Performance of Marine Vehicles at Sea

Prof. S.C. Misra

Prof. D. Sen

Department of Ocean Engineering and Naval Architecture

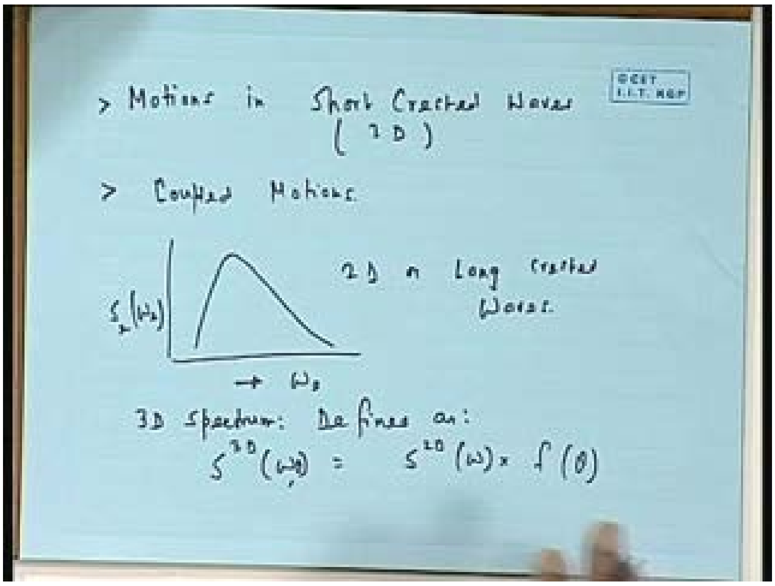
Indian Institute of Technology, Kharagpur

Lecture No. # 31

Motion in Short Crested Sea, Coupled Motions

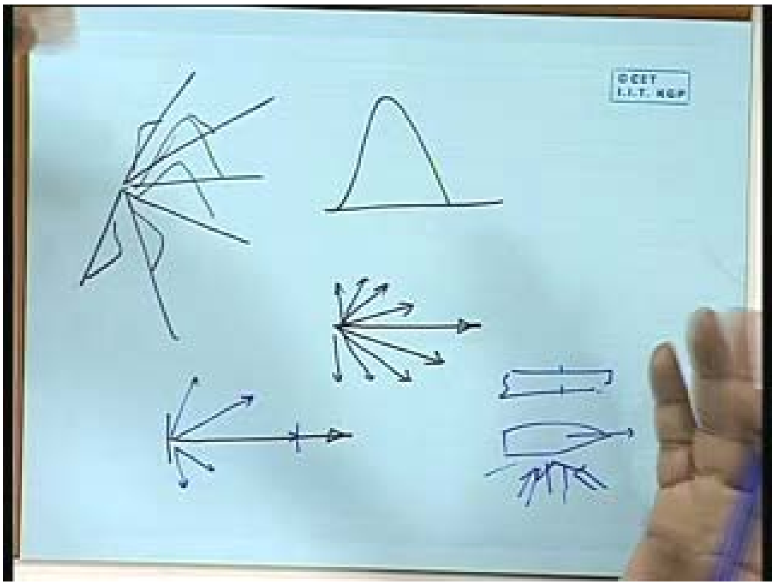
Today, I am going to talk on two of the first lecture on two topics: one is what I call Motion in Short Crested, basically 3 D waves. This I will talk first, and later on, may be what you, I am calling coupled motions. See, the reason I want to talk about motion in short crested wave is that, so far what we talked earlier was a motion in what is called a 2 D or long crested waves. We have briefly discussed earlier, how we can describe a short crested wave by simply multiplying this with a spreading function and taking also the fact that waves come from all directions.

(Refer Slide Time: 00:59)



Basically, what we have defined, that 3 D spectrum S, well 3 D omega is equal to S 2 D omega if there is a theta here multiplied by f theta, like that we described. In other words, if you recall, what we have done is that, we say that if we assume waves are coming from all directions, in addition to all frequencies in one direction, then the spectrum that we get is a bell shape spectrum.

(Refer Slide Time: 02:36)



In other words, the spectrum that will look like is something like, you know, coming well from all direction, and it will look like this, then you have to draw here, then you have to draw here, you have to draw here, etcetera; it becomes kind of a bell shape graph. In other words, let me put it this way, if you have drawn this, and if you rotate that this way, suppose there is a graph here, rotate that along with pulling it down (Refer Slide time: 02:50), that would imply that you are actually simulating or representing an irregular waves where all the waves are coming from this direction, plus this direction, plus this direction, plus this etcetera.

That means, you are assuming that all the waves that are, you know the the the wave height or elevation that you are getting is because of some of all waves coming not only from one direction of all length, but also from all directions, but the directional spreading gets sort of thin down. In other words, there will be much larger waves coming this side, little small at this side, little smaller this side and ultimately no waves this side and like that, and this was simulated by kind kind of multiplying this with a spreading function.

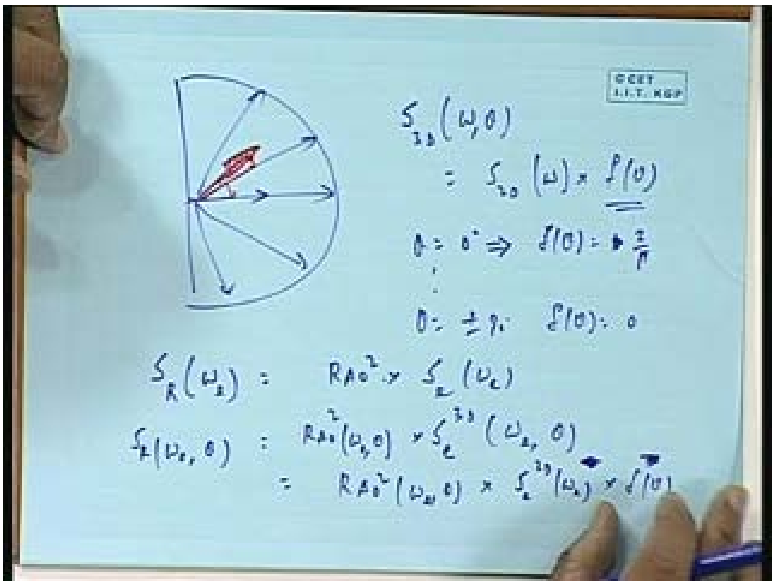
So, in essence what we are doing is that see, that we are saying that there are waves coming just give give a three example; in one case I have all the waves coming from this side and it gives some energy, but in this case, I will have all the waves coming from this side, but of course, it will cannot have all the energy, so it will be may be little smaller you know that if this is and then there will be all some waves coming from this side, some coming from this side like that.

Now, what is happening? Now, if this is the wave representation of spectrum, now if my ship is moving in such a wave, obviously I should have some mechanism to find out the response in such a wave and this is more realistic because just think of it, a ship is moving and I am trying to find out ah you know irregular wave coming all from the, you know if you rather draw this wave all coming from beam waves.

Now, actually all waves came from this side, then the ship would have rolled heavily, but if supposing in reality, then you will find that not all waves can ever come from one side, they will be quite some wave from one side, but there will be some from this side, some from this side, etcetera, etcetera (Refer Slide Time: 04:45). The waves are not normally all coming from one side and if that happens obviously, it effects the response clear case here, the role if you assumed all from one side may be it would roll say 30 degree, but if you assume it is spread, then it will be probably less.

Of course, from the design point of view, what we still do is that, we take long crested wave because, if I have designed the ship to withstand roll for all waves coming from one side, obviously if it is spread it will be less, so we are on a safer side that is a different issue, but we should still know, how we can calculate in short crested wave.

(Refer Slide Time: 05:37)



Now, the idea is very simple, see short crested wave goes like this, that I mean the principle goes very simple. Let us say, I will draw just draw a case of waves coming from all side, this is a wave direction, wave direction, wave direction, etcetera waves are coming from all side and for each side I have a spectrum.

So, I have S 3 D or rather any how omega theta is given by S 2 D omega into f theta, f theta is a non function, therefore when theta equal to for example 0 degree, when I know that you know like f theta becomes 1 like that, when theta becomes you know plus minus 90 degrees, f theta becomes equal to 0 and in between that it goes. That means, what you are multiplying the spectrum, see this is my wave direction here, what you are multiplying with that? Essentially, is a if not one sorry, it will be some factor, some factor may be 2 by pi, I forget, some factor so that when you sum them all up actually, the area under this and area under this becomes same (( )) theory.

Let us say it is known, what I am trying to say here is that, we have a representation of short crested wave, when we know how much of waves are coming from direction 1 and how much from direction 2, how much from direction 3, how much from direction 4. Let us say you break it in say 10 degree interval, so you know all the waves coming from the dominant direction if I call that to be say 0 degree, then all the waves coming from 10 degree, 20 degree, etcetera, etcetera.

Now, let us say on this the ship is moving in this direction, the ship heading is like this. Obviously, what would happen is that, it is very simple, now I would know now all the waves coming from this direction to it. Then in other words, I know I know how much of waves come from this direction, so now, but all the waves of this direction make some heading of so much angle, so I would know that, look the ship as if ship is going with so much angle in all this waves. Then, next time I will say that as if the ship is going with this heading angle with all this angles like that, in other words what happens now? I have to all that I have to do is, I have to spread it out over the angle also.

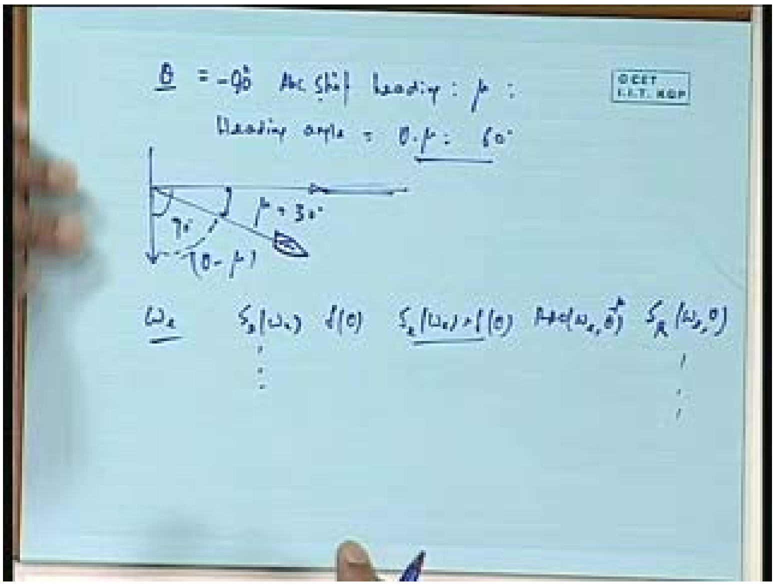
See if you look at the principle, now response spectrum this is response, if now let us say the (( )) this changes omega e etcetera response spectrum in omega e is RAO square into S e omega e is my formula, but now what is happening? My S e is 3 D which has omega e and theta in a 3 D case, so this becomes obviously, I have to have RAO square which will be omega e and theta and this is going to be my S R omega e theta, basically I have to have this, I have to simply add that.

Now, when you do that, you see what you have to do is that, you have to actually do the calculation of this part; if you now break it further, this is RAO square omega e theta into now this part is S e 2 D omega e square into f theta square. So, now what we have to do obviously, the procedure becomes some what simple, we have to go one by one for various angle, I will just show you what we should do.

D square and f S square

Yes, yes because this 3 D is 2 D plus no there is no square here, sorry sorry square square is not there, you are very right, this is just this. This is S 3 D 2 S 3 D is equal to S 2 D omega into f theta, that is right, this square is not there here very correct, just this part becomes square.

(Refer Slide Time: 09:33)



So, what we are doing is that, we are simply actually doing the calculation repeatedly, see what would happen here? We will have this thing, say wave angle theta equal to let us say 0 degree, we will we will start from that or let us say you can start from minus, you have to do it from minus 90 to plus 90. So, let me say we start from or let me say, we will may be we cannot go this way, theta is so and so wave heading angle, ship heading.

Now, you see here ship heading angle is say mu now, now here this is interesting see this is my x axis, so my wave here is 90 degree, this is the wave and my ship heading angle is actually you know this say mu. Therefore, the relative angle of that becomes actually you can say theta minus mu, so it becomes theta minus mu. So, then we have, this will tell me what is my sort of heading angle with respect to the wave.

This heading angle is absolute heading angle, this is absolute heading angle because, you know you are having a reference system see again, this is my x axis, see let me I am giving the principle only, this is my axis. So, now this is my dominant wave axis, let us say that is my dominant wave direction, therefore most waves are coming from this side, you can take any of the reference that is always much better, because if you take this dominant then integration becomes minus 90 to plus 90 because, all the waves will be between this sector.

So, wave is my coming from this direction, but the ship is my moving in this direction. So, now I take say all the waves from this direction, I must find out now all this wave from this direction implies that, all the waves are making an angle of theta minus mu, say this is equal to say 30 degree. So, that means, I am trying to find out all the waves when the ship is moving in all this waves at an ((angle)) of 60 degree, what is my response? So, now this becomes equal to say theta minus mu equal to 60 degree, so for that, for this now I will figure out omega e S e omega e which I have got, we know f, in fact there has to be f theta so, therefore, I have S e omega into f theta that becomes my component for 3 D part.

Here, I have got RAO omega e theta, remember theta actually is not theta, this is theta minus mu here, this is this is theta minus mu, that that particular heading, that is that RAO. Then, here the space, see here I can say S R omega e into I can call that theta also does not matter it is a function of that, mu will remain constant after all 30 degree will remain constant, so I can call it theta here, so I get this.

That means, what I have done is that, I have found out what is my response spectrum for all the waves coming this side. Now, similarly I have to repeat that for all the waves coming from this side and for all the waves coming from this side. So, in other words actually it is nothing but an algebra, we do not go through this detail, but it is nothing but, an algebra of simply taking 2-dimensional spectrum for various angles ok.

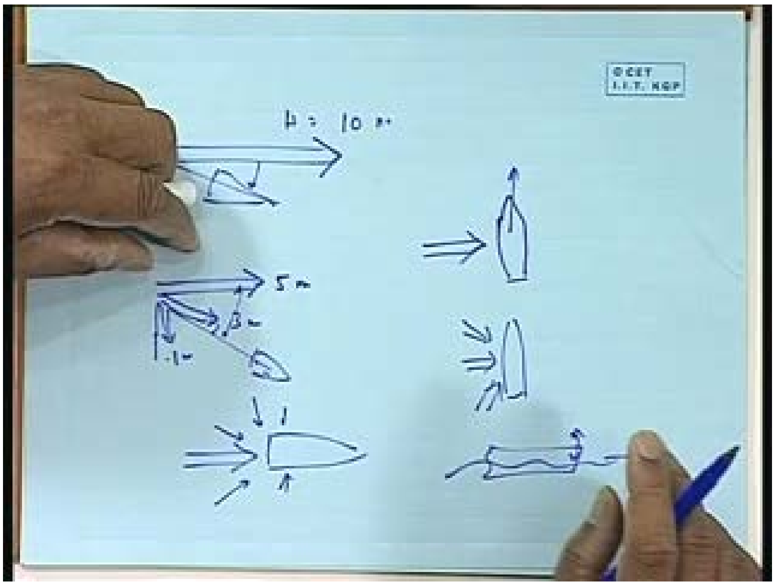
So, what would happen now? If I end up doing that and then add this area, I will again end up getting a spectrum which is spread. See, earlier I have got a spectrum, which was actually in one direction, now I get a spectrum which is also spread.

See, I what I for example,

Only RAO square (( ))

No, no no R RAO square is always there, RAO square is always there, but RAO is now has to be taken for (( )) direction. See, we will give an example of just one wave let us say, just for illustration; supposing in a 2 D case you assume that, there was a wave coming, only one wave was coming of some height.

(Refer Slide Time: 13:25)



Let us say, some wave was coming of say height of 10 meter in this direction, my ship was moving in this direction, so what I did? I would find out instead of frequency, I am trying to say that, the as if there was only one wave coming this side 10 meter height, it will have certain energy so I would find out if this wave, what is the ship response? That is it, but now what happen, I am trying to say that this 10 meter is energy is not all 10 meter, there is actually as if a wave coming of 5 meter here, there is a wave coming of say 3 meter here, there is a wave coming from 1 meter like that.

As if the wave energy is spread, no I say that no the wave that is coming in this direction of 10 meter I assume, is it not so. What is happening is that, there is a total energy so much equivalent to a 10 meter high wave, but the direction of the wave is spread; that that means, that that energy is spread due to a wave of 5 meter high this side, due to another wave of 3 meter side, due to another wave of 1 meter high this side, etcetera so that when you sum them together, the energy becomes same as the energy of a wave coming in 1 meter.

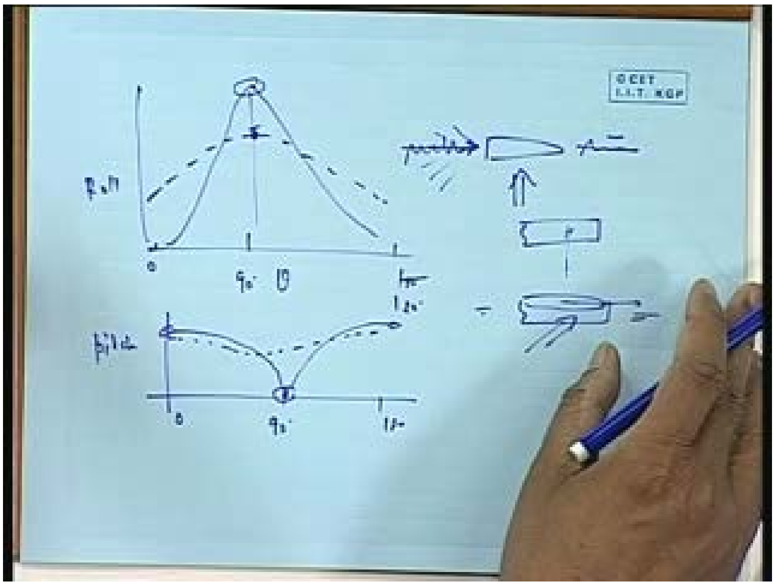
Now, if you see that, now now the ship is moving in this direction, now what I have to do? This 5 meter wave gives a response to that, this 3 meter wave gives a response to that, this 1 meter gives response to that, but they are different because, this 5 meter wave gives a response because of this angle whereas this gives because of this angle, etcetera, etcetera.

Simple case, supposing there is a ship moving in beam wave, let us see. Suppose a ship was moving in this direction and all the waves were coming this side, you have got a very large roll, but instead of that, if you assume that there is a small wave coming, another wave coming, another wave coming, you will have less roll angle.

Conversely, supposing another thing suppose all the waves are coming from this direction, you have no roll angle, but if you assume that no, some waves are coming from this, some waves coming from this, etcetera then you have some roll angle. So, what happens? See when you assume that the waves are not from one side, but from all sides coming then, you have got different kind of response, it is normally mellowed down you know like it is spread.

So, this is what is how we can find out motion in 3-dimensional spectrum, it is turned out that most motions in 3-dimensional spectrum would be lower than corresponding maximum you might get in one side. For example, say pitch if the ship is moving in this wave, it may have a large pitch, much large pitch, but if you assume the waves are coming from all sides, then little bit less pitch, so normally what happen? The maximum you get from 2-dimensional spectrum is always more than what you get from 3-dimensional spectrum, a typical example I show you here, what would happen.

(Refer Slide Time: 16:15)



See for example, if you if you say heading angle here, you know some angle ship angle here, long crested wave say 90 degree you will find out that the roll motion is very small here 0 degree, very small 180 degree, but it is very high like that, isn't it. See, if you take short crested sea, long crested sea waves are all one direction, if wave is coming in all direction to which ship makes 0 degree, that is following on ahead no roll, there also no roll, beam there is much larger roll you understand, this is roll.

See if you assume all the waves were coming in one direction and if they are all coming from 0 degree, that means the ship is like that the waves are all coming like this. What is the Roll? No, roll I mean this is all irregular wave coming roll is 0, if it is coming on this side roll is 0, if it is coming from this side roll is very high. But, now if you assume that look, although my dominant direction is this, my waves are actually spread over. If I take a short crested assumption, then although it is 0 but there will be some roll here and when it is 90 degree also, there will be maximum but less than this. So, the roll curve will shape look something like that if you take a short crested spectrum.

What is this angle? This angle is normally defined as the dominant angle of the dominant wave direction. In other words, in case one, all the waves was in that direction only, in case two, most of the waves in that direction, but there are quite some waves also from other directions spread around the direction.

So, even though that direction of wave do not give roll at 0 degree, because there are other waves from other direction will give you some roll. On the other hand, if there is a it gives maximum roll because of all the waves in 90 degree, but now here it is not all that, so much waves are 90 degree they are spread so it will be less, so it will look more spread. Say pitch for example, if you see the pitch again in say this is 90, this is 0, and this is 180. If you see pitch, you will find that the pitch you know like at 90 degree it does not pitch, so the graph will look something like that and again something like that. Because at 90 degree you know when the waves are coming at this side, the ship does not pitch. See, when the waves are coming beam, see waves are coming in this direction, the ship does not pitch isn't it, when it is coming this side or that side it pitches maximum, so it is maximum, maximum like that.

But, now if you take a 3-dimensional spectrum, you will find that this actually will come down more spread. Actually, the drastic difference that comes in roll, but as you can see if you want to do a conservative design this is maximum, again this is maximum, I mean this is higher.

So, therefore,

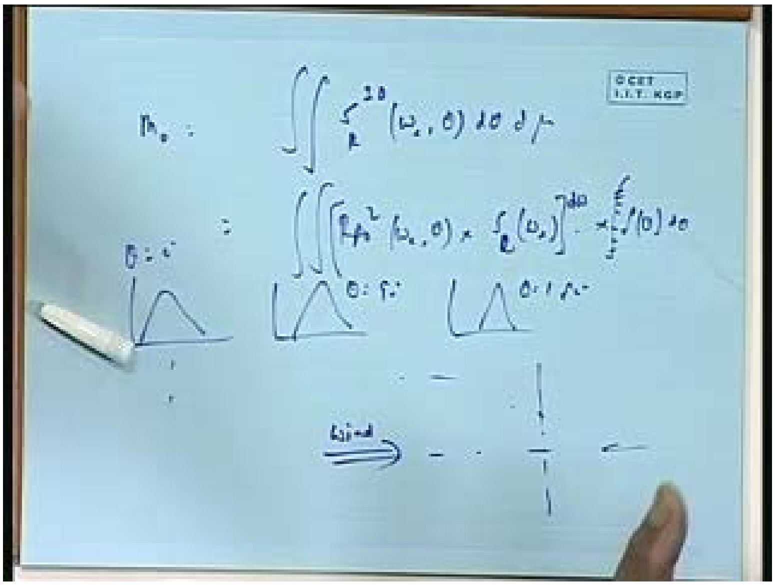
f T count for that

No therefore, what happens? In a design what you can do is that, if you have designed a ship which survive at long crested waves, then you are actually design it for a worse condition, you are on a more conservative side. But some people will say, you see it is a debate will go on because although, suppose you are doing and you say that you know the ship is basically surviving so and so, but your prediction show that the ship might have maximum roll of say 25 degree or 20 degree, but then you know the designer would like to convince owner that look actually it is less than that 15 degree only.

So, they want to insist on doing may be 3 D spectrum, what I am trying to say is that, if you have designed it to survive for a long crested sea 3 D spectrum will always be lower normally. So, from a design point of view, if it is good for long crested wave it will be good for short crested waves. Short crested waves are more difficult to calculate, more complicated it will looks different. However, the procedure for calculation is simple, basically repeating the long crested wave a number of times.

See long crested you have one table, here you have to have say table for various angles that is all like it is a difference between hydrostatic and cross curves. One you do for one angle, another you do for all angles and add them up. So, when you try to find the area also you have area under each angle then add them up over area, you see you want to find out area.

(Refer Slide Time: 20:42)



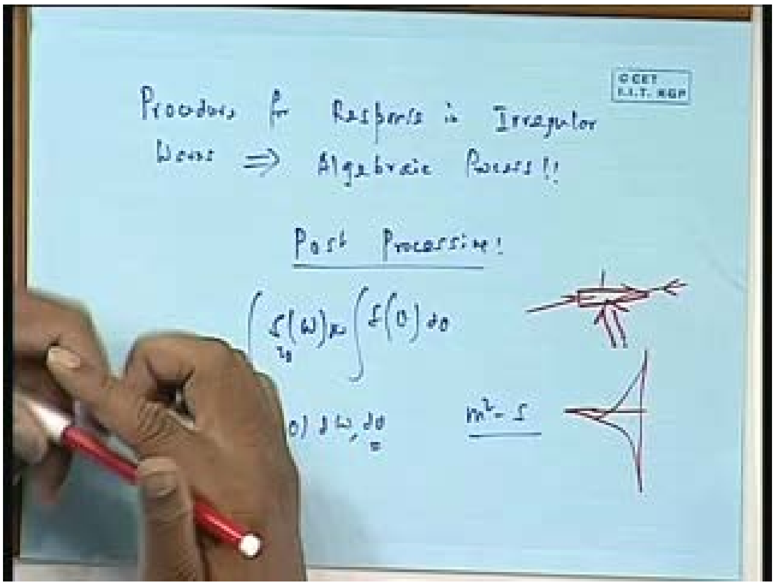
So, I have got area here, S 3 D R omega e theta, D theta D mu the 2 angle. So, you will do in 2 phase, you will do actually this is taken know at RAO square omega e into say theta something like that into here we have got S e omega e and again f theta so you have this part taken as D omega and this part taken as D theta and you know, you separate the multiply. So, it is actually it is simple, what I am trying to say, it will become very, I mean you do in 2 fashion because it may not be like this, you can do that first this one and then this one.

So, it is a very simple thing, you see you are doing it for one angle theta equal to 0 degree, then you are doing for theta equal to 90 degree, then you are doing for theta equal to 180 degree, you find this areas here you again integrate them along the theta angle, you will get a total value that is all. So, the conceptual difficulty is very straight, you will actually learn that this kind of thing always when you really do a problem, otherwise you know in the class room we cannot see the numbers, so my purpose of telling you in the class room is that, it is not complicated, it is simple algebra.

In fact, it is absolutely pure and simple algebra that is all, just you have to keep your head and all that we need to know is to just keep that in mind, that in the short crested sea, all we are saying is that, the all the waves are coming from various direction, so for each direction I must find out what is the ship response and again add them up. So, the wave is broken, in earlier case, various waves coming from one direction, but now we have to spread that also various directions. And the spreading is also well defined, how much it is spread from other direction, the energy is very well defined, if the all the waves are coming from this direction is of unit one, then direction two will be you know that into some function of theta.

Because we know that if the waves are coming from if the wind is blowing this direction, see if the wind blow this direction, most of the waves will be in this direction, no waves will come on the opposite direction and as you go to 90 degree, it will be 0. So, when the wind is blowing in one direction, you will expect most waves are this direction but will be spread that is why up to 90 degree it is taken, the logic is very simple.

(Refer Slide Time: 23:23)



So, we will not be doing problems of that, but I wanted to just tell you how the short crest wave as I said the full procedure, for the procedure for response in irregular waves, it is an algebraic process, pure algebraic process some people call it actually in offshore industry post processing not even processing. Because you know there is nothing, it is the RAO getting where the hydrodynamics comes, the other part is simply break it you know like, it is a like 100 is broken into 5 plus 5 plus 3 plus 2 plus 1 and again add them up, this table.

So, that is what I keep on insisting the class that, this part is the most simple part, although the first paper that came out gave gave this idea of how to find out motion in irregular wave, this paper was called determination of ship motion in a confused sea, it is a very famous paper (( )) in 1953.

But I keep telling that there is nothing confusing about confused sea, it is most straight forward thing, and it is just algebra, pure algebra. Now, tomorrow say somebody wants to find out bending moment not heave, all you need to do is to take the RAO for bending moment that is all, so you want to find out mid ship bending moment in irregular waves.

So, you find out bending moment for regular wave 1, 2, 3, 4, 5 add them up, same way bending moment divided by per wave amplitude that is the RAO, any response amplitude by the input wave is the response amplitude operator. You want to find pressure fine, you take pressure at any point divide amplitude, so anything that you want to do is some pure and simple algebra and there is just nothing to it.

So, I what I that and then, 2 D is a more simpler algebra, 3 D is little more one step ahead algebra, because you have to add also the directional component, so we call it short crest. Saying that also I am trying to tell that normally, even I mean I repeating this that in any design, if you have not done 3 D never mind because if you have designed the ship which is good in 2 D for all direction, it will be definitely even better in 3 D. Because, if it is suppose maximum pitch, when if head sea is say say 5 degree in a in a short crested 5 degree will be 4 degree. So, you do not worry if the ship is surviving 5 degree fine, so this is what we need to know, this is pure algebra as I said, some people call post processing ok.

We will be doing some problems perhaps, very simple problems ah because what happens some time is that, you remember that, the unit becomes important see when I do again just mention this little bit, omega into f theta, this way the integration comes you know, this is D omega, this D theta, you see this area, I mean when you do S omega theta, e omega D theta.

This is written as 2 D omega D omega and 2 like that, now now what happen? You see that, the units are same because, integration over D theta does not contribute to unit, D theta is radian which has no unit, therefore the unit is still meter square second. This unit all remains meter square second because this is meter square second, this is no unit. So, it is unit is still meter square second for the wave part and for the response part it will be response, the unit of a response square into second.

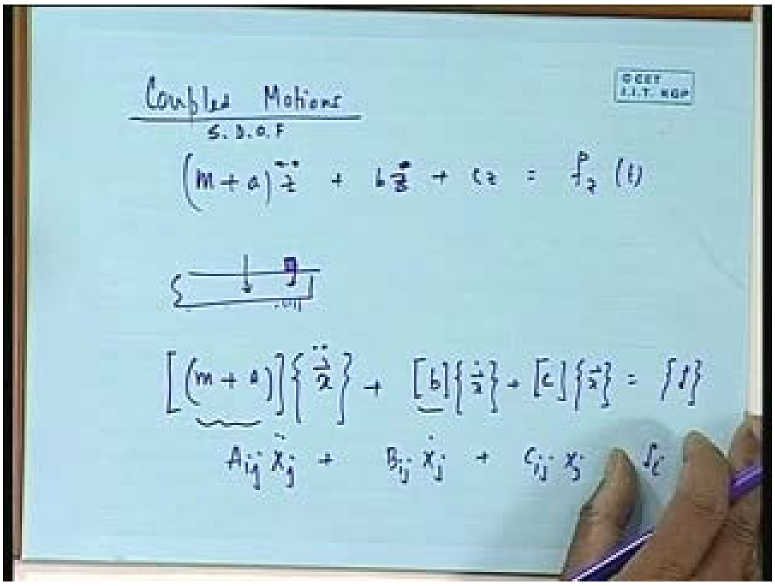
So, it will there is no contradiction what so ever, when you do over the angle, because angle does not have any unit, so that also should be remembered very clearly. This is very briefly, in fact one problem that will occur that, that numerical complication that will occur with is that, if you do a spreading see supposing here I want to tell you this very briefly, supposing there is a ship going this way and I was trying to do all the waves coming from say this side.

So, I have done the calculation all, but if I want to do now spreading, a 3 D then I have to consider all the waves coming from up to this direction to this direction. So, there will be some waves that will be following, although the primary wave is may be at 85 degree, there will be some wave which is coming at 175 degree.

Then, in the following waves you may encounter all funny, funny looking transformation you know all this, this, this, this type. So, you have to be careful in algebra that is all even though the dominant wave is head, there can be some component coming from the following waves. So, therefore, you have to be you know careful about this algebraic process of going to negative frequency, positive frequency etcetera, but it is pure algebra.

So, that you know calculation you may do, you write a program, you have not bothered you certainly find that something as infinity your program stopped, because some number has become 0 here, that you know S omega by theta. So, all I am saying that, you have to be more careful that is all, there is nothing difficult in concept. You know because, yesterday we talked about following waves; in a following wave the transformation becomes very different, some time the wave transform spectrum ordinate may become infinity, such situation may occur when you do 3-dimensional spectrum, so one should be sort of careful about that.

(Refer Slide Time: 29:19)



So, this is about the short crested wave, now I want to change the topic now, little bit on what I mention before as what we call coupled motions and then of course, I want to talk little bit about so called natural periods, which is important because I will come to that about natural period. What is meant by coupled motion that see when I did RAO, see we have written this I will just tell in a very simple way, we have written equation of m plus added mass into say heave this thing plus b heave plus c this thing equal to f, f z like that we wrote, if you recall and amplitude may be this is f z T like that we wrote.

This was an equation we wrote when ship is only heaving, but in reality what would happen is that, when it heaves, supposing you take a ship that is heaving.

(( )) single z single z

Oh sorry sorry yes yes yes you are very right, here single single dot that was b z very correct damping.

Damping

This is the Heave velocity, this is a this is what we call S single degree of freedom equation.

Yes sir.

What I want to mention is that, in reality though what you would have done that in a vibration course, most likely its very famous thing, but what happen in our ship motion is that, sometime heave motion in direction one will induce a force in another direction. Given an example, suppose you take this body here, now you are trying to pitch, pitching; as you pitch here, you see there is a pressure field created and that net pitching may give rise to net heave force, these is exactly what happens to in the hydrostatic trim and parallel sinkage.

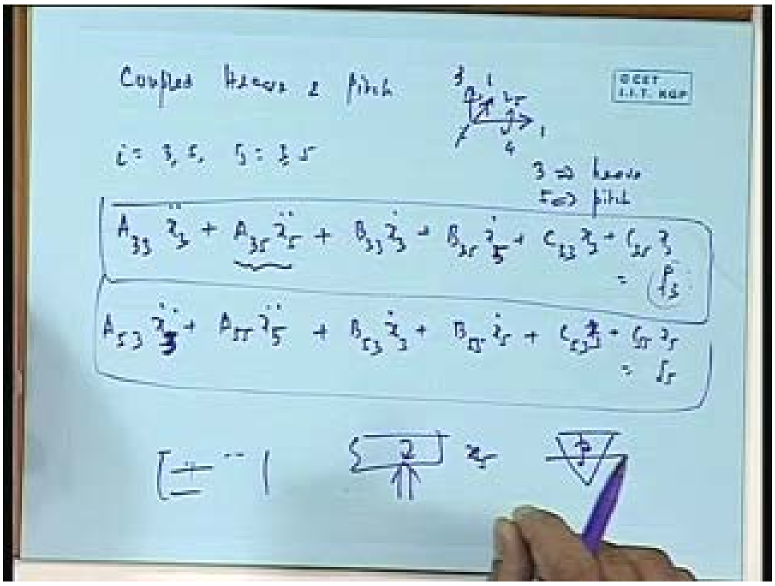
If I put a weight here, what happens? It undergoes trim, but it undergoes also parallel sinkage. In other words, if I wanted to trim it, I will find out that trim will necessarily cause a parallel sinkage and you have seen that because when the ship trims trims about l c f, so there is a, if you just trim the body, its mean draft has changed.

This means that, when I want to trim it, just pure trimming is not possible, unless of course l c f is just at the mid ship, so if you trim it necessarily may induce heave. So, moment you (( )) trim it there is trimming moment, but there is also a vertical force, that vertical force will cause the ship to have vertical velocity. So, in short to be very correct, you have to assume that all motions have influence on all other motions and that is done by taking what is called coupled equation motion.

Then, in that case without sort of writing in detail I want to say that, what happen is that, this equation of motions becomes more complicated and this m plus a becomes matrix now, it becomes mass plus added mass matrix into here the response. Let me write general response as x vector dot dot plus this will become b vector into x vector dot plus c into x vector equal to f vector.

What happens is that, it becomes complicated, more complicated, I will just explain that to you in terms of two, so called coupled heave and pitch. What is the coupling between heave and pitch, actually we can expand that more, I will just write that in a minute. So, it becomes something like this, in fact one people can write if I call this to be a, it can be written as A i j into x j plus i if I call this b, B i j into x dot dot x j dot plus C i j into x j equal to f i, like that one can write because you see this will be 6 by 6 matrix m plus a 1 it is a matrix 6 by 6 matrix. This is a 6 by 6 matrix, this is 6 by 6 matrix; obviously, you can write them as a 11 X 1 dot plus A 1 2 x 2 dot plus A 1 x 3 dot etcetera, etcetera you probably know that right, you you probably know that.

(Refer Slide Time: 33:38)



So, what I mean is that, if you look at this, let us look at this coupled heave and pitch only and how it looks like. Now, heave is actually, we are we are calling this axis system, this is 1, this is sorry this is 2, this is 3, this is 4, this is 5, this is 6; so, index wise heave is 3 is heave, 5 is pitch. You know the motion if I call x 1, see it is a convenient thing to write actually in index form x, y, z and moment about x, y, z you can call it x 1, x 2, x 3 and rotation about x 1 x 2 x 3 as x 5 x 6 6 7 x 4 5 6 axis.

So, index wise if x 1 is my search, x 2 is my sway it is like that. So, coupled heave and pitch may be 3 and 5 in this equation will look something like this, see if I just assume that there is coupling heave induces pitch, then I will have a see I equal to 3 and 5, j equal to 3 and 5, this two will actually exist. So, I will have A 3 3 x 3 dot dot plus A 3 5 x 5 dot dot plus B 3 3 x 3 dot plus B 3 5 x 3 dot plus C 3 3 x 3 plus C 3 5 x 3 equal to f 3 and here A, this is i is in 3 3 3 5 and 5 3 x 3 dot plus A 5 5 x this is not 3 dot this is 5 no no sorry sorry 3 dot this is let me see this matrix this 3.

We have this matrix here 3 and 5, 5 and 3 only. This is I think I am making a mistake here, this 5 here, this is 3, this is 5, dot dot plus B 5 3 x 3 dot plus B 5 5 x 5 dot plus C 5 3 x 3 dot plus C 5 5 x 5 oh sorry not dot is not there like that. Let me just write this and explain what is this, see let me just take one case, let me take this case, this is my coupled the heave motion this, there will be 2 equation of course, coupled heave and pitch. What is happening? This is my heave exciting force, that is force coming in direction 1; I mean heave direction, direction 3, I mean force coming in heave direction.

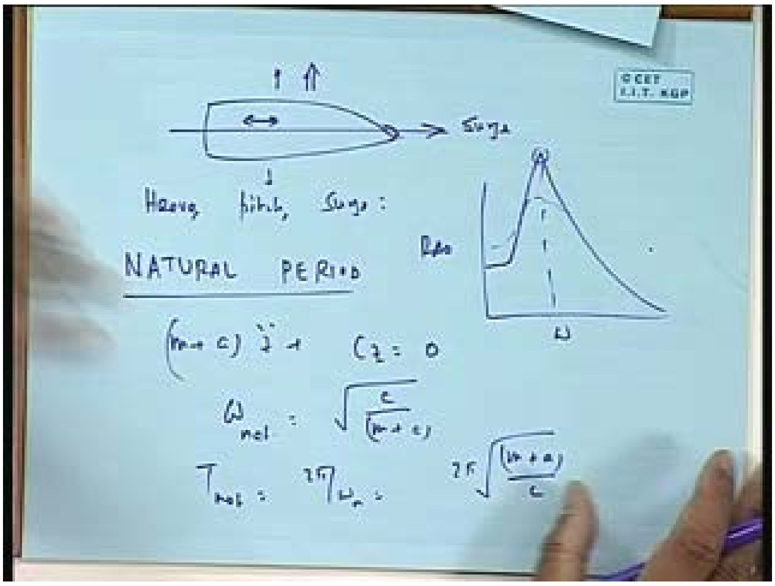
Now, this will produce obviously an added mass for heave and what is called here, coupled added mass force for pitch. What does it mean? It means that, even though I have given a heave, see supposing I have a pitch motion x 5, supposing I I explain that, supposing I pitched it I give an x 5 motion, that x 5 motion may give a net force on the upward direction.

See, I have to add all the force in upward direction. So, this of course, if this is the force in upward direction, now pitch can also induce a force in upward direction. Similarly, pitch velocity can induce a force in upward direction like that, so what is happening is that, we are adding now a coupling that although ship is heaving and pitching together.

So, now supposing it is heaving and pitching same time, what is my force in the z direction? There will be force coming because of heave, but there can be force in z direction because of pitch also, this is what is called couple. Therefore, you cannot solve heave by itself, you have to solve heave and pitch together because, heave is influencing pitch, pitch is influencing heave, this is what is called coupled motion. You see what I wrote before hand is a general general one, where you know you have you are assuming without knowing that, every motion is coupled with every other motion. We will find out for that ship because it is symmetric about you know like central plane heave pitch and roll are coupled and surge sway and yaw are coupled like that.

Because, heave and pitch are always coupled with roll also because, see when you give a heave it gets pitched also and when you give a heave, it can also roll because you see when you like this, when you try to heave it or when you roll it will cause a cause a heave. So, roll heave and pitch gets coupled actually you know, but that is I mean let me give this example little bit in a different form.

(Refer Slide Time: 38:28)



Supposing I am having this ship here like this, now this is my Motion here sorry I mean this is my surge motion here. Now if I push here this side, whatever way it does not give any force this side. So, surge that no sorry actually what is happening is that, I i should say this way, heave, pitch and surge they get coupled why because, if I give a see if I heave it or pitch it, specially if I heave it can cause pitch, if I pitch it can cause surge because you know the forward aft is (( )), but if I surge it will not cause this side.

If I surge means, supposing the body is symmetric, so see if I move it this way that way, will it cause any force this side? No, because it is exactly symmetric both sides whatever I do, whatever suppose a fluid gets pushed equal fluid gets pushed here. So, the net force in the y direction because of x direction motion is 0, this is called that having no coupling between sway and surge. In other words, there will not be a surge motion induced because you have given a sway or otherwise vice versa.

But, that is a special case for a ship which is symmetry, you know that we need not worry, but what I am trying to say is that, in general you will have (( )) when you say coupled motion, you need to actually have all this together, assuming everything is depending on everything else, but in a ship it turns out that, that is not always true you can break it down to three, three parts. But you cannot have only heave, heave and pitch, and surge; surge is not normally common, so this is the most common thing, heave and pitch is the most common coupling that is always taken and this is very important, therefore you cannot solve a problem usually only heave, but you have to have heave and pitch together. Because you see, to say the ship is heaving by 10 meter only, in reality, actually if you oscillate by 10 meter it will also have some kind of pitch motion this is what is called bi-coupled motion that is what sort of we need to know.

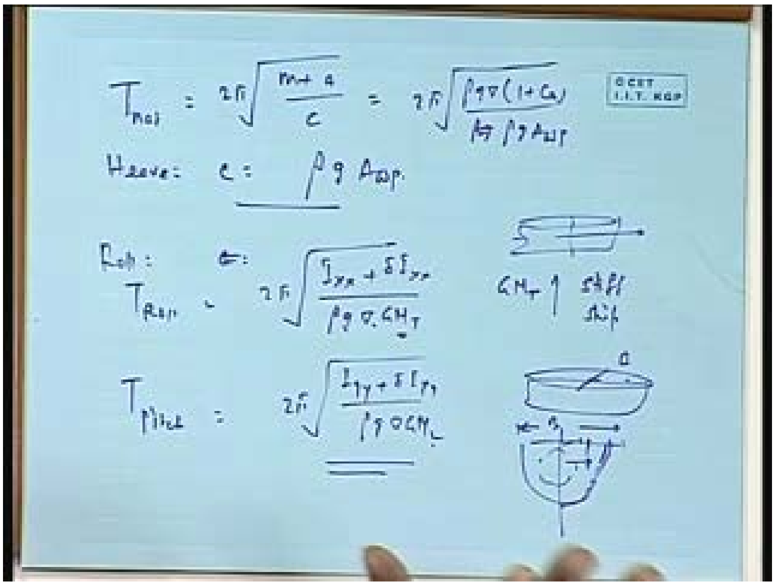
Now, having said the couple motion, I want to just very briefly tell, so in other words therefore, when you want to solve this equation of motion, you have to end up actually solving this equation not 1. See earlier we say that you find out this, you find out this, you find out this, you can solve it, single degree of freedom. But in reality you have to actually find all this which is got 36 elements 6 by 6, this is know like it is a big matrix, this full thing will be 6 set of equations, coupled equation with number of couple of n, so you find out all the added mass, all the damping's, all the restoring forces then you get this, this is how it is.

This is just for the sake of telling for completion, especially as I said heave pitch is a (( )). I just want to briefly mention a very important thing in another few minutes time is with respect to this natural period, because this natural period has a very large influence on my response of the ship. Now, this is something I want to tell little bit, what is natural period? If you know this, if you draw an RAO with omega there is a there is a place where it goes like that, this is my natural period you know. You know that there is high one and this is important because if my spectrum is falling here I mentioned yesterday, then I have got a very large wave.

What is natural period means, the ship is going to have a very bad motion in a single wave if that wave is of that frequency. Supposing a ship has a natural heave period of 10 second, and then if the encounter period of that wave is 10 second, then it is going to very badly heave. So, supposing a spectrum has a large number of waves of 10 second period, then it is going to heave very badly.

So, what is natural period? See, the formula for that is given by see, if you go by this equation actually, what is called, this equal to 0, it is from the omega natural is given by root over of c by m plus a or T natural is 2 pi by omega natural equal to 2 pi root to the power of m plus a by c. I mentioned that, why I want to say that is because I want to right now discuss little bit about the natural period's values approximately.

(Refer Slide Time: 42:58)



Now, for heave values for typically roll heave and pitch. Remember that, there is no see restoring force in unstrained motion of sway, surge and yaw, why if you push a ship it stays there nothing is bring it back. If you shift it on y direction it just stays there. It is like a car, you just move there nothing is holding in back only in a heave roll and pitch if you push it down it gets pushed up, if you tilt by an angle theta you know 10 degree it will come back. If you pitch it by 2 degree it will come back because the fluid hydrostatic will put back. Now for heave the c is given by rho into g into a w p and this is m plus a mass plus added mass.

Now, for roll see actually roll, this formula will become for roll, this m plus a will become replaced by I x x plus a, because you know moment in it is now rigid body moment of inertia. So, this will roll T roll 2 pi of I x x plus delta I x x. what is I x x? I x x is basically nothing but about x axis the mass moment of inertia divided by now this see here becomes rho g v into G M T this is the formula for that, it becomes rho g, actually it gives weight into G M T because weight into c z is a restoring force, c z is G M T into theta therefore, it becomes C, you know weight into G M T that is what it becomes. Now, you know this is how it is here and T of pitch will be 2 pi of I y y plus d i y y plus rho g into G M.

Now, if you look at that, I just want to say here very briefly, we may not have much time, see this will become 2 pi of the rho g into v is the mass into 1 plus see added mass coefficient divided by rho g into sorry rho into g into a w p. You see, look at these formulas, heave natural period becomes something like this.

Now, yesterday I i mentioned that, if you can calculate and we will do that in our assignment that if you have A W P high compared to delta, then this becomes small and in fact, for a typical barge it becomes 6 to 8 seconds, for a ship may be 12-14 seconds, for a semi submersible much more, this is how it becomes this numbers, we should have a feel about the numbers.

In here, you see T roll if g we can find out, if G M T it is inversely proportional to G M T, if G M T is low, T roll will be high. So, you will find out that if I have a very stable ship, G M T is high. If I see you want G M T to be high for high hydrostatic stability, you know G M is high means very stable, but if G M is high, then T roll becomes low and you can find out this values, that the typical ship this values will again come out to be for very small ship with high G M T may be 8 to 10 seconds, for a you know like a you know bigger ship may be 12 14 second. It has been found that, people are comfortable if this period is about 15-20 seconds, you know or may be 15 16 seconds.

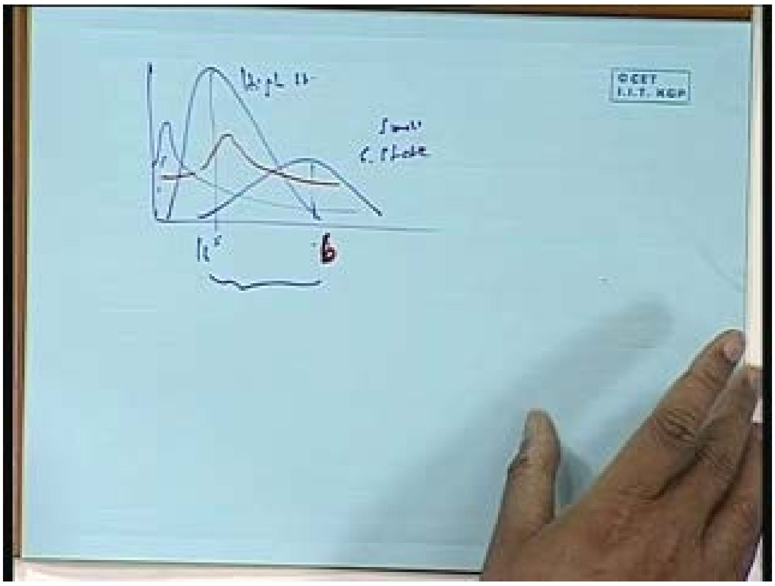
But to do that, you have to actually have G M T to be equal to less. Now, it is there is this contradiction that comes in passenger vessel, passengers will not ride your ship unless they are comfortable. Therefore, they want somewhat good T natural 15, 16, 18 seconds, but it is a passenger ship you have got a higher equivalent for G M T because it should be more safe. So, there is a contradiction that a designer has to sort of figure out, a ship with a G M T high is known as a stiff ship with G M T low will be called a soft ship, a stiff ship is more stable, so you have to be compromise.

But we will not work it out, but you can work it out from this typical number of values because it is easy to estimate. D I X X is very small, I X X is mass into moment of inertia, mass is given by the displacement, moment of inertia about x axis will be something like about 30 percent of breadth. Because you see if you take a ship here, and if you take a moment of inertia over this line, this is my b here, all the all the hulls are on the side, if you can approximate it can be between 25-30 percent of the breadth because you see like this, the moment of inertia will lie somewhere somewhere here, approximately this kind of distance.

If you rotate that, you know you can it is between a solid cylinder and a hallow cylinder. You can make it out because obviously, the weights are more weights are there on the shell, more side frames are there, so it is more like a hallow cylinder shifting little outside, but not fully, fully hallow cylinder because quite some weights are also scattered inside. So, it is in between that and people find out that it is about 30-35 percent of breadth; see, if it is exactly half will be 25 percent because the breadth is full (( )), but this is my b.

So, if it is halfway of b, it will be 25 percent it is little more than that. So, you can estimate that, you know the G M T value, G M T may be you know our requirement is only 1.5 meter, but most ships have 1 meter or so. So, you can find out what is T roll and you will find out that, this comes out to be around 10 seconds. Similarly, you can find out this also, this also actually becomes of similar order because you see G M l is high, but I y y is also equally high, so they get canceled in same order. Why I mention that, because it turns out that for a typical ship, all natural periods happened to be somewhere between 8 to 16 second widely let us say 8 to 16 second.

(Refer Slide Time: 49:16)



Now, you take a small sea state and a large sea state, this may be about 8 second, this may be about say 16 second. What I mean is that, very small sea state to very high sea state. The pick is between 8-16 and it turns out that my natural period is also within that range, you see I it is not that is very for a typical ship, but a semi submerge it is outside, for a semi submerge when it comes see my comes here, but for ship it comes unfortunately all the picks come actually in between somewhere. So, therefore, what is happening is that, ships do undergo large motions in the typical what we call everyday weather that you meet.

(( )).

More or less, more or less see 8 second will be about 100 meter long wave or may be 6 second it can be actually, it can be even lower to 6 second at a very small sea state, you know like 6 seconds means may be like 80 meter long wave. This is why, now again as I mention yesterday, smaller boats and stiffer boats, suppose this ship is stiff will have a roll and all on side.

So, they if you take the barge with a high G M, then it has got a low natural roll period and even in a smallest wave it is going to have a very large roll, so it is not comfortable. So, this tuning this tuning is very important in any motion and that is why I mention this because it turn out unfortunately for a ship, that the natural periods are not widely far from the kind of peak period of the waves that occur.

See, had it been finish in a second if you take a ship of 100 meter long length, you know that ship would behave badly between may be 3 by 4th of the length to twice the length, between 75 meter to 200 long width. And it turns out that the waves that occur in ocean are of that order, they are not 1 kilo meter so many, they are not 20 meter so many, they are mostly 150 meter.

So, you see that the this factor, therefore you have to have in the design tuning, this is why this sea keeping is becoming more and more important with time because, you do not want a bad ship with respect to sea keeping. Anyhow, so this is about this natural period, we will we will talk little more later on and I will I will end it for this lecture now here.