Strength and Vibration of Marine Structures
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Lecture - 16
Theory of Column – I

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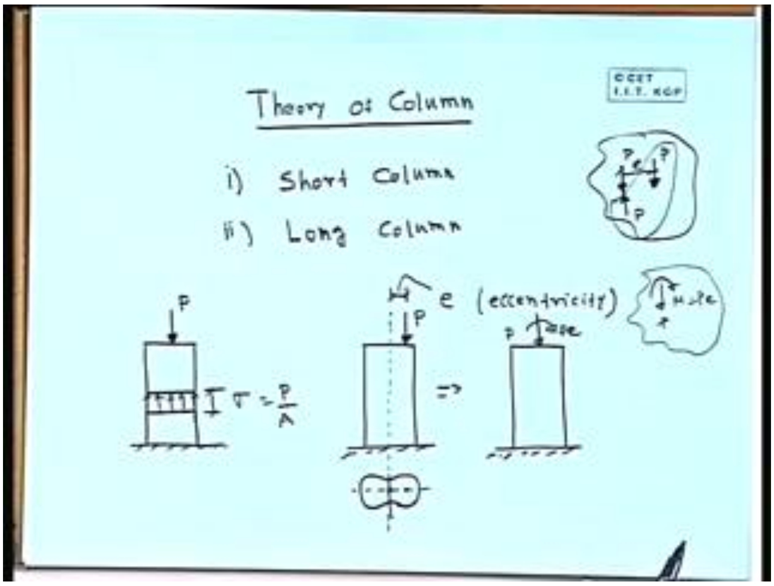

So, today we will be talking about theory of column. Now first of all we must define what is column. Column is geometrically looking like a beam. So, one of its dimension is quite long compared to the other dimensions. So, the length is a predominate dimension here compared to breadth and width; that is more or less like a beam if we think in terms of geometry. Now the basic difference of column with beam is the loading. In beam, loads act perpendicular to the axis of the member, but for column load acts along the axis of the member, number one. Number two, we are much more interested if the load is compressive in nature. So, first of all load should be along the axis, and it should be compressive in nature.

Now column or rather the theory of column it is divided into two groups. So, one group we say it is a short column; another group we say it is a long column, right. So, we can say case one it is short; number two, it is basically the behavior of this short column and long column is entirely different. So, our primary concern is with the long column; how it will behave under the action of compressive force, but before that we can discuss something on short column then switch over to the problem of long column.

Now for short column and long column, the primary difference is the ratio between the length and its lateral dimension. So, say, b d if it is a rectangular section, there will be width; there should be a thickness, and there should be a length. So, the ratio between the length and the other two; that will give the guideline whether the column is a short column or long column, right, say if we take say this pen, it is a cross sectional area. So, if we take that length, the phenomena will be something like that and if we cut a single small piece from here; the cross sectional area is same, but the length is rather a small or we could take a much longer stick having a same cross section.

So, structural behavior of a small piece and a long piece will be entirely different. So, the ratio of the length compared to the lateral dimension; that will give the guideline whether the structural component can be handled either short column or a long column. Now if we start with short column, say, we can take a problem like this, say, there is a piece of column, length is not a very big; definitely, it is a short column, say, it is resting here. Now if we apply a load at center or rather the centroid of the cross section of the column, at any station your stress distribution will be something like this. It will be P by A, say, P is the force, and A is the cross section. So, that part the stress sigma will be P by A, right.

Now if the application of load is bit different, say, same problem if we pickup, say, that is the center line. Now load is acting not at the center, say, it is acting little bit away from the center. So, same P and say the distance from the centroid of the load if we say it is e; e sometimes we define as an eccentricity, right. So, if the load is little eccentric not exactly on the centroid of the cross section. Now the cross section may be a rectangular cross section; it may be a circular cross section; it may be some electrical cross section or any arbitrary cross section, say, if we take a section what will be happening, say, we will get say if I draw the section here, there might be a section like this also.

So, that is arbitrarily I have drawn. Main idea is this part should be symmetrical about this. So, this will be the centroid, and the load may be acting not at that point; it is acting at some other point. So, the distance between these two we say it is the eccentricity, right. Now if the load P is acting like this, the load P we can shift at the centroid but we have to take a moment, right. Now, say, there is a load P, say, on a structure it is acting. Now if we take a force like this, say, if it is P if it is P, this is also P. So, initially it is P.

Now this part we can just add. So, it is just like we add something subtract something. So, this set of force is not going to affect the structural deformation. So, it is it will be identical. Now next step we can take this load and these two load we can take separately. If this P and that P they will form a moment couple, right, and force wise they will cancel each other. So, that problem we can think in that manner, say, this is the structure, the load is there plus there is a moment. So, moment is equal to P into e if the distance is e and this is the P.

So, the force here P is not acting at centroid; it is little bit away from the centroid at a distance of e. So, we can shift the load at the centroid, but along with the force there should be a moment that will be P into e, right. So, this problem we can just think in that manner. This is the column. So, the equivalent force here is the load plus a moment. So, this is basically your force P and this is P into e. Now the effect of P you know P acts at the centroid; that will be a uniform distribution of stress and that will be P by A, and there is a moment that is P into e. So, if we apply a moment here, we know it will generate a bending stress. And bending stress at the neutral level it will be 0, and it will vary in a linear manner, right.

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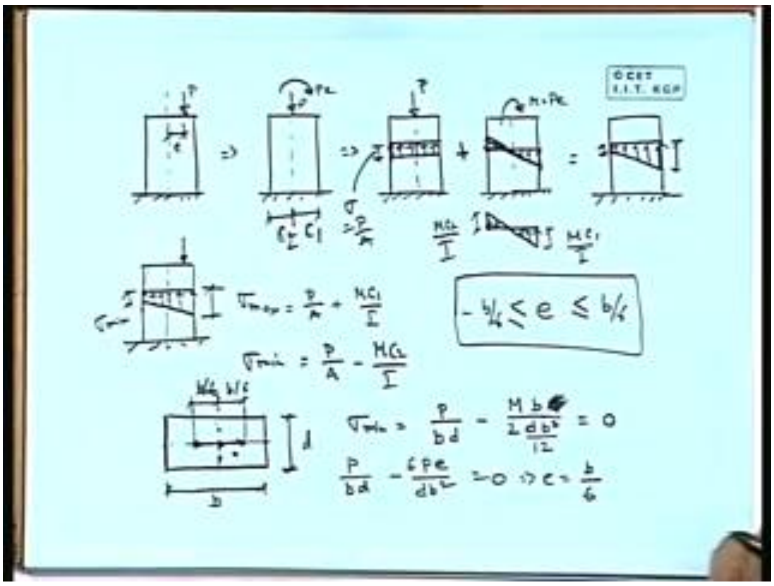

So, if we take the problem in this manner. The stress block became little erratic if we put it in a correct way it should be like this, right or if we just draw the final form. So, the actual load is P here. So, that distance we have defined as e that if the load we have shifted at the centroid it is P into the moment P e. So, that part we can just add in a separate way. So, there is a load here P that will give the stress distribution. So, that part already we have defined; it will be your sigma equal to P by A and here if you apply moment P into e. So, it will be a just bending problem. So, there will be a bending stress.

So, this part will be your maximum bending stresses, right. So, this part will be your M Y by I. So, if it is this side is, say, it is C 1; this is C 2; it will be C 1; this will be MC 2 divided by I, right. So, it is a C 1; this is C 2. So, it will be M C 1 by I M C 2 by I. Now if we add that, this stress will be how much? There is a stress M C 1 by I; there is a stress P by A. So, both we will add together, and here this stress will be this minus this. It will be P by A minus M C 2 by I, right, this small value.

So, due to the direct load there should be a direct compression uniform, and due to the bending, it will be also the normal stress; one will be plus, another will be minus. So, this side it will be compression; this side will be tension; here it will be entirely compression. So, this part and this part will be added together and this part and this part, because this part will be deducted from this side, right. So, here the stress block, finally, whatever you will get here. So, if I draw that block again from this side. So, that is the P.

So, the stress block whatever we are getting here, this part you can say it is sigma max; it will be P by A M C 1 by I and this part if we say sigma minimum. So, sigma minimum will be P by A minus M C 2 by I, right. Say, if you take that member, apply a moment. So, this fiber will be in tension; that fiber will be in compression, and if we apply direct load, all the fiber will be in compression. So, this side it will be compressive additive; this part will be compressive minus that part you are getting in a form of tensile stress due to the moment. Say, it is very simple, say, you are standing straight. So, you try to think about the pressure on your foot, right. It will be uniform if you try to just slightly bend.

Student: So, it is possible that at one particular part there will be no compressive stress, sir?

Yeah, so that part I will come little later. So, if you just try to bend little forward. So, here you will get more pressure, because you have a tendency to bend that side; means the load is on the forward side; means there is a moment in the forward direction. So, that side you are getting more pressure, whereas your heel part will get less pressure, and sometimes, your heel may lose contact; what you are talking about there will be no stress there, compressive stress, right.

Now here it will not be no pressure, some pressure will be there in a negative form that will be your tensile stress not a compressive stress. Now here in bending M C 1 by I and M C 2 by I they are the bending stresses; maximum one is compressive, another is tensile, and here P by A that is the direct compressive stress. Now if it depends on that magnitude entirely, say, this side there is no doubt it will be always compressive force, but here if P by A is more, say, P by A is 10, and M C 2 by I equal to 5, then it will be still compressive but value will be 10 minus 5 will be 5, but if this is 10 and if it is say 15, right. So, bending part is more compared to direct mode.

So, this negative part will be more. So, 10 minus 15 it will become minus. So, minus 5 compressive stress means there will be a tensile stress of 5. Now there are some situations; we do not allow this tension to be developed. In some cases there is no problem; it may be tensile, it may compressive, but there are some cases, say, particularly I was giving the example of pressure below our foot. Now if we try to design some foundation of some structure or some machinery; now, say, it is resting on something some concrete block or somewhere, now if eccentricity is more. So, here there is no contact. So, it can take direct compression; it cannot take tension, right.

When it is a continuous one, it can take tension as well as compression but here only that pressure it can take. So, the tensile stress it cannot take. So, definitely if there is some foundation. So, this part there will be no tensile. So, automatically there stress entire pressure will be 0, and stress block will be entirely different; that part sometimes we try to avoid, or there might be some other type of example where this part we try to make is at least 0 or a small positive value.

So, definitely C 1 C 2 I is the property of that cross section. So, M E is P into E. So, E E is the controlling factor. So, if E is very very high if you go on increasing E, moment part will be go on increasing, and this part will be more, and automatically it will switch over to 0 and the other side negative side. So, there should be some restriction of this E eccentricity, and beyond that if you go, then that problem will occur means.

Student: It could reach a position where P upon A is equal to M C 2 upon I

Yeah, yeah. So, that is the limit.

So, that should be the limit. If it is that critical point then each your stress will be equal to 0, or this particular stress will be equal to 0, right. Now let us consider a rectangular cross section and try to investigate how long we can go at it. So, what should be the maximum limit of E? Now you have rather uttered the solution that sigma minimum it should be equal to 0, and we have to put the provident value of I and C 2. Now if the cross section is a rectangular section, say, this is your b and that is, say, your d, right. So, that is a centroid hull level, say, E is somewhere here, okay.

Now if we take sigma minimum. So, sigma minimum will be P by A, A is how much? It will be b into d and this one will be M, what will be C 2? C C 2 will be b by 2, and I will be your? Bending will be along this direction. So, it will be b q. We are taking moment if we put the force here; there will be a moment, right. So, you try to think there is a column and it will try to bend. So, bending about this axis, so it will be d b cube. We could take b this one, d that way again; anyway we have taken it will be d b cube divided by 12. So, this should be and that should be multiplied by e, right.

So, M into e is the moment. M C E is C 2 is b and b by 2 and I is b d cube by 12, right. Now that will be the minimum value. Now if we take it is a limiting case, say, this stress is equal to zero. Means this stress should be equal to this stress equal and opposite. Now here if we simplify it; no, it was M. We have written M as well as E. So, it will be b into E or M; rather, E part we just eliminate from here. Next step we will put it. So, this b b one part will cancel; here d will be there.

So, we can write, say, P divided by b d minus say P e at this level we put b and here it will be 6, and this part we can make, say, d b square equal to zero B by 2 because we are calculating stress here, right. So, this is C 1; this is C 2. So, both are equal to your b by 2 b by 2; rather, we could take d and b this side. So, I have given b d in such a form I think. So, it is getting little confusion anyway. So, b d here it is 6. So, 6 it is placed here at the top, b will cancel one b; it will be b square, d is there. So, b d part also we can cancel, and P we can cancel. So, from here will get e equal to?

E we will get; we will get B by 6, right. So, E will be your b by 6 maximum. So, this side it can be b by 6 and in a similar manner it can go here, right. So, that is the range. So, it can go up to here. So, you can say there should be a limit b by 6 b by 6, right, or we can write e should be less than equal to b by 6. So, eccentricity if it is within minus b by 6 to plus b by 6, the entire cross section it will be compressive stress; there is no chance of occurring of any tensile stress, right.

So, it should be less than or equal to. So, if it is equal to the extreme end it will be 0; if it is less, some value will be there compressive stress. So, that problem if we put in the reverse manner; so, b if it is in the minus side. So, if it is b by minus b by 6, shear stress will be 0 and from there it will generate. So, there it will be a triangular stress; if it is here b by 6, it will be a triangular stress. If it is less than that, it will be some value; it will be a taperial type of stress, right.

Now eccentricity we have made along b; it may be along d; it may be acting somewhere here. So, same thing it will be valid. So, it will be at d by 6 or minus d by 6. Now if it is here, what will be happening? Simultaneously, extensity will be there along x axis along y axis, right. So, first we have taken eccentricity along x axis. Now same logic will work if extensity is along y axis; only the directions we have to change.

Now it may be here again diagonal. It may not follow 45 degree or a line joining corner to corner any arbitrary, say, if it is 1 millimeter, it may be 5 millimeter there in the other side. So, there extensity will be in both the direction. Now such a problem can be handled in that manner.

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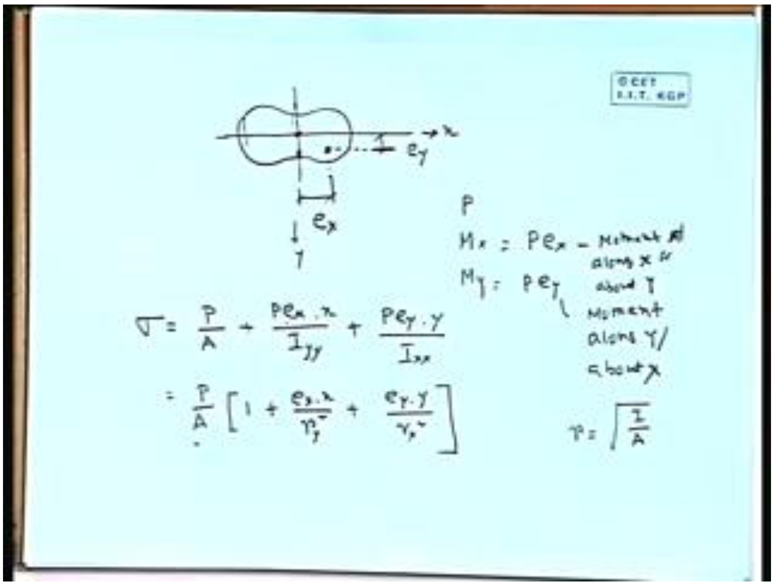

Say if I take any cross section. So, any cross section is quite important here; I will discuss little later, say, this is the centroid or a cross section of a column. Now if the load is acting here, say, this distance we can define it as e x and that distance we can write as e y, right, say that side if we write x and this side if we write y. So, along x the extensity we are defining as, say, this is the load point. So, this is e x and this is your e y, right.

So, this load we can shift here. So, it will come here with a moment P into e x about y axis. Now next step if you shift that load, there will be a moment along x axis P into e y, right. So, what will be happening? So, if we shift here here ultimately at the centroid. So, we will get p, and there will be a moment about x, and there will be a moment about y, right.

Now this m x; m x means moment about x. Sometimes, we can take moment about x or we can say moment along x. So, about is the direction of the vector and it will try to give a bending along x, sometimes we say along. So, anything we can call along or about, right. So, if we take along at least this e x part it will be followed. So, we can say P into e x and it will be P into e y, right. So, here if we shift from here to here; so, it will be P into e x that moment will be moment along x or moment about y.

So, you can say it is moment along x or about y, whatever you can define. Similarly, this is moment along y or about x, right. So, once you will bring it here next step if you go there, there will be moment about x or moment along y. So, this part is P into e y. So, there are two moments. So, there is a moment along this; there is a moment along that plus there is a force. So, it is just like this the force was here; we have shifted there. There is a moment along this; there is a moment along that. So, it is a biaxial moment, right; it is m x m y plus P.

So, P will contribute straightaway P by A and m x and M y that will generate M y by I. So, sometimes I have to take along this; sometimes you have to take about the other direction because moment will be along this or along that. So, we can write stress at any point. So, stress will be P by A. So, that will be a direct stress and your M x; M x will be your P into e x is M x. So, here it will be that into x by I y, right, or I y y you can say. P into e x that is M x; M x is along this.

So, if we give a moment at any distance. So, M x by I y y plus your P into e y, so moment about this; so, at any point y it will be I x x. This along about x y that we can change little bit. So, there is no unique relation that we have to take in that manner. So, what will be happening there should be uniform level of stress due to P by A and P e x x I y y. So, here there will be a variation of stress along x. So, p x I y y they are constant quantity, only the variable quantity is x. So, along x there will be a variation.

So, you can say there is a stress block having a top surface like this inclined along x and here it is P e y y y I x x. So, there will be a variation like this, right, but here due to that moment, this side will be the compression, and this side will be tension. And due to the moment this side, here it will be compression, other side it will be tension. So, in the positive direction of y, it will be plus; positive direction of x, it will be plus.

On the negative direction of x, it will be tension means minus. On the negative side of the y, it will be tension of compression due to the bending. So, that will be the total expression. Now this part we can express in this form. In this P by A we can take common P divided by A, right. So, it will be 1, and here this P we are taking common. So, there it will be A and A and e x it will be there. So, this part sometimes we write that there is a term called radius of gyration, right.

So, the definition of radius of gyration is it is written as r and it is square root of I by A, and normally I the unit is, say, meter to the power 4, and area is meter square. So, ratio will be meter square. So, r will be in the form of meter. So, it is called radius of gyration. Now here we can say A above. So, I y y y a if we put. So, this part will be r square, right.

So, we can write e x x by r square. So, this part we can write as Y, right, and this part we can write e y y divided by r x square.

So, this r y and r x we have defined because about y we are calculating I. So, it was defined in that manner. Now that is the general expression. Now if we want to express the stress at the extreme fiber. So, this x we can write, say, C 1 x; this is say C 1 y, other end it is C 2 x or C 2 y, right. So, for some specific case we can write the extreme values. Now here we can take the example of a rectangular section and try to see what is happening. So, sigma equal to P by A 1 plus e e x x y r y square e y y y; this is the stress at any point. So, it is x y.

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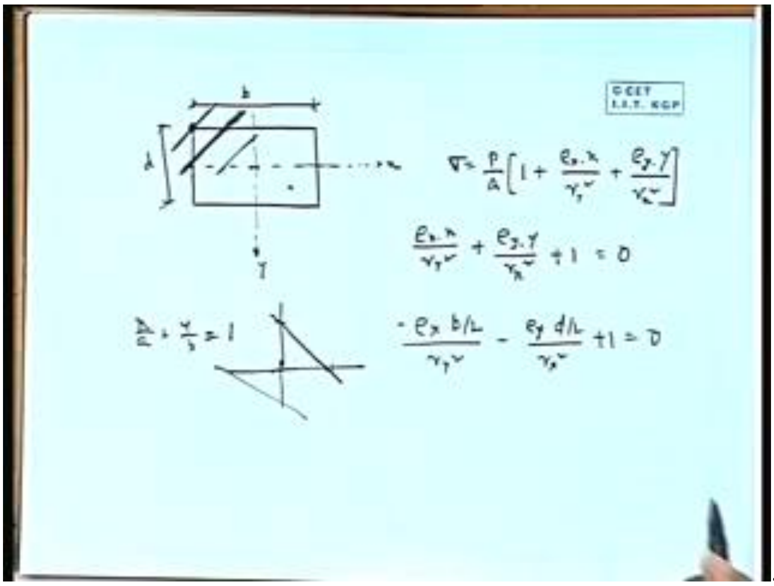

Now let us take one rectangular section it will be, say, this is x, and this side is y, right; that is b, and this is d. Now we repeat that expression it is P by A 1 plus your e x x by r y square e y y by r x square, right. Say, the load is acting somewhere here. So, this is your e x; this is e y whatever we have defined, and stress at any point will be something like this. Now we are interested for the limit of e x and e y, okay.

Now earlier we have seen if the load is at P by 6. So, here the limiting line was passing along the outside length and if it was here it was along this line. Now if it is here, it will be somewhere some line; there it will be 0. If the load is acting here, say, at b by 6, then your 0 stress line will be here. If it is beyond that, it will be inside. Now if we take this side is d by 6. So, 0 line will be here. If it is more than that, it will be here; after that tension will be generating.

Now if it is here. So, the 0 line will be along this, right, because there is a moment in that direction resultant moment. So, there will be some line that will have a 0 stress; beyond that it will be tensile stress, and this side it will be compressive stress, right. So, this e x e y anything if we pickup, there should be some line. So, this side will be compression and that side will be tension, and there should be a limiting line. Now if we do not allow the tension within that section, this line should pass along this; that will be the extreme, right.

So, if we are interested that everywhere it should be compression. So, there will be no tension. So, this line will not allow to enter inside. So, if you give high value of e x and e y, definitely tensile stress will be generated, and there should be some line beyond that it will be there, right, or we can say this sigma r this line is basically the 0 line. So, if that is 0 e x e y, say, r y r x, these are the section properties, right, or we can write your e x x by r y square plus e y y by r x square plus 1; that should be equal to 0, right.

So, that critical point if sigma equal to 0, then this part should be equal to zero. So, what is that? That will give the equation of that line, right. Now see there is an e x and there is an e y and for the section r x r y are known, and due to that situation we are getting the general expression of sigma is this. Now on that section, say, there will be some point where sigma will be equal to 0, say, if we equate to 0, then this part will be equal to 0, right.

We can understand if the load is here or there should be a line; along the line it will be 0. If the load here there should be a line along that it will be 0; if it is like this, there should be some line like this, and on that line the stress should be 0. Now the stress if we make 0 that should be equal to 0, or rather this will give the equation of this line.

Not necessarily it should be diametrically on that line, it depends whatever will be the e x and e y value.

Student: Directly opposite to the central.

We can say it is, say, this part if you bring it down. So, x by r y square by e x y by r x square by e y, right; that should be equal to minus 1. So, any straight line if we have an equation, say, x by a plus y by b equal to 1; what is that line? So, this is x by a; this is a. This is y by b; that is equal to 1. Now here x by this quantity, y by this quantity is equal to minus 1 or if you make it that is plus 1 means it will be minus minus some line, okay. So, if it is x y this is minus, this is minus, right. So, there will be some line. So, x x intersect is this quantity minus y intersect is this quantity minus y is this way positive. So, both will be minus minus that will be equal to 1.

So, it will give basically the equation of that curve and what will be the value of this one or this one; that we can get from this equation, right. Now we want that line should not be here or here; it should be on the edge, right. So, that intersection it should be equal to it will be compression, no tension. Now in that case what will be the x y coordinate? So, this line; so, there is a point x y on that line that will be minus b and minus d. So, we can put here e x b by 2 minus r y square minus e y d y 2 r x square plus 1 equal to zero.

Now from here we will try to get the value of the eccentricity e x and e y, say, sigma general expression we have written. Now if we make sigma equal to 0, then we are getting this is equal to 0. So, that will give the equation of that line. Now that line if we want to keep outside the column section, the extreme point is here. So, on that line there is a point and it has a coordinate x equal to minus b by 2 y equal to minus d by 2. So, that part we have substituted that extreme case.

Now if it is there, from here we will try to get the equation of e x and e y. So, this is e x; this is e y. Now e x and e y, this r y and r x now we can generate. So, what is r x and what is r y? It is square root of that I and all those. So, those values we will substitute here, and finally, we will get the expression of e x and e y and try to get the limit or we can keep up to this level; next class we will continue the remaining.