if we put here x equal to 1, we will get this one.

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Handwritten derivation on a whiteboard:

$$0 = -\frac{Rl}{P} \cos Kl + \frac{R}{KlP} \sin Kl$$

$$\frac{R}{KlP} (-Kl \cos Kl + \sin Kl) = 0$$

$$-Kl \cos Kl + \sin Kl = 0$$

$$\tan Kl = Kl \quad f = \tan Kl - Kl = 0$$

Iteration table:

$Kl = 1$	$f = -0.157$
$Kl = 1.1$	$f = -0.047$
$Kl = 1.11$	$f = -0.01$

$$Kl = 1.11$$

$$Kl^2 = (1.11)^2$$

$$\frac{P}{Rl} Kl^2 = (1.11)^2 \Rightarrow P = \frac{20.2 R l}{1.11^2}$$

Say, 0 is equal to minus R l by P cos K l plus R by K P sin K l, and this part will be equal to 0. So, if we come here, if we put here, it will be 0; it will be sin K l cos K l R l by P R by KP. So, whole quantity will be equal to zero. Now here this R by P we will take common; some K and l also we can take common, K we can take common, right. So, it will be K l minus cos K l plus sin K l. So, R by K P you have taken common; it will be sin K l, it will be cos K l.

So, R by PK is there, l will be there. So, we can multiply K here, K here, so K. Now here the centre part should be equal to 0. So, if there are some load, R will be there and if P is not there. So, P should be at least some value and K will be P by i. So, we are not bothered about what will be the value of that. So, we can write the central part that should be equal to 0 or from there we can write tan K l equal to K l, right.

We can write minus K l cos K l plus sin K l is equal to 0 or sin K l equal to K l cos K l or cos part if you take here tan K l equal to K l. Now this is one of the tricky equation; earlier we got K l equal to sin K l equal to 0 or cos K l equal to 0, K l equal to either pi or 2 pi, something like this or pi by 2, 3 pi by 2 or something. So, here K l equal to 1; it is just like x equal to tan x, right.

So, there is no direct solution of this. So, we have to solve it in an iterative manner. So, we can just put here 1 tan K l will be 1, and you say this value is 1 on not; after that you can put say 1.1 and 0.9. You see if which one is going in a better way or we can say tan

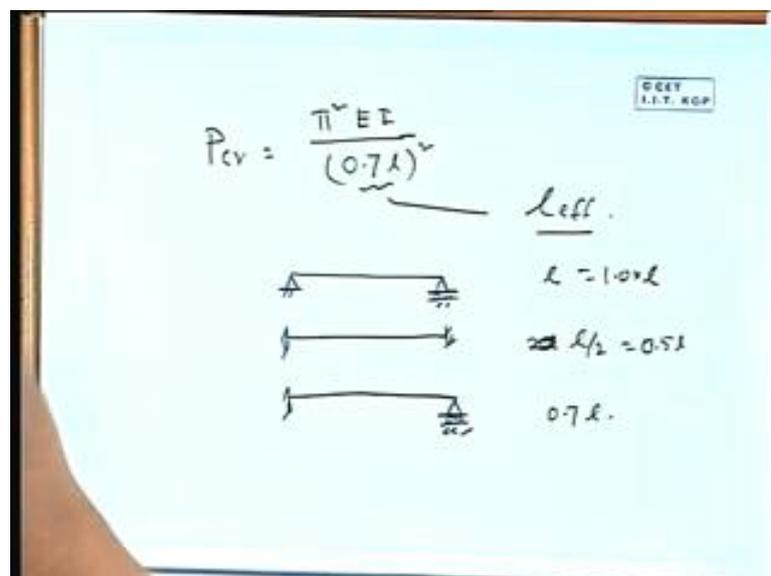
$K l$ minus $K l$ equal to zero. So, we can put, say, $K l$ equal to 1 and this is some function f . For $K l$ equal to 1, function equal to how much and 1.1 how much, 0.9 is how much? So, it should tend to the 0.

So, which one is tending to 0? So, like that if you just calculate substitute some value of $K l$. So, $\tan K l$ minus $K l$ should be equal to zero. So, $K l$ you can start from, say, 1, then 0.9, 1.1 or 1.5, 0.5. So, initially you can go with a large interval; after that it will give some neighborhood. So, within that we have to refine, and we will get the value of $K l$; that should be equal to 4.49.

It is not the exact value; if we match we will not get identical, small fraction will be there. So, it depends how many significant derivate it will pickup. So, if it is $K l$. So, we can make square square. So, K square l square equal to 4.49 square. So, K square equal to π by l . So, it will be π divided by $EI l$ square equal to 4.49 square, right. So, 4.49 if we just from here P equal to 4.49 is equal to 20.2, right, EI by l square roughly the value you can take more significant digit.

So, $K l$ equal to 4.49 π square l square will be square of that; that is 20.2, so here P by EI . So, if you rearrange, again it will be 20.2 EI by l square. Now we have the standard form π square EI by l square. What is π ? π 3 point something square. So, that part if you take common, there will be some parameter; that parameter in an inverse form if you put below and within l .

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So, P can be nicely represented as P or P critical; it is $\pi^2 EI$ by $0.7 l$ square, right. So, this 20.2 divided by $\pi^2 EI$ if you make something we will get. So, that if you put it below, you will get some 0.49 or something else; so, that if you put in the bracket it is 0.7 . So, what is this? This is basically l effective, right. Now l effective here it is how much? $0.7 l$ square, right; now you come to the different cases. Here l effective is equal to how much? L , this is $2 l$, not $2 l$. It was l by 2 or we can say $0.5 l$; this is how much? $0.7 l$.

Now here it is hinge. Now if you make both the end fixed; so, your l means $1.0 l$. So, $1 l$ became $0.5 l$. Now if both the end is not fixed, one of the end is fixed. So, it should be in between 1 and 0.5 . So, value came 0.7 , you should be happy if we put 7.5 . So, it is not 7.5 but anyway it will be some intermediate value if you take 0.75 . So, you are going to take more effective length.

So, more effective length means it will be much more weak more weak, right. So, from your intuition if you take 0.75 ; so, you will be on the little safer side in the design, but actual value will be 0.7 , okay. So, all the cases it is basically $\pi^2 EI$ by l effective square. So, l effective it is for cantilever it is $2 l$; for fundamental case it is l ; for clamped clamped is it $0.5 l$, clamped hinge it is $0.7 l$. For single span more or less, these are very standard type of form. So, that can be utilized, right.

Now in the tutorial class we will carry some numerical example and try to find out, say, some section will be given, what will be the limit of load, or load will be given, you have to find out the dimension or cross section of the member. So, it will be a reverse problem a design type of problem.

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$$P_{cr} = \frac{\pi^2 EI}{\lambda_{eff}^2}$$

$$J = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{\lambda_{eff}^2 A}$$

$$= \frac{\pi^2 E}{\left(\frac{\lambda_{eff}}{r}\right)^2}$$

$r = \sqrt{\frac{I}{A}}$
 Slenderness ratio

Now before finishing today the buckling part this $\pi^2 EI$ by all the time we are talking about E ; this P critical equal to $\pi^2 EI$ by l effective square. Now here direction we are not talking about this way or that way. So, it may deflect like this; it may deflect like that; it may deflect in a different manner. Now it depends on the cross section, say, if it is a rectangular section, say, there is a column. Now if you calculate I in that direction you will get I in the direction you will get.

Now I about this is much higher compared to that. So, here it will be I minimum, right. So, if it is $I_x I_y$. So, I_x is greater than your I_y . So, I_y will be taken here I think. So, because if there is a thin plate. So, it will buckle like this not it will buckle like that, right, or if it is a very ordinary section, you have to find out which direction your I is minimum number one, number two this l effective is a very important.

Now if there is a plate like this, now there might be a pin. So, this hinge is hinge in that plane; other plane it may be, say, top end there is support condition like this. There are two plates inside it is there and there is a pin. So, it can nicely rotate, but it cannot rotate in that manner. So, here if you take the bending in that manner; so, it will be a fixed support more or less. So, l effective will be $2l$ then I may be less.

So, this side I may be less, but l effective may be more; other end I may be more, l effective may be less. So, different combination you may get, and from there we have to find out which side it will buckle, right. So, there is a possibility it may vary it may vary.

If it is a ball socket type of joint, all end it is identical situation, but if it is a plate type case. So, one side it will be hinge, other side it will be clamped, right. So, that part we have to remember and in the form of stress if we write this is one of the familiar stress will be your P critical by A. So, it will be $\pi^2 EI$ by l effective square and here A.

Now it is $\pi^2 E$. Now this I if we take below. So, I will come here. So, you can say l effective by r square. So, r is what? R is root over I by A. So, square means I will be there I will go of it will be A. So, for a member r is radius of gyration and we have to take the minimum radius of gyration, right. And if it is minimum this rather l effective by r is important; I was talking about r may be minimum this side, but l may be more. Other direction l may be, say, l and r it may be in a reverse situation. So, l by r it is called a slenderness ratio. This is very important.

So, your allowable stress critical will be $\pi^2 EI$ by slenderness ratio square and E for a particular material it will be fixed. This is a material powered Young's modulus and π^2 is fixed. So, if you fix up, still this part will be fixed. So, it will be depended on that slenderness ratio; what is slenderness ratio? It will be l effective by r ; l effective is actual l into some factor 2 or 0.7, 0.5, 1 and r is your radius of gyration of root π by A.

But not necessarily in one direction r may be minimum, but l effective may not be minimum. We have to check the different directions, and from there you will get a minimum l by r ratio. So, that should be your guiding slenderness ratio, right. So, l by r should be maximum. So, when it will be maximum, this value will be large. So, denominator large means your capacity will be less, right. So, this is the basic guideline. Now we can carry out some problem in tutorial and try to explain how these parameters can be applied for designing a long column where length is quite high compared to the dimension of the member. So, we can complete here.