Strength and Vibration of Marine Structures

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Lecture - 12
Statically Indeterminate Structures VI

So, we are talking about unit load method, and we have applied one beam problem and another frame problem; both are statically determinate type. So, next job is we will apply the method to a statically indeterminate problem. Now we can take a beam problem at the beginning; later on we can switch over to a frame type of problem. So, that will be more or less a wide coverage of the application of this method for statically determinate as well as indeterminate type of problem.

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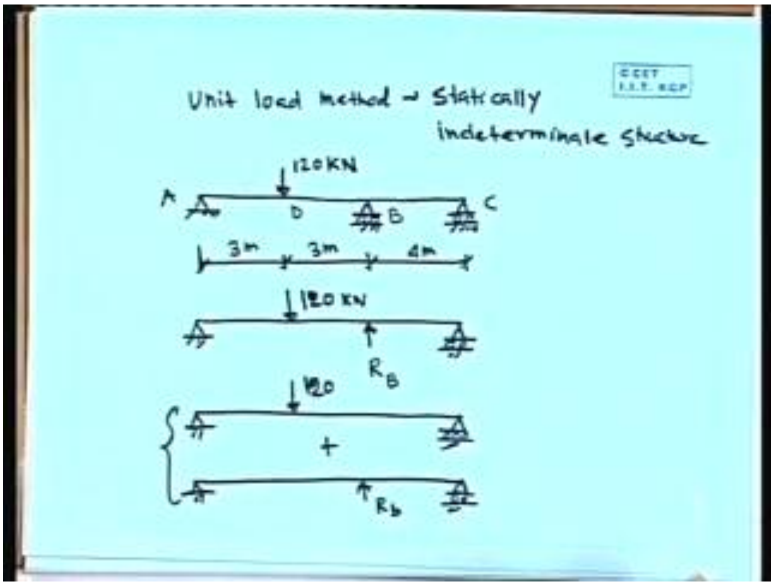

So, we can say this unit load method applied to your, right. So, statically indeterminate structure we want to solve by unit load method. Now what we can do? We can take a beam problem like this, right. So, say, this is 3 meter; this is 3 meter; this is 4 meter and the load, say, 120 kilo Newton, right. We can say this is A; this is B; it is C, and the other load it is D. So, this is a two span beam AB one span, BC another span, three supports. This support is hinge, they are roller.

Now here we will get two unknowns horizontal and vertical; horizontal in that case it will be zero. So, one vertical reaction will be there plus at B and C we will get vertical reaction. So, three vertical reaction plus one horizontal reaction; so it will be three plus one, four and we have the equation of statics three, right. So, one extra or we can say this support at C we can remove. If we remove it will be a simply supported beam with a load; only one overhang part will be there.

So, that problem is a standard typical simply supported beam problem with a central load; only some overhang part will be there without any load. So, that problem we know it is a determinate problem. So, this reaction is the additional reaction at C or other option the support B we can remove. Then it will be a simply supported beam with some load at any point. See this reaction is the only additional unknown. So, actually this problem is not a determinate problem because one of the reactions is becoming extra. So, that may be in the form of C or in the form of B. So, this is extra reaction.

We have three equations of statics; if we have three unknown reactions, it will just match; it will be determinate one and any increase in number of reaction. So, three two, it becomes four. So, extra here one means one degree of statically indeterminacy is there in the structure. So, that one additional unknown if we can determine, then we can solve the problem. Say this reaction RB produced by the support B if we know numerically, say, it is 120 if someone tell that reaction will be, say, some value 90 kilo Newton.

So, we can just remove that put 90 kilo Newton, the problem will be determinant one. It will be just like a determinant one, we can solve it. So, our job is we have to find out the value of the reaction that additional support reaction; if that is known we can solve the problem. Now this problem we can handle like this. So, this is the actual structure. It is 120 kilo Newton, and this is RB. So, this is the actual structure; only the support at B I have removed. Instead of that I am putting a reaction and that reaction whatever this support will supply, it is RB; unfortunately, this RB value is not known at this level, right.

So, that value anyhow we have to find out; now how will we find out? So, this problem again this is defined in that form. So, this problem can be defined again. So, if we just combine together we are supposed to get this one on the actual problem. So, it is a simply supported beam subjected to 120 kilo Newton, and another case it is RB. So, both the case if we superpose we are supposed to get RB 120 together, and this is the actual problem, or we can say the actual problem we have splitted into two part; one is due to 120, another part is due to RB, right.

Now 120 no problem we can go ahead; this RB part is still a difficult thing. Now here we have to take the help of deformation of the structure, because earlier also we have tried to handle a statically indeterminate problem. We have tried to find out deflection of the structure at some important point, but the deflection is known and what are those points? Those points are basically some point, but there are some supports. So, displacement should be 0 there due to the different load.

Now here this is the actual support. So, there should be no deflection along the vertical direction. Now due to 120, there should be some deformation here. And due to RB there will be some deformation, and this deformation and this deformation should be equal and opposite. So, that it will cancel together, and the deflection should be 0 at B because it is a real support. Now the numerical value of deflection at this point due to 120 we can calculate numerically, but this value we cannot calculate numerically. But we can put one unit load instead of RB and whatever deflection will be there; that multiplied by RB will be the deflection there.

So, this structure is subjected to RB. So, deflection will be a function of RB or we can say there is a one unit load. So, whatever deflection we will get that multiplied by RB will be the deflection due to RB. So, that deflection and this deflection should match, and it should be equal to 0.

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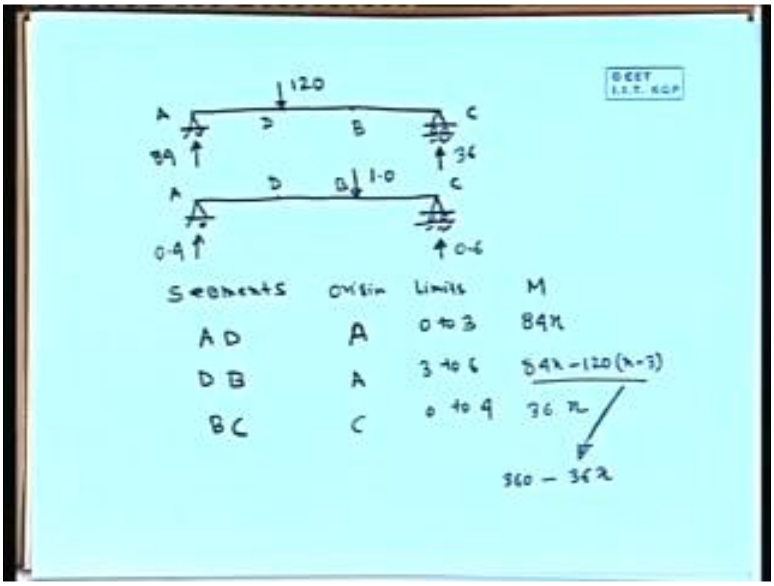

Now this part if I take in that manner, say, again I shall draw. So, it was 120, right, say if it is A. So, there was some point it was B; there was some point C, and this point we have defined D; say, same point it is A D B and C. Now we know B; there is a support and deflection will be equal to 0. Now due to 120 kilo Newton, we can find out the deflection, and here we will find out the deflection using unit load method. So, what we have to do? We have to again calculate M small m divided by E I d x integration.

So, under 120 kilo Newton, we will get a set of bending moment that is capital M and due to one unit load there will be a set of moment small m. If we combine, we will get the deflection at B plus if there is a one unit load and if we want to find out deflection under this load. So, capital M small m will be small m actually this is not the one unit load; here rather there is one reactive force RB. So, it will be one into RB into this one; if we multiply we will get another set of deflection. So, those deflections should be equal and opposite, right.

So, we can just write segment wise, then say, you write segments. So, one part is AD, another part is DB, another part is BC, right. So, you make origin limits, okay. Now you make the origin as 0. It will be 0 to 3 meter, and origin is A and 0 to 3 and DB, which origin you will take? You can take D, you can take A.

Student: We will take A, sir.

You can take A. So, origin will be 3 to 6, and BC definitely you will take C, and it will be 0 to 4, right. So, from this side we will take, and if it is 120, what will be the reactive force here? This is 120 into 3, and this will be three three six ten. So, this reaction will be 120 divided by 10 into 3; it will be 36, right. So, if you take moment 120 into 3 divided by 10; so it will be 0 will be there 3 into 12 is 36 and this side it will be. So, 20 it will go, another 16 will be minus 16 means 84, right, and here it will be if you take moment out here.

So, this is 4. So, it will be 4 divided by 10. It will be 0.6 or from this side it is 6 into 1 6 divided by 10.6. Now capital M if we put here, so it will be 84 x and the next segment 84 x minus 120 x minus 38, 84 into x, here also 84 into x minus 120. This is x minus 3, and this part will be 36 into x. So, this part we can redefine; we can write 84 x minus 120 x. So, it will be this side 16 plus 20. So, this part we can redefine as minus minus plus. So, 360 minus 36 x, right. So, this part we can write in that manner.

Now segment wise we have written the expression of capital M. capital M is produced by the actual applied load; here there is only one load. In other situation it will be more than one load. So, due to the actual applied load M we have generated. Now this part for the unit load at B, the reactions are known. So, we can find out the expression for small m. Now small m we can write it here or we can make a separate table.

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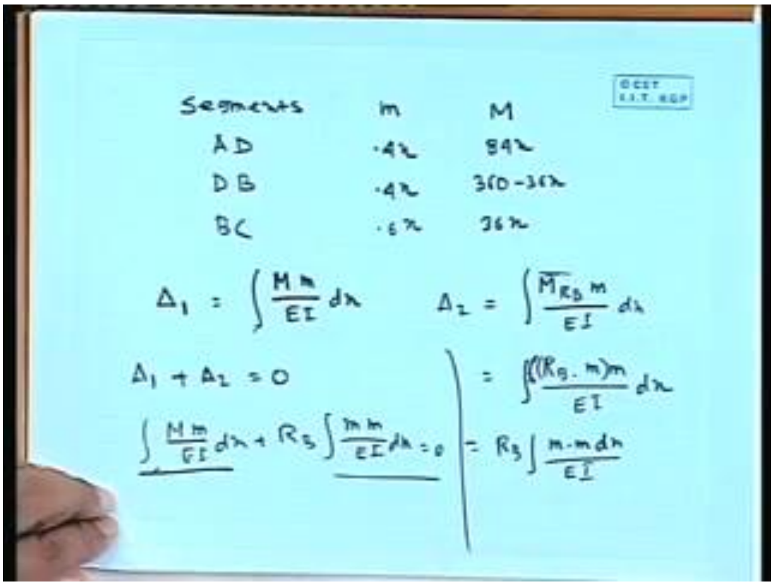

So, it will be again segments. So, it is AD, DB and BC, and this is small m. Now it will be how much? It will be 0.4 x, second part is also 0.4 x, third part is 0.6 x, right. So, if you come to that it will be 0.6 x; this will be 0.4 x, okay. Unit load it is 0.4 up to here it it will be. So, we are not crossing that limit; here the value will be 0.4 into?

Student: 6

At this point, it will be how much 3 3 6, so 0.24. So, this side will be 0.6 into 4.24. So, both the end it should match. Now this is one set and M if I put here already you have written; just for convenience if I put 84 x, this part is 360 minus 36 x. This was 36 x; that already you have taken down. Now we can say delta due to applied load, right, say if it is delta 1. So, it will be M m E I d x integral. Now M m already we have in front of us capital M small m. So, if we just multiply this into this 0 to 3, this into this 3 to 6, this into this 0 to 4, integrate, you will get delta 1.

This is what M small m. What is capital M, what is small m? Capital M is due to this, small m is due to that. So, means we will get deflection at this point due to 120 kilo Newton, right. So, there is a load 120, and we are interested for finding out the deflection at B. So, that will be obtained by M generated by this, and small m generated by one unit load at B, and whatever we have written and this is this part. Now we will get some deflection.

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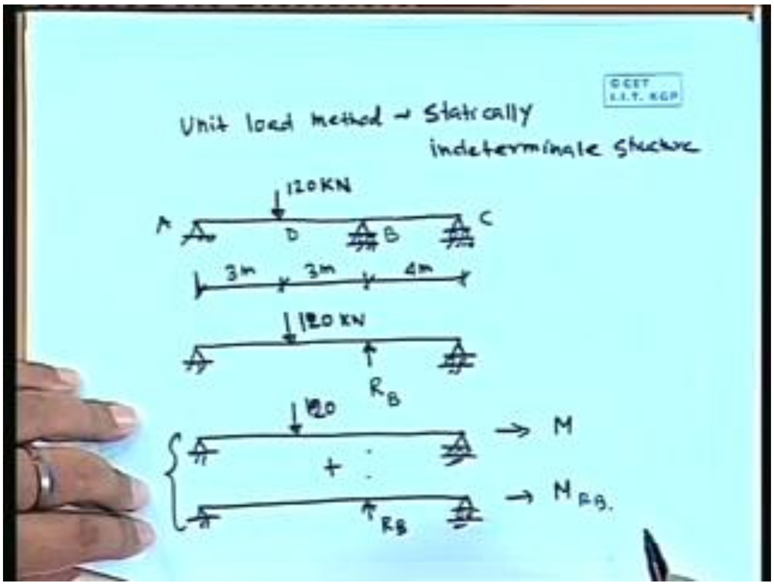

And that deflection should be our actual problem was like this. There is 120 load, and that should be balanced by this RB, right. Now RB is not known, but what would be the value of deflection due to RB? If we put one unit load here that deflection due to one unit load if we magnify that by RB; that will be the deflection. So, deflection produced by RB will be equal to deflection produced by one unit load here multiplied by RB, right. So, that part we can say though the directions are different if I put the reaction in the other way. So, that deflection it is a matter of sign conversion.

So, delta 2 it will be. So, capital M is the moment produced by 120 kilo Newton. If we put RB and get the moment expression that moment expression will be MRB. That if you multiplied with M by EI that will be the deflection at that point. Here this side is giving capital M. Now this side if we give MRB; now this M into your small m we are getting deflection produced here, and M and unit load if you produced you will get the deflection there.

Now, unfortunately, this part is not known. Now this part we can say RB. Now MRB is moment produced by RB. Now what is m small m? Moment produced by unit load. So, instead of RB if we put one unit load, we are getting the moment expression small m. So, actual load is RB. So, if you multiply with that that should be equal to this one MRB, is it clear this part? On the structure here it is RB; if we put naught RB one unit means whatever effect we will get due unit load due to RB it will be just RB times.

If it is one unit whatever moment you will get if it is 10 it will be just ten times; if it is 20 it will be twenty times. No, no that part will take care, okay. So, if we put RB in the reverse manner. So, here the problem is simple; we are taking RB interlinked in the upper direction. So, this value and this value automatically it will be opposite. So, numerically we can put the value, but if you follow mechanically, we will put all the load in a downward direction, so RB.

So, that deflection and this deflection should be just we will add algebraically and that should be equal to 0, right. So, if you take the RB in a reverse manner or if you are serious about the sign, we can take the sign here again, okay. This small m is due to a unit load applied at the downward direction. So, if the load is not one unit if it is RB in the opposite direction. So, it should be multiplied with a minus RB, okay now. So, here we can multiply it with a minus one or if the RB is in if we take in a downward direction, automatically sign part will be taken care of…

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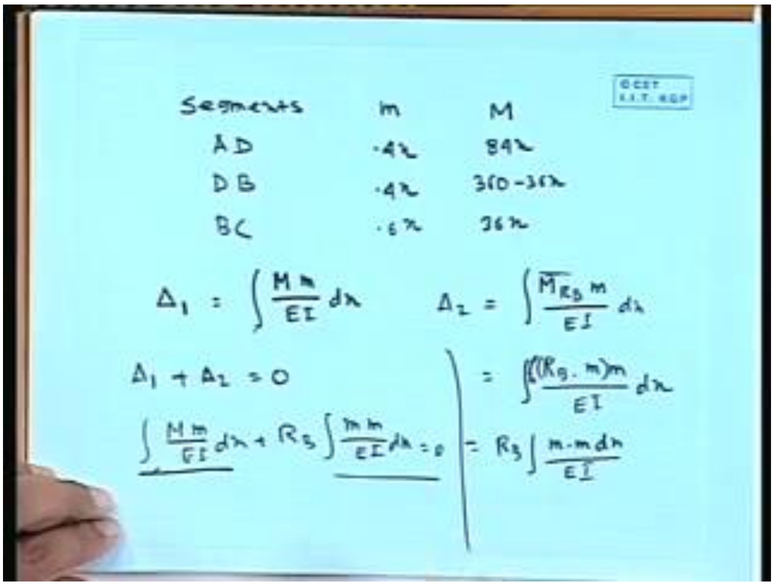

So, we need not take the minus. So, the whole quantity we can write; RB is the unknown part, here we can write M m d x divided by E I, right. Now this delta 1 plus delta 2 it should be equal to 0 there. Now if we put here it will be basically M m E I d x R B m m E I d x that will be 0. Small m, already we have that; it is there again front of us, right. So, basically we have to take what we have done? We have removed that support, put the support reaction. Then we have written the expression of capital M due to really applied load externally without the support reaction.

Now another set of M we have written for putting a unit load corresponding to RB in the positive direction, means downward I have put. Now some expression for that we got. So, this capital M small m E I will be delta 1; that will be produced by the really externally applies load without the support frame reaction. Now del 2 is the deflection due to the support reaction only. So, total deflection if we combine the deflection should be 0 because it is a plus part; there is a real support. So, it will not allow to move upward or downward. So, delta 1 plus delta 2 should be 0.

Now what will be delta 2? It will be the capital M generated by RB and one unit load there.. So, one unit load is downward. Now this MRB for the support reaction that part we cannot readily get, but that we can express one in two RB plus or minus form, where

I think RB into m. So, RB is taken common. So, it will be m m small m small m d x E I. So, this part you can take minus if we exactly follow that case, or later on we will not follow the direction; say, here it is 120, we will just put the same direction without knowing.

So, if it is wrong we will get a negative value, right. So, this one if I put in that direction, we are supposed to put this is minus. So, if we put here minus. So, RB that side if it will go, it will be automatically gives some positive value, but if we just put that direction this direction. So, it will be plus and automatically RB will be negative; negative means it will not be this way it will be this way, automatically it will be taken care of. So, RB this value we can calculate because M small m is already known. So, numerically we can calculate through integration of the different zone, here also we can calculate.

So, these two products this quantity and this quantity will be some numerical quantity and this part will go that side and this part we have to divide; automatically, we will get some numerical value of RB. Now this plus minus part already I have defined here, okay; so this is a simple problem we can get. So, we are putting it here. So, in that case we can put a negative one negative one already we will get; otherwise, if you just follow in a mechanical way not bothering about the sign. So, we are putting one unit load RB also in that direction it will all plus plus automatically it will minus. So, there it will give the integration the way you have taken all downward; it is not downward, it will be upward, okay. Now this part I am not explicitly calculating putting capital M and small m because this part I think with that you can calculate.

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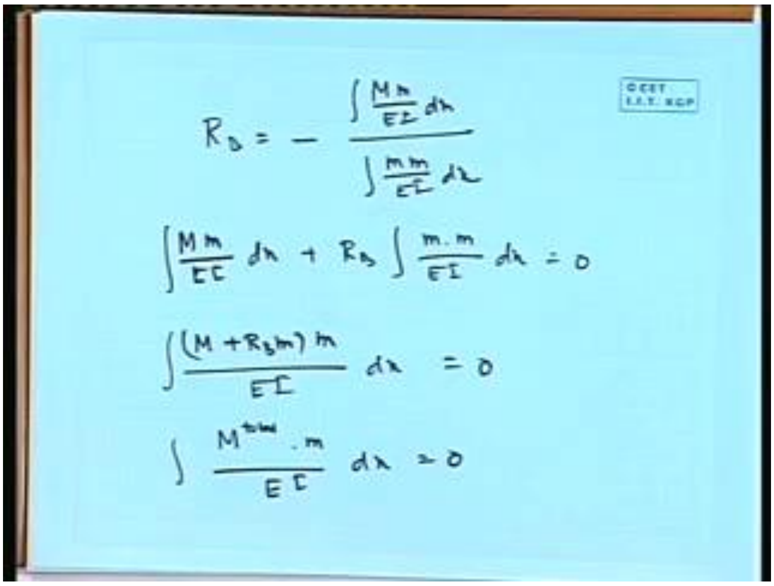

Now this RB we can say it is in that case minus your M small m EI d x m m E I d x, right, you can calculate. So, once RB will be available, definitely you can go ahead for the solution of the actual problem. See, this is the problem 120 numerically if you know that. So, it will be a simply supported problem with two loads; you can find out the reaction for the two loads, calculate shear force, bending movement; everything you can find out. Now here I want to explain that say this equation I want to define again delta 1 plus delta 2 is equal to 0. So, we have written here M small m E I integral d x plus RB.

Now what we can do? The small m E I that part is a common part. So, if we just take common, it will be M plus RB small m by E I d x is equal to 0. Now we can say this is M total. Now these two quantities basically we obtained in a separate manner; this part we have defined as the deflection due to the applied load without the resistance at B due to the support there. Now this part is the deflection produced by the reaction given by the support at B. So, this deflection and this total deflection delta 1 plus delta should be 0, because there is a real support.

Now from there we came. So, if we combine. So, it can be written in that manner. So, this small m E I d x small m E I d x that part we have taken common, and this part is M. RB is small m. Now M RB small m, what is capital M? It is produced by 120; that is externally applied load. What is RB m? RB m this small m is due to unit load. So, RB m will be produced by this is basically the moment produced by RB. So, if you take a structure, there will be some actual load plus RB. So, actual moment will be we are defining it is m total.

So, m total is the reaction as well as the applied load, total effect is m total. So, this total we have splitted into that and again combined, right, but we cannot take the total form because here total if we split; one part is m and other part is RB into m, and this part we can calculate, not this one. So, it will be in a common form from there we have to. So, we have to start from here also; instead of coming from different component combining, we can take the effect everything, say, due to the load, actual load due to the reaction. It may be more than one reaction, whether we will take a problem the number of reaction will be more.

So, due to the external load we will get m; due to the reaction we will get two, three components. So, if we combine we will get m total. Now this m total into m E I that will be 0. Now that part we cannot use in this form; we have to basically split, and we will get this form of equation. From there this RB or other reactions we have to find out; we can determinate it. Now this is basically the procedure. Now I will take an example of a frame. The idea is the technique can be applied to the frame type of case number one; number two, the reactions it may be more than one, it may be two, it may be three.

So, for a problem having statical degree of statical integral means if it is more than one that type of case also we can handle. So, we will consider the same total frame problem what we have taken in our earlier class, but it was determinate one; we can change the support, it will be indeterminate one.

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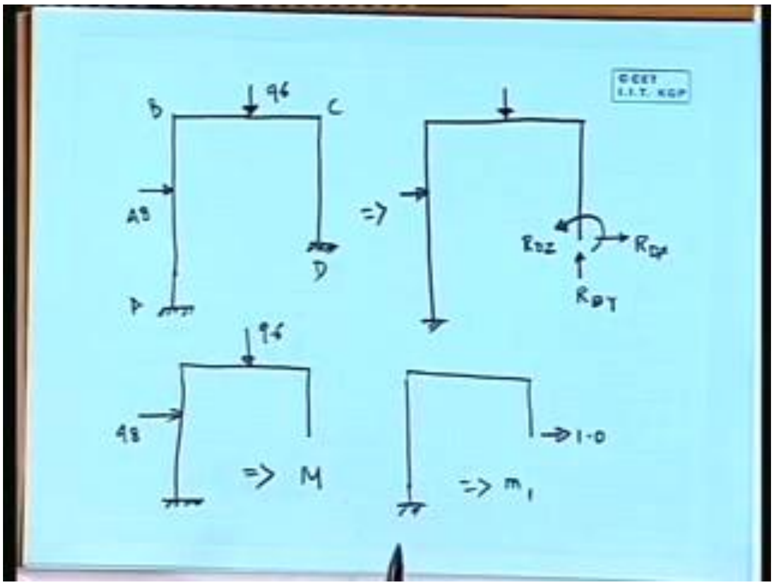

So, this is 5 meter, 3 meter, 3 meter, 3 meter, 4.5 meter, same problem; only the supports are clamped, not the hinge and the roller. The load was how much?

Student: 48.

This was 48 horizontal 48, 96, right. Now here this is a clamp means three reaction components will be there, horizontal vertical, class movement; there will be three. So, total reaction is three plus three six; so three extra. So, we say degree of indeterminacy means how many extra?

Student: Both the side is fixed

Both sides fixed. So, after 3 it is another 3. So, 3 extra 3 is the degree of indeterminacy. So, the problem cannot be solved, because once the reaction cannot be determined we cannot go inside. So, what we have to do? We have to release some of the restrain and instead of that we have to put some reactive force. And initially those reactive forces will be unknown, and our job is to find out those reactive forces. Once that will be known we can go inside. So, three restrain we have to replace and put some reactive force. So, one of the simple option is this end all the restrain we can release and put some reactive force there, right.

So, this problem we can just define, right. So, we can say this is RD Y RD X, and you can say it is RD Z, right. So, these are the applied load real load plus this clamp support we have just removed, but we cannot remove; we have to compensate that with support reaction. So, it was A B C and D it is D; we are writing reaction at D, reaction at D, reaction at D; it is along X direction. So, we are putting X; it is along Y, it is Y; it is along Z, it is Z. Z means there is a moment.

Moment is also a vector, and vector is defined by if we take a right hand screw, it will move along Z direction. Sometimes, we put a force also with two arrows to define a moment. So, here there is a reaction like this. Now this problem will be solved in that manner. We will take the frame; put the actual load 96 and 48 without support reaction, and that will give capital M, right. So, this is it will give the expression of capital M. Capital M is moment generated by actual load without this reactive forces. So, here to here it will be moment 0, it will be 0, it will be 0, right. So, here 96 means we will get 96, here 48 means 48.

Now if you take moment 48 into this side 96 into that one. So, moment will not match. So, that moment will be there. So, here you can start that moment into this force into that distance. So, like this you will get some moments, some moments, some moments here; this part will be zero. So, you can calculate the expression of the capital M. Now we have to take a frame like this. We have to put one unit load, say, it will give small m. So, if you give one this will be one, right.

So, one will be one and vertical force there is no force; it will be 0, but this one and one gap is 7.5 minus 5 to 0.5. So, 2.5 into 1 that moment will be generated here, right. So, if there is a force that force will generate a moment here. So, definitely we will get some moment inside that. We can go for the detail of this.

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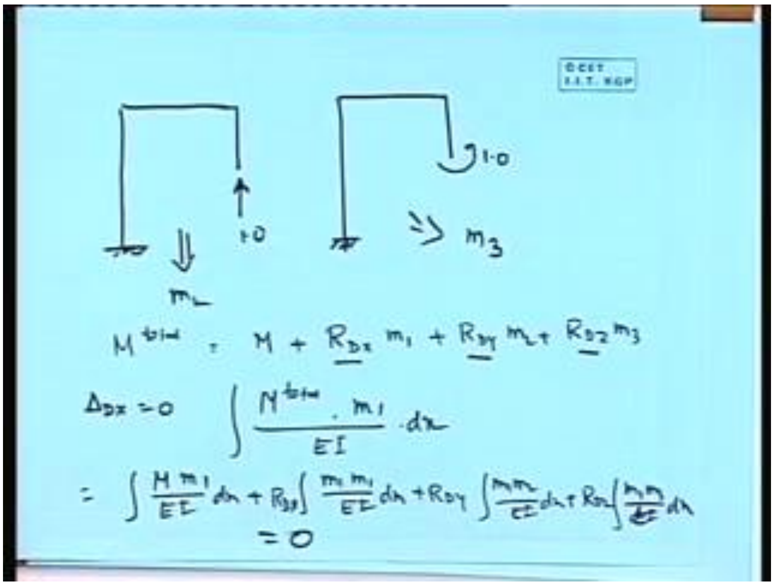

Similarly, rather we have to apply one unit load; we have to apply one unit moment and that will give your m 2; that will give your m 3. So, try to come to this problem. So, it is due to the external or m, there are three reactions. So, first one we will get m, another m 2, one unit moment m 3. Now due to that load due to this moment, what will be the expression of this m 1 m 2 m 3, capital M. That part we can determine segment wise, because here one of the end is clamped. So, we can find out the support reaction including the moment; with zone wise we can make the nice table capital M small m, m 1 m 2 m 3 like that.

Now here there are three unknowns. So, what will be the total M? So, M total it will be capital M plus RD x into m 1 RD y m 2 RD z m 3, right, because this case this is the case. So, due to that it is m due to this, this multiplied by RD x. This one this one multiplied by this one, and the last one this one multiplied by RD z, right. So, these are the quantity we multiply with m 1 m 2 m 3 plus m. So, that will be the total M, right. Now what will be the deflection along horizontal direction of D? It will be 0, vertical direction it will be 0, rotational will be also 0.

So, we can say delta D x that is 0 that is how much? Your m total into m 1 by E I d x, right. Now M total if we substitute, it will be m, take your m 1 E I d x; it will be m 1 m 1 E I d x. It will be RD x, it will be RD y m 1 m 2 E I d x. RD z it will be m 1 m 3 E I d x, right. So, if you expand that. So, it will be m m 1 E I small m 1 RD x m 1. So, RD y m 2 m 1 E I R D to m 3 m 1 E I d x, right. So, if you put. So, whole quantity will be equal to this will be equal to 0, is it okay. Now the integral you can calculate; only the factor RD x, RD y, RD z; they are basically the unknown. Now this is deflection along x is 0.

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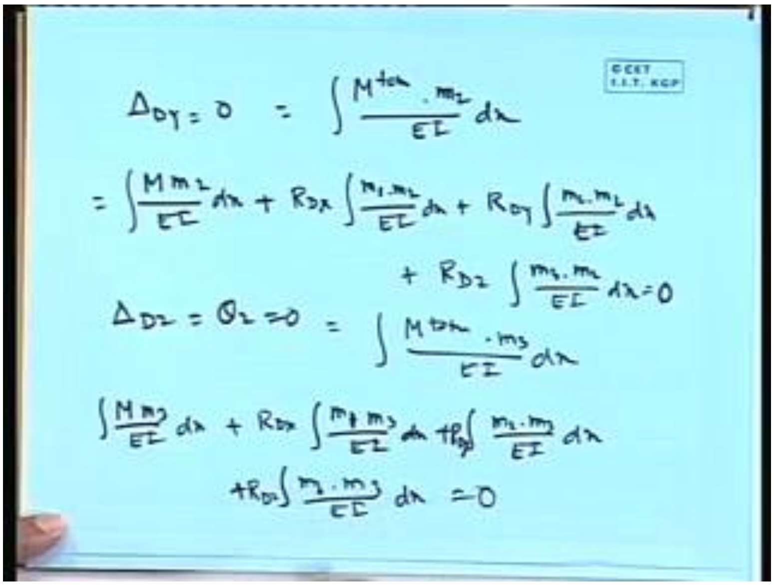

In a similar manner we can write delta at D along y equal to 0. So, it will be how much? It will be total moment into m 2 by E I d x. Now if we put the value it should be your capital M small m 2 E I d x plus your RD x. ere it will be m 1, it will be m 2 E I d x plus RD y m 2 m 2 E I d x plus RD z m 3 m 2 E I d x will be equal to 0. Similarly, if we make delta D z it is basically theta z; that will be equal to 0, right. So, that part will be your M total into m 3 divided by E I into d x, right. So, that should be equal to zero.

So, if you expand again you will get, say, M m 3 E I d x RD x m 3; it is m 1 m 3 E I d x. Here it will be RD y; it will be m 2. M 3 is a common factor E I d x plus integral of it is RD z it will be m 3 again m 3 divided E I d x; that will be equal to 0, is it clear what is going on, because capital M is due to the load without the reaction, but reactions are there, but reactions values are not known. So, we are putting one unit of load or moment corresponding to the reactions and we are getting m 1 m 2 m 3.

So, those moments if we just multiplied with the actual value of the reaction, we are supposed to get the moment contributed by the support reactions. So, all the component if we add due to the actual load plus the reaction that will be the total M; so that multiplied by m 1 will be the deflection along m 1, m 2 along m 2 means second direction, m 3 means in the third direction the rotational direction. So, all the quantities are zero but that form we can keep. So, capital M part to we are splitting and we will get more or less this form. That m m 2 by E I, this part we can calculate.

So, this is basically the external load will produce a deflection along your direction to means along y, and this is m 1 m 2 means if we apply one unit load along x, how much will be the deflection along y direction? So, actual load along x is RD x. So, that multiplied by this factor. Now this part is m 2 is one unit load along y and multiplied by m 2 means in that direction, what will be the deflection? But load is not one; it is RD into m 2 into this one.

Similarly, m 3 is one unit moment, actual moment is this one. So, that is the effect. So, if you multiply it by m 2 means deflection along y direction. So, total effect should be equal to 0. So, if we just write these three equations. So, this is one of the equation; this is one of the second equation, and at the beginning we have written this equation. So, these three equations you will see these components are all known; only the multiplier part is unknown. So, three unknowns and three equations; only calculation of this part is quite involved. Just it has so many integral and one integral if you divide if you take that frame one two three four five segments, right.

So, if we expand it will lead to time integral, 0 to 4.5, then u 4.5 to 7.5, 0 to 3, 0 to 3, 0 to 5. So, five components are there, but in many cases we will be fortunate if we get 0 either in capital M or small m. So, five will reduce to somewhere four three one two; we are very unfortunate, all five we have to calculate. So, it will be one one component. So, all these components you can calculate, but there are some similarities, say, m 1 m 2 here, here m 2 m 2. It is m 3 m 2, say, this is m 3 m 2, m 2 m 3 here it is a common type of calculation.

You can multiply m 2 m 3 or m 3 m 2 it is same; more or less if we write in a systematic manner, you will find some of the parameters already you have calculated for these integrals. So, if we write in a systematic manner. So, what will be happening? So, whole it will be three equations with three unknowns, but calculation of this part definitely it will take little time, and if you solve that, you will get the reactive forces, right. So, here there are two objectives. Number one, the unknowns are not one; earlier it was only one, and directly you have tried to match.

So, here mechanically we are putting the three reactions, replacing that, putting one one unit of force or moment in a separate level, getting m 1 m 2 m 3. And straightaway in terms of M total that was splitted and we got in this form plus problem is applied to a frame type of case. Now how we will get the detail expression of capital M small m 1 m two m 3; that part, okay, we can handle in the tutorial class. Now with that we are closing today. Next class it will be a different type of problem.