Performance of Marine Vehicles At Sea

Prof. S. C. Misra

And Prof. D. Sen

Department of Ocean Engineering and Naval Architecture

Indian Institute of Technology, Kharagpur

Lecture No. # 03

Dimensional Analysis

Good afternoon, gentlemen. Today, we will talk about dimensional analysis.

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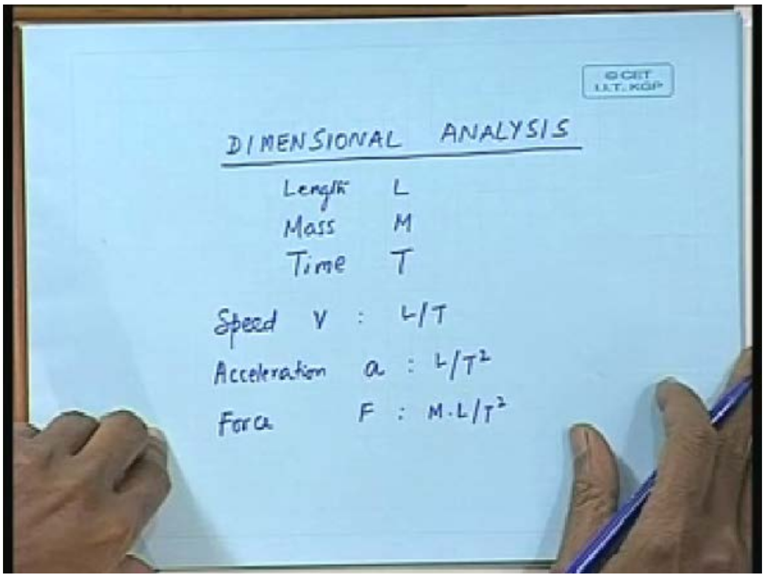

Dimensional analysis is a technique used to understand a phenomenon when the exact solution of the problem at hand is not known; in other words, when you have only partial knowledge of a problem you try to find out in an empirical manner what could be the nature of the problem in a mathematical sense. It is in case of a resistance (( )) we know that resistance is a complex phenomenon, it depends on certain variables, but we are not in a position to find out the functionality of resistance with regard to variables exactly in that case, it is possible to find out an approximate relationship between the variables and the quantity- that is, in our case resistance- through a dimensional analysis procedure. Basically, this is, dimensional analysis is a technique when the result or the quantity- in our case, resistance- is expressed in terms of certain variables, known variables; we try to achieve the dimensional homogeneity between resistance and the variables, by that process we can find out the relationship between the variables and the quantity at hand, that is, the dimension of the left hand side of a quantity must be equal to the dimension to the right hand side of the quantity.

This is sometimes very helpful as we will see in case of resistance, this gives very interesting results. In general, in case of mechanics of fluids and solids there are 3 basic dimensions on which all quantities can be defined that is, length L or length dimension L, which is any linear dimension, Mass dimension M and time- these are three basic dimensions based on which you can find the dimension of any other variable. Let us take an example of speed, speed V, the dimension of which is as we know the distance travelled over a period of time, so it will be distance is length dimension over time dimension that is, L over T. Similarly, if you take acceleration, a, is speed per unit time, rate of change of velocity per unit time that is, speed divided by time come to L over T square. Similarly, if you take the force, f, is mass into acceleration therefore, it is M L over T square- and like this we can define all dimensions.

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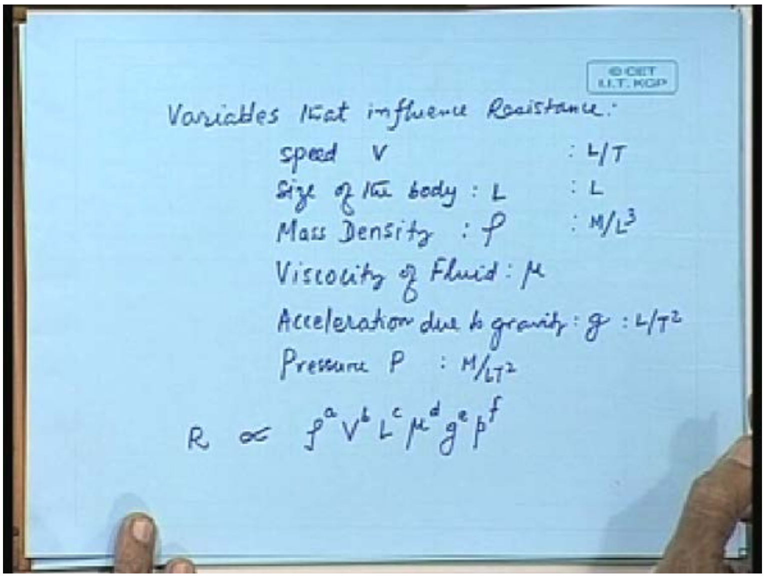

Now, if in case of resistance of ships the variables that affect resistance, that influence resistance can be written as, we can list them: first of course, is speed V; size of the body, size of the body can be expressed as a linear dimension L that is, length dimension is L, area dimension is L square and volume dimension is L cube; then, you have density of fluid in which the body is moving, mass density, rho, which is mass per unit volume; viscosity of the fluid, mu; acceleration due to gravity, we have seen that the wave making resistance is a function of gravity, g; and pressure, we have talked about pressure in some detail of pressure that is, p. Now, if we write this in a equation form, we will say R is a function of, or R is proportionality that means, a constant of proportionality, we can write rho to the power a v to the power b l to the power c mu to the power d g to the power e and p to the power f.

Now, this is a very general solution, we are just writing down here the variable, v l rho mu g p raised to different power which we do not know, and we have said proportionality, so there could be a proportionality constant somewhere. This is a general form of expressing resistances as a function of variable which we think affects resistance; understand that this is only an approximation relationship, we will try to find out if we can get a better relationship than this by equating the dimensions of left hand side with those of the right hand side. Therefore, it is now necessary for us to find out the dimensions of all the variables that we have written here. Now, speed dimension is L by T; size of the body, length dimension L; rho is mass per unit volume, M by L cube; mu, we will find out in a short while; g is L over T square; and pressure p is force per unit area, it will be M over L T square- is that right?

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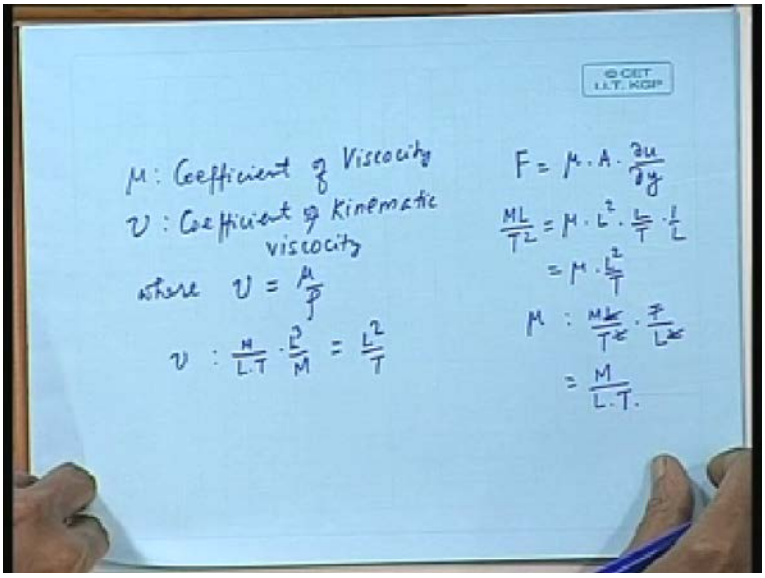

Now, the mu, what is mu? Mu is the coefficient of viscosity, representing the viscosity of the fluid. How do you know what is the unit of mu? Mu is, we have seen that mu is the shearing force between two layers of fluid, or solid and fluid basically, it is a shearing force, so it will depend on the force, viscous force will depend on- what will it depend on?- we can say mu, coefficient of viscosity, area into the rate of change of velocity in the transverse direction is the one layer of fluid moving over another layer of fluid, the difference between the velocity that is, du by dy, y is the (( )) in the transverse direction- is that clear?

Now, let us do a small dimensional analysis for this- F is, we have seen F, ML over T square- is it not?- mu a is L square, del u, del y, we can say this is velocity, this is distance, so L over T into 1 by L. What we get? Yes, tell me- Mu L square by T. So, what is the limit of mu therefore? M L by T square into T by L square that is, M by L into T- is that right? One more quantity we will use slightly later is the coefficient of kinematic viscosity- this is, mu is coefficient of viscosity and nu is coefficient of kinematic viscosity where nu is represented as mu over rho. So, the unit of nu therefore becomes, you can see M divided by LT into (( )) that is, L square by T- coefficient of kinematic viscosity is L square by T. Now, let us go back to our resistance equation and see dimensionally whether we can find something.

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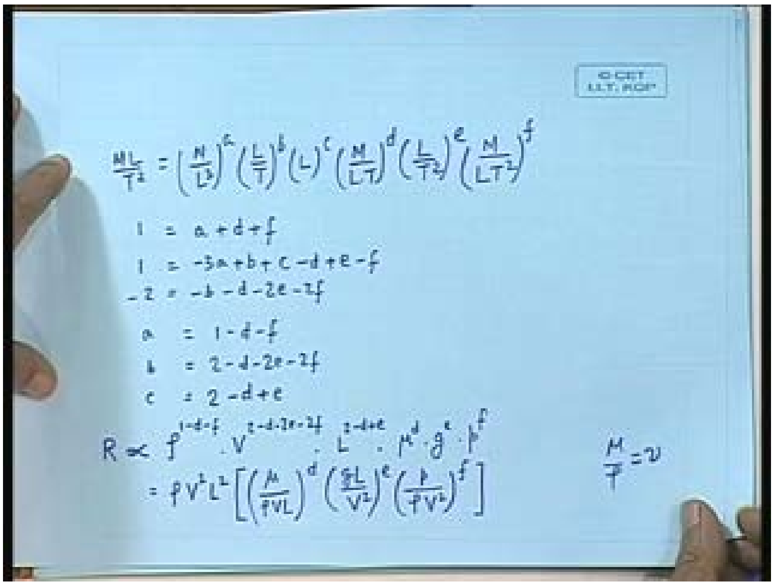

We have got this equation R is proportional to rho to the power a V to the power b l to the power c mu to the power d g to the power e and p to the power f, now, if you put it, put them in the dimensions that we have defined now, M L by T square is equal to M by L cube to the power a L by T to the power b L to the power c mu power d and L by T square to the power e and mu by LT square to the power f. So, now, if we equate the coefficients of M L and T separately between the left hand side and right hand side, we will arrive at the equation, coefficient of mu, m (( )) m, left hand side is one, right hand side will become a plus d plus f. If you take the coefficients of L is 1 equal to minus 3a plus b plus c minus d plus e minus f- is it right? And if I take the coefficient of c, you see minus 2 is equal to minus b minus d minus 2e minus 2f.

We can reduce these six variables to three by redefining a, b and c. You can see here from equation one, first line, you can see a is equal to 1 minus d minus f, from the last line you can say b is equal to 2 minus d minus 2e minus 2f, and c is equal to, you can now do what, for c from here, from the middle line, you can put c and put the values of a and b as we have found, and c will come out to be 2 minus d plus e- simple arithmetic, you can just put the values and see. So, you get a, b and c in terms of d, e and f. So, then, R can be written as proportional to rho to the power 1 minus d minus f V to the power 2 minus d minus 2 e minus 2 f L to the power 2 minus d plus e mu to the power d g to the power e and p to the power f. This is equal to, if I take out, you can see I can take the rho V square L square, if I take out the constant quantities and put all the quantities which are raised to d separately, those which are raised to e separately and those which are raised to f separately, I will get mu divided by rho VL to the power d g L by V square to the power e and p divided by rho V square to the power f- this equation is what we find by dimensional analysis, am I clear? Yes?

Very interesting equation, I will spend some time on this. What does it indicate to us? Resistance is a function of three quantities- the first quantity is mu divided by rho VL, second quantity is g L by V square, third quantity p by rho V square. It is a function of 3 quantities and the function nature would perhaps depend on the shape of the body. Therefore, if I have two bodies which are geometrically similar that is, the shape is same, only the linear dimension of one body is exact proportion to the linear dimension of another body, then you can see that each quantities are dimensionless and constant, and the functional relationship will remain same therefore, this quantity multiply with this, sorry, these quantities will, this functional, function d, e and f will be same for two geometrically similar bodies. Is that clear? Perhaps that is not very clearly, we will explain it further.

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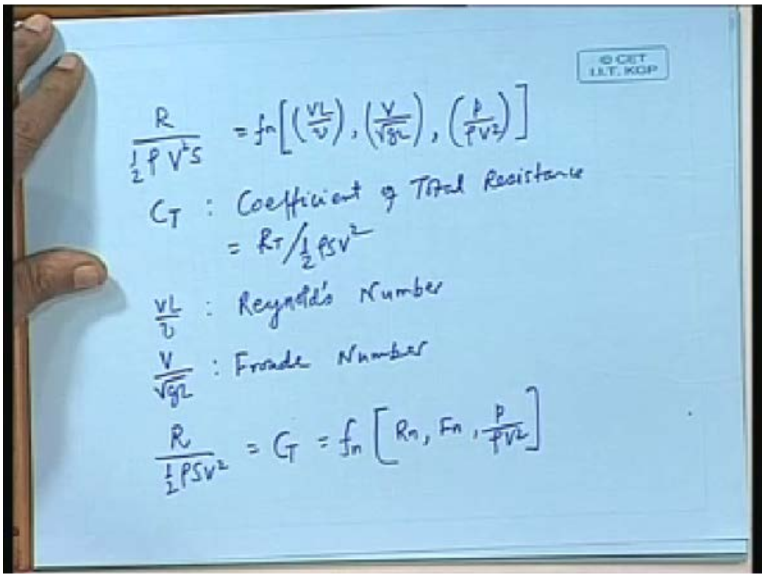

Let me write this equation again. L square, let us look at this other equation, L square is an area function by our definition, we have said area is length into length, so this we can write as the weighted surface of the body, or the surface of the ship that is weighted by the water. We write in general, this is a function of- what is mu by rho? Mu divided by rho is nu- is it not?- kinematic coefficient viscosity- I can also write it in the reverse manner, what is shown here is mu divided by VL, I can write the same thing as VL over nu- since my function is of general nature I can write this, yes? Next quantity g L by V square similarly, I can write this as V square by g L, or I can write it in the form of square root of this quantity that is, V over root g L is the same thing- you agree with me? - I have written here g L by V square since, my function nature is general I could also write this as V square divided by g L or square root of this that is, V over root g L. Clear?

And the last one. Does anything look familiar here? What is R divided by half rho V square into S? This is general drag coefficient of a body in water. A drag coefficient is defined as total drag divided by half rho L V square, this is generally represented as CT that is, coefficient of total resistance, is equal to RT divided by half rho SV square. VL over nu is same as Reynold's number. And V over root gL is the Froude number. So, what we get here, this R divided by half rho SV square that is, CT, is a function of Reynold's number, Froude number and pressure coefficient, p by rho V square.

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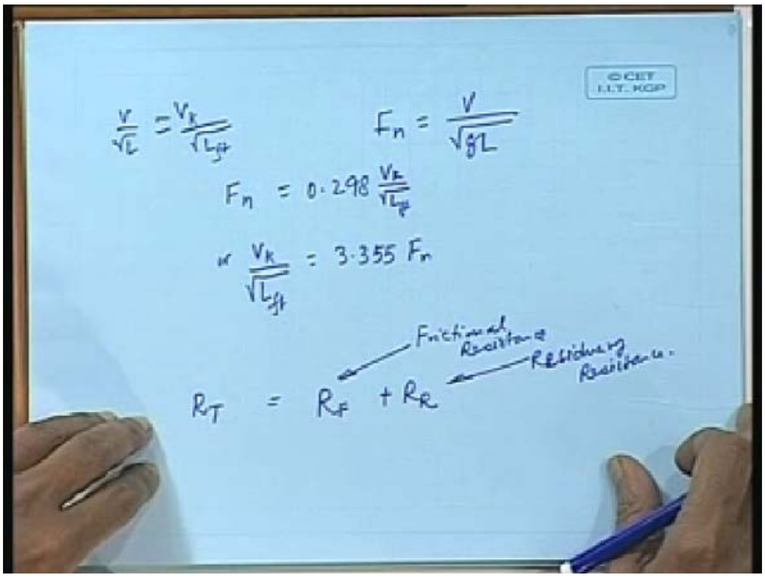

Froude number is very interesting. Normally, in ship terms we use a term called V over root L -have you heard, V over root L?- when somebody represents a quantity called V over root L normally, V is expressed is knots, V over root L is generally given as V in knots divided by L in feet- this is a common use of ship terms, we will see why this term has been generated, which is so unscientific, it is not dimensionless, and V is in knot and L is in feet, but this is very commonly used, this V over root L. And we can see that between Froude number, V over root gL and V over root L, there is only difference of a constant- can you see that?

Normally, we have decided at the beginning of this course that we will be looking at all the problems in SI units. Then, in SI units V should have been, in V over root gL V should have been meters per second, g should have been meter per second square and L should have been meters, and so this becomes dimensionless, Froude number becomes dimensionless. Now, between this quantity V over root L where V is written in knots and L is written in feet, we will find between this and this there is a difference of a constant only. So, that constant, I will give you that is, Fn equal to 0.298 VK over root L, L in feet, or VK over root L in feet is equal to 3.355 Fn. So, Froude number is normally represented V over root gL, and it is (( )) V over root gL when V is in meter per second, g is in meters per second square and L is in feet, L is in meters, but if somebody refers to a quantity V over root L, we doubt further elaboration, it is to be taken that V is in knots and L is in feet- is that clear? So, if no dimension is mentioned, somebody refers to V over root L, it is not a dimensionless number- it is V in knots and L in feet.

If you recall, when we looked at the components of resistance, we have said that they are basically two major components- wave making resistance and viscous resistance, or pressure resistance and viscous resistance, this is what we had said; of the viscous resistance, we had said that the major component is the fictional resistance component. William Froude way back in 1868, 1868 means about 130-4, 5 years back, laid down a postulate of the law which we still use- surprising that a law without a proper scientific background was demonstrated long back, it was so well laid out that we still use it today. He said the resistance of the ship can be divided into two parts, frictional resistance and all other resistance, which he called the residuary resistance that is, RF plus RR where RF is frictional resistance and RR is residuary resistance. So, in RF, looking at it now, not then, but now, RF will consist of 2 dimensional frictional resistance and 3 dimensional resistance, and RR will consist of frictional resistance and all other component of viscous resistance- is that clear?

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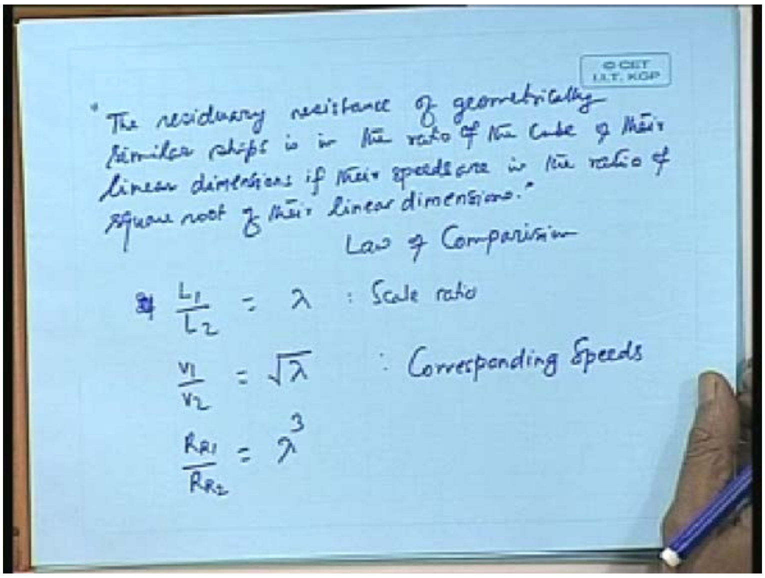

Now, Froude then, laid down a hypothesis which is known as law of comparison, this law stated that, I am writing down- the residuary resistance of geometrically similar ships is in the ratio of the cube of their linear dimensions if their speeds are in the ratio of square root of their linear dimensions. This is the famous law of comparison enunciated by Sir William- no, I do not know whether he was knighted or not- by William Froude in the year 1868. What does it say? It says that if the speeds of two geometrically similar ships are in the ratio of their linear dimensions, then the residual resistance component of their resistance will be in the ratio of cube linear dimensions. So, if the ratio of linear dimension, if I have two ships designated by suffix 1 and suffix 2, then if they are geometrically similar, then L 1 by L 2 can be written, can have a ratio of say lambda, which can, which we can call scale ratio. If that is so and the two ships have moved at speeds V 1 and V 2 in such a manner that V 1 by V 2 is equal to square root of lambda- this is what it says, is it not?- then RR1 by RR2 will be in the ratio of lambda cube. Is that understood?

This is the law of comparison, not at all complicated it is very simple, it says if two ships are geometrically similar and their ratios are in, the length ratios are a constant, any length measurements you take is a constant, then you can understand the areas will be in the ratio of lambda square and volume will be in the ratio of lambda cube therefore, displacement will be in the ratio of lambda cube. If that is so and they are moving at the so called corresponding speeds- these speed are normally called corresponding speeds that means, the speeds are proportional to square root of lambda- then the residuary resistance is proportional to lambda cube. The big advantage coming out of this is very simple, instead of making two big ships as geometrically similar you can make one small ship and one big ship geometrically similar, and the small one we can test in a model tank and find out the residuary relation by some means, we will see how we can do that, if we can do that, we have a good idea of residuary residence. So, what is, what does this mean? (Audio not clear from 32:23 to 32:31) V over root L is constant

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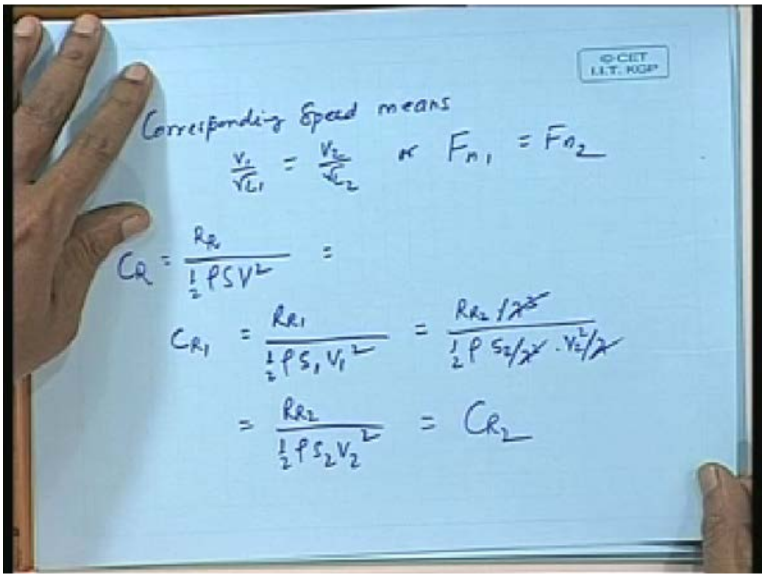

So, this corresponding speed gives us, corresponding speed means, V over root L of first ship is equal to V over root L of second ship, or Froude numbers are same- is that correct? So, what William Froude said as corresponding speed is nothing but at the same Froude number; if two ships are geometrically similar, their residuary resistances proportional of lambda cube if they are moving at same Froude number- you understand?

Now, residual resistance, CR, what is CR? (No Audio from 33:39 to 33:43). Now, look at the CR1 and CR2, let us compare, CR1 will be RR1 divided by half rho S1V1 square, that is equal to RR2 into, RR2 divided by lambda cube, RR1 by RR2 you have written as lambda cube- is it not?- so, RR2 by lambda cube divided by half rho is same- S1, what will happen to S1?- S2 divided by lambda square- and V1 square... What is, what does it give us? This (( )) lambda cubes will cancelled- am I right?- and you get half rho S2 V2 square, which is equal to CR2. So, what does it mean? If two ships are geometrically similar and they are moving at the same Froude number, then the residuary resistance coefficients are constant- is that correct, is it understood, no doubts?

Now, we have got the residuary resistance constant, let us look at the equation we had formulated using dimensional analysis. Let us look at this equation (Refer Slide Time: 18:27). If for a moment I consider the fluid as non-viscous, then the dependence on Reynold's number will vanish. So, we will have only dependence on friction, Froude number and pressure coefficient, and this CT will become CR because CF will not be there anymore if we consider a non- viscous fluid- you understand that, clear?

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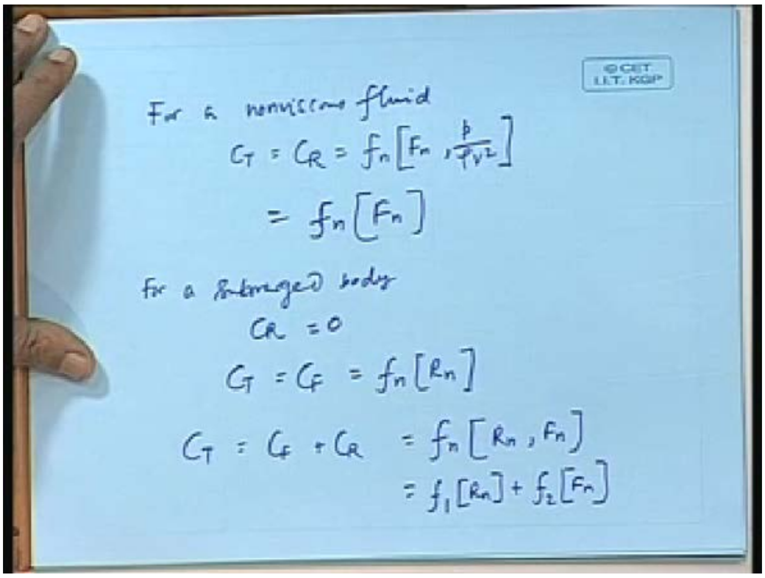

So, for a non viscous fluid dependence of the CT is equal to CR because CF is 0 and this will be a function of Froude number and p by rho V square. Now, let us look at this p by rho V square a little bit. What is this p? P is the (( )). We have seen last class that p will be equal to atmospheric pressure plus the static water head plus dynamic water head. Between a ship and model this similarity of this total pressure is impossible to be achieved, because again, whether it is towing tank or a ship the atmospheric pressure is constant- is it not?- therefore, you cannot reduce the atmospheric pressure in a towing tank by scaling it down. But fortunately for us atmospheric pressure does not affect resistance very much in normal circumstances; we will see the abnormal circumstances a little later. So, what happens therefore, that you neglect from this p, subtract from this p the atmospheric pressure, so you have only the static water head and any dynamic pressure that between two geometrically similar bodies that is proportional to the length dimension itself; we can say the static water head represents this if you have h1 for the big ship and h2 for the small ship, it will be in the ratio of the length.

So, therefore, p is proportional to length- mind you, remember that ignoring atmospheric pressure- and rho is same and p is proportional to square root of lambda therefore, p by rho V square is same as Froude number, V over root L. If I represent p by L, then it becomes L by V square, rho being a constant. So, this becomes, this dependence on p by V square vanishes if we are ignoring the atmospheric pressure. So, we can write this as a function of Froude number only- is that clear? In which case will atmospheric pressure become important? Therefore, we have to find out, is there a case when atmospheric pressure becomes important.

Atmospheric pressure will become important when the pressure falls to very low values of the order of atmospheric pressure or less than that, that is a case that occurs in cavitation. If there is any cavitating flow, then this assumption we have made cannot be right, but in normal merchant ships we do not consider cavitation for resistance purpose. Therefore, it can be written as a function of Froude number only- mind you we have taken non viscous fluid. If I have non-viscous fluid, the ship moving in non viscous fluid and the model moving in non viscous fluid, then the total resistance I measure for the model I just multiply by lambda cube, I get the total resistance of the ship- do you understand this? But unfortunately water is not non-viscous, so what do I do? If, going back to our formula analogy, if we take the ship down completely submerged in water, then wave making resistance will vanish, there will be no wave making as we have seen before therefore, dependence on gravity will vanish, or wave making resistance will vanish, so CR will become 0, for a submerged body CR equal to 0 therefore, CT will become equal to CF- by separation of resistance into two parts frictional resistance and residuary resistance, the residuary resistance primarily consisting of wave making will vanish and therefore, CT will become equal to CF- and that will become a function of (( )). Is that clear, yes?

So, in general now the ship is neither submerged nor moving in a non viscous fluid. So, generally, we can write CT is equal to CF plus CR and a function of a combined function of Rn and Fn, or by this analysis if we consider this is equal to this, we get CF and this we get CR then, we can also write F1 of Reynold's number plus F2 of Froude number- we can separate the effect of Reynold's number and Froude number. Am I clear? T his is the fundamental relationship of surface ship resistance we obtained from dimensional analysis- and this is justified. This justifies that Froude's law of comparison between two geometrically similar ships. So, for geometrically similar ships if we can find out CF by some means and move the model at corresponding speeds, then I can get the CR of the model and I get total resistance coefficient for the ship- very simple.

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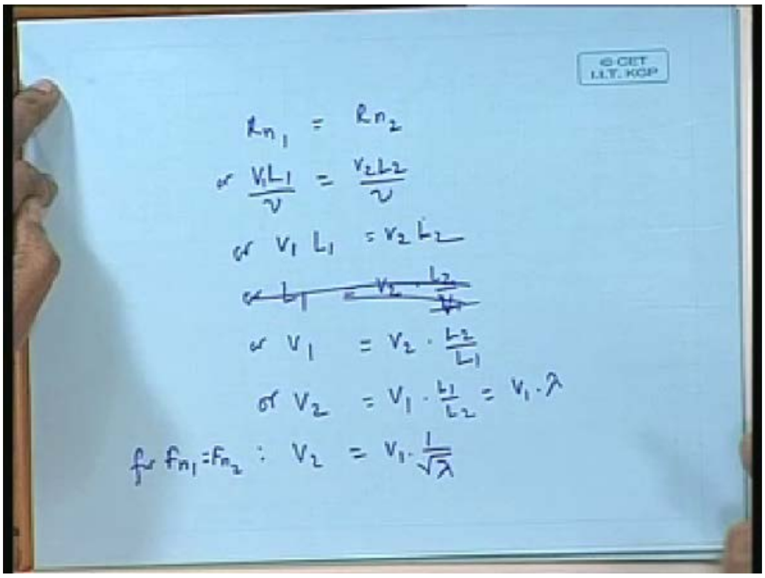

How do I get CF, how do I get CF of a similar ship? Just like I made a law of comparison for residual resistance I can also form a law of comparison for frictional resistance. That would mean that Rn1 should be equal to Rn2 if Froude number of model and ship were same and residual resistance coefficient are same. If I can make the Reynold's number of the ship and that of the model same, then I will get the frictional resistance coefficient same- you understand? Now, let us see, this is what, V1L1 by nu, coefficient of viscosity is same, or nearly same, so we can ignore that, or V1L1 equal to V2L2, or V 1 is equal to V2L2 by L1. (Audio not clear from 44:15 to 44:30) Sorry.

V1 equal to V2 into L2 by L1, or using our precious (( )) V2 is equal to V1L1 by L2, L1 by L2 we have said lambda- am I right? What does this mean? This of course means that in one assumption we have made we have cancelled (( )) left side and right hand side; that means, they are moving in the same viscous fluid. If I want to move my ship, my model now in water having the same viscosity, to have Reynold's similarity I must move the model at a scale, at a speed of the ship multiplied with the factor lambda. But for Froude number, what did I have? I had for Fn1 equal to Fn2, what did I have? V2 is equal to V1 into 1 over lambda, square root lambda. This is very interesting, you see now?

Suppose, I have got a ship which is 100 and 25 meters long and a model 1 is to 25 scale, 5 meters long, I want to move this model at 5 meters at a particular speed and predict the resistance. Suppose, the model moves at 2 meters per second then what is V2? V2 will be V1 2 meters per second, if we say, we have not defined which is bigger and which is smaller in this whole this thing, let us say V2 is the model speed that is, 2 meters per second, that will give you 1, lambda is 5, V1 will be V2 into 5, lambda is 25, 125- let us write question properly.

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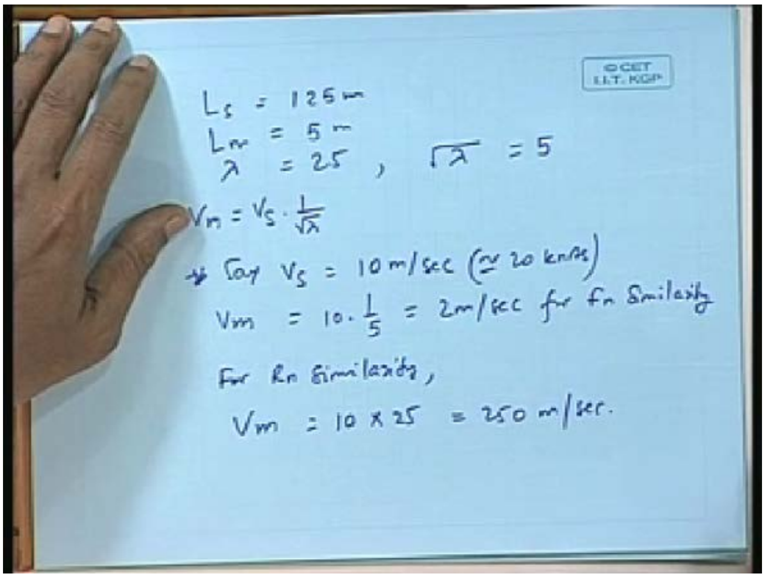

Length of ship is 125 meters, length of model 5 meter, lambda 25 and square root lambda 5. Now, Vn is equal to VS into 1 over square root of lambda. Say, VS, say, VS is equal to 10 meters per second, 20 knots let us say, approximately 0.5144 (( )); then, what is Vm? 10 into 1 by 5 that is, 2 meters per second for Fn similarity.

Now, for Reynold's similarity, Rn similarity, what will be Vm? 10 into 25, 250 meters per second in water, mind you. So, you see, you cannot attain Reynold's similarity physically in a towing tank- that is out of the question. So, what are the alternatives available to us? Alternative available to us is we can have a fluid of different viscosity and then maybe speed will come down a little bit, or we can adjust the speed- typically, different viscosity means you come to air, a wind tunnel test perhaps. But then, the test in air is quite different from test in water because air is compressible and water is incompressible, other characteristics change therefore, strictly you cannot test the ship in air and extrapolate Reynold's similarity to get the (( )), because air apart from being a fluid, it has different physical characteristics than water, kinematic similarity may not be attained, because there are other problems that air surface one, water air interface you cannot generate, so there will be other problems.

So, normally, what is done by trying to predict the resistance of a ship from a model is that if we can have the frictional resistance coefficient for a large number of 2 dimensional bodies or 3 dimensional bodies not going up to the ship, but at a smaller length and smaller size and we could perhaps extrapolate it to ship length and say that this is how the frictional resistance will be, then perhaps we can do this. For this purpose, this is the basis on which the present model tests are conducted.

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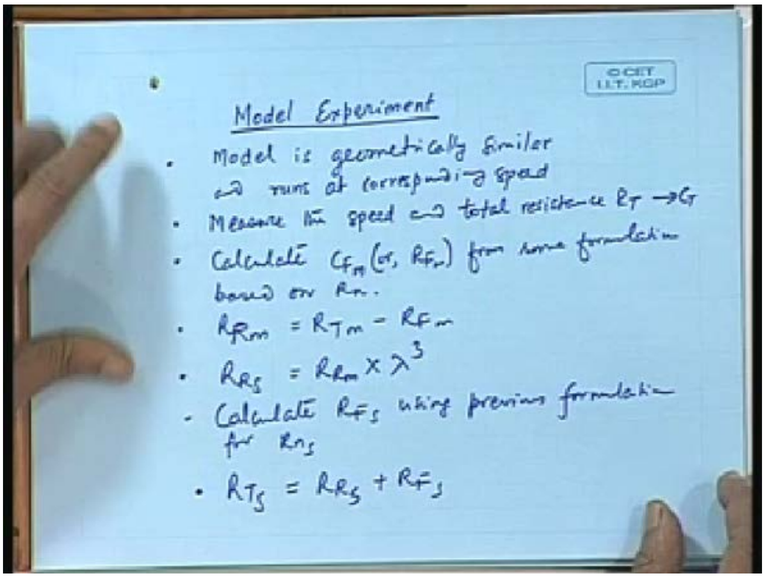

Now, model experiments as enunciated by William Froude, and as we have discussed now, can be written down in the following steps: model is geometrically similar is the first step and runs at corresponding speed; second, while running a model we have to measure the total resistance, that is the easiest thing to measure, measure the speed and total resistance RT- from there get CT; calculate CF model, or RF model from some formulation based on Reynold's number, Reynold's number of model- formulation is same for both ship and model; then, we will get RRm is equal to RTm minus RFm; then, you get RRS, which is RRm into lambda cube; calculate RFS using previous formulation for ship Reynold's number and finally, you get RTS is equal to RRS plus RFS- we have got both of this RRS and RFS. So, Froude's similarity law actually gave us a procedure for model experiments and extrapolation of the resistance data from model scale to full scale. Thank you

Preview of next lecture

Lecture No. # 04

Frictional Resistance

William Froude gave a formulation for frictional resistance based on planks experiments, he did a number of experiments on planks that is of a particular length and height submerged in water and towed them and measured the resistance.

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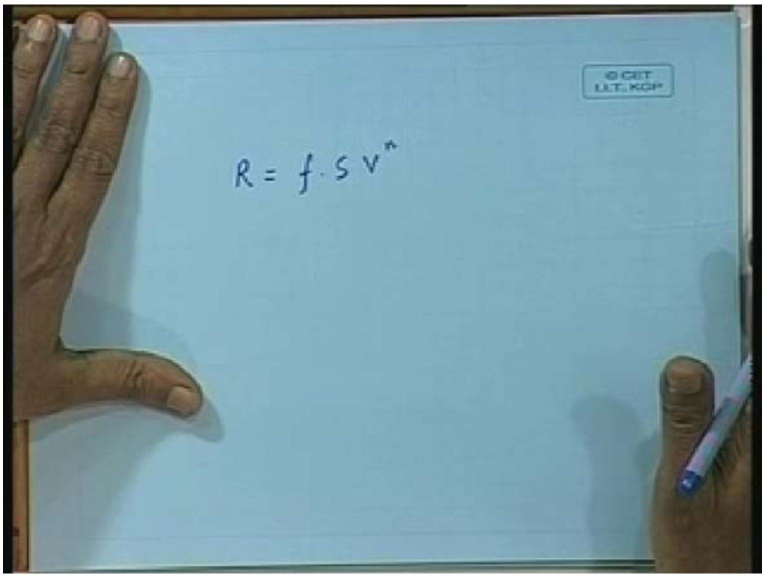

Based on the planks experiments he gave a formulation: R is equal to- mind you, this is purely frictional formulation- f S V to the power n, this was the original formula; in the original formula resistance was given in pounds, pound force, S was in feet square and speed was in feet per second. But now you can convert the formula to metric units, f and n being two constants where f n was dependent on length of the body- length of the plank- and f was dependent on the surface finish of the plank; that is William Froude was the also the first person to detect that frictional resistance would depend on the roughness of the surface.

So, since, Reynolds's number was not explicitly used in this the n factor varied based on the lengths, for which we conducted experiments for various lengths, and lengths were not very large, the lengths were up to a limit when you can tow it in a towing tank, mainly the towing tank at Torquay where he did most of his experiments in England.

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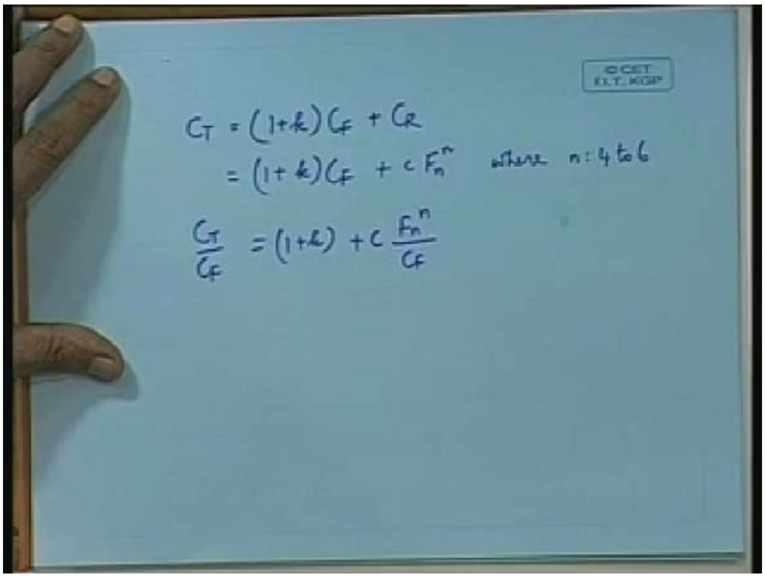

If I write down CT is equal to 1 plus k- now, I can drop CF0 and write CF always, meaning CF means 2 dimensional or I T T C line and when I write 1 plus k it becomes three dimensional- CR, we have seen is a function of Froude number, it can be roughly said to be a function of a power of Froude number between 4 to 6. I have this, I divide the whole thing by CF, then I get CT by CF is equal to 1 plus k plus CFn to the power n divided by CF. So, then, if I know this n value and this C value, then I can calculate 1 plus k. Now, this is now possible using some mathematical technique, you can fit a curve to the equation, to a resistance curve find out the value of n and C to the residuary resistance and calculate the value of k.

This is other method which is also recommended by I T T C. But as you know since the errors are not constant over the entire speed range, measurement range and neither are there positive errors- always the error can be positive or negative- this type of fitment, regression equation fitment to find out what is the value of n and c can also give erroneous results.

So, you are on the safe side. If you take form factor, then you will become more accurate and your power prediction will reduce, so, that is beneficial from the point of view actual economy- you get it?- but it is fraught with the danger, if it is not estimated properly, then you may underestimate it. So, the k has to be estimated very well, if you are unable to estimate k properly, do not use it- that is the advice that can be given. So, if you look at the towing tanks world over, you will find that sometimes you use a form factor, sometimes you do not use a form factor- I I T C has left it open by saying that the phenomenon of 3 dimensional frictional resistance effect are not very well known. Is that clear? So, now, you do your model experiments, the procedure is known to you and you can extrapolate from model scale to full scale. We will stop here. Thank you.