

Performance of Marine Vehicles At Sea

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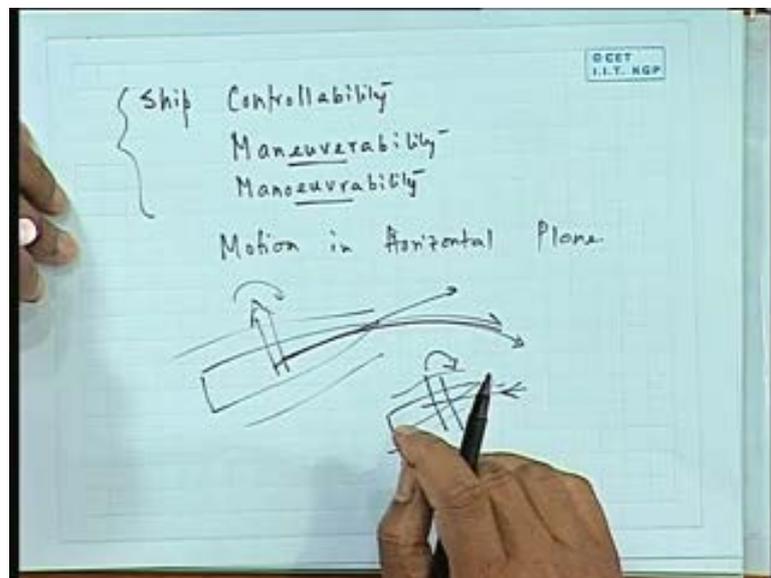
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Lecture No. # 33

Ship Controllability: Introductory Notes

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Today, we are going to talk about a topic of Ship Controllability or you may say, this maneuverability **well** (No audio from 1:18 to 1:26), this is the American spelling, this is the British spelling a u r and o e u v r a, I am just writing, because we may change one to other so that, we have no confusion.

Basically, this thing are more or less same topic, we are going to be discussing this part of the **you know** ship area which is related to you may say, maneuvering, controllability, course keeping, etcetera. Everything related to a ship is trying to move in the horizontal, motion in the horizontal plane. Earlier, I spoke about more of motion in the vertical plane that is under wave action.

But here, what we are going to talk about is that the sea is calm, there is no waves etcetera, ship is moving and it is trying to turn, it is trying to, it is when it moves along a straight line, you study the subject of resistance, but here we are trying to find out how it turns? Should it turn? What forces is necessary to make a turn easily or what should be the characteristic, so that it does not turn for a simple **you know** like simple forces external forces or if it is a turn, because of a wave or some disturbance should it come back to its original line.

All this things are a part of what we call maneuverability, controllability and this some time they are kind of, we have to understand the beginning contradictory, a **a** ship which is highly maneuverable means, very highly means you can turned it very easily becomes less controllable, because it is always trying to turn it just like your scoter or a vehicle which is always trying to turn and if you want to make a steady course you have to hold it tight.

So, there is two kind of aspects again, I mean if it is something is **you know** it is a tanker going on a straight line, you cannot you get a rudder, it does not turn its absolutely having a very strong directional stability, but then it is not controllable, this is the part we will talk in general; before that, let me tell you about this, why the force has come.

See, again looking about that **at that**, now what happen as you try to turn a ship? See, when you are going on a straight line, the force is symmetry there is no **no** force coming in y direction, no movement coming in this direction, force is absolutely along x direction only, so the ship is moving exactly on a straight line.

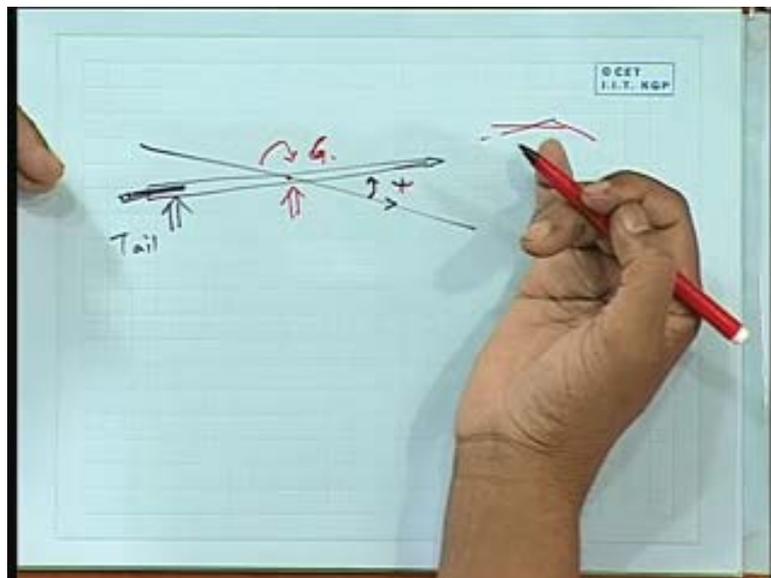
But here, what happened as you try to turn naturally there is a asymmetry developed, the ship is not symmetric anymore as if it is having a motion and the flow is coming in this direction, and then, what happened obviously the flow goes various ways and there will be different set of pressure and therefore, there will be some kind of force coming some kind of moment acting on that.

And it is that, the which will cause the ship to turn, in other words the only mechanism that you had to cause force on a ship which is not restrain is fluid forces, because of the flow, because the way the flow going passed it.

These are the forces that will cause it to turn, you see you keep a rudder what happened you change the rudder, but what happens, because you change the rudder the flow pass that develops a different pressure system and that is why there is a force. Why I am saying, that is because in order to study controllability and maneuverability we have to necessarily study fluid forces, because it is that forces which will cause the vessel to turn or whether the forces very large etcetera, we will decide if the vessel would remain steady.

A simple example is given by this weather (O) stuff, which is what I will just briefly mentioned.

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See, suppose there is a initial trajectory of a line and there is a particular you know like small line with a what you call a large tale, let us see, there is a small thin plate with a large tale here and the tale is such that that is along that gap produces a a force, what happened? It is supposed to go this side, but now it is turn by some means, it has actually turned by an angle ψ .

Then what would happen, that would obviously cause, even it is going on a straight line the flow was going passed, but now the flow is going passed, I am just considering this this part it measure, it is going to give some kind of a force on this direction. This force is equivalent to, if let me say c_g , this force is equal to this force plus a moment you

know any force can be translated back to another point or rather this force will give a moment about this central point, let us say c g point, this is say c g.

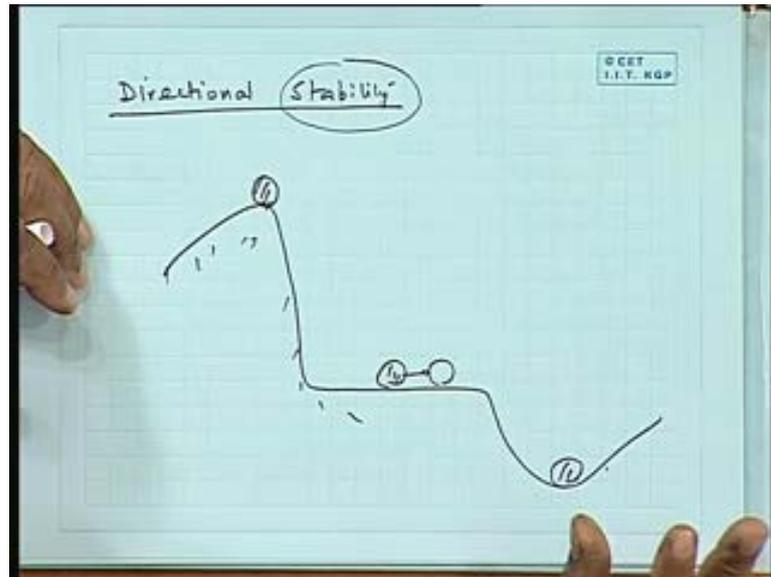
What it does therefore, this part this direct part of the force will cause the thing to move up, because there is a force were acting on the sea, so it is going to go up and this moment will go into make a turn like that so that you will expect the vessel to actually I mean, this body to actually gone slightly up and then begin to turn, begin to turn as it turn. Now, think of that, as it turn this angle becomes small, so at some point it has moved up and then begin to turn. As it turns, this angle **as become small** as it becomes small the force become less, so ultimately there will be a point, when the forces become 0, so it becomes straight line.

So, therefore, you are then, you are the behavior is guided by the fact of how much force is coming on that, which depends on obviously angle of attack, which of course, will keep on changing as the angle of attack reduces. So as an introduction, we have to realize that we have to study, the force is coming for a general body, moving in a horizontal plane, if you want to understand even the elementary, maneuverability later on of course, we must talk about rudder, what does rudder do?

The **the** most common mechanism for keeping the ship in force is rudder, as we know and if you want to turn you might have an additional device like a bow thruster or thrusters on the centre plane etcetera **etcetera**. In other words, you have to introduce a force on external mechanism that we will come later on, but to do that **we have we should** we should understand what kind of force come on the body.

Let us, go to a very basic definition of what is called course keeping; now there **there** are certain thing that can occur for a course keeping, I will explain some one.

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First of all, let me introduce the word directional stability and various definition of directional stability for a ship, what kind of stability the ship might possess, obviously you would want a ship to be as stable as well as controllable as I said highly stable is less controllable or less maneuverability.

Now, the very basic definition of the word stability as I mentioned, I mean **we have** we have talked that in an earlier class, is related to the fact that, if on a body an external disturbance acts for some time and causes the body to move from its original position equilibrium.

Now, if you have removed that force, should the body come back to its original position of equilibrium, should it stay where it works, or should it keep on deviating, this is what the standard definition of anything called stability, which is the classical example is given by this sort of a hill, with a **with a** ball here, this is obviously unstable, because the moment you change it, this is an equilibrium position, there is no force and **moment** you change it is going to actually go away from that (Refer Slide Time 9:13).

So, it is unstable, it is deviating, if you give a momentary disturbance, it deviates from a state of equilibrium, you keep that here it is neutrally stable, because you shift it here it stays here and here this is stable.

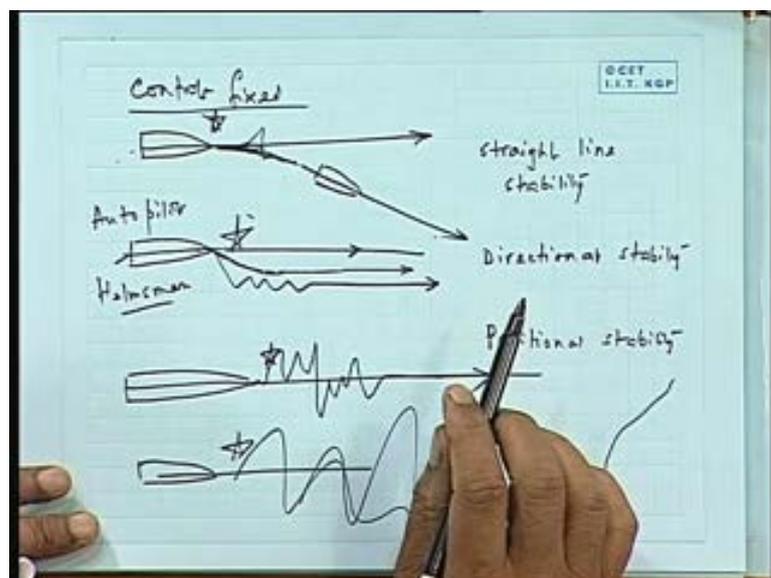
Similarly, for a ship you would expect that it is going on a straight line maintaining a direction, now you give a disturbance, so it has changed its heading, should it come back to the same **you know** straight line or should it keep deviating etcetera **etcetera** that would be the concept of directional stability.

Same concept you have a state of equilibrium, which is in this case the ship moving along a straight line, you give a disturbance which might have come, because there was a sudden gust or wind or whatever and there is always a wind coming going to cause disturbance.

On that does it keep deviating this with that way, this is actually a concept which is very valid in many cases, if you think of those old days, I remember my younger days, there used to be this corporation car for **you know** dart, there will be an engine with number of bogies, that when they go in a turn it go like that and everything follows behind that.

Even there is a concept of directional stability because if they are not directionally stable that there is a design consideration, then when it turns here you see, along that one bogie you try to go this side, other will try to go this side, other will try to go this side but, it does not happen you see, that when it turns in a number of those **those** bogies just follow it, this is also same as directional stability, so it is connected to the fact of whether it actually follows a path.

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Now, let us see, for a ship what are the possible definition of directional stability, you can have two or three or three actually kind of cases there is a ship here, always we are assuming that its going on a straight line, now what happened you gave a disturbance there is a some kind of disturbance here, **you know** some disturbance here, so the ship can go along it gets disturbed here, say it gets disturbed here whatever.

And but, when the disturbance goes away it follows now another straight line, it can have that or if you have given an angle here, it can actually goes in a circle but, that is not very common, say this **this** is what will be called straight line stability. Now you can have another case, there is a ship here, suppose to go in this line, you get a disturbance, so what happened it gets disturbed but, ultimately it goes to this line along this line, it can have this **this** will be called directional stability.

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I will tell you, this actually can also happen in a two ways if the disturbance is given it actually comes like that and goes like that, see, that second case, ship is going in a straight line at this point of time, there are certain disturbance given that means suppose you have pushed it, a wave came and give a big push it has now change direction, now what happened it can follow, now a path which is going like an oscillation it has introduced some kind of oscillation but, even see, the disturbance is given in one moment it is like a bang you have given and removed it let us see, just for demonstration purpose in reality will be for sometime.

So, it can introduce an oscillation and then actually ultimately when the disturbance goes it, actually follows a straight line parallel to the original straight line, this would be means, what it is maintaining direction after a disturbance is over and here also there can be two cases normally it can actually, the disturbance would have cause in oscillation and then it goes to a straight line or it could have simply deviated that and it goes to a straight line, these two are basically both the cases are example of directional stability but, you know the **the** mathematics of the forces differ, one can tell from the hydrodynamic characteristic, whether it will be this type or that type whatever these are directional stability.

The third one is that, going on a straight line you give a disturbance here, so it actually goes to this thing but, ultimately it comes to again straight line when the disturbance goes away. So in other words, in case one you **you** have disturbance when is given as cause the ship to go along a straight line after you have removed it, so it is having a straight line stability, in all this cases it is possible then I have given a disturbance, it simply goes to some other direction then it is unstable ship.

Supposing a ship cannot be controlled there is a disturbance there it simply introduces some force and in fact, what will happen it actually goes larger and larger then it is unstable ship we are not talking of that, I am talking of the definition of various kinds of stability that a ship have instability is obviously when a disturbance is given you remove that it simply goes wherever you **you** have no control, then it is unstable we are not I mean that is obvious, so I am not discussing that right now.

Now, the supposing in stable, what kind of stability the ship may possess? Is the question, so it can possess like this, it can possess like this, it can possess like this (Refer slide time 15:09). Now it, now I tell you that this two that this **this** can be called positional stability, the ship can have also position **you know** it maintains the same line.

Now, remember a ship has now restraining force, so if you if I have a ship is going some **some** place and you have given a disturbance it is change heading, nothing can bring it back if you have not made a correction, so if you have an auto pilot or a helmsman **helmsman nay** or a rudder if you have no rudder, you cannot have the situation. Because if you do not give a rudder correction action, you cannot maintain the heading you can only have this stability, in other words a ship without any control, this is what is called control fixed, and if you do not have a control surface operating at best what you can have is straight line stability.

In other words, you have a rudder frozen just ship is going you give a disturbance **you know** it **it** takes another direction and it keeps going on a straight line, that is the best you can have for a ship if you do not keep rudder, because obviously the ship has changed its heading, there is nothing to bring it back **you know** the ship as going in one line it **it** suddenly become this angle, you cannot bring it back to unless you get the rudder.

But at least what happens it is maintaining this straight line, this is what is called straight line stability, which is controlled fixed, these two are possible only with the rudder acting or autopilot acting, on that also you see because, otherwise whether heading is changed you cannot bring it back to the original line but, so these two are basically acting with rudder autopilot, so these two are kind of stability.

Now, normally what happened initially we will be discussing about obviously the characteristic of the ships inherent stability, because this part is connected to the hull, you want to know whether the hull has a intrinsic stability property, because if the hull is of this type, then it is much easier to control that with the rudder.

See, if the ship tends to give a disturbance, tends to go this way, you can correct it by rudder but, you need probably much more rudder action, you have to continuously make moving the rudder most ships you see, especially larger tanker primarily goes along a straight line and only if you make a turn, you will make with a large circle so large ships you do not need very high maneuvering **you know** quickly following that you might need for a petrol craft or something.

So, this is the one that we will be talking little bit initially and try to find out what is a characteristic necessary to ensure that, the ship possess straight line stability which of course, can be done only in control with of course, I can also tell you that, many, many ships in reality, do not have straight line stability or have slight lack of straight line stability. There are various measures of extent of stability, because they get corrected by this, you can also see, that, if the ship is very strong straight line stability, that means it always wants to maintain a straight line, then it is difficult to turn it.

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Yeah **yeah yeah.**

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Yeah **yeah** this case.

First case.

Yes.

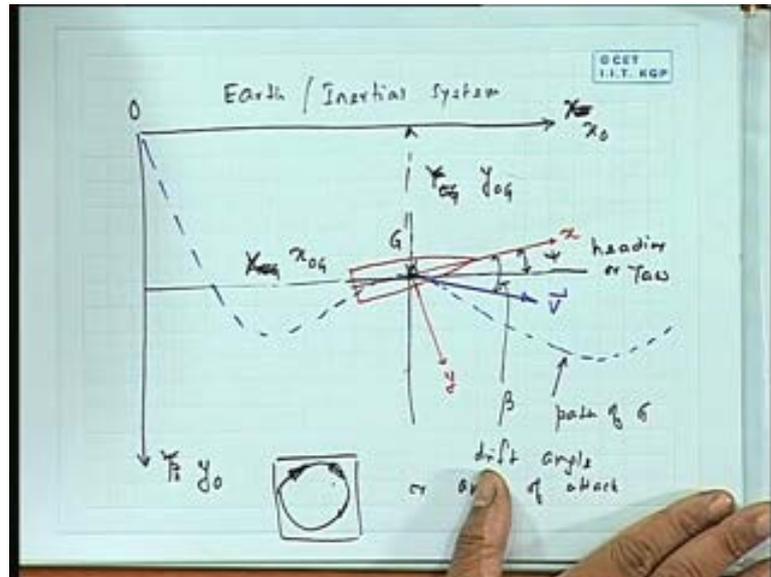
Or it does not have a rudder.

Or it does not have a rudder, you can raise the control controls is not operating you not have a rudder, so that means that, this is guided by the kind of forces are developing on the hull itself, because it has got turned. See, that, the **the the** question to answer is that there is a disturbance it has cause the hull to turn, the turn has caused some forces this forces that has got created, because of the turning should it diminish with time or increase with time.

We would wanted to diminish with time, so that it becomes a **you know** like it does not yeah just like **you know** hydrostatic stability, I give it a hill angle that possess a moment that moment does it cause the hill to reduce or increase, that is what we want to only thing is that, they are hydrostatic force setup here, I have got a complicated hydrodynamic force setup.

So therefore, I need to talk little bit about this equation of motion type of thing, so we will talk about that although I frame that as a next lecture but, then it is combined topic. So, we understand that essentially for ship maneuvering we are looking at the ship motion in a horizontal plane, you can say surge sway and yaw, those three motions that is on the on the plane, where there is no restraining or hydrostatic **you know** spring force nothing is holding it back that is what we are talking about now.

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So what I need to know, do is that we need to actually come up with a kind of equation of motion, after all why equation of motion, you see, equation of motion tells me mass into acceleration equal to force, I have to find out if I have given a force it has a set of an acceleration. So it has have a velocity have a displacement does that velocity and acceleration reduce with time or increase with time, that is what we want to investigate therefore, we have to have an equation of motion.

We will not go through the detail but, I will try to tell you what we normally study, so when we want to equation of motion, obviously you have to define an axis system, so I will try to draw this as cloud be as possible. Let me say keep somewhere here, this drawing takes little time that is why I am trying to first draw, use various colors it will be easier (no audio 21:25 to 22:05) oh [fl] we have shut down this, let me just **just** draw that you can just follow it little bit.

See, we will now go through that, first of all let us say that we are going to only use x y coordinate system, because we are thinking that it does not have any motion in the z plane, although in reality there can be surge, little **little** heave and if you want to talk of submarine you have but, we are talking of ship here, so I have a global coordinate system o x_0 y_0 , this is my earth or inertial, what we are trying to do is that how do we define the ships motion in the horizontal plane.

Now, I have got a ship here and I have got a center of gravity here, normally what happened now a days it is the path of the center of gravity which is this blue line that is the path of the c g, that is defining the path, the way of the center of gravity is moving in fact it is a it was the same thing in sea keeping also.

Now, I have a body coordinate system that we are defining here, as g let us use this think x y and see, this is the path therefore, what is the velocity **velocity** is obviously tangential this line is tangential to this line, see, this line is a tangential to this **this** path line.

So at any point, obviously the ships g has a vector this way tangential to the path obviously that means here, the ship is moving in this side but, having an heading angle having an angle. Now the **the** various definition that comes in see, no other colour is there no, see first of all, I can draw another line here, which is parallel to this x x 0 y 0 there is black line this one, this line is just parallel to that **you know**.

So now, I have defined a axis system x y on the body, so g x y is fixed on the body, so g x is the longitudinal axis g y is your star boat side standard fixed, this is what you are you when you are sitting here you are seeing x is your **you know** like bow and y is your on that side that is what you see. This is my reference line fixed on the globe, so my heading angle is this angle what it makes that is what we called heading angle actually normally in **in** navigation you talk in terms of heading one 80 etcetera, where you would have decided that with respect to another frame of reference perhaps with the art system **you know** like longitude latitude etcetera but, that is different, you have a reference line here fixed on the earth on the shore with respect to the ship has an heading angle of this.

This angle is what is called the drift angle, because what happened the ship is moving this side, so the flow comes you can see, if you are sitting here, you will think the fluid is coming this direction to you, **fluid is coming this direction to you**, making an angle of what we have, we can write beta here, drift angle or one can say also angle of attack.

So you see, what the velocity vector makes with the x axis is the what is called drift angle or angle of attack, because that tells me which direction the flow is coming to; in fact, the fluid force depends on only that whereas, heading angle just tells me the a heading angle or you can say this is heading or yaw, heading is same as yaw, that tells me which direction it is heading **right** and I have got of course, this x y a system here, so

you know if you see, this **this this this** vector is this to this is going to be y_0 of g this one, because this is this is y_0 this is x_0 this is my g point, so what is my coordinate of g , y_0 of g is how far it is from the origin and obviously this much is x_0 of g this is capital X .

That heading will yaw that angle.

Oh no **no let let me** let me now write this separately here, it is let me make it small x **sorry** let me make it small x_0 of g , let me make it because, small y_0 of g , let me make it y_0 let me make it x_0 , because I am going to use x_0 .

See, again I will explain, I mean this was only a question of nomenclature, I have got o_x o_y o_z , which is the earth fixed system, I have got g_x g_y which is fixed on the body, g is the center of gravity, so where the g moves with respect to this earth fixed system tells me where the ship is located.

So, the ship is located, say so much **you know** say y direction so much x direction that tells me where it is located on the earth surface point, there is a coordinate point and obviously it is **its** heading angle is given by where this x axis located with respect to this. In fact, this is nothing but, actually the you may say, you may sense sway surge and yaw this is actually sway, this is actually surge and this is actually yaw heading angle, you only need this three you do not need here, roll pitch and heave, because they are on the other **other** axis system, so here I have got x y and a moment about this axis going through the vertical.

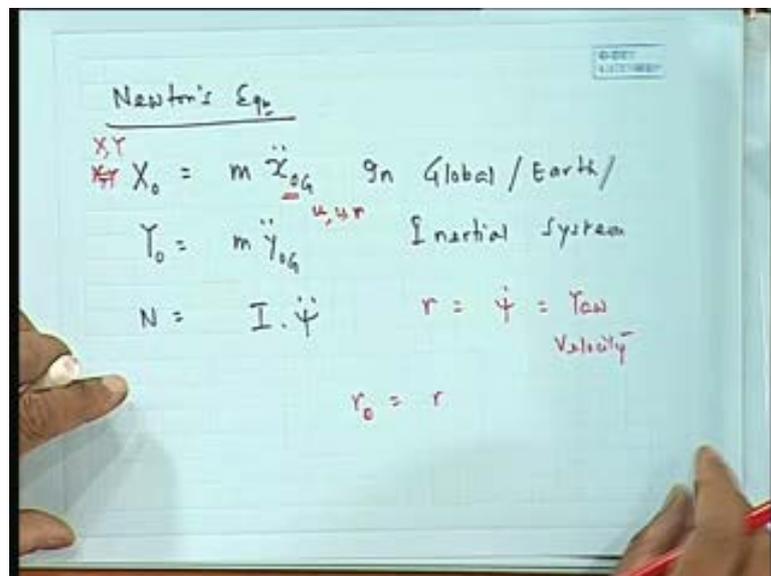
So, this definition should be very clear in the mind, because many thing is depend on that, so you see, when the ship turns this is a very common thing, no ship can ever turn when it is going exactly a path line is same as heading line, you have to have an angle of attack β , when the ship turns you know that it turns like this like this as **you know** you all know it cannot turn like a car exactly following like this it just cannot turn, because no forces can be produced if you turn that way on the y direction.

So, you **you** cannot turn like that, what I mean is that you cannot have a turning here when the ship is exactly parallel to that, you have to have an angle of attack, why because without angle of attack you cannot have the force on that. So I will **I will** come to that later on that is an interesting part unlike car or any other things, that you will find

because here, there is no external force the only force is speed forces acting, that cannot act unless there is an angle of attack, flow must come with an angle because the flow with come from this side there was no force on this side, if the flow came on along x axis, there is no **no** force on this side that would be like a ship moving on a straight line.

Now, what we need to do is to find out equation of motion what does Newton said, force equal to mass into acceleration. So I must find out force equal to mass into acceleration, f equal to m **you know** into acceleration, what is acceleration here? This I will now, we have to refer to this equation number of times but

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Let me say, that I had this the Newton's equation basic form, we will not go through much of this equation I want to as I said we **we we** will try to contain that, to just bring out the essential features, not really work of the **the** equations etcetera to that extent as you **you know** I mean we would not make it complicated.

See, it now we call here looking at that, we will call the force coming, now I have to write the force so I am going to call the force coming in this side, the force remember as capital X 0, that is what we will have and the force coming on this side as capital Y 0 and the moment coming like this will be moment N.

Now this I **I** now N, I am not calling N 0 etcetera, because whether the moment is above this vertical line which for x y system and x to the y system is same, because this line is

same whether it is g_x or g_y , because it is a display so it **it** is n state but, never mind the n part so we **we** have this. Now why I writing that you see, x_0 should be equal to mass into this is x_0 my force in this direction, is mass into acceleration in this direction, what is the acceleration in this direction, is double derivative of this distance, so I can write x_0 equal to m into $x_0 g$ dot, dot y_0 equal to m into $y_0 g$ dot, dot and n equal to I into ψ dot **dot** I am **I'm j** just going to write that and then this thing.

So we can say X_0 equal to m , m is the mass into $x_0 g$ dot, dot Y_0 equal to m into $y_0 g$ dot, dot, N equal to I into ψ dot, dot this is my Newton equation of motion in global earth whatever you call or inertial system, this is my global or earth or inertial system again coming back to this we have to refer to this many times this way.

Now, **now** the question comes like this see, what happens is that, this is a very sort of simple algebra, now you do not want to find force on this line, because at some point the ship is making an angle of say 10 degree, next time it may be making a angle of 20 degree, next time may be 30 degree, so to find force along this line is always difficult.

See, at one point you just **just** think the logic, again the **the** physics behind it why we want to change it to another system because, if the ship was going along x say if this by inspection it would be so much easier to find the force along the small x axis and small y axis, because small x axis of force would be the resistance force and y axis will be the force along the say you can say sway direction force, because sway direction is defined as that **that** way. What happens it is not depending on the geometry, instantaneous position of the geometry, it depends on the geometry for a particular hull for a for but, **but** for a given hull it will not depend on geometry any more, if you are defining the forces along the body systems, because the body geometry along this g_x y is not changing with time.

Whereas the body geometry about this one is changing, because today it is like that, tomorrow is like that, tomorrow is like this (Refer slide time 33:28) therefore, it would be so much easier, if we want to define all the parameters and equation of motion in along this line, that means I want to find out what is my force here, this force here is mass into acceleration, so let me call that the velocity this side is u here if **if** I want to take a velocity here and velocity here is v here.

See, if I, now this v was a vector, so I got a velocity this side some that is $x_0 \dot{g}$ but, I can, let me define this velocity as to be small u **right** I am defining, I am calling that a velocity this side is small u , velocity this side is small v and rotation of course, remains same which will be $\dot{\psi}$, which **which** I can call r , normally it is called r when I r equal to $\dot{\psi} \frac{d\psi}{dt}$ by the rate of change of this heading angle, we will come to that afterwards, I will **I'll** come to that little later on; but understand this that I let me call this to be x this to be y . And now I am going to call the force on this side to be X and force on this side to be Y , so what we are doing think of that, there are two systems one is that along this line I have got, a velocity which is $x_0 \dot{g}$, you can say if I want, I can call it u dot velocity and I can call this to be v dot as velocity.

So I have got here u dot again we go slowly I along this line **along this line this line.**

Heart axis.

Heart axis I have velocity u , dot which is equal to $x_0 \dot{g}$ **you know** the d by dt of $x_0 \dot{g}$, I have got velocity v dot v_0 I have not v dot v_0 , which is $y_0 \dot{g}$ and I have got a rotation which is obviously $\dot{\psi}$ which is r , I am calling it r **r** does not change, because r axis is remember vertical axis, rotation above this axis. Now similar and that the forces I am calling here $x_0 \dot{y}_0$ and n similarly, I am defining here now small x small I forces as capital X , capital Y velocity as small u , small v .

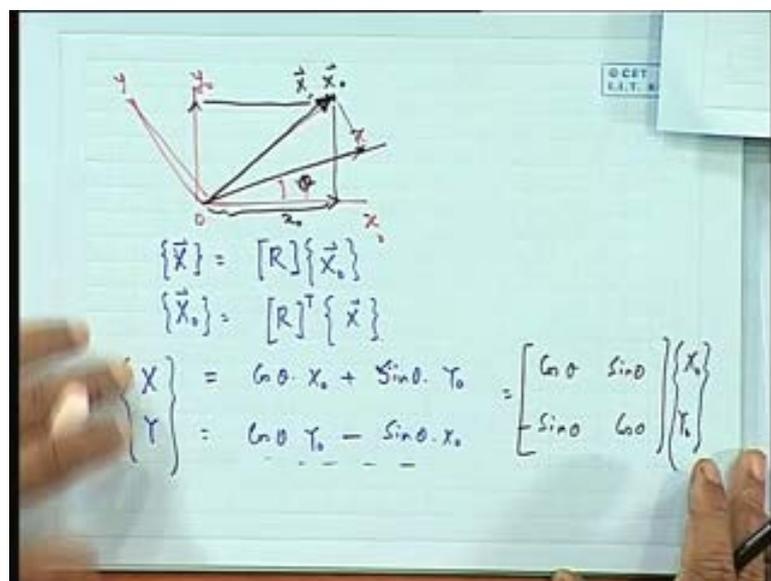
Now, I have an equation of motion which we just, where did we do that where I have got in heart system X_0, Y_0, N , **these are in heart system** this is heart system, I want to change it all to body system, that means I want to write this **this** thing in terms of u and v and r and this thing I want to write in terms of x and y .

Sir r and small s and y sir

No the forces are capital X and Y see, forces are capital X_0 , capital Y_0 , as of an x and y , so all the o system, let me put it this suffix o refers to heart system and small x y are coordinate and big x y are forces and why r , **r** is nothing but $\dot{\psi}$, dot is nothing but, yaw velocity that is invariant whether it is X_0 or Y_0 , there is no r see, there is no r_0 if you call r_0 then in r_0 is r , because the rotation is the velocity you see and this angle $\dot{\psi}$ the rate of change.

See, for example, it was let **let** me say that, the it was ten degree and it is rotated next second its becomes 11 degree, so it has changed to 1 degree per second, whether you are refereeing with **with** this axis or that axis it is 1 degree per second. So r the yaw of velocity, the reason is because yaw velocity is the rotation above this axis, this **this** vertical axis that is **that is** same whether it is here or here, this is why there is no change in the yaw velocity. So, I need to know basically make a change of this equations form this axis to that axis **all right**.

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Now, I will not go to this **this** much detail or that but, what I will try to tell you here looking at that it turns out that, if you have a system here x y and if you have got a another system x_0 y_0 see, if you have got here something as say x_0 y_0 and you have got x and y and if you have got an angle **angle** ψ here and if you take any point here it turns out that see, if **if** this **this** vector.

let me write in a different angle if this is a vector, this is x on vector, so if you take this vector in this coordinate system or that coordinate system and you call by different name **you know** x or x_0 or you can call it also x_0 bar, it turns out that the **the the** coordinates in one is related to the coordinate in other by kind of transformation, that in other words you can say that, X in system one can be related to a **a** transformation matrix into X_0 in system two.

And similarly, X_0 , R it become inverse but, inverse is transpose become X_0 let me not get into that so much but, I will just give an example, that what happened our X force and Y force, **will become** this will become equal to $\cos \theta$ into X_0 plus $\sin \theta$ into Y_0 , this will become $\cos \theta$ into Y_0 minus $\sin \theta$ into X_0 or I can write see, it becomes like that.

This is a very simple coordinate transformation I am just trying to think if I should explain this term in more detail, let me **let me** say I am trying to explain this in more detail see, this is a vector **this is a vector** of a point p here, now this vector is when I am representing in terms of.

Coordinates.

Coordinate x_0 y_0 , then my this becomes my x_0 , this becomes my y_0 , so this is my x_0 this is my y_0 and when I try to express that in the other coordinate system my this becomes this distance becomes my y and this distance that is **you know** but, you can say this distance **this distance** becomes my x , so this is my x , this is my y , this is my x_0 , so y_0 this is my x_0 so this **this** is my x_0 this is my y_0 this is my x this is my y (Refer slide time 40:32).

What it says is that, by coordinate transformation mean is that how do you express these in terms of this and that, in fact we have done that actually if you recall in our cross conceptuality, this becomes nothing but, this into $\cos \psi$ plus this into $\sin \psi$, the $x_0 \cos \theta$ if **you know** in this case say.

Psi.

This is θ here let me call we are just trying to tell about a general transformation if you have represented a vector in an axis system same vector in another axis rotated by an angle θ , then how they are related to each other they are related by a simple transformation like this, very simple and in fact, you can write in a vector form if I want to put that, I mean if I just neglect that this is becomes actually, $\cos \theta \sin \theta$ minus $\sin \theta \cos \theta$ this multiplied by x_0 y_0 .

This is actually so elementary that **I** I have need not actually talk so much but, what I want to say is that, there is a very standard fixed relation available to transform one axis

to the other axis. Now what we are doing you see, we are now going back to this again equation here, now we have got $X_0 Y_0$ they we need to transform them to $x y$ and here we have got acceleration.

In fact **in fact** I can write this term as $u_0 \dot{}$ and this term as $v_0 \dot{}$, I have got here $u_0 v_0$ but, with a dot, so I have to first change $X_0 Y_0$ in terms of $x y$ and I have to change $u_0 \dot{} v_0 \dot{}$ in terms of u and v , the two things are there I will **I'll** come to that in a minute **in** may be systematically here.

(Refer Slide Time: 42:31)

The image shows a whiteboard with handwritten mathematical equations. In the top right corner, there is a small logo that reads "GCET I.I.T. KGP". The equations are as follows:

$$\begin{Bmatrix} X_0 \\ Y_0 \end{Bmatrix} = [R]^T \begin{Bmatrix} X \\ Y \end{Bmatrix}$$

$$\begin{Bmatrix} X \\ Y \end{Bmatrix} = [R] \begin{Bmatrix} X_0 \\ Y_0 \end{Bmatrix}, \quad R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\begin{Bmatrix} u_0 \\ v_0 \end{Bmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$$u_0 = u \cos \theta - v \sin \theta$$

So, you have got here see, $X_0 Y_0$ this turns out to me R of $X Y$ and when R equal to we have said this R actually **you know** the **the** value this is R^T according to our definition or rather $X Y$, we found out R of $X_0 Y_0$ when R is given by $\cos \theta \sin \theta$ minus $\sin \theta \cos \theta$ something like that these are all standard.

Now, think of this, now **now now** that is very simple, now I can always go back to this **this** equation and I can write $X_0 Y_0$, instead of $X_0 Y_0$ I can write them in terms of $X Y$ see, I can write X_0 to be $X \cos \alpha$ plus you know like I can write this X_0 as I have mentioned here in the previous slide (Refer slide time 43:41) I mean I can just do the opposite I can write $X Y$ in terms of that or $X Y$ in terms of the other one not a big problem you see.

So I can basically express X_0, Y_0 in terms of X, Y or X, Y in terms of X_0, Y_0 , let us say that I understand, now we **we we** need to understand little bit of u and v that is the little more complicated see, I have got u_0 and v_0 these are also vectors velocity vectors in which axis.

Heart.

In heart axis, that will become obviously here, R which is let me write it of in terms of this thing $\cos \theta$ actually this is $\cos \theta$ minus $\sin \theta$ and $\sin \theta$ $\cos \theta$ into u , I will explain that little bit one of them say u_0 becomes u into $\cos \theta$ minus v into $\sin \theta$. Now, comes the ball game, now I want to do u_0 dot.

(Refer Slide Time: 44:41)

The image shows a whiteboard with handwritten mathematical derivations. At the top, it states $u_0 = u \cos \theta - v \sin \theta$. Below this, it says "Eq. of Motion" and shows $(X_0) = m \cdot \dot{u}_0$. An arrow points down to X, Y . Then, the derivative is calculated: $\dot{u}_0 = \frac{d}{dt} u_0 = \frac{d}{dt} (u \cos \theta) - \frac{d}{dt} (v \sin \theta)$. The final line shows the expansion: $= \cos \theta \cdot \frac{du}{dt} + u \cdot \frac{d}{dt} \cos \theta - \dots$

So, I **I** will start with this **this** is an interesting point, so I have got see, u_0 equal to $u \cos \theta$ minus $v \sin \theta$, so what happened is that we have got this u and v terms properly. But now my if you look at my **my my** equation of motion, I will just give me bring back my equation of motion is, X_0 equal to mass into u_0 dot, so I now I have expressed that in terms of X, Y I have expressed u_0 in terms of u and v but, I want to find out what will be u_0 dot in terms of u dot and v dot, I must try to find what is u_0 dot in terms of u dot and v dot.

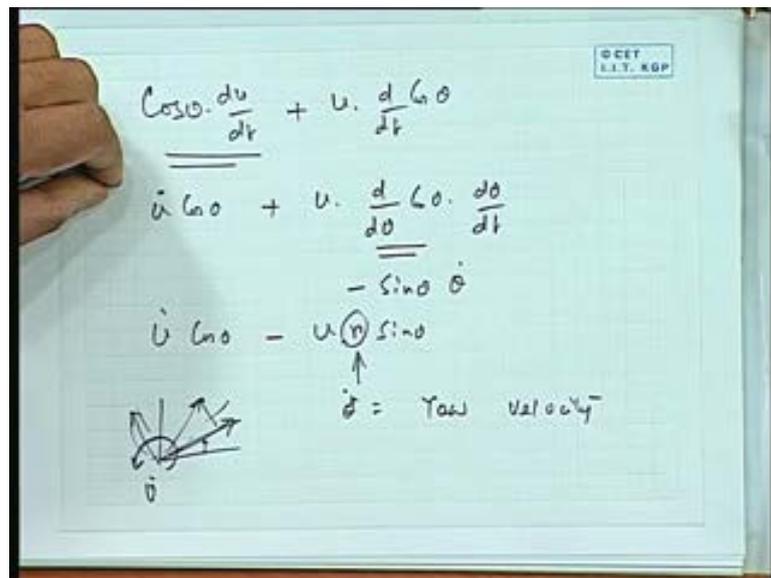
Now you see, form this equation if you do u_0 dot, what is this **this** is d by $d t$ of u_0 this is going to be d by $d t$ of $u \cos \theta$ minus d by $d t$ of $v \sin \theta$, why am writing this

you know, because this has two parts cos theta **theta** angle itself is changing with time, so I have got here cos theta into d u by d t plus u into d by d t of cos theta.

Both are dependent.

Like that, now if you do this d by d t of cos theta you get two terms, because I will **I'll** come to this **this** just I am not going to the exact mathematics.

(Refer Slide Time: 46:20)



Now **I I I** have got here see, one term was cos theta into d u by d t, one term u d by d t of cos theta I have got that, now what is this **this** is u dot cos theta, because d u by d t is u dot straight forward. Now what is this **this** is u, now d by d t of course, it is d by d theta of cos theta into d theta by d t by **by** chain rule this is equal to minus sin theta and what is d theta d t is theta dot, what is theta dot? It is yaw velocity.

So, this becomes minus u dot cos theta minus u r sin theta, what is r, r is theta dot equal to yaw velocity this is all complicate may be to some of you here, but all that I want to tell you never mind this mathematics, all that I am trying to tell you when you are changing an axis system, this to this axis remember that this axis is rotating this axis is not fixed this axis this angle is changing.

So there is a rotation of the axis, because of the rotation of the axis an additional term appears depending on the rotation r, rotation velocity r that is all I want to know this is

what is called actually lateral acceleration one can called coriolis acceleration, various terms are there, whenever you are having a rotating system, system itself is rotating, if it was not rotating then r will be 0, so you end up getting same sin theta cos part but, the system itself is rotate, so you think of a point here.

Now, this point this vector this vector is not just simple one but, tomorrow this **this** is changed, because the axis on which you are actually defining itself is changing, so the **the** principle that is important here, to understand in your mind is that, when I take acceleration vector then I have got an additional term, because there is an additional rotational velocity of the coordinate system itself, this coordinate itself is rotating at a velocity theta dot see, I am defining with respect to ship coordinate system a point.

But that point, the coordinate system itself is rotating, that gives rise to an additional acceleration term this is what we have to know, we need not sort of go through the detail now all that but, you see, what I will tell you that that this equation force equal to mass into acceleration.

(Refer Slide Time: 48:57)

The image shows a whiteboard with handwritten equations. At the top, there is a vector equation $\vec{X}_i = r \ddot{\theta} e_i$ with a downward arrow pointing to a boxed set of equations. The boxed equations are:

$$\begin{aligned} X &= m(\dot{u} - v\dot{\psi}) \\ Y &= m(\dot{v} + u\dot{\psi}) \\ N &= \mathcal{I}\ddot{\psi} \end{aligned}$$

A hand is visible at the bottom right, holding a marker and pointing towards the equations.

Now, if I rewrite all that in terms of x and u, what I will end up getting see, in other words I have got this equal to this into x o g dot **dot** if I rewrite that I will end up getting this side as X, this side I will get mass into u dot minus v psi dot and I will get Y to be mass of v dot plus u psi dot and n of course, remains same.

See, this is what I will get, now here I will tell you earlier it was X equal to mass into acceleration see, $X \ddot{0}$ is mass into acceleration no not $X \ddot{0}$ **sorry** z equal to mass into acceleration here also X into mass into acceleration but, the acceleration is now yeah now no longer just $u \dot{}$ but, $u \dot{} - v \dot{\psi}$ because this part is arising because of the fact that the axis is rotating, this is a very common thing in any rotating system you go to rocket or something it is much more complicated more term.

But this term arises which is very important, this $v \dot{\psi}$ or you can say, $v \dot{r}$ because of the fact that the axis is rotating, obviously if it is not rotating my side that will be 0 and if it is 0 it is $N u \dot{}$. So I will kind of we will pick up from that, this is simply what we have done is we are still on the introduction you may say we have all we have done is the writing Newton equation of motion, mass equal to force and the acceleration but, now I am defining the forces and the accelerations and the velocities all with respect to the ship system.

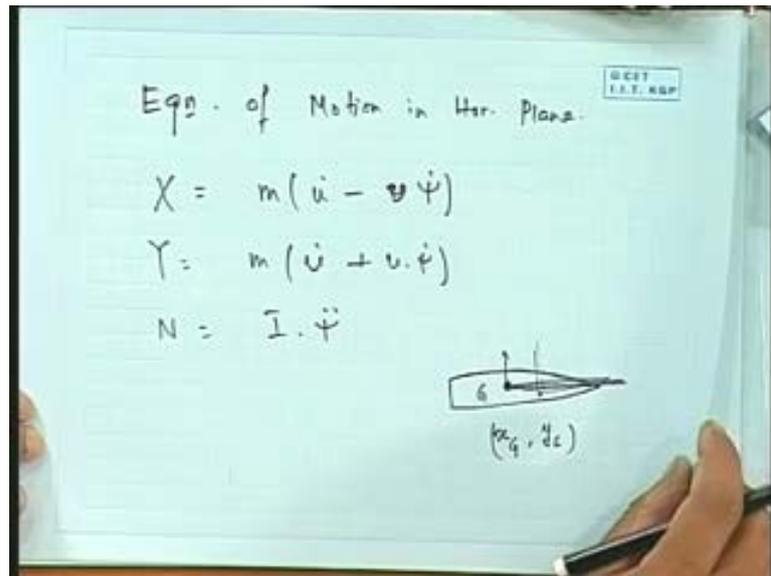
That means, it is the u is the velocity along the ships instantaneous x axis, which is itself changing and because of that you get extra term, so we have end up getting this equation of motion which will be the starting point for our next part of lecture simply by writing equation of motion with a moving body fixed coordinate system that is all, thank you.

Preview of Next Lecture

Lecture No. # 34

Equation of Motion in Horizontal Plane

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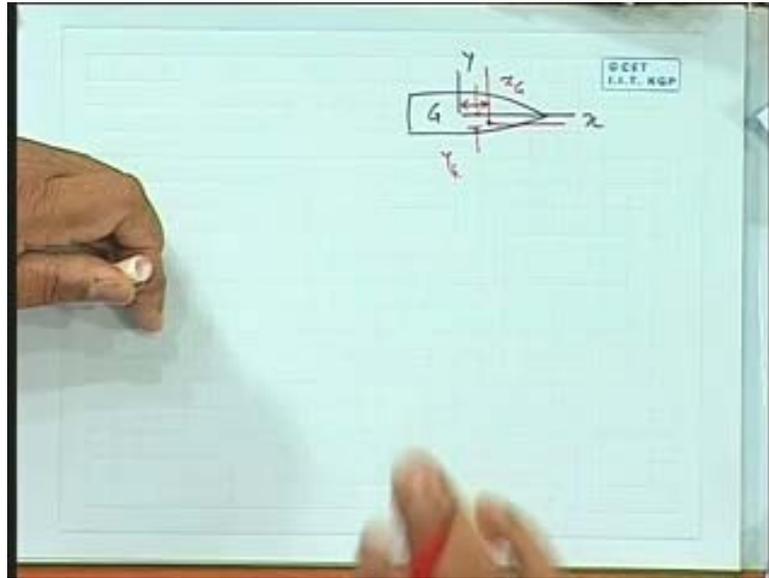


We will continue on this Equation of motion in Horizontal Plane. See, we had this equation just in the last lecture, I just write that v into ψ dot and Y was m of v dot plus I think u into ψ dot and N equal to I into ψ double dot.

Now you see, there is one thing that is that modification, we can do on this is that some time what happened this equations are written when the axis was the centre of gravity **you know** g , sometime what happen it is easier to define that, when the for an arbitrary axis system somewhere else here.

See here, I had taken a this equation of motion are taken where the body system is having its axis as centre of gravity but, some time supposing it is having an axis which is some other location a **o** here, so that the centre of gravity is the coordinates are x_g and y_g In other words I **I** define it with respect to another coordinate system, then an certain extra terms coming in fact, it simply gets modified, as I will just write down that some extra terms will come in here or rather let me do in a second one.

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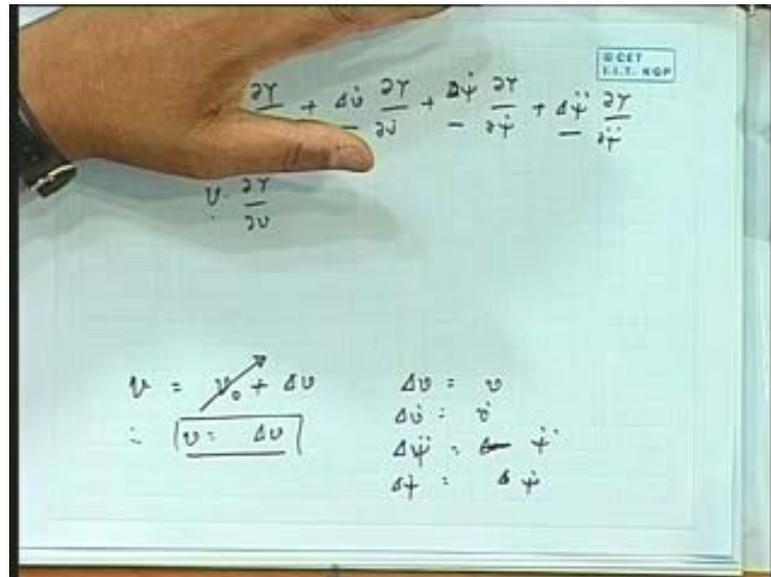
Let me **let me** draw it here, this is my **this is my** y_g this is my x_g this is my y but, if I took a axis in a general sense, somewhere else with this to be the coordinate axis, so that this is my x_g and **you know**.

In other words, if I took let me put it, if I took a body coordinate system somewhere else so that the centre of gravity has a coordinate of x_g , y_g in other words you can say centre of gravity is say 5 meter forward of the mid, say I took a mid ship, so centre of gravity is five meter forward of mid ship.

The reason is because, sometime some of the definitions are easier for a hull, with respect to a geometric fixed point, then the centre of gravity when you are defining lines plan you **you you** are better of taking mid ship as the origin point not x_g is it not, not the centre of gravity, because centre of gravity can be 2.354 meter forward of mid ship, so it becomes difficult.

So, as a result we if it is easy, if you want to want to we can choose the body coordinate system as some geometric point and then say that centre of gravity is x_g meter forward of that, to y_g meter start board of that.

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This is a very, very important and interesting part of that, so what I end up getting is therefore, is that I end up getting that y force, becomes equal to Δv into $d y$ by $d v$ plus $\Delta v \dot{d y}$ by $d v \dot{d v}$ plus $\Delta \dot{\psi}$ $d y$ by $d \psi \dot{d y}$ plus $\Delta \ddot{\psi}$ $d y$ by $d \psi \ddot{d y}$, now this is what we end of getting.

Now, here I want to have to say something more at this point of time, now you see, what is my Δv $\Delta v \dot{d y}$ $\Delta \dot{\psi}$ this thing these are actually the small changes, now there is no point of writing them as Δv , because after all initially you see, I have a v velocity of 0. Now, I have got v velocity of small value of say 0.1 meter per second but, that 0.1 meter per second is just change in v and I call that Δv but, I can also call that as a v itself, because after all the ship was going at a velocity here, without any sway velocity.

Now, I have got a sway velocity small number regardless, so I can call that as instead of Δv I can call that as also v because Δv makes it more confusing so I can call directly this as a v , so I can call that as a v of $d y$ by $d v$, because when the v is understood to be a small number a ship cannot be see, a ship goes at 10 meter per second here it cannot go 10 meter per second this side, it this side is only small number it can only be a change with respect to the 0.

And that I change initially to and illustrate that we call Δv but see, in other words you can see this way, v equal to $v_0 + \Delta v$ but, v_0 is 0 therefore, Δv is v , so

whatever small number you get is the sway velocity. So, I use this nomenclature because Δv is now point of $(())$ so whatever sway velocity I get small number is the change of sway velocity, because my initial sway velocity was 0.

Initially, I had a 0 sway velocity as well as yaw velocity similarly, you will find out that I can call Δv as $v \Delta v \dot{v}$ as $v \dot{v}$, $\Delta \psi \dot{\psi}$ dot at $\Delta \psi \dot{\psi}$ dot and $\Delta \psi \dot{\psi}$ and $\Delta \psi \dot{\psi}$ sorry $\psi \dot{\psi}$ or this is actually $r \dot{v}$ and $r v \dot{v}$ and $r \dot{v}$ and those v and $r v \dot{v}$ and $r \dot{v}$ set up my forces.

Now, this force equal to **you know** a motion I now I want to see, this equation of motion will tell me the solution of that **that** forces that **sorry** that velocity is that was generated because of some disturbance which in turn generates a force does it generate the force at an increasing order or decreasing order, because if that force you think of a loop type for some reason I have given a v , that if this v gives me a y force. If this y force is such that this y force increases my v , then it will have made the my v larger, then the this v would have made my y even larger, that y would have made my v even more larger, it will be unstable but, if it is the other way round I have gotten v which produce an y force this y force in such a direction, that it tries to reduce my v , then that v would have become little small, then the y would have become still smaller.

Then, that would give me my v even more smaller it will go to 0; so this is what is **what is** called the study of stability, that we will **we will** talk next class is given by the characteristic **characteristic** solution of this equation.

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Combine the two

$$Y_v v + Y_u u + Y_r r + Y_i i = m(v + r u + z i)$$

$$N_v v + N_u u + N_r r + N_i i = I_2 i + m z (v + r u)$$

$$-Y_v v + (m - Y_i) v - (Y_r - m u) r - (Y_i - m z) i = 0$$

$$-N_v v - (N_u - m z) u - (N_r - m z u) r + (I_2 - N_i) i = 0$$

Here, I have got v , \dot{v} , r , \dot{r} and all these coefficients, I want to know v , \dot{v} , r , \dot{r} do they grow with time or do they reduce with time, I have to see and my ship would be stable if they reduce with time, this is exactly why you need to find out the forces, you we could have done that ship is stable unstable we cannot do that in this case, for the very simple reason that the forces are necessarily created, because of the velocity in acceleration which are fluid forces.

And it is the force and the velocities interpolate that tells me whether the ship is stable or unstable, so we will pick up from this point and next class study the properties of or the characteristic of or the requirement for a ship to be stable, from that solution of that we would not go to the detail solution just some preliminary ideas, I will **I will** stop it here for today, thank you.