

Performance of Marine Vehicles at Sea

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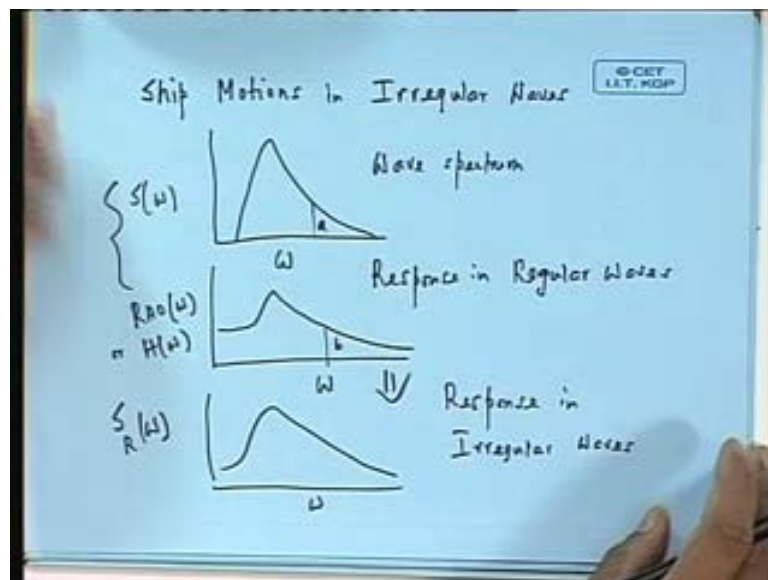
Indian Institute of Technology, Kharagpur

Lecture No. # 29

Ship Motion in Irregular Waves – II

We are going to continue our discussion on Ship Motions in Irregular Waves.

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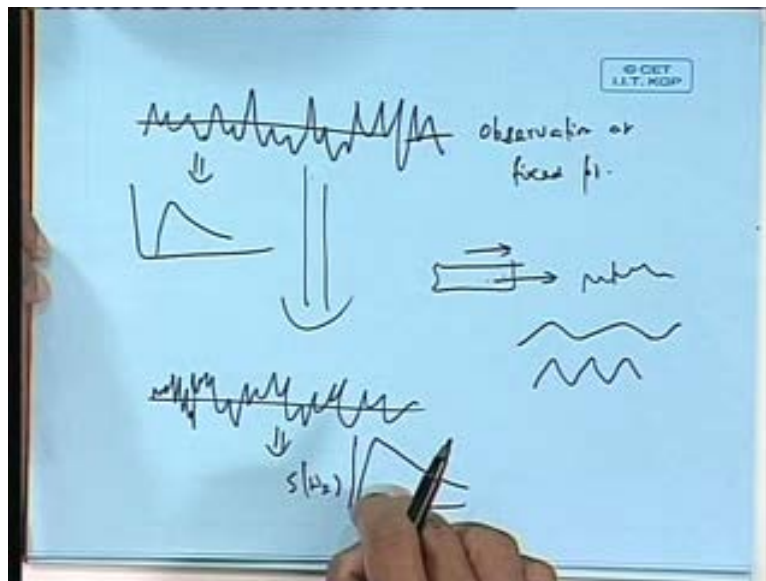


If you recall in the last class what we said, we have actually told about a usual procedure of how we can get motions in irregular waves by saying that we have first a description of the wave, a wave spectrum.

Then, we have got a description of you may call RAO here or transfer function or H some people call it whatever symbol you give; something like this may be, and this and this together combining the two, if you **if you** call it a, this is called b, you have got this S_R omega, this is response in regular waves, so this plus this two together joint **give** gives rise to this response in **(No audio from 2.17 to 2.26)**.

This is what we said, but there was one thing that was missing here is that, this particular description or this method is strictly valid for a ship which is not moving, why? Because, this description that $S \omega$ $y \omega$ you see, it is a description of the ocean surface, as we observed moving from a **from a** stationary frame of reference, we are standing at some point and then what you see, you break it down you get this.

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In other words, supposing you are an observer standing then you would have got a response like that, this is from an observation from a fixed point. So, this has been broken down to a spectrum.

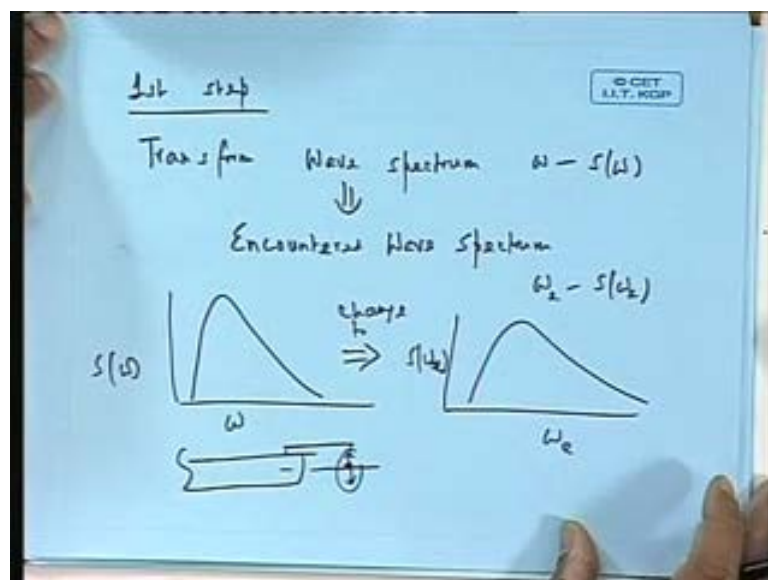
Now, the same observation supposing same **same** point you want to observe the ocean surface with respect to moving from of a **(())**, you will see that, this thing will look something else, it may look something like occurring much faster. In other words it is probably getting squeezed, because things are now occurring faster, if you are in a head sea. Suppose **you are moving see suppose** you are moving in to the waves, **and the** at this point the waves are occurring whatever as you move in, you will look, it will appear different to moving from of reference. Because, we mentioned that a single wave would appear, a single wave which was like that to moving from a reference it will appear to be faster, is not it?

Because, you see, **if you are** this crest is passing every 10 second, if you are sitting on the ship you will feel it to be passing at every 8 seconds, if you are heading into the waves. So, the in other words, there is a feeling that it is getting squeezed. So, this one is the spectrum

that if now, if you make a four analysis that you suppose to get a spectrum which is ω_e versus $S \omega_e$, what is called an encounter frequency spectrum? In other words, what is the description of the irregular waves when **(())** moving or ship frame of reference? First of all, the force that comes on the ship is because of or at the frequency of encounter.

So, therefore, firstly I need to describe the environment as observed from the ship. So, my first picture that I have here, this was the environmental description of the wave systems as observed from a fixed frame of reference, which is what it would be if the ship was not moving or let us say an off shore structure in knot when v was equal to 0, but when you have a v , obviously this spectrum will get changed.

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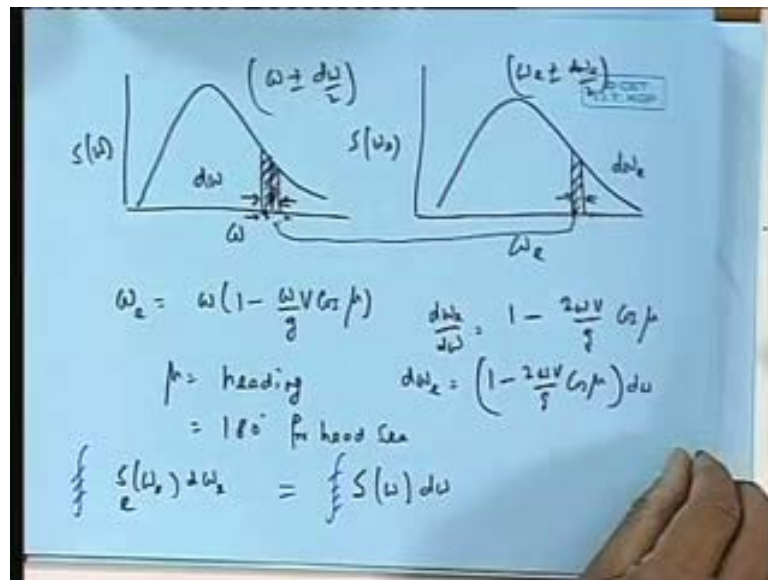


Therefore, first step for us, the first step is to **for is to** transform the wave spectrum to encounter, what we call encounter wave spectrum? Wave spectrum is actually, ω versus $S \omega$ we have to change it to ω_e versus $S \omega_e$.

In other words, I have ω versus $S \omega$, I must change **(No audio from 5.58 to 6.06)**, remember that this is only a transformation, the wave has not changed; it is simply seeing the same wave from a different frame of reference. It is not that the wave has changed, **you have a** see if you have a buoy for example, it is fixed point is an actually observing taking wave records, same wave record, if you have something like this, if you have a say meter here and you are measuring this wave height here, **you know** the meter is fixed. The record that you will get up and down is not same, if the wave was at a fixed point it has got changed.

So, now obviously, the one that is recording at a moving form of reference is the wave that is being felt by the ship. So, I must change that, so I must make a transformation from this to this and the first we should speak about how we can make a transformation? It is basically algebra, we will see that it is absolutely state forward algebra.

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Now you see we again draw this picture, now we have this, we are going to see the head sea case mostly this is omega versus S omega. Now what? Now, our formula is that omega e equal to omega into 1 minus omega by g v cos mu is heading 180 degree for head sea I think that is what it is. So, that omega is become more, you know it becomes exactly 180 degree to the power omega is omega 1 plus omega y etcetera, etcetera, this is my formula. Now, you see is one interesting part is there, so I need to transform that to this omega e versus S omega e.

First of all, each omega gets transformed to a corresponding omega e that we know, an any given omega has an omega e, because you can use this formula. Now, let us look at this small part, where this is actually let us say S omega e, this is my S omega e ordinate, what is this? Now, let me say that this is my d omega. What is this area, this shaded area, what we discussed here, this shaded area is actually energy of all the waves between this frequency bands. Now, this from here I can find out that d omega e.

If I use this formula, see differentiate that d omega e by d omega if I do that, it will simply if we (()) differentiate that d omega e by d omega n you know, it will turn out to be 1 minus 2

ωv by $g \cos \mu$. You can just see, that is ω minus $\omega^2 v$ by $g \cos \mu$ **d**
omega by $d \omega e$ by $d \omega$ will become like that. In fact, some time people do it other
way down, you say $d \omega$ by $d \omega e$ that will become like this, it is a simple
transformation by just taking **you know** y equal to x^2 , so dy by dx get to be x like that.

So, why I do that? Because that $d \omega e$ equal to $1 - 2 \omega v$ by $g \cos \mu$ into $d \omega$
 ω . In other words, the corresponding band width here, from here I can make out here,
that means all the waves between **omega plus minus d see** ω plus minus $d \omega$ by 2
this frequency is equal to ωe plus minus $d \omega e$ by 2 .

All the waves between this frequency bands is equal to this frequency band. Now, happening
is that, this area is suppose to represent energy of all the waves in this waves, no energy of all
the waves between ωe , this $d \omega$ band is actually given by $S \omega e$, **I can** let me
write $S \omega e$ into $d \omega e$, and energy of all the waves of this, **of the** same waves is
equal to $S \omega d \omega$. See, you think of this, there are waves, they are which are in this
frequency band, these are the same waves which are in this frequency band.

Because, this all that is happening is that ω has become ωe and $d \omega$, let me
say **say** 10 to 12 second waves; now, 10 second has become 8 second, 12 second has become
9.5 second, so 10 to 12 second waves are 8 to 9.5 second waves; $d \omega$ here is 10 second, $d \omega$
is 1, I mean **you know** 2 second this is 1.5 second etcetera. So, basically it is a same
thing, but all the waves between 10 to 12 seconds have an energy that is $S \omega e$ into $d \omega$
 $S \omega$ into $d \omega$.

(())

Yeah, that same energy is obviously, for all the waves as **(())** from the ship for 8 to 9 and half
seconds, because those waves are the same waves. So, therefore, this $S \omega$ is $d \omega$
represents nothing but, $S \omega d \omega$. Actually speaking if you look at the other more
carefully, the area under this whole thing which I can **I will** put it separately.

If I **if I** were to see the area under the full thing, it is nothing but integration of that **and area**
under this full thing is integration of that. Obviously, what is happening is that, area under the
full thing is the total energy, total energy is constant, therefore these two are same not only
that, area in the given frequency band equivalent also must be same. Therefore, **you know** all

though this is same I do not write that initially, I say that this is equal to this, $S_e \omega$ is $d\omega$ equal to $S \omega d\omega$.

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The whiteboard contains the following handwritten text and equations:

Energy of all waves in $\omega \pm \frac{d\omega}{2} = S(\omega) d\omega$

$\Rightarrow \int_{\omega_2 \pm \frac{d\omega_2}{2}} = \int_{\omega} S(\omega) d\omega$

Energy is Same!

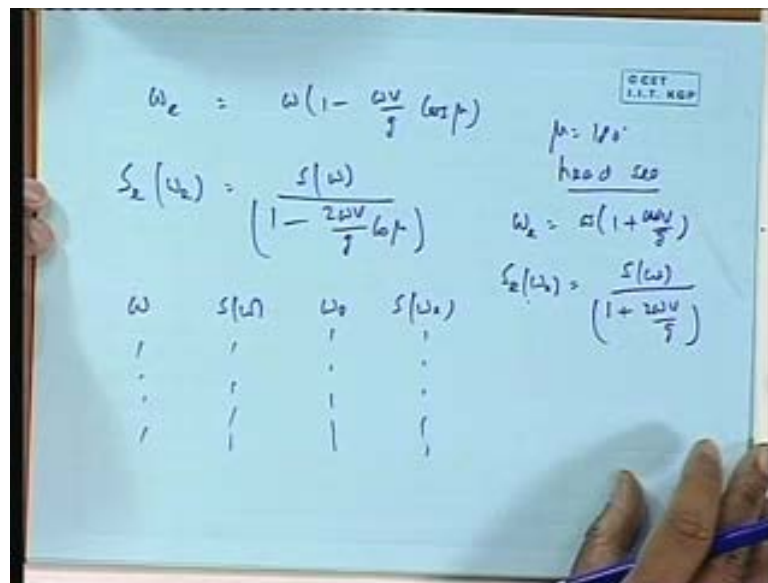
Thus:

$$\int_{\omega_2} S(\omega_2) d\omega_2 = \int_{\omega} S(\omega) d\omega$$

$$\Rightarrow \int_{\omega_2} S(\omega_2) = \frac{S(\omega)}{d\omega_2/d\omega} = \frac{S(\omega)}{\left(1 - \frac{2\omega v}{g}\right)}$$

In other words, I can write it down more carefully here that, this is $S \omega d\omega$ energy of all waves in ω_e plus minus $d\omega_e$ by 2, they must be same (No audio from 11.58 to 12.12). Because, after all ω_e plus minus $d\omega_e$ by 2, represents the same waves as viewed from the ship in the frequency band ω_e plus minus $G \omega_e$ by 2 now energy must be same (No audio from 12.39 to 12:52). This gives me into or other that formula is there, $d\omega_e$ by $d\omega$ is $1 - 2\omega v/g$, so this become simple transformation.

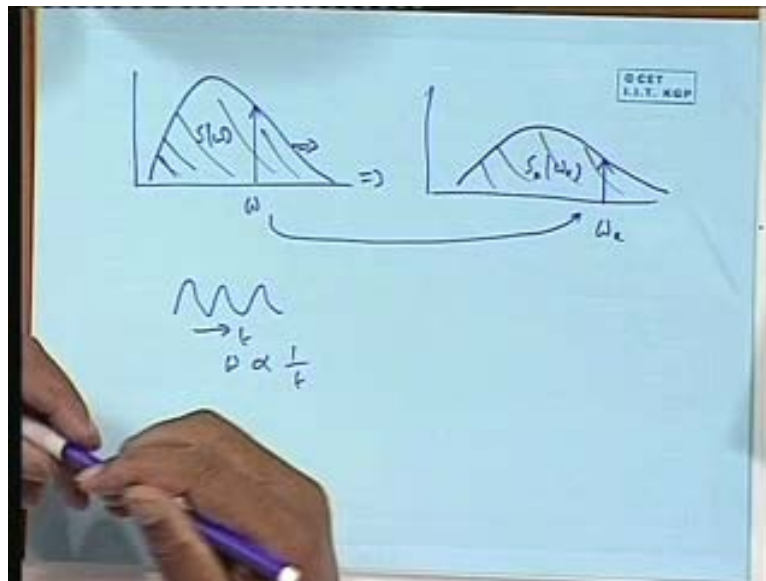
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So, actually now I will just summarize this two and tell you, so what happen omega e is omega into 1 minus omega v by g cos mu and S e very simple. So, you have got an omega here, S omega here tabular form, you write omega e here, S omega e here that is all. Now, let us take a particular case here, see mu this is the very simple transformation, one line algebraic transformation of course, there are little problem that will come afterwards that I will just physically explain.

Now, mu head sea mu equal to 180 degree in head sea. So, what happen? Omega e becomes omega 1 plus omega v by g, omega e becomes more than omega. So, omega e has become omega into 1 plus omega v by g, and S e omega e will become S omega divided by 1 plus 2 omega v by g. So, you can see that omega e now goes up and S e omega e comes down; that means, if I **if I** make a plot here.

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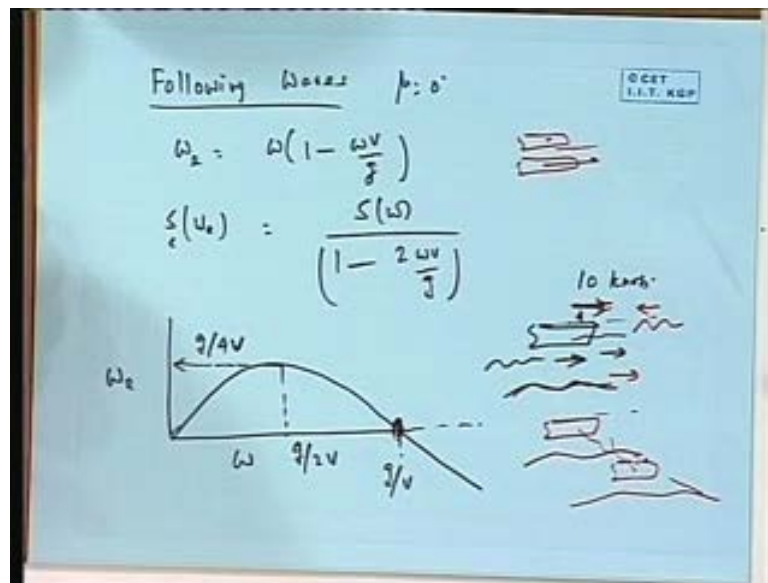
Another one, then I **ii** end up getting something like that you see I have this graph here, this is actually $c \omega$, this is $S \omega$, this will transfer to this point ω_e , this will go to ω_e and this height will become $S \omega_e$, ω_e this is less than this is more than that, that means, as if you are stretching as if you are taking this and pulling out.

(())

No, **no no** see it gets compressed in time, this is frequency actually that **that** is our the problems comes always you have to always think, it gets compressed in time, see it gets compressed in time, but frequency exactly is opposite of **(())** 1 by time. So, it becomes opposite, so we are the in fact, if you wrote t it will be that compression. So, since we are writing in a frequency domain it appears as if we are pulling it, and obviously, that make sense because, the area under that remains constant, so that will be remaining constant.

See area under that, area under that will remain constant because, this is now being pulled out. So, this is increasing, this is decreasing, that is very fine of course, a special problem will come and the opposite will happen, when supposing it is now a following wave let us see following wave, what happens? That becomes a much more complicated scenario.

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In the following wave your omega a becomes omega into 1 minus omega e by say mu equal to 0 degree by g like that, and S omega e S omega by 1 minus 2 omega v by g. In fact, the transformation in following wave is that if you have omega and if you have omega e it goes like that, in fact **you will** I will just draw this value here, this is g by 2 v and this is g by v a. See what I wrote is that here, you will find out the let us look at this transformation, when omega v by g is equal to 1 or omega you know actually when this is this inside part is g by 2 v when omega is g by 2 v what would happen is that, you will find out that omega e will become actually this.

There is a quadratic equation, if you solve it you will find out that when omega is equal to g by 2 v omega e is g by 4 v when it is further, that when omega what happens up to some time as omega increases omega e is also increasing but at lower rate; **beyond that** if you go beyond that, you will find that omega e will begin to decrease. And when omega becomes g by v as you see omega is g by v this becomes 0, then this omega e will become 0 and if it omega S more than that, that is omega is more than g by v, then this part becomes negative omega e becomes negative.

So, the omega is variation with omega becomes like that, it is confusing. Let me say the physical explanation what it means is that, when suppose you are going this side, waves are following, initially there would be some waves which would be appearing to be still going forward because they are travelling faster rho frequency.

When the **when the** long waves are there travelling very fast, **so** some of them will appear that they are going faster, some of this if there is a long wave, it will appear that it is moving faster than you, so if you are standing here, you will think the wave is moving forward. See let us say, let me give an example, say the ship is moving at 10 knots, so there are some waves **which is** which are moving at more than 10 knots so it will appear that they are moving forward; say 20 knots, it will appear that it is moving forward at the speed of 10 knots.

Now, you take another wave which is slightly shorter it will go at slightly lesser speed. So, there will be one particular wave, which will actually travel at the same speed as the wave and that would be this. When supposing you take a particular wave which travels at 10 knots then if you are standing here, you will think that **that** particular wave is not moving at all it has a 0 speed. So, its frequency is in a, so called 0 it will come here omega e omega e will appear as 0 to you. You see, if you are again there will be some waves which are travelling faster than both are traveling in same direction, It is not encounter, this is **this is** also encountered.

(())

See, what is happening now? It is encountering from a different side, what is encounter? **encounter** only means; as you are standing on the ship how you think the waves are coming past you is encounter speed. Now, when **when** you are standing here and waves **waves** are coming from this side they keep crest keep passing much faster to you, but when it is on the other side crest keep keeps passing much slow than you.

Because, you see supposing give an example, you are here, the crest is here. Now, next second you would have gone somewhere here and the crest would have gone somewhere here. This would have moved here you would have moved here.

So, from you; say in 1 second, you would have moved say 10 meter and the and the ship would have moved 20 meter. So, you would appear that it has moved from you 10 meter ahead, this is encounter speed.

Now, problem is that, longer waves travel faster. So, the wave which has very low frequency long **long** waves say 500 meter long wave which is travelling faster 20 knots. You are travelling at 10 knots; 500 meter long wave is travelling at 20 knots. So, you think that, there

is a wave travelling at 10 knots ahead of you, take a 200 meter long wave it might be travelling at 12 knots. So, you will think it is moving at 2 knots ahead of you. Now, take another wave of 100 850 meter it will travel at 10 knots, you are also travelling at 10 knots.

So, you think that wave is not moving, so you are here. Now, you take a wave which is hundred knots a hundred meter it will travel at 8 knots, you are travelling at 10 knots. So, you stand here, so you think as if the wave is going backwards from you at 2 knots.

(()) you encounter (())

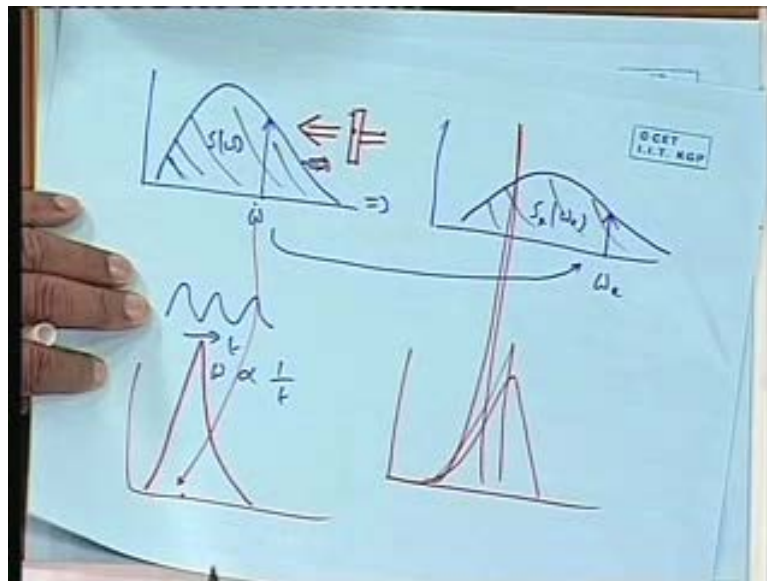
No, you are encountering always that is what is the idea of negative frequency comes in. see

(()) train train (())

It is like train like actually, exactly, so the two trains moving you can say, so you see if the two moving you know if this is moving faster than you, then you think that it is moving ahead of you, is not it. See, when two trains are going you see that local train you sometimes see this is moving faster than you, think it is moving faster by a small rate. It is moving slower than you think that it is moving backward from you.

And if it is same you think it is not moving at all that is the idea here exactly same idea here. And that is why it comes like that, but the problem is not that there is a big problem for that what would happen? Obviously, you know that when the frequencies are at in this region it is basically increasing.

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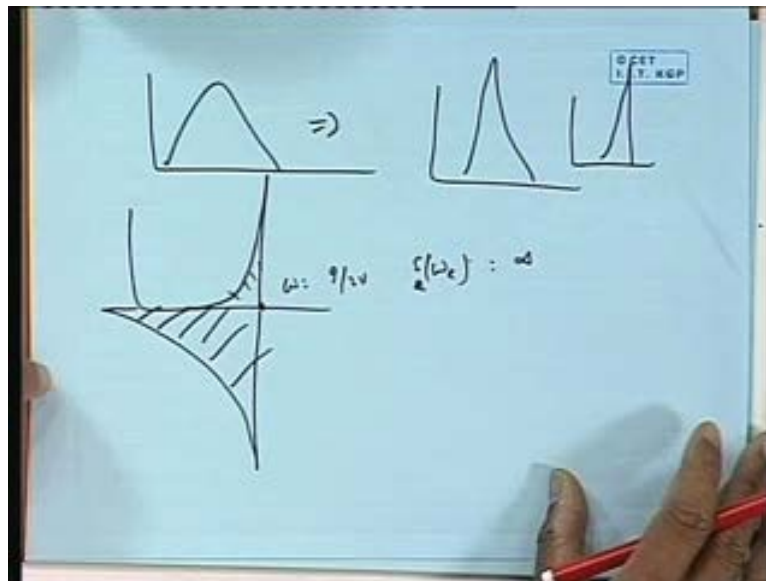
So, what like? In this spectrum, I pushed it out this each omega has become more than omega e, but here in the following see each omega will become each omega will become smaller. So, each omega becomes this omega has come down to omega e smaller.

(())

That means, it your no crest (()) as if you are pushing it the shape of that, as if you are pushing it if you push from here like suppose there is some kind of a you know like, say you are just you are pushing the full craft what would happen? It is going to become sharper right, but it can happen now there is a very big problem that comes in. You look at this, there will be a case when omega has become 0 number 1 forget the case before that before that at the speed of omega equal to v by 2 g that is at this speed, you will find that, when omega is equal to g by 2 v this part is 0.

If this is 0, what happens to this? Something by 0 it goes to infinity, so it is something like you are pushing it so much that, this graph gets squeezed more squeezed, at one time it goes to infinity. Then, you are still having more waves then you will find that, beyond that this is negative. So, then the graph has become negative, so this is, this becomes very complicated to explain me to you, but I just want to tell you the (()) that happens.

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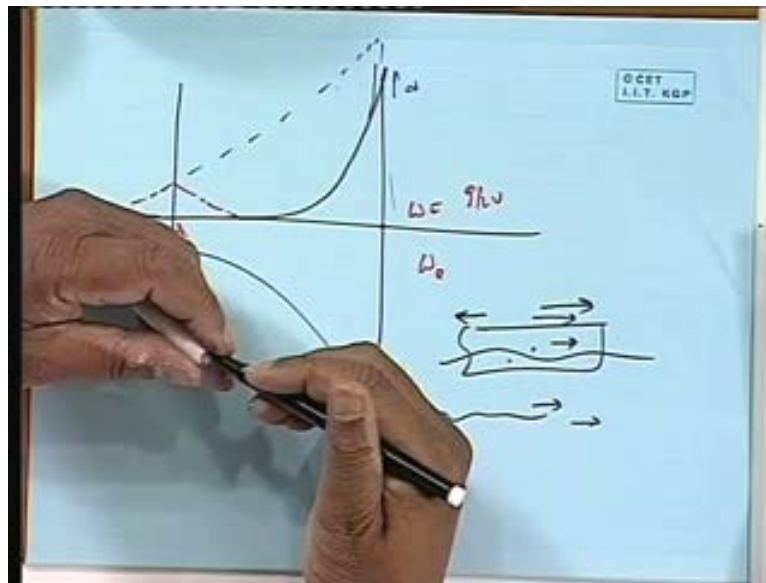


See, this 1 initially has got squeezed like that, further squeezed like this, you are making further squeeze. So, it goes to infinity at point then beyond that point omega negative and then it becomes go like that, because what would happen? This point is going to be omega equal to g by $2v$ where S omega e is infinity. Beyond that it becomes negative because you see this (∞) beyond that this is at 1 point this is 0.

See, first of all, this is becoming smaller and smaller than 1 as becomes smaller and smaller than 1 this is become more than this, because something by point 1, something by 0.05 etcetera. Then, it becomes something by 0, and then it is infinity. Then, it becomes something by minus a number minus now point you know 2 because from 0 it is going now negative then this minus means, this value will become a large negative number. So, it the graph will look something like, infinity it goes to negative side negative then it will begin to go like that.

So, the spectrum will look very awkward, so you see why I mentioned that because spectrum is not always nice looking following sea you have a very odd looking spectrum it will look something like this, like this, somewhat some people will do is that, you can actually I mean I will just draw a another one that better way.

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So, you have got 3 parts of a spectrum you will find 1 part going like that to infinity, then the second part goes from here this is going to infinity actually. From infinity going like this, and third part will become you will find that ω has now become negative also so it will (()) like this. See, there will be a point where ω becomes also negative, you see here this is when ω equal to v y **sorry** g by $2 v$ then it is infinity, but when ω equal to g by v then it is 0 because at that time ω e becomes 0 this is my ω e axis **right**.

So, here ω equal to g by v this is ω equal to g by $2 v$. So, you see what happen? And then beyond that it becomes negative what people do is that, after all what is important is the area under the graph what is the area under the graph? It is the entire thing. So, what people can do that you can fold it upwards; if you fold it upwards say it can look like I can fold it upwards go like that. So, I can have this area plus this area and this is negative also people do not like because after all this is an area, so you can again refold it this red part on this side.

Its, So, you can make it this way because after all the area under this is same as area under this. So, you have Area under this plus Area under this blue 1 plus Area under this 1. And then you can actually add them all up and you can get a composite curve. See the what is important point is something like this see here, that **that** I like to tell you this from this, other this graph this where is that graph here from this 1, see what is happening? think of this, now think of a point here.

You see this ω let us say this ω is equal to some value, this can occur because of such an absolute wave it can also (ω) such an absolute wave. In other words, you can have same encounter frequency of let us say 8 seconds for a wave absolute value of a wave which may be of 12 second as well as some other value. So, you see there are two possibilities; because when you are sitting on the **on the** ship a long wave travels and a short wave both may appear to you to be at equal speed, it equally encountering you know the crest are passing by you at equal intervals, but they are completely different waves.

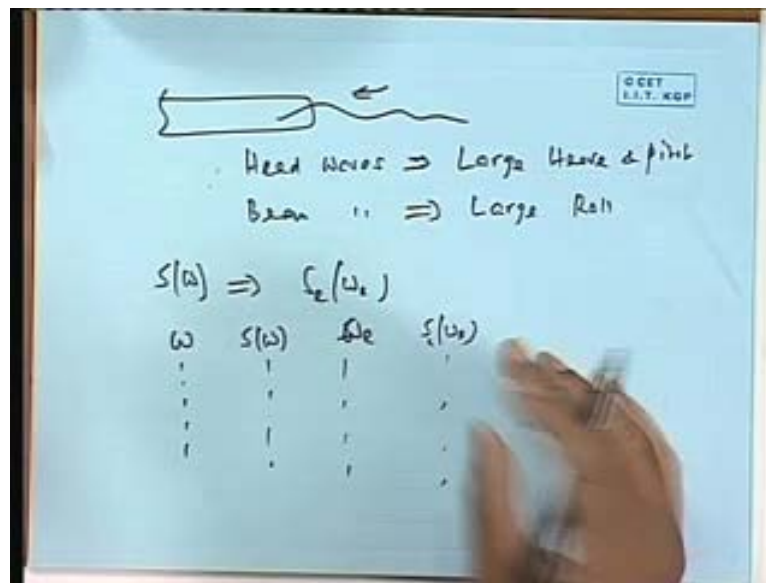
And if you actually fold this some people fold this black part this way, you get a third wave encounter is same, but it is moving in the other direction. See in other words, this I will explain in a **in a** diagram again. You have a ship here you are travelling a number of waves are travelling behind you. See, there is 1 wave which is hitting you at every 10 seconds.

There **there** can be another wave also hitting you at every 10 seconds both moving forward there are 2 waves there can be wave number 1 here, one here wave number 2 both waves will have a same encounter frequency possible if there is a possibility. There is also a third wave having the same encounter negative encounter frequency, which means; that there is also a third wave meeting you at every 10 second, but that wave is moving backward.

So, as long as you are there force is there is a push at every 10 second it can occur for 2 forward moving wave 1 waves behind going on the other side. So, there are 3 kind of waves, this is why encounter frequency do not give you a absolute picture. If you just solve the problem that I am going to say that my wave excitation is every 10 second you do not know which one, and that is why, all the more last class I mentioned following sea 0 encounter frequency is very **very** dangerous because there is a wave here, you are also going the wave is also going same speed.

You think that, you know you are static water, but actually not so. Because, you are moving this wave also moving just that it is moving at the same speed. So, when you look at down you think it is moving at a same speed. But then there is a dynamics involved, so it is like you are trying to walk on a (ω) **on a on a on a** floor the floor is moving and you are also walking on the floor. So, you have no grip you try to fall, just same thing happens in the following waves it is not same as hydro statics. Anyhow, so this is I just want to mention, but we normally do not get into this complicated stuff of this, what I mean? Following wave because, fortunately it turns out that most ship motions are important in head waves.

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Because you know, only when **when** you have got this waves coming this way, you have got large you know Head waves gives **gives** you large heave and pitch you know large up and down motion. And beam waves, beam waves gives you large roll, so as far as sea keeping is concerned as far as for example, strength say I R S or any society wants to find out strength they will try to analyze mostly head wave conditions. Because, that is when the ship has much more large motion you know it can break down in 2 etcetera.

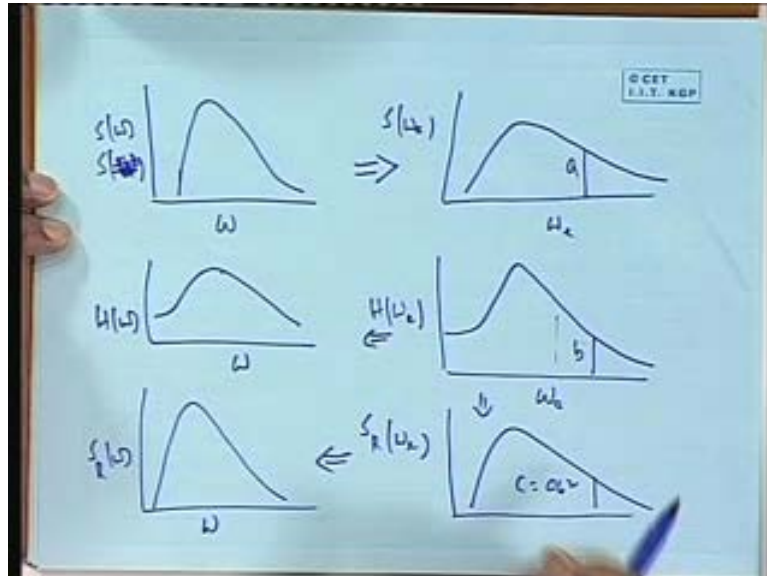
So, normally the following waves we do not discuss so much in sea keeping, but I wanted to tell you that because it is a typical phenomenon; in a very typical phenomena and it takes little time to be clear in your concept. **Why** what is the meaning of negative frequency? Why the spectrum looks so odd if I use the formula, anyhow forgetting now. Now, we have got this idea about first step of $S(\omega)$ to $S(\omega_e)$ this transformation if its very simple, you have you have a table of ω and $S(\omega)$. You simply calculate $S(\omega_e)$ line by line this $1 S(\omega_e)$.

Even if negative you can find out only thing is that there you will find that is going up at, but this will go up and then begin to come down. If you do negative it will see that this **(())** it will start from 0 like you know 0.1, 0.2, 0.3, 0.4, 0.5 then again 0.4, 0.3 etcetera point 0 minus.

So, far as numerics is concerned you have no problem, you can directly use a formula only thing is that, there can be a situation where this becomes infinity. Our number will show 10 to

the ninety nine something like that that is a different thing but operation is very simple. Now I will tell you this transformation part therefore, here.

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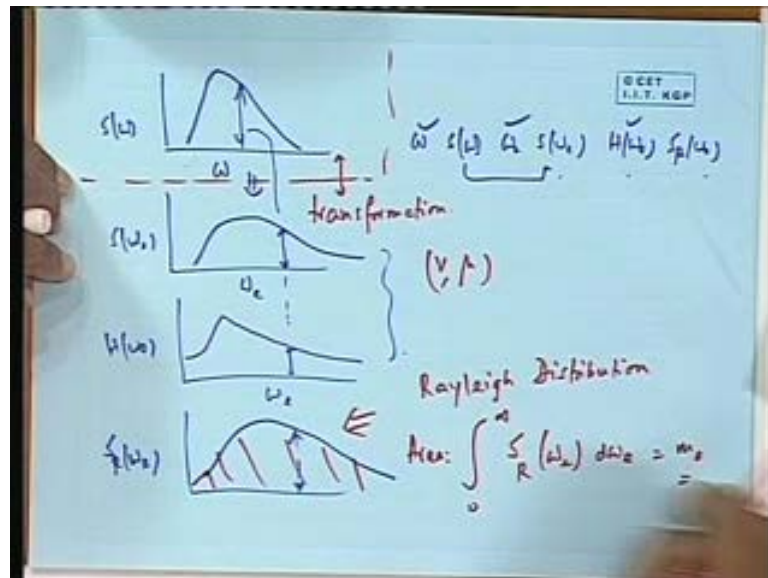
So, now, we have a difference in the calculation procedure; initially I had $\omega S \omega e$ oh sorry $S \omega$. First of all I must transform that, to ωe which will look like that. Now, I have this RAO which is actually the $H \omega$ so called RAO $H \omega$. This also is actually has ωe remember that see RAO also should be at ωe because as I said earlier in the regular wave response the response is always calculated at encounter frequency because, ship is moving and then it is a move moved ship on which the waves are coming and hitting you have to find out the response it will say may look like that, so this is encounter frequency spectrum. Then, you combine that two to get what is known as here the response spectrum this and this together that is if you take the here a, if you take here b, you have got this c equal to a b square.

If you want you can transfer back that to that nobody does it, you can transfer that to this if you want, but that is not normally done. This side procedure is not normally done, but if you want you can in fact, you can also transfer back this to you know like, whatever $S \omega$ versus $H \omega$.

That is simple because, what you do is that for each ω you see, you find this value and just put a plot it here. Because, here there is no transformation after all what you are doing is that you are finding out this value just horizontally shift it. You can do that, but normally it is

not done, so normally what we will do is that, see you will have to go from here to here and just go up to this much that is all. So, you go from here to here and here this way or some people do not like this side. So, I can in fact draw it this way in one line.

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So, you have actually you can say $\omega S \omega$ looking like this then transform that to $\omega_e S \omega_e$ which will look like this, then you have got $\omega_e H \omega_e$ which may be looking like that. Then, you combine this and this together to get $S(\omega_e)$ like this may be like that. So, basically this 1 is transformed back to this 1, which multiply with this square gives you this 1, so this you have to transform up to this square of this. In other words, now you have got $\omega S \omega \omega_e S \omega_e$ then $H \omega_e$ then $S R \omega_e$ from this you get this, from this you get this, is of course, separately available then square of this into this gives you this. You see from this to this in other words this part this first step is transformation this is transformation.

Now, you may **you may** ask why am I bothered about this transformation? I will **I will** tell about that this is extremely important, very, very, important we will find out this is exactly because of transformation, you will see that a particular wave condition which is very bad to a ship, you can change in speed you can actually reduce **(C)** because by changing speed you are actually change the way your encountering waves.

I will discuss that, that is our practical thing that will be very important to discuss how we can in fact, like sort of get away from tuning of the ship that is very important, that I will

discuss later on, Speed wise as well as spectrum wise so called tuning. Now let us, so let us say, we all understood this part this is **this is** a step, this is an algebraic step you need to find out. This is basically an algebra there is nothing much to it, but it is essentially an algebra that is what is important?

(())

This one yes, **yeah** adding means, not algebraic this in this square gives you this see, what happen? Earlier we said this, but actually is the operation forget this is **this is** a separate part let me another column would have been nice arrangement.

You leave that part actually the steps were this 3 and the steps were to be done in $S \omega e$ zone because its encounter wave, what you have said, only thing is that I started the oceanographers will obviously, tell you this. So, here to here there is a transformation you have to transfer and this transformation is obviously, depends on v , as well as μ . This to this transformation depends on v and μ . Therefore, same wave say sea state number 3 would appear different to different ships.

Is the derived value of?

See this is given by oceanographer you have as I said there are formulas. So, if you want to find out what is sea state 3 you go to the find out from table what is my $H S$ and what is my period simply plug in you get this graph, that is fixed for a given spectrum formula. See I T T C 72 spectrum fixed.

But this is not fixed, because now a ship is moving in that particular wave condition it is moving at 10 knots, it will look that. If it moves at 20 knots, it will look something else. If it moves at say 5 knots, it will look something else, let me put it this way, that let us say this wave appears that there are let us say by numbers 10 number of 8 second waves, and 20 numbers of 10 second waves, and 10 numbers of say fifteen second waves.

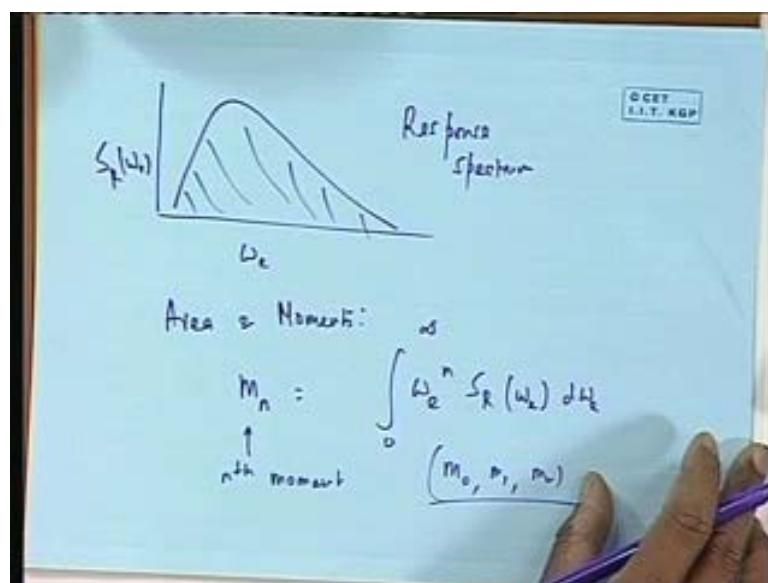
So, let us say that maximum numbers of waves are 10 second waves. When it is going at 10 knots those 10 second will appear as 8 second. So the ship would feel that it is meeting maximum number of 8 second waves. Now, suppose the ship behaves very badly in 8 second waves. So, it is going to have a very bad motion now suppose, other way around now ship is when it goes at 10 as 10 knots it is meeting maximum 8 second waves where the ship is bad.

So, what you do? You simply change the speed; you make it 5 seconds you travel at 5 knots. At 5 knots those 10 second waves will appear as nine and half seconds, but a nine and half seconds the ship does not behave badly, so it is now behaving much better. This I will explain better see this is what is called tuning. You will try to detune see there will be in every ocean some waves occurring most number of times the peak one and you would like to make sure that your response is not so bad in that particular frequency.

In other words, that is not connected to your resonant frequency that is what your aim would be because you know, you move very badly in resonant frequency. I will explain that, little later, but let me say that I have found out this out now you see after finding out this. This also it turns out that, it actually follows; what is known as a Rayleigh distribution? That means, let me put it in a simplest term that the whole lot of your analysis of past many, many, many years have found out that the spectrum follows a particular kind of distribution statistical; that means, you can fit a particular kind of formula.

It turns out the response also follows that distribution somewhat. Then, what happens? If you find the area under the response curve that is area under the curve, which will be given by of course, $\int_0^{\infty} S_R(\omega) d\omega$ then it we can call it to be m_0 again then it turns out that, every statistical quantity can be found out by this area or moments of the Area I will talk about the moments of the area.

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Now, let us look at the response curve and how to find out statistical quantity. After all, you have now got is response spectrum you have got a response spectrum this is my response spectrum. However, you would want to know not only see, engineers do not see you can have a graph, but you want to know the number you want to say look; I want to know in this particular ship environment if my ship goes what is my significant heave amplitude. You want a number what is my significant response roll amplitude. You know see you do not want to give a graph because, you do not you want to tell you want to kind of get a 1 lump sum number.

Now, point is that, this Area and various moments under the graph can be given by say I can call m_n to be given by just like, what is this? This is my n th moment. If you make n as 0 it becomes Area, if you make n th 1 it will become first moment of Area, if it is m_2 second moment of Area etcetera. It turns out that m_0 , m_1 , m_2 are more than sufficient to tell everything that you want I will give you some formulas for that for example, for example, the average value you know all this values, the significant value, etcetera all connected to this numbers $(())$.

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Handwritten notes on a blue background showing formulas for RMS value, significant value, and response amplitude in terms of m_0 and a known factor. The text is written in black ink on a light blue background. In the top right corner, there is a small rectangular stamp that reads "© CEE I.I.T. KGP".

$$\begin{aligned} \text{R.M.S. Value of Response}_{\text{Ampl.}} &= \sqrt{m_0} \\ \frac{1}{3} \text{ Significant Value of Response Amplitude} &= 2\sqrt{m_0} \\ \frac{1}{n} \text{} &= f \sqrt{m_0} \\ &\quad \uparrow \\ &\quad \text{a known factor} \end{aligned}$$

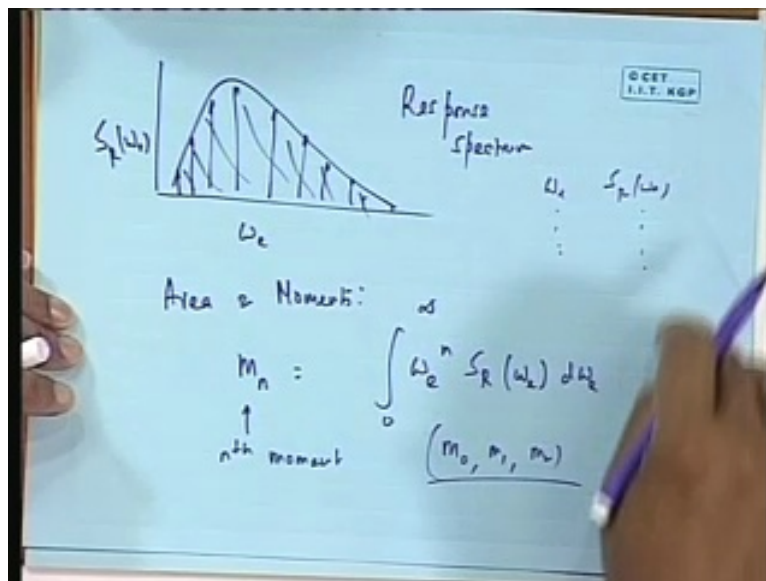
Now, you see what happen, it will turn out. For example, a R root means square becomes simply root over of m_0 , a significant 1 third value significant amplitude actually value of response amplitude let me write amplitude value of some factor into root m_0 . What I am trying to say here, see without going to the much detail of all the numbers no its no point of

trying to go through the numbers everything that you want to know statistically becomes now a function of the Area under the graph.

In other words, you find the Area on the graph of course; sometimes you will find out that you need to know also the m_1 and m_2 . If you know this 3, you can actually find out everything that you want to know. Normally, people will want to know things like, you know that first moments of you know things like what is my significant amplitude? See suppose, the ship is heaving a given example, you want to know that in my sea state 3 or sea state 4 what is the significant amplitude of heave motion. For a given ship when it is moving at 10 knots and a in a particular angle.

So, you do this calculation and find the spectrum, find this you know graph, in a numerically you have this you know that **that** chart you have made and then simply **(())** and find the Area like that I will show you that.

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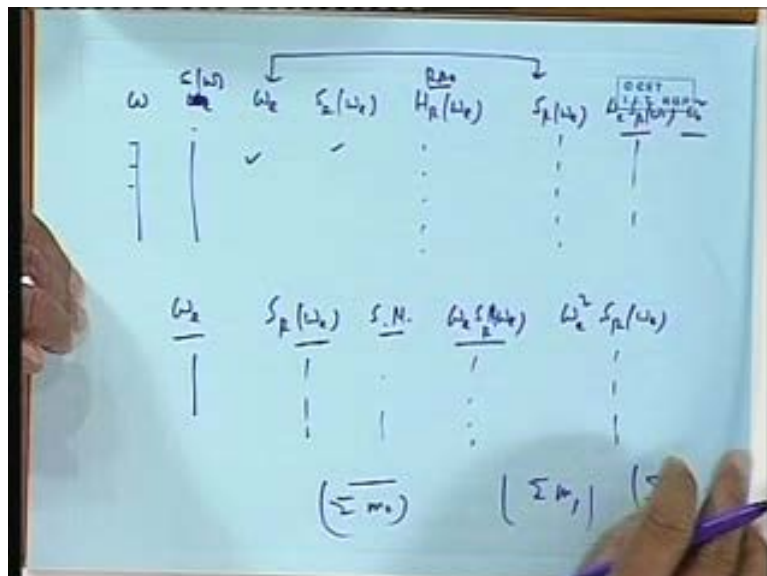
Basically you have this spectrum, this spectrum what you have done? You have found out actually for each value what is this value? You know; you have found out all these values. Various ω_e you have found out what is my response. You calculate that in a table it is a very trivial calculation as I showed just few lines. Then you of course, you Simpson you want to integrate the area, so you have Simpson are same way what you have done? Simpson's multiplier or whatever add then up, you find area that is it, so you get the first moment of Area.

So, finding out Area is a very simple job on that in a **in a** say few lines of computer coding, people call that post processing not even this thing. So, you find that m_0 if you want you can find out also first moment and second moment. In fact, I will show you tabular form how you can do that having found that? Out then in next line you write significant amplitude is equal to $2\sqrt{m_0}$ you see that is it.

You will see that the units and all will match I will also explain the unit part. So, everything becomes part of this Area, suppose you want to find out what is my **(())** significant amplitude it turns out this factor is I forget the number may be 2.53 to double m_0 , there is a known factor given in a table its all in tabular form.

Suppose, you want to find out what is my peak period or what is the probability that **that** heave will not this response will not **(())**. So, much or what is my probability that this. So, much of heave amplitude will occurs once in hundred years, everything that you want to know becomes function of this, and they are given in the table. You know probability of heave between 3 meter to 4 meter, there is a table which will there is a formula which will involve this m_0 m_1 m_2 . So, much m_0 into, so much m_2 etcetera, so everything becomes very simple, so you see you are making a table here.

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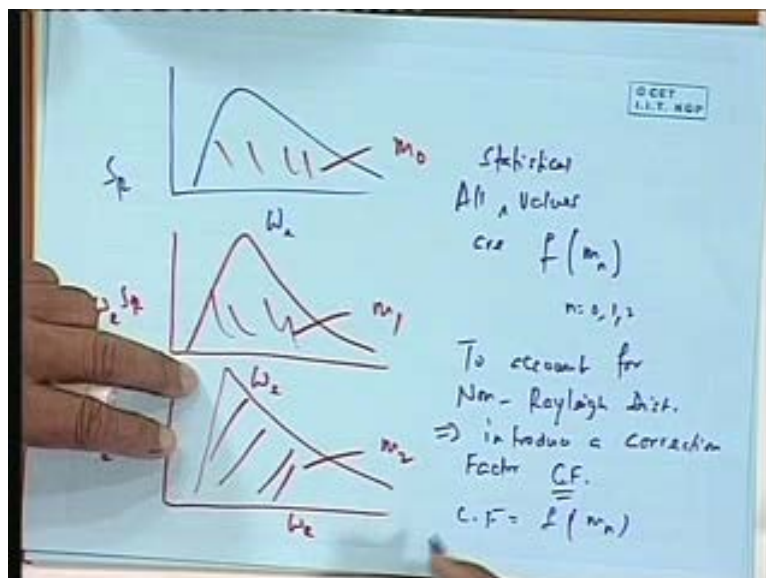
So, you have got in a very more simple way **omega** **omega** e **sorry**, no let me write this **omega** and **S omega** this is start with you have got this numbers. So, this **omega** to **omega** e you find this to this, by that formula $n S e \omega e$ you find. So, here to here you have a formula, you

know ω_e is equal to ω_1 minus whatever this to this you found out. So, this is given, this is given, this is found out, this is found out. Then of course, this will be treated as given say Response is RAO it is given to you; then square of that into this give you $S R \omega_e$.

Now, you have to integrate that remember the graph is this and this ω_e versus this. So, now if you want you know like now separately you can do or if you want a moment for example, you have another one $\omega_e S R \omega_e$ you can **you can** write here. If you want to find first moment, you have here $\omega_e S R \omega_e$ you make a table you have ω_e square **(())** ω_e I mean its very simple, I mean this does not look, but what I mean is that if I now take it here ω_e is here, $S R \omega_e$ is here.

Simpson's multiplier; if you want now second 1 you write here $\omega_e S R \omega_e$ sorry, $S R \omega_e$ that is the first moment. You write ω_e square $S R \omega_e$ multiply this with this, with this. So, you end up getting you know so called here if you do here you will end up getting this m_0 part here you will get is m_1 , here you will get m_2 , this is very trivial. In other words, all you are doing is finding out the areas; area under the $S R$ curve if you want to find the first **(())** Area under the $\omega_e S R$ curve or Area under ω_e square $S R$ curve.

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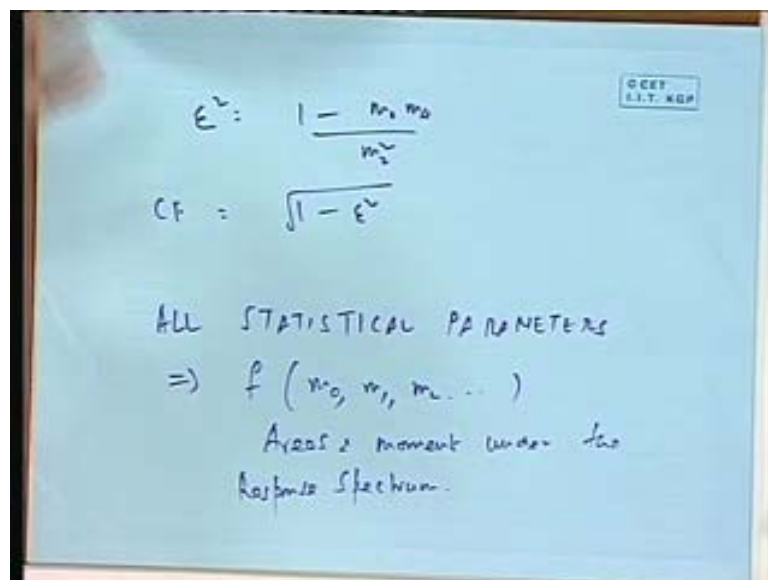


In fact, point is very simple that if this is my $\omega_e S R$ then ω_e square $S R$ curve **curve** is going to be looking like, ω_e into here $\omega_e S R$ will look something like may be something like that, I don't know how it is. And you can have a third 1 ω_e into

omega e square S R, this will look even more. This will be the Area under that is going to be m 2, area under that is going to be m 1, area under that is going to be m 0 simple as that. If you look, you know more sort of graphically more algebraically it becomes like that, so it is **it** **is** very simple. In fact, you can find out m 0, m 1, m 2 and everything can be found out.

In fact, there is more to it see it turns out that all this values, all values, all statistical values, I will write are function of m, n, n equal to 0, 1, 2. But some people say that sometimes this graph does not look like Rayleigh. So, what happen there is a correction factor to an account for Non-Rayleigh distribution, you can have to introduce a correction factor. You can multiply all the number that you get by the correction factor and the correction factor also becomes a function of m n, there is given 1 minus m 0 m 4 by m 2 square.

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In fact, I mean one introduces that **I i** do not recall, but something like epsilon square equal to 1 minus m 0 m 4 by m 2 square and correction factor becomes 1 minus epsilon square root over something like that. You can what I mean is that, you can make a correction also if you want a modification by taking this various moments that is all you see. So, these formulas apply for a Rayleigh distribution strictly, if you do not have to.

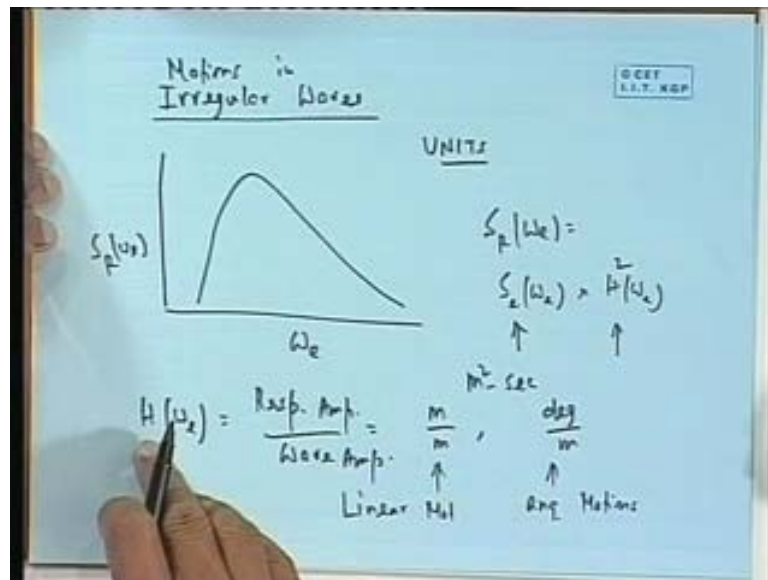
In fact, have to use the correction factor, but the point remains I will close this lecture by saying that, all statistical parameters I could say you could ever want to know I am not writing for the lecture here like they say all statistical parameter that you would ever want to know are becomes function of m 0, m 1, m 2, sometime you know for correction m 4.

Basically, the Area and moments under the spectrum **under the response spectrum** you can write. And you will check that, the units also will check next probably part of the lecture, the units are all matching **units will all match** exactly, what you get? So, I will close this part of the lecture now, and we will pick up from here for the next part thank you.

Lecture No. # 30

Ship Motion in Irregular Waves-III

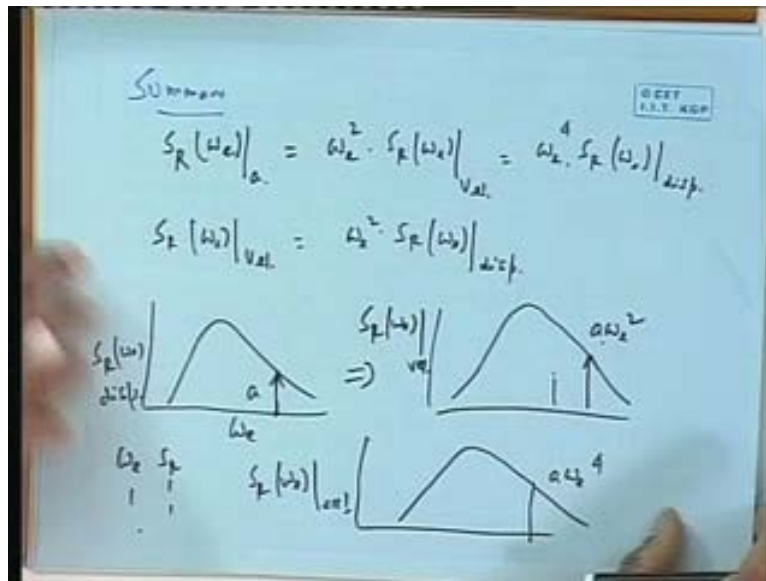
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We will come on that later. See we will continue on Irregular wave motions in irregular wave this is the. We will continue the discussion on that, the very beginning now I want to tell you about these UNITS just for checking because it is always a good diagnostic see we have got this omega e and say response. What is S R omega e? This is given by S e omega e into H omega e square this is the formula, what is this unit? This is meter square second.

Now, this unit you will become now H omega what is H omega e? This is actually the amplitude of the response divide by the amplitude of the wave. So, it is response amplitude by wave amplitude. So, this is going to be meter by meter for heave roll and pitch or it is going to be say degree by meter for the angular motions. It is going to be this for linear motions, this for angular motions. Because, it is response by wave. Now, supposing this is meter by meter then this becomes meter square second into meter square by meter square, so this becomes meter wave.

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So, I will just write it again here, because this page is the problem is that; $S_R \omega_e$ equal to $S_e \omega_e$ into $H^2 \omega_e$, so this is meter square second into meter square by meter square gives you meter square second, this is for linear motions or this is meter square second into degree square by meter square give you meter square degree for angular motion that is what it is now. So, it is simply multiplying by a square of the amplitude that is all. This is a very interesting phenomena, that we should know and then we will find out why it is important I want to tell you, this there is a mathematics is very simple Summarize it in Summary that one.

SRI am writing this way acceleration equal to ω_e square into $S_R \omega_e$ velocity equal to ω_e 4 $S_R \omega_e$ displacement and $S_R \omega_e$ velocity equal to ω_e e square S_R by displacement. So, now you have got this S_R displacement, say displacement take any value here, just replace here this into square of this. So, you get this graph; that means, say this value say a say this value is ω_e .

So, here that equivalent value is a into ω_e square you will get here $S_R \omega_e$ velocity and take here a into ω_e 4 then you will get S_r . So, you see you can get this to this spectrum just one more step that is all, you do is that corresponding value multiplied by see you have got ω_e . So, you have got ω_e versus S_R simply write this is for displacement S_R velocity will be square of this into this.

Acceleration is going to be square of this, I will I will make a table next step, but this is very simple, very, very, simple you know like if you look at this the mathematic part of it, absolutely straight forward and simple (Refer Slide Time: 56:21). So, now what we have done is see, that you have. So, you see what happen, a lower sea state I end it here soon the lower sea state just simple, lower sea state means, sea states at 3 or something. When the wind is blowing at 20 knots, you are having more number of waves of 6 to 8 seconds but that 6 to 8 second waves the barge behaves badly.

Therefore, even in a very low sea state barge behaves very badly, higher sea state it. Obviously, it behaves badly because it is (()); that means, it is simply goes up and down with the wave. Now, you take for example, a large tanker lower sea state it may not respond because lower sea state you know this this peak goes on the other side of that, where it is coming down. So, it may not Respond lower sea state at higher sea state means, a sea state 5 or 6 where the period is 10-14 second it will behave very badly, still higher it will behave very badly. So, a large tanker that is why do to respond to small waves, but large waves will respond.

If you take a semi submersible, it is by design the semi submersible is designed such that its peak is much away from any peak see 20 second no waves exist except very small, there are simply no waves to cause excitation, so it has very low exertion. Why it is designed for this, see there is a (()) in a minute there is a big reason for that, the reason is that ship has the privilege of running away if you if you encounter that motion you can just change. Semi submersible or (()) designed for one place if you have a bad weather you simply have to go up and down. You cannot afford to have bad weather by design of the hull that is why you have a semi submersible design.

This is why semi submersible concept and similar concept has come for swath hull, small water (()) twin hull I will talk about that next class, but this is a very interesting part of a small boat barge or you may say a small petrol boat, pilot boat, everybody say the pilot boats going to this thing is not comfortable (()) down. Same reason no large waves, but till (()) up and down, but you go in a tanker small waves does not (()). So, day to day life you are not bothered in a tanker you rather be assigned on a tanker then on a on a on a small pilot vessel and semi submersible is (()) better.

So, any how, I will end it here. We will have more to discuss next class on the same topic of now coupled motion etcetera, thank you every one.