

Performance of Marine Vehicles at Sea

Prof. S. C. Misra

Prof. D. Sen

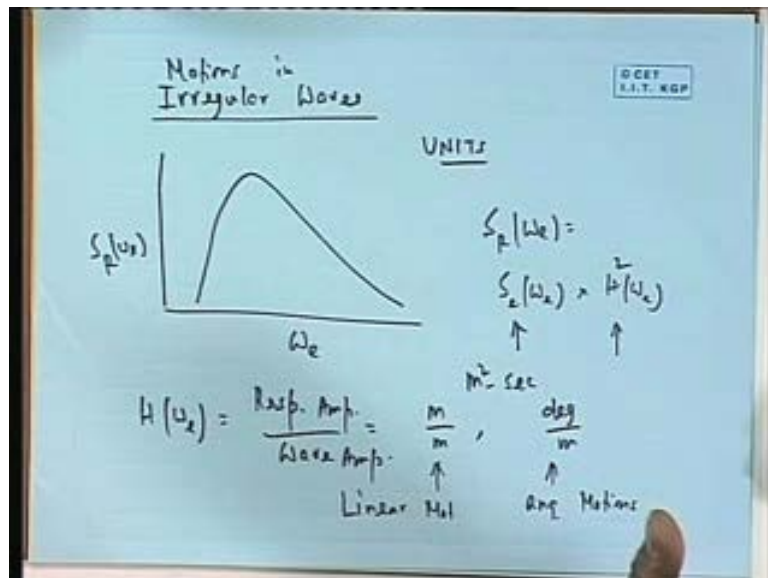
Department of Ocean Engineering and Naval Architecture

Indian Institute of Technology, Kharagpur

Lecture No. # 30

Ship Motion in Irregular Waves – III

(Refer Slide Time: 00:56)

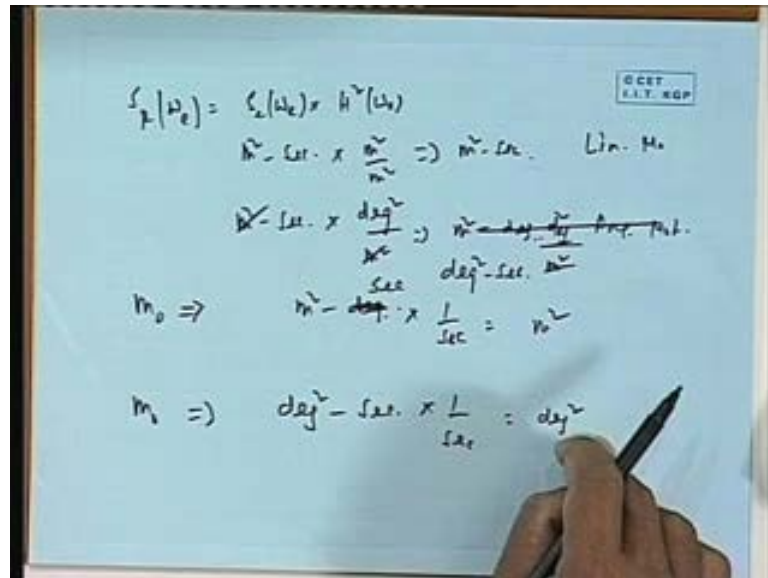


We will come on that later on, see we will continue on **irregular wave** Motions in Irregular Wave ok, **this is the** we will continue the discussion on that. At the very beginning now, I want to tell you about this unit, just for checking because it is always a good diagnostic, see we have got this omega e and say response.

What is $S R \omega_e$? This is given by $S e \omega_e$ into $H \omega_e$ square, this is the formula, what is this unit? This is meter square second, now this unit U will become now $H \omega_e$, what is $H \omega_e$? This is actually, the amplitude of the response divide by the amplitude of wave, so it is response amplitude by wave amplitude. So, this is going to be meter by meter for heave roll and pitch or it is going to be say degree by meter for the angular motions, it is going to be this for linear motions, this for angular motions, alright because it is response by wave. Now, suppose this meter by meter, then this

becomes meter square second into meter square by meter square, so this becomes meter by....

(Refer Slide Time: 02:35)



So, I will just write it again here because, this page is the problem is that, S R omega e equal to S e omega e into H square omega e. So, this is, meter square second into meter square by meter square gives you meter square second, this is for linear motions or this is meter square second into degree square by meter square gives you meter square degree for angular motions.

That is what it is, now area under the graph is going to be these into omega e, so m 0 unit is meter square degree into **sorry no his no there is a second, there sorry first one, no no, I made a mistake here, meter square second, no this is meter square second into degree square by meter square, see it is meter square second into degree square by meter square, that is going to be sorry that is going to be sorry, I am working in, degree square into second, this what it is, yes yes yes.**

This is meter square second, multiplied by 1 by second, this is meter square for linear motions, for angular motions this is degree square into second into 1 by second that is degree square. So, root over of m 0 give me meter, root over of this give me a degree, simple as that, actually it is more simple that to see, that because this is response amplitude unit square divided by sec, in a more general way it is something like that.

(Refer Slide Time: 04:22)

Handwritten mathematical derivation on a blue board:

$$\int_P(\omega_e) = \int_2(\omega_e) \times H_P^2(\omega_e)$$
$$\Rightarrow \cancel{\text{m}^2} \cdot \text{sec} \times \frac{(\text{unit of Res})^2}{\cancel{\text{m}^2}}$$
$$\Rightarrow (\text{unit of Res})^2 \cdot \cancel{\text{sec}} \times \frac{1}{\cancel{\text{sec}}}$$
$$m_0 \Rightarrow \text{unit of Res}^2$$
$$\sqrt{m_0} \Rightarrow \text{has a unit of response. !!}$$

This equal to S e omega e into H R square omega e, this is actually unit of meter square second, this is into a unit of response square by meter square, if this gets cut, so this becomes actually unit of response square into second; so, the integration because multiplied by 1 by second, so m_0 becomes unit of response square, **so m_0** therefore, root m_0 has a unit of response (Refer Slide Time: 04:22).

So, so, that is why the area under the response curve always has a unit of response, so when you took m_0 it will have a unit response, but if you take m_1 , it will have a unit of response into omega, so response into 1 by second. If you take m_2 , it will be response in the 1 by second square, if you take that, now this there is a very interesting thing with respect to those and velocity etcetera **that I will,** I want to say about that part.

(Refer Slide Time: 05:48)

Velocity & Acceleration Spectra.

$$\eta = A \cos(\omega t + kx + \beta)$$
$$\dot{\eta} = -\omega A \sin(\dots)$$
$$\ddot{\eta} = -\omega^2 A \cos(\dots)$$

Amplitude of wave	⇒	A
Amplitude of vel	⇒	ωA
...	acc	⇒ $\omega^2 A$

The next part, before I can go to this, velocity now you see, we will take a simple case of wave only, now a wave was η was given as an amplitude into $\cos \omega t$, let's say $e e t$ plus something $k x$ plus some β something, so η dot is velocity, this is going to be $\omega e A$ ωe into say minus sin of this thing and η double dot equal to minus ωe square A ωe cos whatever. Now the point is that amplitude of wave is A , amplitude of velocity is ωe into A , amplitude of acceleration is ωe square into A , that everybody knows that you just have to multiply with that, this is for wave.

Now, if you take rest now if you if you take, this instead of η , if you take it has a response say h , h is also an amplitude into $\cos \omega e t$, what is h velocity? ω into amplitude into sin of this thing, so let me take a response, you take take a response say h , so $z t$ is given by amplitude of course, at a given frequency into $\cos \omega e t$ plus something, this is my h amplitude. Yeah, amplitude of h response, everybody understand that, what is my z dot t , it is going to be $\omega e z A$, ωe into sin of something. This is, what is this? This is my amplitude of velocity response, h velocity, this is amplitude of h velocity, now what is z dot dot t , this is going to be minus, forget the minus part ωe square $z A$ into say cos of something, so this becomes my amplitude of h acceleration.

(Refer Slide Time: 07:24)

Take a Response, say Heave, z

$$z(t) = \underbrace{z_A(\omega_e)}_{\text{heave amplitude}} \cos(\omega_e t + \dots)$$
$$\dot{z}(t) = -\omega_e z_A(\omega_e) \sin(\dots)$$

Ampl. of Heave Velocity

$$\ddot{z}(t) = -\omega_e^2 z_A(\omega_e) \cos(\dots)$$

Ampl. of Heave accel:

What I am trying to say is that, now this is very important, whenever you have a sinusoidal motion, see if there is a displacement amplitude of some amount, then velocity amplitude becomes omega times that amount and acceleration becomes omega square times that amount. So, the amplitudes of velocity and acceleration are simply multiplied by omega, omega e ok, so the response amplitude of velocity is omega e times displacement amplitude like that.

(Refer Slide Time: 09:31)

Heave displ. ampl. = $z_A(\omega_e)$

→ vel. " = $\omega_e z_A(\omega_e) = z_A^V(\omega_e)$

→ accel. " = $\omega_e^2 z_A(\omega_e) = z_A^A(\omega_e)$

Rao for displ. = $\frac{z_A(\omega_e)}{A}$

→ vel. = $\frac{z_A^V(\omega_e)}{A} = \omega_e \frac{z_A(\omega_e)}{A}$

→ accel. = $\frac{z_A^A(\omega_e)}{A} = \omega_e \frac{z_A^V(\omega_e)}{A} = \omega_e^2 \frac{z_A(\omega_e)}{A}$

Now, comes an important question therefore, **I can**, I can write it like that, say heave velocity displacement amplitude is $z A$, heave velocity amplitude is $\omega e z A$, and heave acceleration amplitude is ωe^2 , let me call this to be heave velocity instead of this thing, so say heave velocity amplitude, I am calling $z A$ of v to be heave velocity amplitude, and I am calling say z acceleration amplitude like that, see $z A$ ωe is displacement amplitude, $z f v \omega e$ is velocity amplitude, $z A a \omega e$ is acceleration amplitude, this is equal to this, this is equal to this, what is RAO for displacement?

That is given by $z A$ by A wave amplitude, **right**, what is RAO for velocity, this is $z A$ the $v \omega e$ by A which is nothing but, ωe times $z A \omega e$ by A , see $z f e$ that velocity amplitude by wave amplitude is RAO for velocity. Remember that, RAO means always what is the amplitude of response per unit wave amplitude, that 2 units need not match, isn't it; because, I have got what is the roll in degree per meter of wave height for example.

What is my amplitude of velocity per unit of wave amplitude, $z A v$ amplitude of velocity divided by amplitude of wave, but amplitude of velocity is nothing but, ωe times amplitude of displacement, so it is this. Now, this is of acceleration is $z A$ acceleration by A , actually it is turns out to be z , **you can** you can see, I will write in two phase now, this will become ωe times amplitude of velocity divide by A or ωe^2 times amplitude of displacement by A .

In other words, what happened? The RAO is nothing but, amplitude by A , so the way the RAO also change, RAO of displacement into ωe gives you RAO of velocity into another ωe give you RAO of acceleration.

See after all, RAO and displacement is only that, you just divide this amplitude by constant A and wave amplitude you get RAO, so the way that this change, this into ωe equals to this, this into ωe equals to this (Refer Slide Time: 12:42). Similarly for RAO also, this into ωe give you RAO of this, this into ωe gives you RAO for this, so the RAO that response amplitude operator for velocity and acceleration also gets available by multiplying with ωe , as simple as that.

All I have done is that see the bottom part is same, you are multiplying this, see velocity is velocity amplitude by A , but velocity amplitude is ωe into $z A \omega e$ by A , the

top body. Acceleration is $z A$ amplitude by ωe that we can show, in fact you can get this by differentiating this once. So, I just stored 1 it becomes $\omega e z$ if e by A or ωe square $z A$ by e , it is very simple matter of taking 1 level higher differentiation that is all. Why we say that, because tomorrow I want to find out, say spectrum for velocity, not spectrum for, let us say for displacement.

(Refer Slide Time: 13:50)

The whiteboard contains the following handwritten equations:

$$RAO|_{acc.} = \omega_e \times RAO|_{vel.} = \omega_e^2 \times RAO|_{disp.}$$

$$RAO|_{vel.} = \omega_e \times RAO|_{disp.}$$

$$S_R(\omega_e) = [RAO(\omega_e)]^2 \times S_e(\omega_e)$$

$$\equiv (H(\omega_e))^2 \times S_e(\omega_e)$$

$$S_R(\omega_e)|_{disp.} = RAO|_{disp.}^2 \times S_e(\omega_e)$$

$$S_R(\omega_e)|_{vel.} = RAO|_{vel.}^2 \times S_e(\omega_e)$$

$$= \omega_e^2 \times [RAO|_{disp.}]^2 \times S_e(\omega_e)$$

So, **now**, now this I will now write this that, it turns out that, RAO for acceleration equal to ωe times RAO for velocity equal to ωe square times RAO of displacement, or an RAO of velocity equal to ωe times RAO of displacement, this is what the standard formula is. Now, let us say how we have done response of something, this is equal to RAO square into $S e \omega e$, this is what we have got, this is my formula isn't it that response spectrum is RAO or H this, we have call this to be H ωe my writing, same as this.

Instead of RAO, we write H

We have written H, but I am here calling RAO because this H will be changing that, so this what **what** I want to say is that, now what would happened, that if you take $S R \omega e$ for displacement, then you get RAO. Let me now write ωe displacement, this square into $S e \omega e$, see **if I**, if I want to find out, what is my displacement response, then that is RAO for displacement square into wave **wave** amplitude, this much we have done always, throughout we have been doing that.

But if I want to find out, what is my this thing of velocity, then I must take RAO square of velocity into S e, that is what I should do, but RAO square velocity is the square of this, which is omega e square into RAO square displacement, now you see RAO square velocity, now here, now carefully see, what is RAO square velocity? This, this is this, so we square that, it will becomes omega e square into RAO square displacement into S e omega e, but what is this, this is my S R omega displacement.

That is S R omega displacement, into omega e square as simple as that, that is what I am trying to tell you, so it becomes this into omega e square simple as that; so, this I will may be, I should write in the next page because it is this thing.

(Refer Slide Time: 16:42)

The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$S_R(\omega_e) \Big|_{vel} = \omega_e^2 \times S_R(\omega_e) \Big|_{displ}$$

$$S_R(\omega_e) \Big|_{acc} = RAO \Big|_{acc}^2 \times S_R(\omega_e)$$

$$= \omega_e^2 \times \left[RAO \Big|_{vel} \right]^2 \times S_R(\omega_e) = \omega_e^2 \times S_R(\omega_e) \Big|_{vel}$$

$$= \omega_e^4 \times \left[RAO \Big|_{displ} \right]^2 \times S_R(\omega_e)$$

$$= \omega_e^4 \times S_R(\omega_e) \Big|_{displ}$$

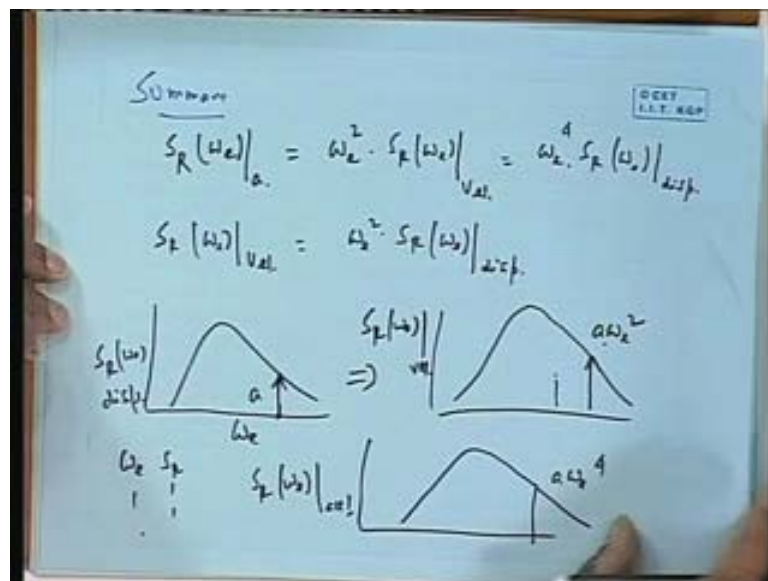
In the top right corner of the whiteboard, there is a small logo that reads "GCET I.I.T. KGP".

So, what, therefore the interesting part that we have obtained here is that, S R omega e velocity turns out to be omega e square into S R omega e displacement; now **same to can** if you see that, S R omega e acceleration will be turning out to be RAO for acceleration into S R omega e, this is square of course and you will see that, this one if you go back to this, now there are two steps, this one is square of this or square of this. That means, this one is omega e square RAO Velocity, that means this one can be written two steps omega e square into RAO velocity into S R omega e, this is like this or it can be written as omega e 4 into RAO displacement square into S R, why its 4? Because you see, if you square that, omega e 4 into RAO square comes in, you know this part.

So, now interestingly you will see here, this part is I kept omega e square, this part is nothing but, S R omega e velocity; see RAO square velocity into S R omega e is obviously, S R omega e velocity and this part, if you see here is becomes omega e 4 of course there into S R omega e see displacement. So, **what you are** now, I will summarize, what you get is that, see velocity response is omega e square displacement response, acceleration response is omega e square, velocity response is equal to omega e 4 displacement response.

So, it is simply multiplying by a square of the amplitude that is all, this is a very interesting phenomena that we should know and then we will find out why it is important, I want to tell you this, that there is the mathematics is very simple, let me summarize it in summary. S R, I am writing this way, acceleration equal to omega e square into S R omega e velocity equal to omega e 4 S R omega e displacement and S R omega e velocity equal to omega e square S R omega e displacement.

(Refer Slide Time: 19:01)

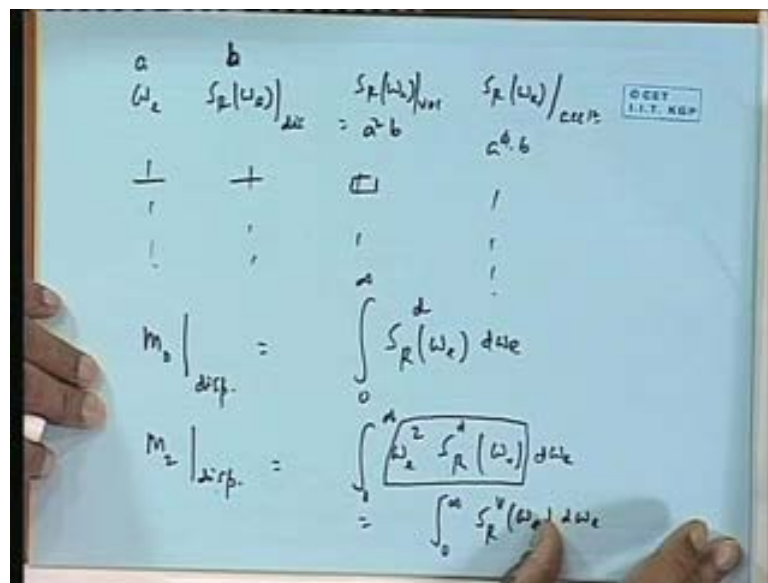


So, now you have got this S R displacement, say displacement take any value here, just replace here this into square of this, so you get this graph; that means, say this value say a say this value is omega e. So, here the equivalent value is a into omega e square, you will get here S R omega e velocity and take here a into omega e 4, then you will get S R so you see, you can get this to this to this spectrum just one more step that is all, you do

is that corresponding value multiplied by see you have got omega e, so you have got omega e versus S R.

Simply write, this is called displacement S R, velocity will be square of this into this, acceleration is going to be square of this, I will make a table next step, but this is very simple, **very very simple**, you know like, if you look at this the mathematics part of it, absolutely straight forward and simple.

(Refer Slide Time: 21:07)



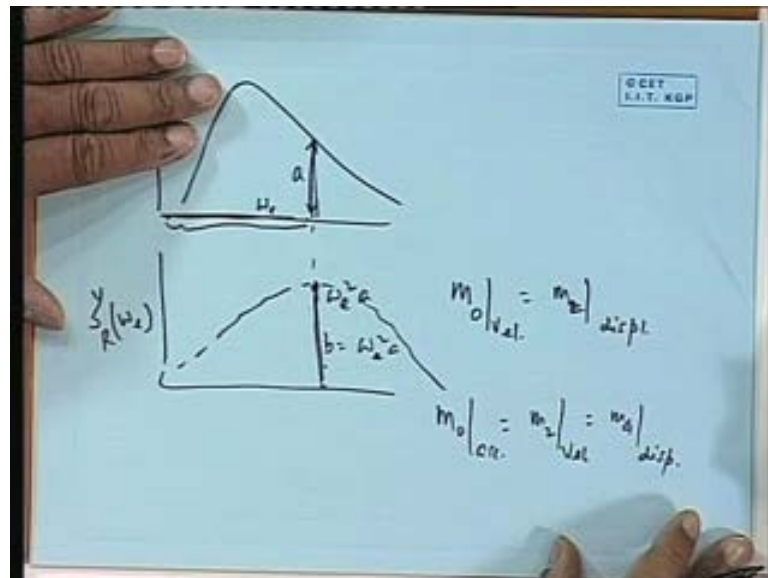
So, now what we have done you see, that you have got here omega e let us say you have got S R omega e, this is for displacement, then you will write here S R omega e of velocity equal to if I call it a, if I say a, if I call it b, this is simply a square into b, **a square into b**, you know, you have got these numbers, see you have got this, **and this, and this** value is square of this into this, here you write S R omega e acceleration, this is a 4 into b, 4 power you know, this 4 into or you can say square of this into this which is the same thing (Refer Slide Time: 21:21).

You see a 4 b is a square into a square b, same thing isn't it, that is what we have done before, so you get this, so you are getting at same time acceleration spectrum, velocity spectrum just from the same table, very simple, absolutely simple you have got the same table you just take one more line, if you are doing computer program you have one more line, you say that S R omega e I equal to a square b, next one is a 4 b that is all.

So, it is that, you know the concept, that the application of it or calculation following this is very simple, all that people get confused is the concept, if you get the concept clear, I tell you in a finger tips, you can give this numbers very **very** fast, there is no **no** problem on that, but this is of the calculation part of it, what I will now discuss little is the **is the** practical implication of this, this velocity in acceleration, but before that there is a very interesting part with respect to m_0 m_1 m_2 .

Now, you see m_0 for displacement is area under the displacement curve, that is $\int_0^\infty \omega e d\omega e$, let me write this as d as a displacement, 0 to infinity, what is m_2 of displacement? This is $\int_0^\infty \omega e^2 S R d\omega e d\omega e$, but this part is nothing but, $\int_0^\infty v \omega e d\omega e$, because $\int_0^\infty v \omega e d\omega e$ is $\int_0^\infty \omega e^2 S R d\omega e$, so this become equal to again $\int_0^\infty S R v \omega e d\omega e$. So, this is nothing that the second moment of area under the displacement is nothing but, area of the velocity spectrum, you get my point, this is very interesting. You see m_2 of displacement is $\omega e^2 S R$ displacement $d\omega e$, square of that, but square of that is nothing but velocity, actually this is more illustrative when we will draw the spectrum.

(Refer Slide Time: 23:57)



See I have got the spectrum here, displacement spectrum, which looks like that, now I took this value. What I do that, I took this value and I plot it here, this is **this is** a here, this is $c R c \omega$, I plotted here ωe^2 into a like that I plotted this graph,

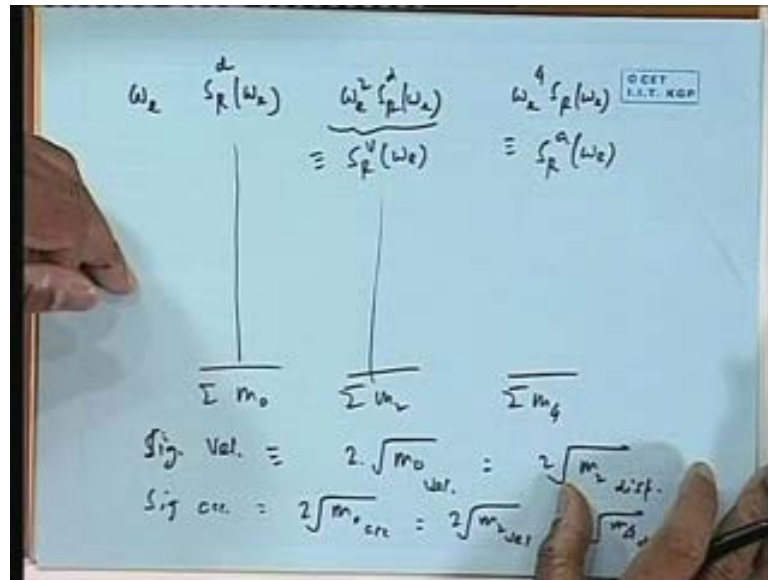
what is this graph, this graph is $S R \omega e$ velocity, this graph is $S R \omega e$ displacement (Refer Slide Time: 24:24).

So, you see the velocity spectrum is nothing but, ordinates are taken square of this, I mean these into square of that, what is that? That is nothing but, taking second moment of area for this, because after all, if you now look at that, see this value is nothing but, second moment of this about this line, because it is these into square of this; see when you take this into square of this, **see if you take this into square of this**, ωe square a is this, it essentially means that this graph is nothing but, second moment of area for this graph, each point is second moment of area, in other words the area under this, if m_0 for velocity is becomes m_2 for displacement.

Because, what we are doing is that, see after all when I have taken velocity, all I am doing a replacing the ordinate by ordinate into square of the frequency, but that is nothing but, the second moment of that ordinate. So, therefore, and that is nothing but, m_2 for displacement, so m_2 for displacement is m_0 for velocity and you will find out similarly, m_0 for acceleration, will be m_2 for velocity, will be m_4 for displacement, I will not going to prove that now, because we are probably going to run out of time.

But you will find out, there was a extremely nice sequence involved here, that I will show you in a tabular form which will be much nicer, you will find out that area under the velocity curve, area under the velocity curve is equal to second moment of area under the displacement curve, area under the acceleration curve is second moment of area under velocity curve equal to 4th moment of area under displacement curve.

(Refer Slide Time: 26:38)



How does it go? In a table form is like this, you are writing here omega e, you are writing here S R omega e for displacement, now here you write A omega e square S R omega e, if you do that and add that, see as I said if I **if I** did and add that I have got m 0, if I did and added I have got m 2, and if I did m 4, I added that I would have got m 4, but remember that, this is nothing but S R, this is displacement velocity omega e. And this is nothing but, S R acceleration omega e, therefore what you get m 0 is nothing but, m 4 is nothing but, area under the acceleration curve, which is m 0 for acceleration curve, what you get under m 2 is nothing but, area under velocity curve.

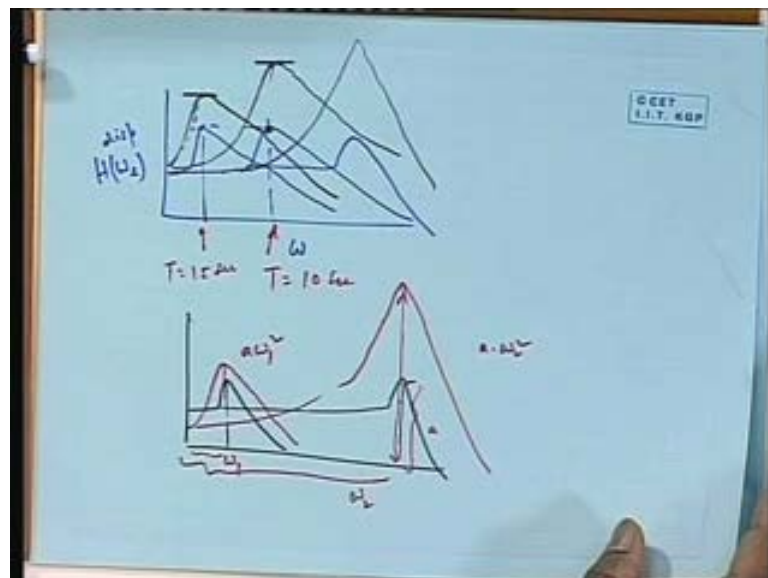
So, you see it becomes very **very** simple thing, so in other words, you do not even have to draw that, you make a table there, and you simply find m 2 m 4 etcetera, and if you found out that, that become the acceleration, so you can get the values. Now, again suppose you want to find out, what is my significant value of velocity, obviously say what my significant velocity is, this is going to be some factor say 2 times root over of m 0 under velocity curve, which is nothing but, 2 times root over of m 2 under displacement.

Because m 2 under m 0 under velocity, that is area under velocity graph is nothing but, second moment ordered displacement curve, similarly if you want to have significant acceleration is 2 time of m 0 under acceleration, it is 2 time of m 2 under velocity, is 2 time of m 4 under displacement, so you can get one to other very **very very** quickly, there

is no problem at all, it is very simple and very straight forward very you know nice, there is no problem, this is what I want to tell you, that this acceleration velocities are very simple, ok.

One is nothing but, the **the the** reason is because in a sinusoidal motion, each of them differ by a frequency, that is the only thing, that is why it becomes so simple, even it is **it is**, 1 meter heave the period is 10 second, then every 10 second 1 meter, so you know actually, you know 1 meter per 10 second is the amplitude and 1 meter per 10 second **per 10 second** is acceleration, so you know that pickups absolute related.

(Refer Slide Time: 29:13)



Now, comes the question about the relation with **(())** as I mentioned before, this is very interesting we should see that. So, let us say, this is omega and I have got 1 response spectrum only, let me say response only about displacement, so it is having a resonance peak here, something like that, now it is somewhere here.

Now, you see what would happen? If you now try to find out the acceleration, **sorry, velocity is** velocity going to square of this, square of this, square of this. Let us look at the peak value, peak value is going to be this into square of this, so it is going to be something like let us say it will go up something like that, and you know like go something like that, this value is going to be, **this value is going to be** this into square of this (Refer Slide time: 29:42).

Now, supposing the graph is not like this, but the graph is like this, same height but graph is like this, you take the second one, now the peak value of that is going to be how much, again this value into square of this, but the square of this is now much less so, therefore, it is going to appear here much less.

So, you see that what is happening is that, if now, this is very interesting, because what is happening, if now what is this, this is a low frequency, means **high** the frequency is low. So, this value has become now, because the frequency is here low here, frequency low means period is high; that means, it is happening every so much later, so when the frequency is low, you would and the maximum value of velocity becomes low, even though that displacement maybe same.

In other words, this is interesting because, if supposing I have a ship, let us look in terms of period, let us say this higher one, so let us say this is T equal to say 10 second; obviously, this is going to be lower period means. T equal to say 15 second, I mean higher period because lower frequency. So, supposing I have a ship at T equal to 10 second having displacement of 1 meter per 1 meter, then I will have certain amount of velocity, but if the same thing happened at 15 second I have got a lesser velocity and if this got shifted in another words, if this was to get shifted here further, then I would get the amplitude much more, I will show an more I mean extreme diagram.

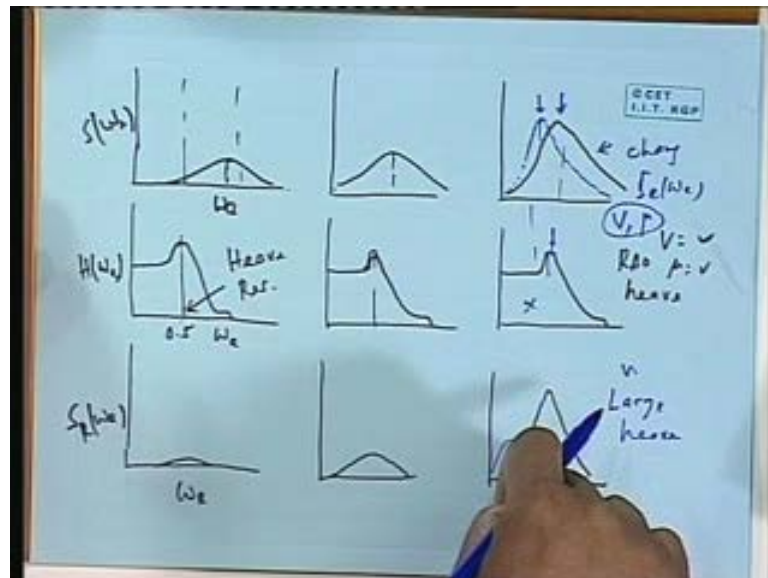
You taken case of this, another case of this, shifted see the response is shifted, one is behaving at low period, one is behaving at high period. This is going to be actually, that the case for a small boats, this is going to be for, **sorry** this is going to be small boats (Refer Slide Time: 31:41). Small boats will be responding **responding** at high frequency, I will **I will I will** mention that later on, but let us say that two case there, now this one and **this one**, if you look at the acceleration of velocity, **the velocity** for this one probably let me draw from here, **it will** may be it will look something like that, but this one is going to be look like that.

Because of here you are taking every value square of the distance, so you see this peak value, this value is this particular value, let me say call this to be a , this is going to be say ω^2 , a into ω^2 square, this is going to be a into ω^1 square, but ω^1 is much less than ω^2 , therefore this is much higher than this. So, you find that if the

ship is responding at higher frequency, then its velocity becomes much higher, acceleration becomes much **much** higher.

And obviously, this is in conversely if you see that the moment of this graph over this is lower than moment of this graph of over this, if you take the second moment of area for this graph, it is much more than this graph. Second moment of area is nothing but a velocity, so you can find out this. So, this is an important part with respect to my **my** responsible, we will now come to see about the tuning of response little bit and how we can see about this **this this** part, the velocity part and all how this is important, you see.

(Refer Slide Time: 33:35)



It is a very nice example given here, I will **I will** try to draw it as much as possible, three cases are there, I mean this is an example to show, how the tuning of a ship with respect to a spectrum becomes important. Now, there is a response graph here is 0.5 second, here if this response does not change it is something like that, this is there everywhere like this, a container ship, this is my $H \omega_e$.

See, this is my RAO in here, it is heave of heave, let us say this is an RAO of heave; obviously, the ship has got some particular, you know resonant frequency is height is peaking up this, this is my resonant frequency it goes up like that, this is my say this is 0.5 something like that (Refer Slide Time: 33:35).

Let us say, this is what it is, see the ship response do not change, I am here showing a particular container ship, how it behaves in different sea states, here we are changing the sea state, but response is same. That means the container ship is moving at certain speed in with certain heading that remains unchanged, it is going at that certain v and certain μ , all of them are going at that, they are not changing, the v and μ is not changing all right.

When v and μ changes of course, we will have a difference there, now there is a spectrum low high etcetera. So, let us say there is a spectrum here, which is low sea state, so what happened in this particular case, low sea state spectrum will look something like that, sea state is low remember this is a lower sea state, where the waves are occurring at much higher frequency.

Now, you see if you look this into this, because this peak is not matching with that, now if you take when it is hull largest number of waves are there, very less response is there, when the large response is there, very well less waves are there; so this area will become if you multiply square of this into this will looks very simple small, because where the largest waves are there, that this is not there.

Now, you take now a sea state, which where this has shifted sea is grown, so obviously it has shift at this way, so now it has gone something like that here you know, now here this is closer to this, so you get slightly more area here. Now, you say sea state, where the sea is even larger, so this has become sea state becomes like that, then here you will find this and this absolutely matching, so this is going to be very large. So, you see here in this case, because the sea state has waves, which is those waves where you are not getting the response when you multiply this with this, you get very small response.

In other words, the waves are not tuned to response, see the ship is responding at this frequency, which means if there are waves of this length corresponding to this frequency, it would behave badly, but this sea state has less of that, so you have got very less; this has got slightly more, this has got most of them, so you can see that you end up getting a

very large response, if you had the tuning of so called the resonant frequency with the peak frequency of the spectrum.

Because peak frequency of spectrum of course, these are all transform frequency remember these are all $S \omega e s$, this side and this is $S R \omega e$ **omega e**, you what I am trying to show here is obviously, this is a very interesting and important phenomena about the tuning of wave with this, so you have a ship which let me give an example, that 0.5 I do not know how much length would be 2π by 0.5 is **2 pi by 0.5** may be 6 by 0.5, 12 second may be this is probably equal to a 200 meter long wave.

Let us say this is about 200 meter long wave, lower sea state I have no 200 meter long waves very less, so it does not matter, slightly higher I have got little more waves, which are closely 200, slightly more higher you have got much more waves close to 200, so you end of getting like that. This is actually what happened to large vessels you see, large vessels what happened, the period is somewhere resonant frequency comes somewhere this side, because I will discuss that again in another class little, the next probably class, but let me take that most ships would have been heave period etcetera may be 12-14 seconds for large ships, that means it is somewhere here.

And in rougher way that you have got more 12-14 second waves, so they are getting tuned, so it is becoming like that. Now, what happened? Suppose, so this is one way of tuning that is, wave with response at a given speed, now what happened? Suppose you are **you are** let me put it this way, now suppose you are in this one, you are going in this sea state and you have got this very large response, you know this; if you take the area under that may be the this is very large heave you are getting, what you do.

Now, what you do, you cannot change this, what you can do is that now supposing, I now go at slower speed, I cannot actually normally speed up more because, speeding up is more difficult, say I change my heading. Now, if you actually if you **if you** remember, if you go at higher speed given v is more in **in in** a head waves this gets stretched out, in fact if you go at I mean if you change the speed, what will happened, if you head into the waves, you have going to actually have this stretched out like that.

But here what you do is that, say you go slower speed, if you go slower speed what would happened, that one you know **you know** $1 - \omega v$ if you see that formula,

1 plus ωv by g , v is less ω to ωe is smaller, so this will actually get, you know there will be some **some** change this will probably becomes like that.

Now, this now may not tune with that, if you do that then this may actually become see when this maximum occurring this is become less now, it is basically other way round so, this may become smaller. Because what is happening now, the peak value which was matching with this you are changing, see earlier what happened? This peak and this peak were matching, that means the 200 meter long waves encountered waves, where causing the ship to heave very much, you change the speed.

Now, as you change the speed, you are not encountering 200 meter long waves anymore you have change encountering some other meter waves, most number, so you have got d tune, this is what you can do by changing volunteer, I mean basically volunteer speed direction. When you meet in many **many** things you can do say, basically what you are doing is that changing the ship, change $S e \omega e$ by changing v and by changing μ .

Therefore, this is not tuning that, you cannot change this remember, this is cannot be change because, this is the intrinsic characteristic of the hull, the ship that you design, behaves badly in a 150 meter long wave period, so if 100 meter wave is 150 meter long wave or I could say in a certain frequency, it has a natural period of say 12 second, it is 12 second, whenever the excitation is 12 second, somebody pushed every 12 seconds, it will behaves go up and down very badly.

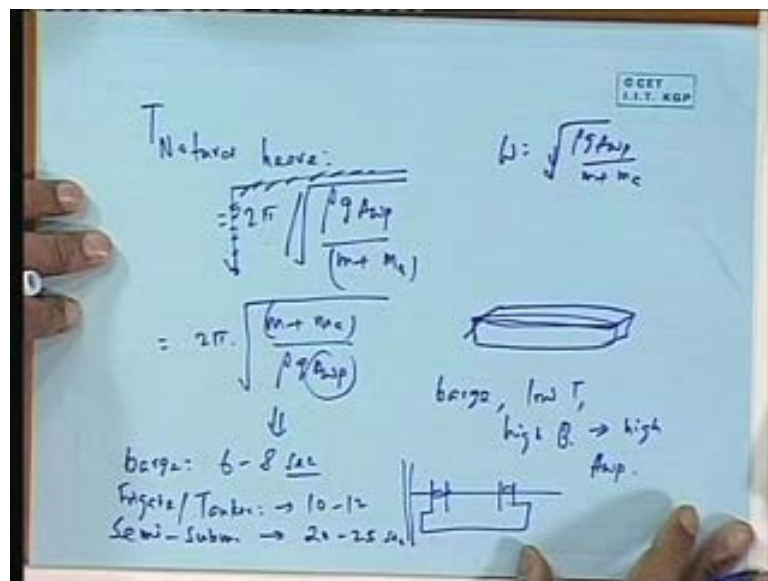
You have to make sure that the waves that exist do not cause that 12 second tuning, how do you do? You change the speed, change the angle, and in fact this is also related if you have a large actually heave and pitch motion, you end up getting what is called large added resistance, large additional power, you know. And that is where comes the concept of ship weather routing, sometime you never go from a to b state, and you might actually change and you may find out that your fuel consumption is overall less, because if you went from a to b, may be you are motions are so large, you have to maintain speed of 15 knots you have to burn more fuel or you are having a less speed, more resistance is there.

So, if you go slightly away, you may reach at the same time, you know burning the same fuel, fuel burning will be same, because your Rpm is same, engine running at the same speed, so then the you know, supposing your engine is at the same speed, you have to go

from a to b, so you have 2 option, you see if the engine is the same speed, your fuel consumption is same.

So, you try find out which is the route where you can reach b at the shortest possible time, now it turns out a to b straight line may not be the case, this is where the concept of weather routing and people actually slightly route, companies are there, who will give you this. A very interesting example I will tell you about, you know like semi submersible etcetera **etcetera**, why it is evolved, that or before that a small ship.

(Refer Slide Time: 43:29)



It turns out that you see, that we have discuss that **natural** T natural, let me talk about this little bit, because that gives you the tuning factor slightly, now this is given by well 2 pi divided by say rho g a w p by mass plus added mass omega root over of that, ok.

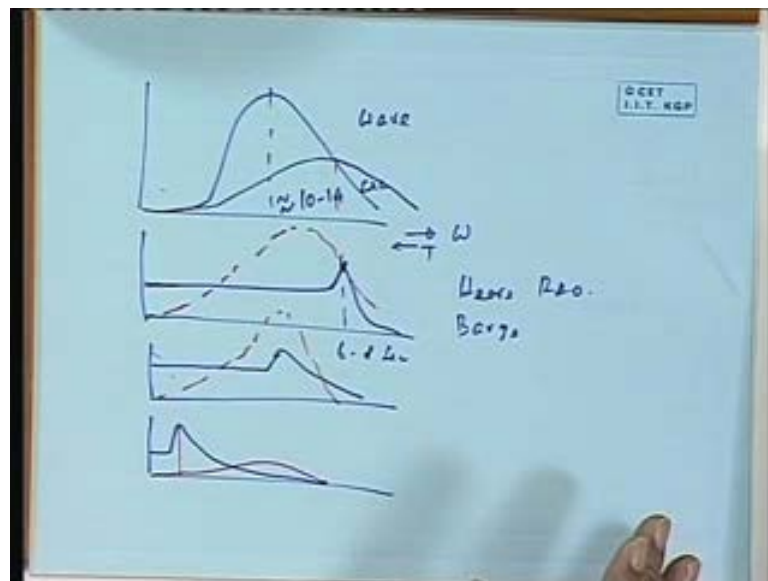
See, in other words omega is this thing or I can write with this way, this formula of that natural period, see omega is root over of rho g a w p by m plus m a and t is 2 pi by omega, so 2 pi, no this is the 2 pi is outside, sorry **sorry** this 2 pi by root over of this. So, this becomes m plus m a by rho g a w p, I think like that, **if you** if this thing something like that, this is my T natural period, I mean I will discuss that afterwards, what I want to say is that see we have told, I mentioned this earlier, that if a ship has a large a w p, but it is not so much large mass, which means the ship is very shallow like a barge, large beam like a barge, low T high B, B gives you high a w p, ok. If that is the case here, so this is high here, this is what would happen to a small barge, **all right**. See you know the one

that goes along the **axis**, you will find that this becomes low, because this is actually high, so this becomes low this might become say 6 to 8 seconds for a typical barge.

For a typical ship, let us say tanker, you will find out that, you know that say frigate for example, where a w_p is the they are normally deeper, t is more, then for a say frigate, for a ocean tanker for example, this may be actually 10 to say 12 second, may be more. But if you look at in semi submersible, I mentioned that perhaps, where a w_p is much lower, because that is the idea, that you lower the a w_p very much, you actually make a semi submersible so that your area is very much lower.

But for safety, you have to have high **high**, so you separate out that is why they are called column stabilize semi submersible, any how here we will discuss that, I will discuss the numbers later on, but typically this might be say semi submersible, it may be 20 say 25 second, let us remember this part it can be quite high.

(Refer Slide Time: 46:38)



Now, you see how the behavior would be in a **in a** large ocean. Now, typically I want to show you here, that typically there is a spectrum here, wave spectrum here, now the three cases we are considering, this is a wave. What is this? This will be equivalent to actually, something like say 10-14 seconds, most of the cases the peak would occur at around that period, you know the ω will be such that peak occurs at that period, remember that this side is ω , that means, this is actually t is this side, higher of that means lower of this thing.

Now, a barge typically would have a response like this, because it is all you know like something like, **may be this is, may be something like this or may be something like this something**, a barge might have, because you know it has got a higher peak, it is have a 6-8 second period **heave**, this is heave RAO, so I am talking of heave RAO for a barge. Now for a typical ship, this will come down this side, so it may be something like that, and for a semi submersible this will be actually much below that, it may be something like **something like** that (Refer Slide Time: 47:47).

Now, you see if you multiply this with this, you see here because this is one here, this is all this is one because, it is actually that means, that it behaves like a wave, so square of that is quite large, so it is going to actually give you response, I just draw the response here more or less. Because this full part has got large part, so you see square of this into this will give you something like a big value, so it will give you some kind of a response curve like that. This one I am going to give you actually even higher responsible over in this case, but this one is going to give you very low response because, here the values are very small, and this is occurring here, so what will happen is going to give something like this. What I am trying say here is that, if you see this interesting part you will see, that in a typical sea state.

In fact, if the sea state is lower, then it would have been like that, so you see what happened a lower sea state, end it here in a soon, the lower sea state just simple, lower sea state means sea states are 3 or something, when the wind is blowing at 20 knots, you are having more number of waves of 6-8 seconds, but that 6-8 second waves the barge behaves badly. Therefore, even in a very low sea state barge behaves very badly, higher sea state is obviously, it behaves badly because it is one, that means it simply goes up and down with the wave.

Now, you take for example, a large tanker lower sea state it may not respond, because lower sea state gone, you know this **this this** peak goes on the other side of that where it is coming down, so it may not respond lower sea state. At higher sea state means, say sea state 5 or 6, where the period is 10-14 second it will be a very badly, still higher it will be a very badly. So a large tanker that is why do not respond to small waves, but large waves it will respond; if you take a semi submersible, it is by design in the semi submersible is designed such that, its peak is much away from any peak, see 20 second no waves exist, it is a very small.

There are simply no waves to cause excitation, so it has very low exertion, why it is designed for this? See there is a I will end in a minute there is a big reason for that, the reason is that ship has the privilege of running away if you **if you** encounter bad motion, you can just change, semi submersible or offshore stock is designed for one place, so if you have a bad weather you simply have to go up and down.

You cannot afford to have bad weather by design of the hull that is why you have a semi submersible design, this is why semi submersible concept and similar concept has come for swath, small water plane area twin hull, I will talk about that next class. But, this is a very interesting part of a small boat barge or you may say a small petrol boat, pilot boat everybody says that pilot boats going to this thing is not comfortable go up goes up and down.

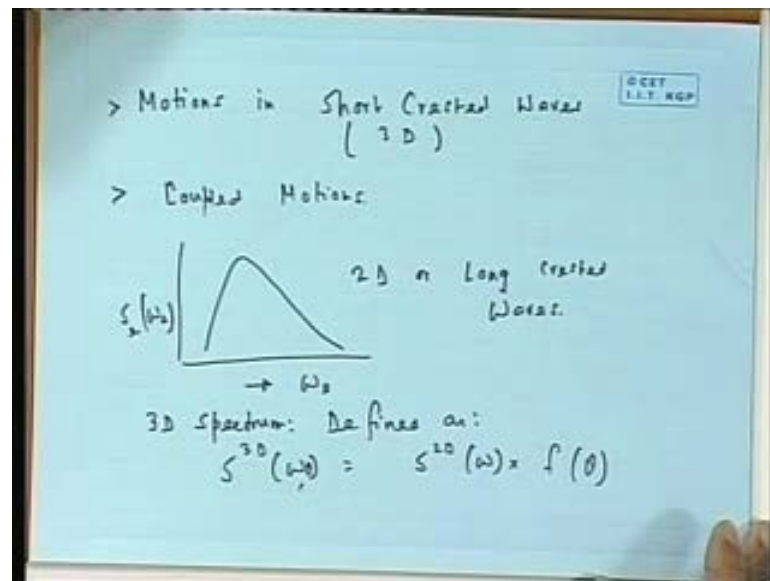
Same reason no large wave, but still goes up and down, but you go in a tanker small waves does not happen, so day to day life you are not bothered I in a tanker, you rather be a assign on a tanker, then on a **on a on a** small pilot vessel and semi submersible is even better, so any how I will end it here, we will have more to discuss next class on the same topic of now coupled motion etcetera, thank you very much, I will end here.

Preview of next lecture

Lecture no. # 31

Motion in short crested sea, coupled Motions

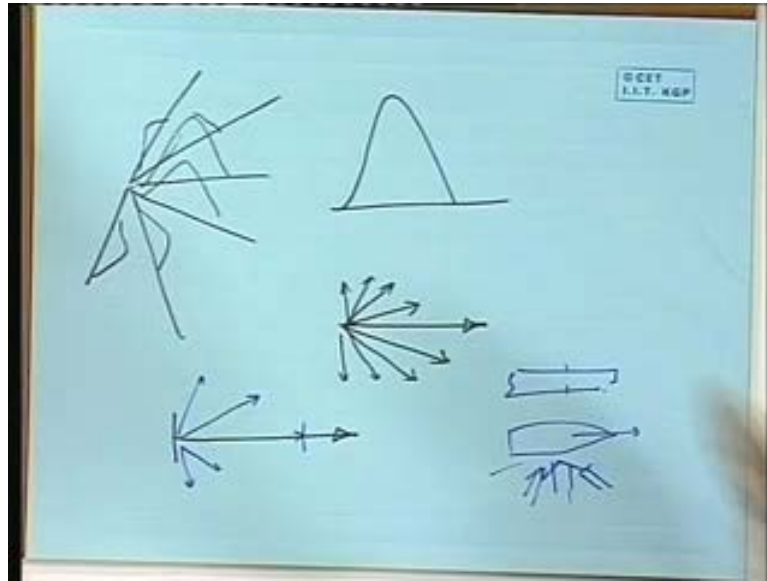
(Refer Slide Time: 51:42)



Today I am going to talk on two of the first lecture on 2 topics, one is what I call motion in short crested, basically 3 D waves, this I will talk first and later on may be what I am calling coupled motions. See the reason I want to talk about motion in short crested wave is that, so far were we talked earlier was a motion in what is called a 2 D or long crested waves. We have briefly discussed earlier, how we can describe a short crested wave by simply multiplying this with a spreading function and taking also the fact that waves come from all directions.

Basically what we have defined that 3 D spectrum, S will 3 D ω is equal to S 2 D ω will be there is a theta here multiplied by f theta, like that we described. In another words if you recall, what we have done is that, we say it that if we assume waves are coming from all directions in addition to all frequencies in one direction, then the spectrum that we get is a bell shape spectrum.

(Refer Slide Time: 53:19)



In other words, the spectrum that we look like is something like, you know coming well from all direction and it will look like this, then you have to draw here, then you have to draw here, you have to draw here etcetera, it becomes a kind of a bell shaped graph. In other words, let me put it this way, if you have drawn this and if you rotate that this way, suppose there is a graph here, rotate that along with putting it top, that would imply that you are actually simulating or representing an irregular waves, where all the waves are coming from this direction, plus this direction, plus this direction, plus this, etcetera.

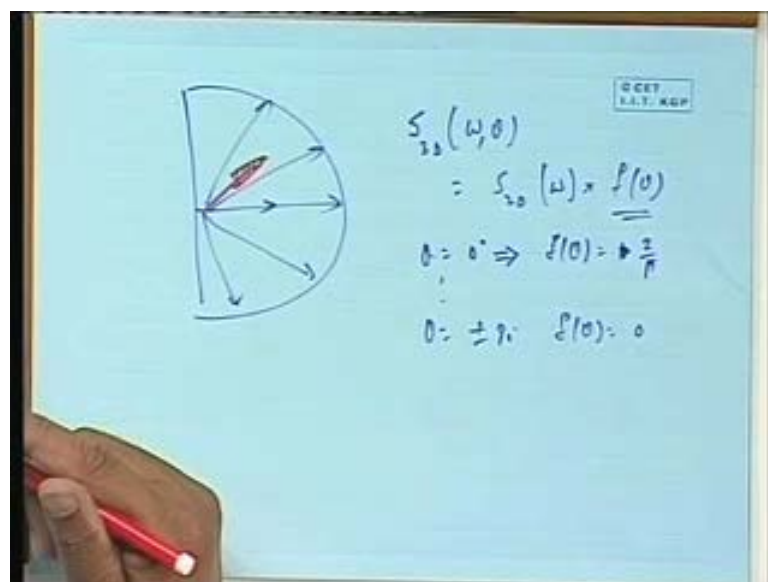
That means you are assuming that all the waves that are you know the **the the** wave height or elevation that you are getting is because of some of all waves coming not only from one direction of all length, but also from all directions, but the directional spreading gets short of thin down, in other words there will be much larger waves coming this side, little smaller this side **little smaller this side** and ultimately no waves this side and like that and this was simulated by kind **kind** of multiplying this with a spreading function.

So, in essence what we are doing is that see, that we are say that there are waves come, just give **a give** a three example. In one case, I have all the waves coming from this side, and it gives some energy, but in this case I will have all the waves coming from this side, but of course, it will cannot have all the energy, so it will be may be little smaller, you know that, if this is and then there will be all some waves coming from this side, some coming from this side like that.

Now, what is happening? Now, if this is the representation of spectrum, now if my ship is moving in such a wave obviously, I should have some mechanism to find out the response in such a wave and this is more realistic because just think of it, a ship is moving and I am trying to be find out, you know irregular wave coming or from the, you know if you rather draw this wave, all coming from beam waves, now actually if all waves came from this side, then the ship would have roll heavily. But if supposing in reality, you will find that not all waves can ever come from one side, there will be quite some wave from one side, but there will be some from this side, some from this side etcetera **etcetera**.

The waves are not normally all coming from one side and if that happens obviously, it effects the response for in the clear case here, the roll if you have assumed all from one side may be it would rolls a 30 degree, but if you assumed it is spread, then it will be probably less. Of course, from the design point of view, what we still do is that we take long crested wave because, if I have design the ship to be stand, roll for all waves coming from one side obviously, if it is spread it will be less, so we are on a safer side that is a different issue, but we should still know, how we can calculate in short crested wave.

(Refer Slide Time: 56:21)



Now, the idea is very simple, see short crested wave goes like this, that I mean that the principle goes very simple; let us say, I will draw just draw a case of waves coming from one side, this is the wave direction wave direction, wave direction, etcetera.

Waves are coming from all side and from each side I have a spectrum, so I have S 3 D or other any how omega theta is given by S 2 D omega into f theta, f theta is a known function, therefore when theta equal to for example 0 degree, when I know that you know like f theta become 1 like that, when theta becomes you know plus minus 90 degree, f theta becomes equal to 0 and in between that it goes. That means, what you are multiplying the spectrum see, if this is my wave direction here, what you are multiplying with that, essentially is a, **no it is not one sorry, it will be some factor some factor may be 2 by pi I forget some factor,** so that when you sum them all up actually the area under this and area under this becomes same of theory. Let us say it is known, what I am trying to say here is that, we have a representation of short crested wave, when we know how much of waves are coming from direction 1, and how much is from direction 2, how much from direction 3, how much from direction 4.

Let us say you break it in say 10 degree interval, so you know all the waves coming from the dominate direction, if I call that to be say 0 degree, then all the waves from 10 degree 20 degree etcetera **etcetera**. Now, let us say on this the ship is moving in this direction, the ship heading is like this. Obviously, what would happen is that, it is very simple, now I would know, now all the waves coming from this direction to it.

Then in other words, I know that **is I know** how much of waves coming from direction, now but all the waves of this direction make on heading of so much.