Strength and Vibration of Marine Structures
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Lecture - 17
Theory of Column – II

So, it is basically continuation of the previous class. We were talking a column problem having x eccentricity in both the direction.

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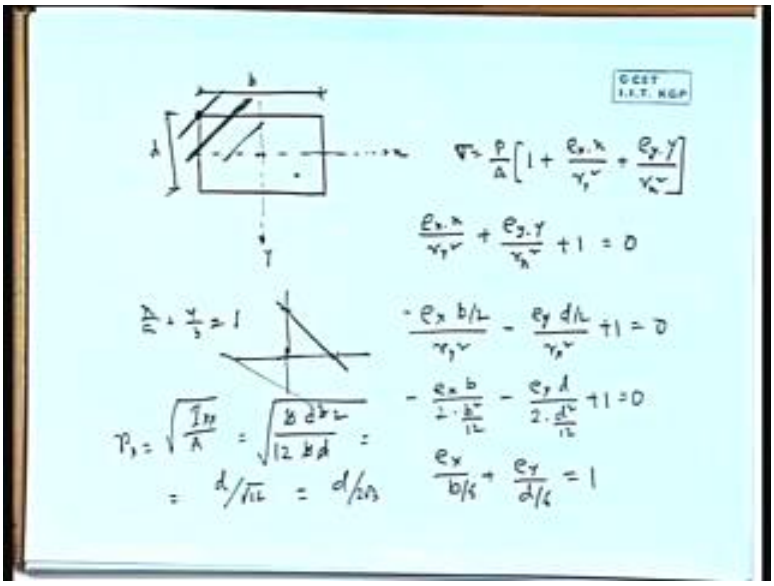

We got the expression of stress, and the stress should be equal to 0; from there we got that equation, fine. Now that equation is basically the equation of the line where stress will be equal to zero; so beyond that it will be tension, this side it will be compression. Now if we want the entire cross section will be under compression; there will be no tension inside the section of the column. So, the line should pass along the corner. So, to the corner has a coordinate of x equal to minus d by 2 y equal to minus d by 2 that we have substituted.

Now at this level the value of r x and r y we have to substitute. Now what is r x? Let us calculate r x from there in a similar manner we can get r y. So, your r x equal to root over I x x by A; now what is I x x? So, second moment of area about x x, so that is b d cube divided by 12 into A is b into d, okay. So, b if you cancel and d is2; so it will be d by root 12 or sometimes we can write d by 2 root 3. So, r y will be your d by 2 root 3 in a similar manner but here it is r x square r y square. So, automatically it will be d square by 12 d square by 12.

So, here you can write your minus e x b 2 you put below and r y will be your B square by 12 and minus it will be e y d divided by 12; it will be d square by 12 plus 1 is equal to zero. Now this 2 12 it will be 6; one of the b you can cancel. So, you can write e x divided by b plus e y divided by d; that is equal to 1, right. So, e x by b plus e y by d that is equal to 1

Student: The limit is such this b and d.

b and d the actual value we have substituted here. Now we want to get the limit of e x and e y, right. So, e x and e y again it is giving a curve, say, e x is a variable; e y is a variable; it is x by A y by b is equal to 1, right. So, we can get a curve you know that is the six part we have not taken here.

Student: By 6, sir

So, by 6. So, it will be b by 6 d by 6. So, it will be e x divided by b by 6 e y divided d by 6 is equal to 1. Now come to the next page.

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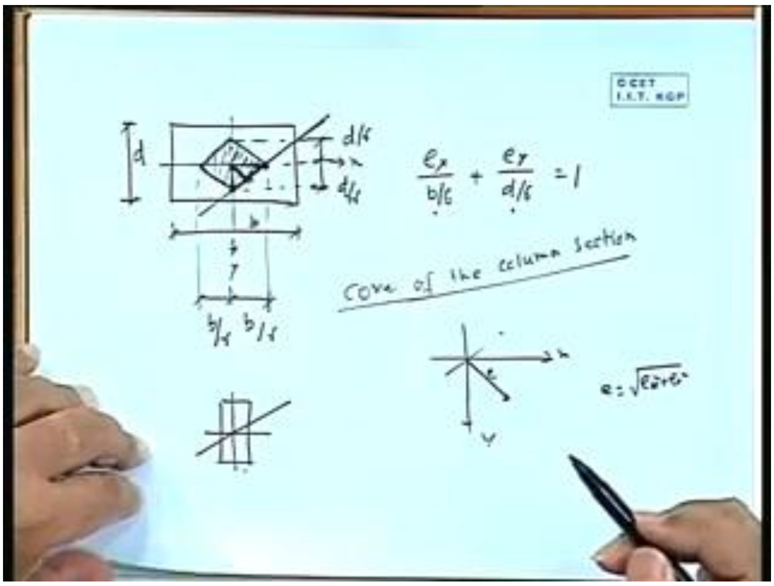

So, we are ultimately getting more or less this form. This was b, and it was d, and just now we wrote e x by b by 6 plus e y d by 6 equal to 1, follow. Now what is that line? So, what is e x? E x we are taking along this. So, here it will be b by 6, right; it is b by 2. So, it will be more or less one-third, right, and here it will be d by 6 one-third. So, we can draw one line. So, this value is your d by 6, and this value is b by 6, right; that will be equal to 1 or we can say, say, e y is equal to 0.

So, it is x; it was y, say, e y is equal to 0 only e x is there. So, e x equal to b by 6; that already you have obtained. If e x equal to 0 e y we will get d by 6; that also you have determined. If both are there, then if you just join these two points, there will be a line and that is the equation of that line, okay. So, if the load is on that line, the transition line will pass along the corner, right. So, if it is here the line will be here if it is here it will be.

So, if the load is here the transition line will be here; if it is here transition line will be there; if we put in between it will be rotating like this, and if it is here any other point it will go beyond, right, and if it is here it will go inside, right. So, in a similar manner if we try to put the load in other quadrant; so we can define a zone, right. So, this will be your b by 6, b by 6; it will be d by 6, d by 6. So, this part we can define b by 6 b by 6 here also this part if we define d by 6 d by 6.

Now here inside the region if I put our load on the entire cross section we will not get tensile stress; it will be entirely a compressive stress, and that area is called core of the column section, right. So, what is core? Core is a region; if the load is acting on that, the column section will not be subjected to tensile stress, right. This is the boundary; if the load is at any point on the boundary, it means one of the corner point or one of the edge will have a stress is equal to 0, right.

Say, if it is here the edge will be 0; if it here this edge will be 0; if it is here this corner will have 0. So, something it will be a limiting case. Now the next part what you have in your mind that why you have taken e x and e y separately; we could have taken if there is a load it is acting at a point having eccentricity at both the direction. So, both the direction we can combine, say, the load is acting here. So, here to here we can take r and r can be determined easily say this. So, this is x; this is y; this is the point. So, you can calculate directly e, right.

So, this e is root over nothing but e x square plus e y square. So, here we could shift a moment and normal load plus the stress distribution due to the bending moment we could take it. Now here then all the time it cannot be met. The reason is the bending equation whatever we use; we use M y by I for finding out the bending stress and the member is subjected to moment. Now there is one requirement there. The bending will be acting about a plane; we should have axis of symmetry. I have tried to draw one figure here.

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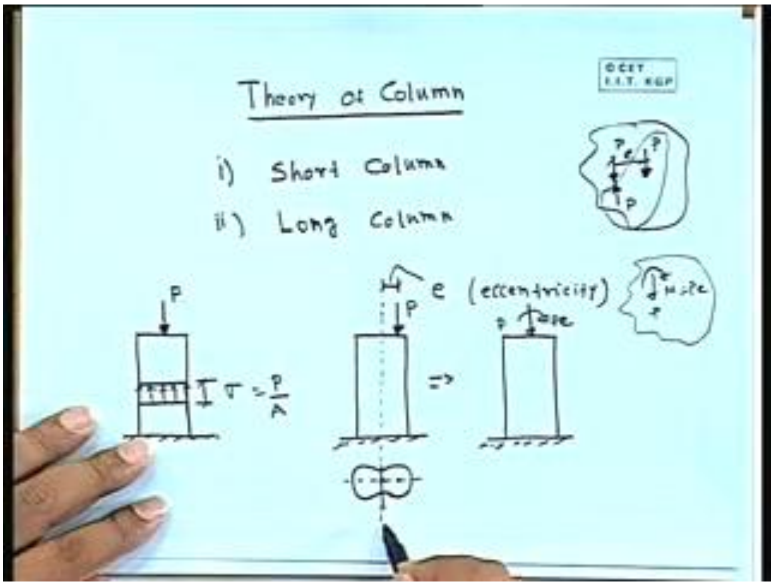

So, this part I have tried to make it symmetric, right. Now I can draw one figure, say, rectangular section if I draw. This section is symmetric about this; this is symmetric about this, but this part is not symmetric about that, alright. So, if we apply a moment about this that formula will not work or that whatever the idea is there it will not work.

Student: The circular section.

Circular section any direction you can apply, but a rectangular section or other type problem it will not be working. The reason is very simple; it can be nicely explained with a section like this.

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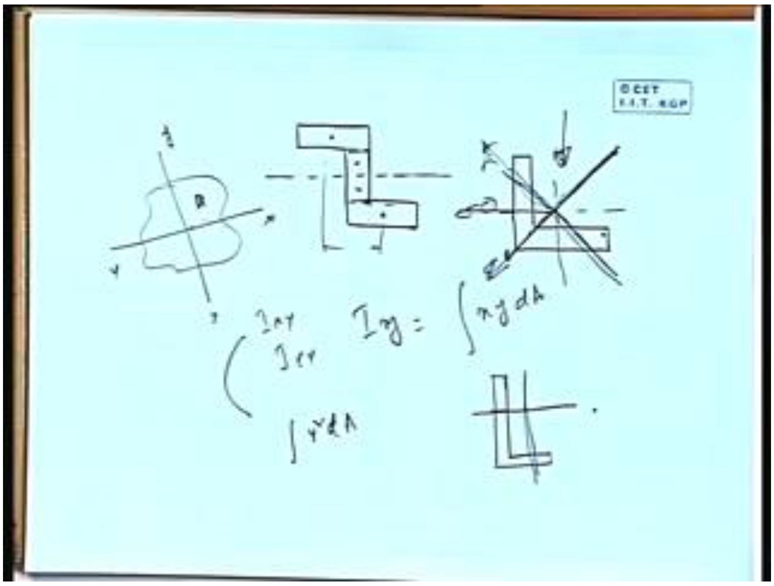

Say, it is a z section, right. So, the neutral axis is this. So, this height is equal to this height, absolutely symmetrical, right, dimension wise, but it is not the mirror image of this. So, this part should be on the other side to get a channel. So, we can say symmetry means if you keep a mirror if you get the image on the other side if it is identical, then it is symmetric, right. So, here rather we have a put a mirror again if you put a mirror. So, it becomes symmetrical, symmetrical, nonsymmetrical.

Now here what will be happening, say, if we apply a moment? Say, this is a cross section of a beam and some loading is there; some moment will be there. So, your stress will be M y by I. So, here M y and M y it is identical. So, stress at this level will be equal to stress at this level. So, stress at this level will be equal to stress at this level. So, on this block there will be a resultant force and there will be a resultant force, right.

Now they will be equal and opposite. So, if it is a moment like this, it will be compressive; it will be tensile, and they will be equal and opposite. So, total summation of force will be canceled, and they will give the moment; that is fine, and central part is as usual, So, there will be something. This force should be equal to this force, equal and opposite and it will cancel.

Now this part there is no problem, central part, but this part they will give a moment. Now this way also there is a distance. Now they will give a moment in that direction also, have you followed? So, there are two forces; if they are on the same line, the equal and opposite force they can give moment but if you keep away. So, what will be happening? Due to this height, there will be a moment, and this side there will be a moment. Now if you keep a beam, load is there; there will be a reaction.

Externally, there is no moment; horizontally, we are not putting any load, but if we just follow the equation M y by I, get the stress. From stress if we try to calculate the moment, we are getting some horizontal moment; means there is something wrong in calculating the stress with M y by I; means M y by I will not work here, because it is not a symmetrical section. This is very important, because the section should be symmetrical; otherwise, you are bending the resultant of the stress block, resultant of the stress block; they should be on the same line, only there should be a vertical gap, right.

Now, say, it will be happening with the angle section also, say, if I draw one angle section, say, the centroid is somewhere here. Now about this axis it is not symmetrical; it is not symmetrical. So, if there is one angle if there is a load, we cannot do the analysis directly; say, one of the leg of this angle is, say, horizontal; another leg is vertical, and load is acting vertically downward.

So, straightaway if you calculate the moment and from here if we calculate M y by I it will not work; now here what we have to do, we have to take one axis system like this, right. So, one of the axis there should be a line which will divide the section into a symmetrical part. So, this part is a mirror image of the other part. Now if we apply a moment, it will generate some bending stress. So, this bending stress will be balanced by this bending stress, right; there will be a counter component, and they are on the same line.

So, if there is a bending stress you have to dissolve into two components, right. So, one of the axis is your axis of symmetry. So, that axis is also not axis; this is not axis of symmetry, but anyway its resultant and its resultant it will match, because here the stress will have a resultant here; here resultant will be here, automatically it will cancel out, okay. Now we are talking axis of symmetry; axis of symmetry is something physical phenomena.

So, from the appearance of the figure of the cross section of the column, we can say this is axis of symmetry, but theoretically it is one of the principle axis, right. So, there is one quantity; just now we have uttered, it is principle axis system, right, any cross section if we take any arbitrary cross section if we take this is x axis; this is y axis. We calculate moment of inertia, second moment of inertia I x x, say, it is x x. If it is y y, we calculate I y y; there is one quantity, it is I x y, right. So, I x y is nothing but it is x into y into d A over the section; just like it is integration of it is I x x is y square d.

So, if we take one element y square d A if integrated, we get I x x, x square d A it is I y y. So, if you take x into y into d A it is called sometimes it is a product moment of inertia. Now there is one orientation where this I x y will be equal to 0 and that direction we define as principle direction, right. Now if we have a section and if we get some axis of symmetry, so axis of symmetry will be one of the principle axis and perpendicular to that it will be the other axis, right.

So, if we have a section, say, this type of section; we are fortunate that there is one axis of symmetry. So, without going for calculation I x x I y y I x y, we can directly get this is one of the principle axis. So, other one will be perpendicular to this, right, but if we have a section like this; this is long, this is short. Your centroid is here; there is no axis of symmetry. In that case you have to calculate I x x I y y I x y and it is not principle direction.

So, all the quantity we will get, and from there we can calculate I x y in an arbitrary direction and that direction if we make that I x y is 0. So, you will get what is the orientation of that direction. So, with the second moment of inertia we have to calculate and get the direction. So, it may be some direction, and perpendicular to that it will be the principle direction. So, any moment when it will be acting if it is applied in the principle direction, there is no problem; we can straightaway apply the elementary bending theory that M y by I.

If it is not along your principle direction, M y by I cannot be taken. So, that is the basic reason that if it is in an arbitrary direction with e x and e y, we cannot combine and take the moment like this, because that moment may not be acting along the principle direction, right. So, our bending theory one of the assumptions we have taken that the bending moment will give a bending of the member and the plane of bending it should be one of the line of symmetry or one of the principle plane of symmetry. So, it should be one of the principle axis.

So, if we take moment in an arbitrary manner if it is not following the principle axis system, the moment we have to resolve in two mutually perpendicular moment; one will be along one of the principle axis, the other one will be following the other principle axis. After that M y by I you can just apply and combine, say, this type of problem. It is very trivial or simply this problem the load is acting like this. So, bending will be in that direction, right. So, the bending part the direction of the bending is this. So, that you have to resolve along this and along that.

So, this bending it will be like this and here bending will be like that. So, individually you can apply M y by I and M y by I, then combine both the effect get the stresses on the cross section. So, that is why I was very careful about drawing the diagram why it is symmetrical and all those and due to that reason straightaway we can calculate with e x and e y, okay. Now you must have realized the circular cross section, it is very beneficial in that sense and if we take a circular cross section.

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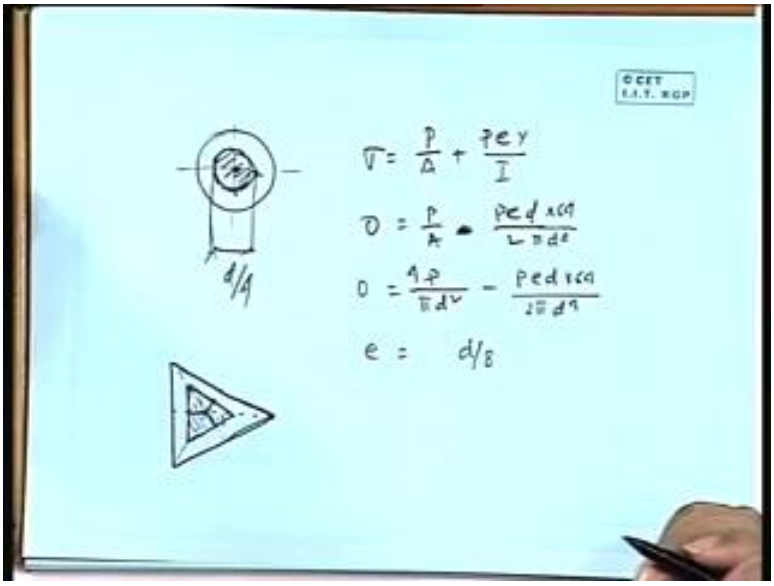

So, this is the point which is the location of the centroid if the load is here. So, it is eccentricity in one direction. So, if we just follow the approach what we have applied for a rectangular cross section with one directional eccentricity, you can find out the range of this e, so that the limiting point will be here, right. Now in a similar manner if it is here because here also it is line of symmetry. So, what will be happening? You will get a core having a circular cross section, right.

So, here if some limit you can find out, say, already we know it will be sigma equal to P by A plus your P into e is the moment into it is Y by I. Now you can write say P by A P e this part if Y is your d by 2 minus and if you make this, this is equal to 0, and I will be how much? It will be pi r to the power 4 divided by 4 or in the form of d if you write d to the power 4 divided by 34 or 64? I think it is 64.

Student: 64.

So, 2 into 2 4, 4 into 4.

Student: 16

16, 16 into 4.

Student: 64

64, right.

So, from here if you try to calculate e, so this P this part also it is P d square, right.

Student: there will be pi d square by 4, sir.

Yeah, we can write P pi d square divided by 4 minus P e d 64 2 pi d 4, right.

So, from here we can calculate. So, pi pi cancel, P P will cancel; here it will be d cube. So, it will be d square d e it will come here. So, e will be d into some factor. So, factor will be how much? This 64 will be 32, 32 divided by 4, 8, right. So, it will be 8, so d by 8, alright. So, it will be e will be d by 8; e is what? It will be d by 8. Now you can put it d by 8; you can it d by 8, d by 8. So, if you follow x y system and it is symmetrical if we take x y in other manner; it will be also d by 8 all the way. So, you will get basically a circular type of cross section that will be the core, and its diameter will be how much?

Student: D by 4

D by 4. Now if I take a triangular type of cross section. So, there is some interest very important cases I am taking here, say, it is equilateral triangle. The centroid will be one-third from the base, right, or we can say it is more or less symmetrical. So, if you extend this one, this will be the centroid. Now if you put the eccentricity here, some tension will be developed this side, right. So, there should be a limit. So, e in one direction you can make it because this is a symmetrical section.

So, if you put e there will be a bending and if you make that limit something you will get. Now you can think in that manner; you can push the load this side. So, there will be a moment, and it will be symmetrical. So, there should be some limit and in a similar manner something. So, there should be some triangular type of core here, right. So, only this distance you have to find out; if it is equilateral all the side it will be same amount, right, and you can enclose that will be the area. So, within that if we apply the load, section will be under compression entirely.

So, for that type of section it will be the core. Sometimes, we require that information how will be the geometry of the core, and what will be the extent, right. Now I can take one example and at least try to explain how stresses will be generated inside.

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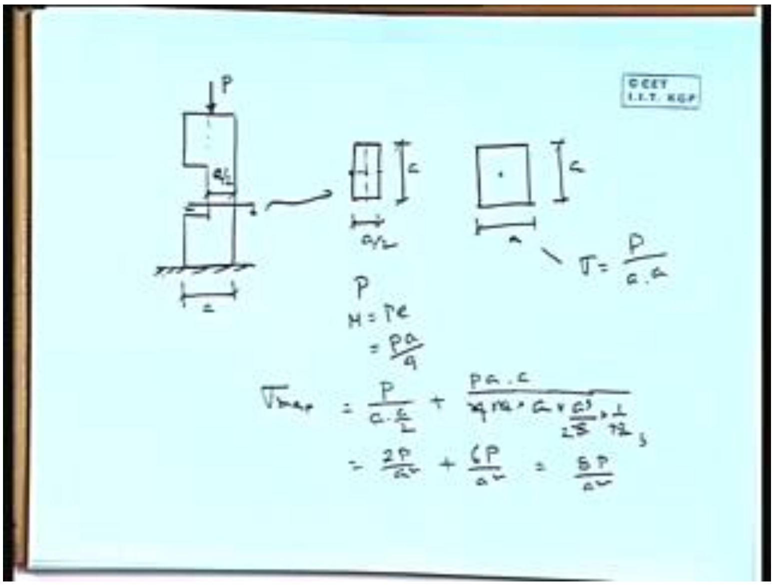

Say, there is a section, say; the cross section is it has a square cross section initially. So, this part is a, other side is also a. Now central part it is reduced to a by 2, other side it is a. Now here this part and this part, the stress will be P by a; a means area, capital A will be A into A a square, but now here the central part stress will be different. Now if we are interested for finding out the stress, what will be the stress? Now if we cut that section it will be looking something like this; this is your a, and this part will be a by 2.

So, if we cut and look from top and the top part it will be square section; it is a, it is also a. So, that was the initial section. Now it became half; it became a by 2 and a, not clear, see here if we cut. So, this section is this one, right. So, this is a by 2; the other side it is a, and here it is a, other side it is a. So, here the load is acting here top part. Now here it is acting at this point, right. Now in that case sigma is straightaway P divided by a into a. Now here for this cross section the centroid is shifted to this point, a load is here, right.

So, here there will be a moment. So, the load is P and moment is P into e; what is e? E equal to a by 4, right, now P into e; so P a by 4 is M. Now for that cross section P divided by area here it will be a square divided by 2 and this P a by 4 is the moment. So, M y by I that part will be 1, right. So, there will be one compressive part that will be maximum and minimum part that will be tensile here, because it is not within the one-sixth range of the core, right. So, it is going to the extreme point; automatically this end will get tension.

So, we can get, say, sigma max it will be P by a into a by 2 plus P a by 4 that is the moment and M y, y will be your a by 4, and I will be, say, M y by I. So, I will be the moment about this; moment about this will be a into, right. Now this part some a you can just make half. So, it will be 4. So, 2 a will go, it will be a square because it will be a square. So, it will be 2 P by a plus P it will be there. So, 4 it will be 2, right, and 4 it will be 3.

So, 3 into 2 it will be 6 at the top and a a it will be a square and it will be a square. So, you can write 8 P by a square and just try to compare. It was P by a square; it is 8 P by a square. So, if the section reduces to half, the maximum stress will be increased eight times, right; definitely, you can calculate the other part. So, other part it will be just minus or here you will get minus. So, there you will get minus 4 P by a square. So, if you want to get stress here. So, this part will be intact; only this part will be minus. So, if you take carry over minus. So, 6 minus 2 this is a minus part. So, major part will be minus.

So, it will be 4 P by a square; you will get half value of that. So, you just try to realize. So, section became half and the level of stress it is increased like anything. So, it not two times; it is eight times, okay. Now this is more or less about your short column. Now next class we will be talking about long column; long column the phenomena of failure is entirely different. Let me try to explain little bit because some time is there. Here what we are doing for short column? Short column basically if it is a case where load is acting along the centroid of the cross section, the case is very straight forward; it is P by a uniform.

If it is some eccentricity, eccentricity will produce some bending moment; we have to take some bending stress in addition to the direct stress. So, some of the compressive stress will increase, some part it will be reduced; it may be 0, it may be in the negative side means tensile stress. So, basically we are getting here some stress in the form of compressive stress or tensile stress.

Now if we want to do that design, we have to check that the tensile stress or the compressive stress whatever we are calculating that should be within our allowable stress; that is our design criteria. Now in long column, your phenomena will be entirely different. So, long column means here the length part we have not considered. So, we are talking about this is a, other side is a; this is a by 2. This length we are not bothering. It may be 5 a, it may be 3 a, it may be 2 a, any a we can take, right, but this height will be not very long compared to this a; that is the restriction.

So, if we study long column, then we will understand there should some restriction of taking this height if we try to handle this problem as a short column problem. Now in long column, the failure mode is entirely different. So, here the failure mode is we are calculating the stress and try to compare with the allowable stress. Now if the stress generated within the section. So, whatever calculation we have made, we found some stress distribution.

Now if the maximum stress is exceeding the allowable stress. So, what will be happening? There should be failure of the material, right. So, yielding may occur or shear may occur; basically, the material failure will initiate, right, but if we take a long column or the length is quite predominant compared to the cross sectional property, say, if we draw here.

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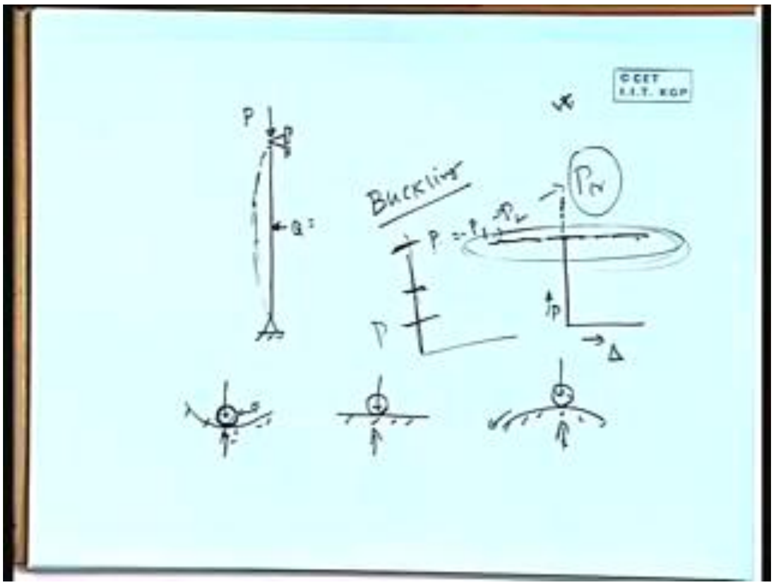

So, it is just P, say, this is quite long compared to the cross section diameter. So, it is so long just I have shown with a line; if there is a load P, what will be happening? It will take care, say, if it is, say, acting absolutely along the centroidal line. So, P by a that will be the stress like a short column, and it will take up to what range? Up to P by a value should be equal to or less than equal to your allowable stress. If we exceed that, then there should be some yielding of the material.

So, material failure will initiate, but if this length is quite high compared to this, the limit we are talking about the P by a it will reach the allowable stress. So, that is quite far off; before reaching that load level, we will get the failure of this member in a different form. What is that form? That is called basically the buckling of that member, right. So, there is a term called buckling. So, what is buckling? Buckling is, say, here if we apply P it will take care, and it will remain straight, no problem.

Now from side if we give one disturbance, say, small load if we apply, it will undergo some bending. Now if you release that load, it will come back to the original position, right; say, P is equal to say P 1 the applied load, and this is Q; Q is some small value. So, if we apply a small Q, there will be a bending because you are putting some load; automatically, it will undergo some bending and load it will be carried by the member, and if you release it will come back.

Now if it is P 1 after that if you go to P 2, say, it is more than P 1 or we can say this P value if we increase gradually, say, if it is here. So, it will be straight. If we apply a transverse load, there will be bending; if you release if we come back again. Now if you increase the load P, it will remain straight. If you apply Q again, you may get little more bending; if you release it will come back. So, like that it will be there. Suddenly, it will come to a point if P is equal to that value if we define this P critical if we apply Q, there will be a displacement; if you release it will not comeback, it will be there, right.

Now you have to bring back with a force you have to bring it back with some force. So, it will be there. Now if we apply more than free critical, still it will remain straight, no problem, because it is below your allowable stress. During that stage if we apply a force, it will undergo bending; if you release the load it will not comeback, it will not be there; rather, it will go on deforming. So, whole it will try to entirely fail. So, I can give one example here, right.

Now there is a surface like this, and there is one ball or one object is there, and it has some load; some reactions will be there to take that weight. Now this is basically P, it is P 1 or P 2 if we give any disturbance Q. So, it will go there; if we release it will come back. Now here it became free critical. So, if you give any force it will come here; if you release that force it will be there, or if you disturb here to here it will be there. It will not come back, but it will not go further, but if it is beyond free critical it will be there, no problem, when there is no lateral force but if we just touch it.

So, if you just help it to bring it here, it will not come back, it will not be there; it will go further, right. So, the stage this is a problem of stability, alright. So, it was this force is balanced by this; it is in equilibrium. This force is balanced by that, it is in equilibrium; it is also in equilibrium. In all the cases it is equilibrium, but this is a stable equilibrium; it will give any disturbance it will comeback. This is it will give any disturbance it will create problem unstable, and there is a case it is neutral equilibrium.

So, if you give any disturbance the disturbance will not be magnified, but it will be there, but it will not comeback. So, the column if we apply a load beyond a limit if we give any disturbance; so long disturbance is there; deformation will be there, disturbance is not there it will comeback. So, it is a stable range. Now after that there will be a critical stage; it will be this particular condition. So, during that load if we just deflect it and you release; so it will be there.

It will not go further; it will not come back, and if we apply further load it will take. So, long it is straight it will take, because this load will be taken by this, but there if we just touch. So, there will be an absolute collapse of this much here. Now, this phenomenon is called the buckling of your member. So, if I draw the load deflection curve. So, this side if it is P and this side if it is delta; so if we apply load there is a deflection load deflection curve we plot.

Now here if the load is axial there will be no. So, it will go up to critical range, right. Now critical range after that it may be this value or this value or this value or any value; it may be in the negative direction also, right. So, you can say it will go like this. Now sometimes in some places you will find there is an extension of this line in a dotted form; means it may go there also, but it is a dotted form means it is in a very unstable region.

So, if you touch, there will be entire collapse; if it is a critical one that displacement may be 1 centimeter or minus 1 centimeter or 5 kilometer or minus 5 kilometer this side, right. So, that is the critical range or the limiting point, and we have to do that design based on that, and our allowable stress will be occurring somewhere here. So, before this your buckling will be initiated.

Now that is the basic introduction of buckling which is related to your failure of the long column. So, here it is not material failure; what is the failure? It is geometric failure, right. So, it is quite long and due to its typicality of its geometry that it is a very long cylinder type. So, it cannot retain its geometry when some load is there; it is not due to the failure of the allowable stresses not exceeding the stresses beyond the allowable stress.

So, the material failure will not be there. It is basically geometric failure, but ultimately material failure will be there. So, if it is beyond critical load if you apply buckling will be there, then there will be excessive deformation. So, large strain and material will start yielding. Finally, it will be material failure, and entire collapse will be generated, but the initial failure or the failure will initiate with the buckling of the member.

Student: The value of P will keep decreasing for the point the extent of buckling that is taking place.

The load will be?

Student: decreasing.

Load will not decrease; here it will be constant, after that it may be like this. Now whatever I have drawn it is very, very theoretical case, right; actual case the phenomena will be little different. So, here it is straightaway it will go and it will be something like this.

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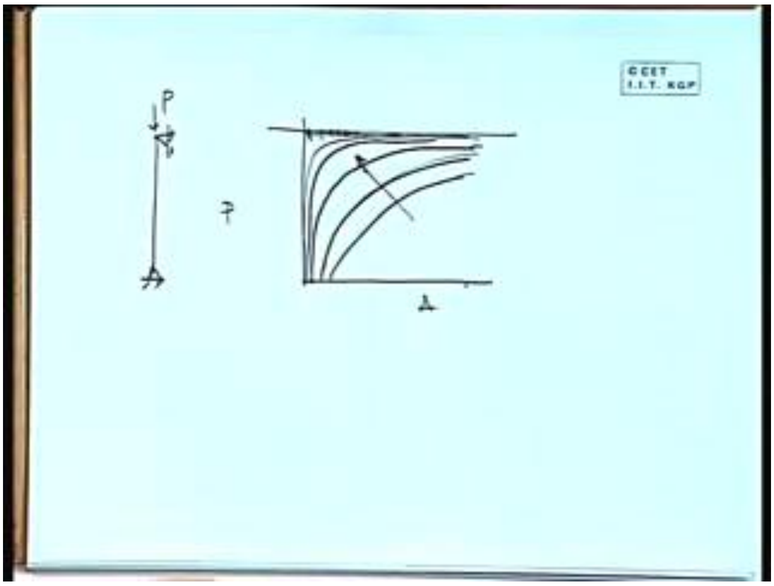

Actual load deflection curve if we draw it will be something like this, say, this is delta; if this is P, say, there will be a column here, there is a P. Now we are talking about an ideal case, say, cross section will be entirely uniform; load will be absolutely through the centroid, support will be absolutely perfect. Now if there is some eccentricity, load is not acting at that point; it is little bit away. So, what will be happening? Some moment will be there. So, there is a load and there is a moment and moment will create some buckling.

So, you go on apply from the beginning some bending will be there. So, it is not that is straight; suddenly it will be there I think. So, there will be a gradual process. So, initially it will be happening; if there is some imperfection, the curve will be something like this.

Now if you increase the imperfection it will be like this; if it is like this if the imperfection is less it will be like this; if imperfection tending to 0 it will be like this. So, theoretically if you try to reduce the imperfection level it will try to follow this; though, it is a theoretical you cannot make the imperfection absolutely zero.

So, depending on the imperfection your curve will be like this, but theoretically if it is 0 it will be in that form, right. Now why we are interested about the theoretical part? Reason is calculation is very easier this one, right. So, that is why we calculate this and that will be normally the upper bound; always actual capacity will be little less. We have the factor of safety to take care that particular aspect; otherwise, if we try to calculate this part, we have to go for a non-linear analysis, because here deflection will be large, and we assume deformation is very small in any analysis. So, that assumption is there.

So, that type of study we can perform if we perform a non-linear analysis; a non-linear analysis is quite advanced type of analysis. So, if it is straight we can take this small deformation and perform this analysis. So, it may be small or large, it is valid. So, in order to make the calculation simpler, we are trying to get a much theoretical case where imperfection is practically zero. Now for perfect case, we put some factor of safety, or we can add something for some level of imperfection some factor to reduce this comes down.

So, that is the main idea. So, actual phenomena are like this and theoretical part is that, and our idea if we get that limit we can access that will be the level; after that we can do some adjustment. So, that is the main requirement or main parameter we want to find out that what is the critical value where buckling will initiate for a perfect case. So, we can keep up to this level today. Next class we will go for calculation of the critical load for different type of problems.

So, we were talking about theory of column. In last class, we have completed the short column part; you must have remembered we have divided the column in two groups. One is short column, another is long column. Short column part we have more or less completed and long column just at what is the mode of failure; how it will behave under the load, what is buckling; all these things we were trying to explain. So, it was just the basic structural behavior of a long column.

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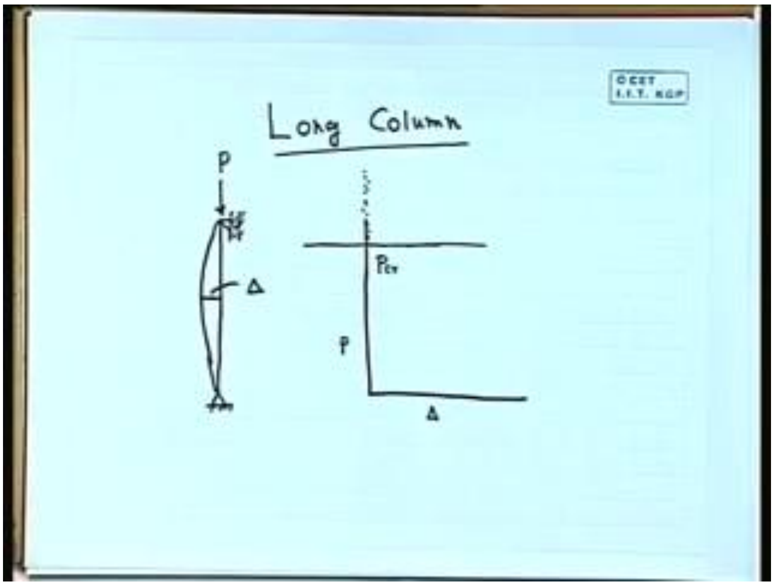

So, we continue with that long column. Now if we draw the diagram what we draw in the last class, say, there is a column like this both end supported. Now the load deflection curve you must have remembered delta this was P. Now if it is a perfect system. So, if you apply the load, it will directly take that load in the actual manner; there will be no lateral displacement. So, this delta is lateral displacement, right. So, if I draw the lateral displacement here, say, maximum displacement if we try to define as delta.

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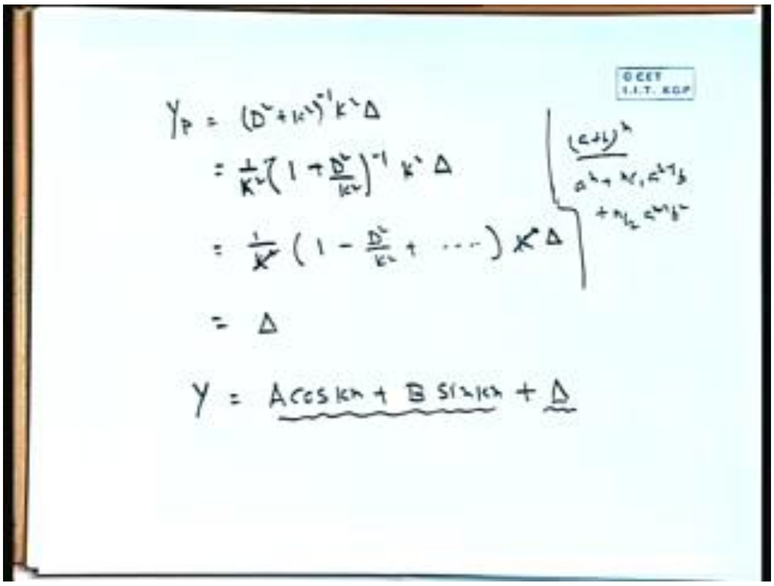

So, it is a to the power n plus n c 1 a to the power n minus 1 b; we write in this form. So, here a to the power n n c 1 n minus 1 b n c 2 a n minus 2 b square. So, it will go like this. So, it will start with a to the power n, next a n minus 1 b to the power 1. So, next gradually it will change. Now, here if we. So, 1 to the power something is always 1, and here first this will not occur; second term it will be n c 1 is n; n is here minus 1, and this part will be there.

So, if you take this term, second term it will be D to the power 4 by k to the power 4; after that D to the power 6 k to the power 6. Now what is D? D is the differential operator. Now this side there is a constant quantity, right. So, if we multiply first of all this k square k square will cancel. Now it will be basically delta into 1 second term if you take. So, delta if you take derivative twice, first derivative itself it will be 0; second derivative there is no way of getting any value, and if you take further terms, order of derivative is much higher.

So, all the term will contribute zero. So, ultimately you will get Y particular is equal to delta. So, we can write the Y equal to A cos k x B sin k x plus delta. So, this part was complementary part; this part is the particular integral part. Now this is the solution of the cantilever beam. Next we will put some boundary condition, find out AB; from there we will try to find out the critical load, right. So, in this class I am closing with that equation. In next class, we will try to utilize try to find AB, critical load and all those things.