Performance of Marine Vehicles at Sea

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Lecture No. # 27

Ship Motion in Regular Waves - III

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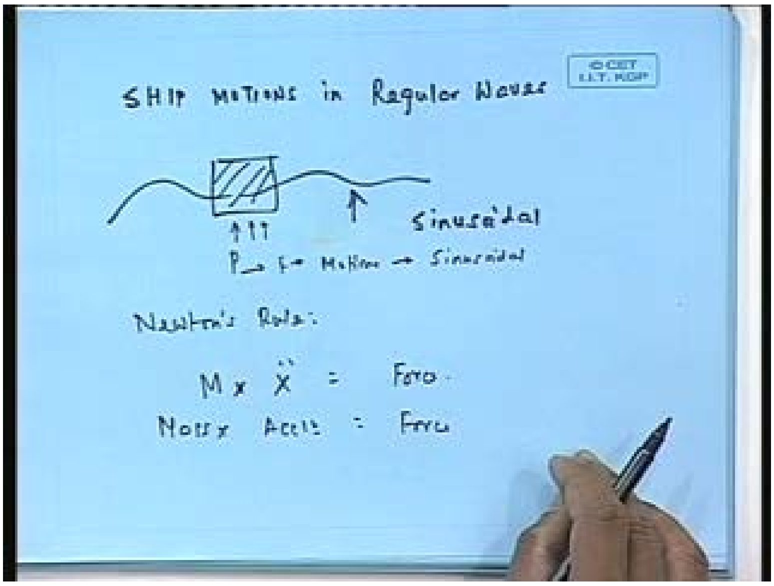

We will continue on our talk on Ship Motions in Regular Waves, see in the last class what I said is that, if there is a body here and if there is a sine wave here. Then, this wave everything is sinusoidal, therefore you would expect the pressure that comes on that and therefore, the forces and therefore, the motions all will also be sinusoidal. If you now, what I will do because we have only a short time, try to explain the physics of it. It imagines this, there is a body here or the ship here, it is kept here, wave comes and hits, as a wave comes hits, sets of pressure field as a pressure field is set up it gives a pressure, as the pressure is given you add them up you get force, as and that will give a motions.

So, if you recall Newton rule, what we have we get is that, mass into acceleration equal to force. This is our standard, you know like mass into age old formula that always applies, we have to invoke that and see how it, you know we can actually figure out this response.

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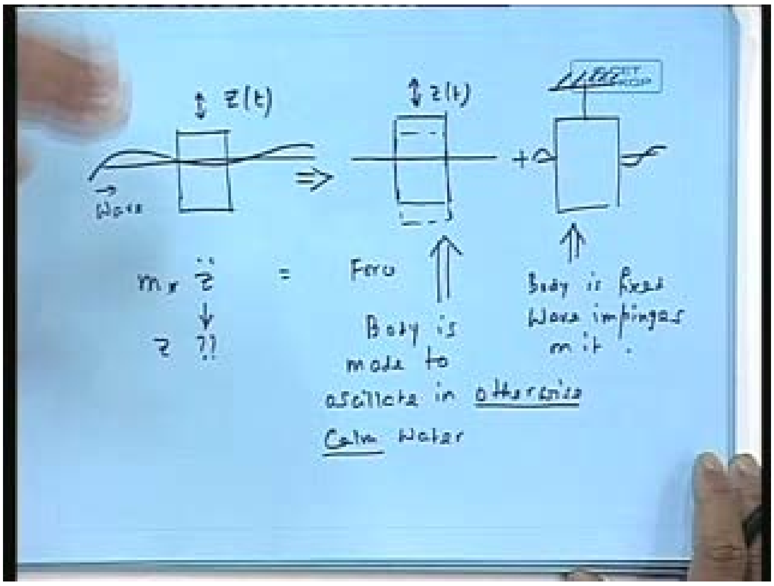

Now, we will only talk very briefly, so what we have to do is see, we will just look at a for a simplistic case, as if the body only give goes in a heave motion, some very simple case to show, how an equation of motion looks like and how one can solve it, because we have a short time.

So, let us say that, I have this body, I have just take a simple case of a cylinder body here and wave comes, so it obviously moves z t, let me call it z t, there is a wave coming. So, what we have is mass into acceleration is z dot dot equal to force, and my job is to find out force. And if I have the next set equation of motion, then what I am looking for, I want to solve, my aim is what z ? That is my, what I am looking for finally, you know given a wave how much it moves.

Now, what happen? This is a hydro dynamically every complex thing force to find out, force is a very complex thing, not a very simple thing, what we do is that, see we assume that, this compose of two pictures: one picture we say in this case, as if the body was some, there was no wave and somebody may be to move like this, we we assume in picture one as if the body as in made to oscillate. That is picture one, plus, so this is going to actually cause the you know the body, kind of oscillates here, plus the second one is that, we assume that the body has been held fixed, but the waves has come and hit there on this tail part.

See, here this this is broken in two pieces, one picture says, when the body is made to oscillate in we write this way, otherwise calm water I have explain this later on and this one is body is fixed wave impinges on it we can say, now this is a little bit difficult for people to understand first time.

The the interesting part is something like this, see when a body moves, you see it creates a what what does it do, if you want to imagine, there is a wave coming see initially there was a wave there, no body. Now, you just put the body inside it, moment you put the body inside it, what happens? The body begins to move.

So, you want to find out what is the force on the body, what is the force? It is the pressure that the wave gives what wave? Moment you have put the body number one is that the the body has disturbed the wave; the wave is not any more what was there. Secondly, the body begins to move, as it moves it creates another system of wave, as you know if you take calm water and if you take a bucket and if you push it down, it will make a wave. And this one, this is going to make some kind of wave, if you take and oscillate it.

So, the scenario that you get the wave field is a very complex scenario, it has got number of waves together and all these waves together, when you add them up, you get the pressure and that pressure should give you this force you see, so it is a complex scenario. All are, we are going to do here is to try to tell you, where it comes from very briefly. So, what is done is that you break it done, you think see when the wave comes it begins to oscillate, you think that as if there was no wave. So, I took it my hand and force it to oscillate, what is the pressure field that gets created? That is one kind of pressure.

Then, I take in a second picture, I say that I am going to hold it fixed and let the waves come and hit that gives another pressure field, because this wave is hitting and reflecting back remember getting scatter, so the wave field is different. So, it gives a net pressure again, that this is another force, so these two forces together make this force.

So, we can call this, let us say this force to be say f some people call hydro dynamic and I call I will write it down later on an f wave, let us say I call this two that means, I call f h to be the force that would be there, if you took the body in a clam water and oscillate. Now, we see the, what otherwise calm, why otherwise calm water? To start with water has calm, but as I shift it down, it will you know creates waves. So, the what otherwise calm water means that, waves do not exist a prior from before it only exists, because of the bodies motion that is why we use otherwise calm.

On the other hand here, I have just holding the wave fixed and the the you know the body fixed not allowing it to move like a fixed body, it comes and hits and obviously, the wave field is remember, if I did not have the body, the wave would be something different, now little different.

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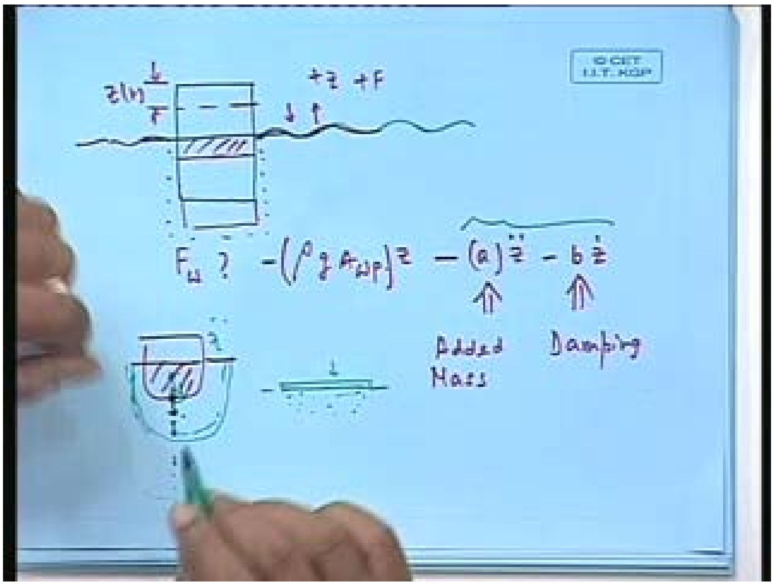

See, I will give you that two examples separately, now how the forces we get. Now, let us see this first part, now we have this body here and I go to this clear picture. So, see this this body another color I take, it was a mean position I have shifted it down by an amount, you see by may be this amount of I mean, I am oscillating it by force, the water is calm it creates kind of a wave here.

So, I am oscillating it by an amount z t, what are the forces that will come? Number the what is my f w here? Number one you will see is that, there will be a force because of the extra buoyancy that is that parallel sink age force that is this much, that is rho into g into a w p into z. You see, there will be a force because of this extra buoyancy that will be the water plane area into the height. This is exactly what we have this parallel sink age force, rho g a w p is my area of that into z that is the volume, in fact no sorry a w p z is the volume multiplied by rho is the mass into g is the weight, so we we get this to be one hydrostatic force.

See, it is very simple, if I have pushed a body down by amount z, there will be an additional buoyancy force coming, which will be equal to this, because it is equal to the extra buoyancy force hydrostatic part, but that is the very simple stuff, everybody knows it.

Obviously it will happen that, if you give the z down stair the force is up stairs, so it is on the opposite side of z. So, you know the sign of that is negative, because if you push it down, see I am giving this as plus z, so this is also plus f everything is up is positive, but here I find that the force is opposite side of z, if you push it down the body the forces upward, so it is minus this, it is very simple.

But the most critical part and that I will just give briefly is that, when you oscillate that, it obviously, creates a pressure field here, dynamic pressure field, because it creates waves outside. So, this part is very complicated, it can be expressed an additional part to be some constant into acceleration and some constant into damp a velocity.

Now, you see this part is very complicated to normally explain, but I can only give you at this class just a physical explanation, this is known as added mass and this is known as damping. Now, the word you will want to know, what is added mass, therefore what happens? See, now take for example, now here separately given example, you have body here, I am oscillating it. Consider this fluid particle here, so I am accelerating, I am accelerating the body, whenever you accelerate a body a rigid body, what is the force, mass into acceleration.

Now, in this case what happen? When I want to push the body, look at this fluid particle, it gets push down, you look at this other particle it gets push down slightly less, so what would happen? As you push up and down some amount of water, became to also gets push up and down along with the body.

So, the water bodies are body would feel as if it is heavy at then what it was. See, when you when you when I it is something like that, if you take a plate just a plate in water, you know this is water surface, you take a plate here you try to push it down, you see you need much more force, much more force than just this buoyancy part of it, it is a thin plate very thin plates. So, there is no static force, but you need more force why? Because, it you have to push all this water down, as you push all this water here must get push down.

This is the added one

Exactly, so here when you are doing that you have to push some water, because the water cannot penetrate, when I am pushing it down this particle has to be get push down by same amount, so what happens? Therefore, when I am pushing as if the mass I am pushing down is more than the actual mass, so there is a feeling like that.

Now, now lot of people will say, it is the mass of the water attached to it really speaking there is no mass you can identify to attach, because this particle gets pushed more, this will get pushed less, this will get push even less ultimately, there is a set of motion over an area where there is pushing down of varying order.

You know, but it can be shown that it is equivalent to some kind of mass into the acceleration, you know some constant into acceleration and you call that to be added mass. In other words, it would appear as if the bodies mass has been increased by some amount, this is what is called added mass.

So, you see when you push down the only, it comes from the fact that that the, the solid particle are pushing the particle, so when I am accelerating this particle also accelerates see, when I accelerate the body like a z z, this particle also accelerates, this is also accelerates lower order. So, there is a kind of mass of water also accelerates along with the body. And if you now take an equivalent mass, which accelerates at the same acceleration as z dot dot then that mass is called added mass.

So, this is a one of the source of force, that comes only in purity, here actually it is so, less that we neglect always is added mass force, now comes this b damping, what is damping? It turns out that is a question of phase gap actually, that the force that you get is not in phase with acceleration exactly, it is with a some kind of it is a difficult concept to tell you, but I let me just explain this way it turns out that as you oscillate it creates waves. And a creation of wave is like a wave resistance, one can show that energy of these waves has to be supplied by the oscillation on their body and energy is work done equal to force into distance.

So, there is a kind of force you are moving by a distance at certain rate, so there is a rate of work done, so there is energy dissipation and that actually shows up, that turns out to be proportional to velocity. So, because when you push body up and down, it sets up a pressure field here, that pressure field when you add you gets a force, that force is actually equal to this force. You see what happen again? I will explain you push the body up and down.

So, in case of hydrostatic this part, it was assuming that water is actually calm although I pushed out it like a batter, water just went down and that is it no motion set up in the water, that is what we always assumed, which will happen if you do very slowly. But, when you do fast, water is having a velocity means there is a pressure, you add the pressure up you get the force, that force it turns out can be written totally that force can be shown to have a component in in phase or in in in line with acceleration and with velocity and you can write it this form.

This is what is called added mass force, this is called damping force. This two together can be called as actually radiation or oscillation force. Basically, it is a force these two together. So, the total force when I have I am pushing up and down, because this plus this plus this this is a hydro static force, which is what you are doing see there is velocity here just displacement.

This is, this is the only thing, that we actually take in my buoyancy course previous course, when you do buoyancy stability you add a weight or you push it down, so it will get pushed up, that is only this part this part is not there. So, we have got here this as well as this part. So, this is one part of the force (Refers Slide Time: 15:10).

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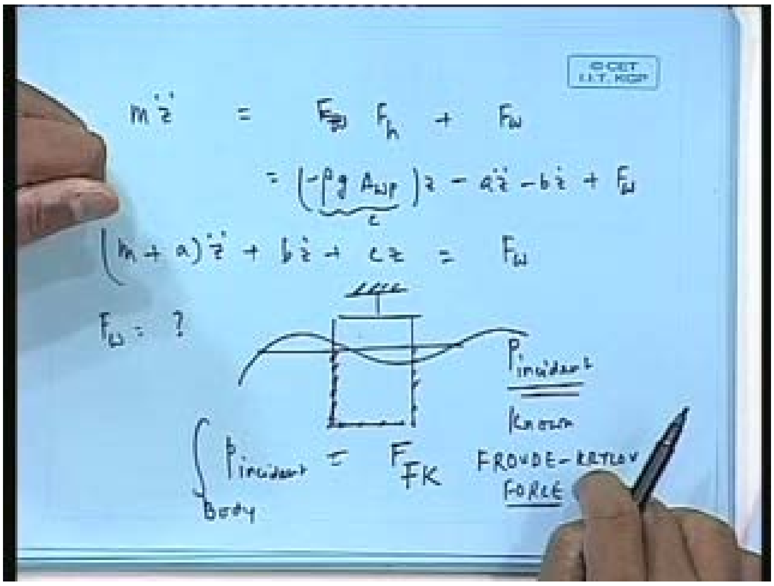

Now, now we will we will come back to this our equation, little bit and then go to the next part after. So, we had this equation of motion, that mass into z dot dot equal to F w oh sorry well F h plus F w, now F h has become minus rho g A w p into z, then minus a z dot dot minus b z dot plus of course, F w now you bring it on this side.

This is minus or plus sir

This no it turns out that it it turns out that you can actually, a is a constant, b is a constant, what happen the force is in proportion to z dot dot and z dot, so you can write them as minus a it turns out that it is minus, it is on the other side of z dot dot actually, this is why I am writing minus, but suppose you do not want to write minus, you write a constant that constant will have negative term number number as it is a constant.

Positive

Yeah yeah

So, this one as if it is not acting down there

No but acceleration in the opposite sides, see the phase is such that when you when you push it down at that time acceleration opposite acceleration is opposite phase of the velocity.

Displacement

You see when here coming back it will coming back, when I am pushing it down my z is actually negative, but acceleration is positive. And the force is on this side and then the mass and acceleration becomes on the other side like that, you see it becomes on the side of this z, so it becomes negative.

So, this is never mind this this integrate maths, but it will be negative because z and z double dot are opposite phase basically. Now, if you now you bring it this side, it turns out that end of getting m plus a z dot dot plus b z dot plus, actually this part people call it c, this is a constant. So, I can call it c z you know this is actually hydro static, this is your t p I turns per inch immersion essentially, becomes f w, this becomes now equation.

Now, I have to find out what is f w? So, this is the you know this looks very in classical vibration equation, you might have done in other course mass spring dash pot they call, there is an acceleration force, there is a damping force, there is a spring force equal to exciting force, this is a this is the we have to find out what is this now?

So, f w now f w is actually more complex to find out, so what is F w, I just explain again see, suppose I have got this body here there is a wave comes out, I am not allowing it to move held it fix know.

Now, supposing the body was not there, suppose I this is very interesting a picture, see initially I think as if this wave passed by through the body, but the body was not there, but as if some hypothetical like body is made of some kind of a plastic through, which the wave can pass by.

See, the body did not disturb the wave, I assume as if it is what you call transparent to the wave, in other words this is this body is there, but it is made of a kind of a material through which waves can pass, let me just assume, in that case what happen now? I know all the pressure of the instant wave; you know remember this is my pressure of incident wave. In the very first class, when we talk about wave equation, this is all known. The incident wave pressure that is because I know phi of that incident wave we have given this formulas of the very first class, I know.

Now, if you take that pressure and assume that, as if that pressure is acting on this, these points and integrate this incident wave pressure, so call. Over the body the force that you get is one part of the force, it will be always existing this we can call it incident wave force, people call this actually this is a famous name, so I write at this f k it is called Froude - Krylov force.

William Froude in England and Krylov at in Russia, independently proposed that in 1880 or something, because we have to find out how the body moves, so there has to be some estimate of the force, so they assume that, I will with a made an assumption. We will assume that the wave is not disturbed by the body, in that case I can know the pressure I can simply add them up. So, this is been since, it is independently done, you know this is now called as Froude Krylov force, both are very giant name in our area, that is Krylov institute in Russia, very famous his Froude's name is very famous in in England, this is a Froude Krylov force.

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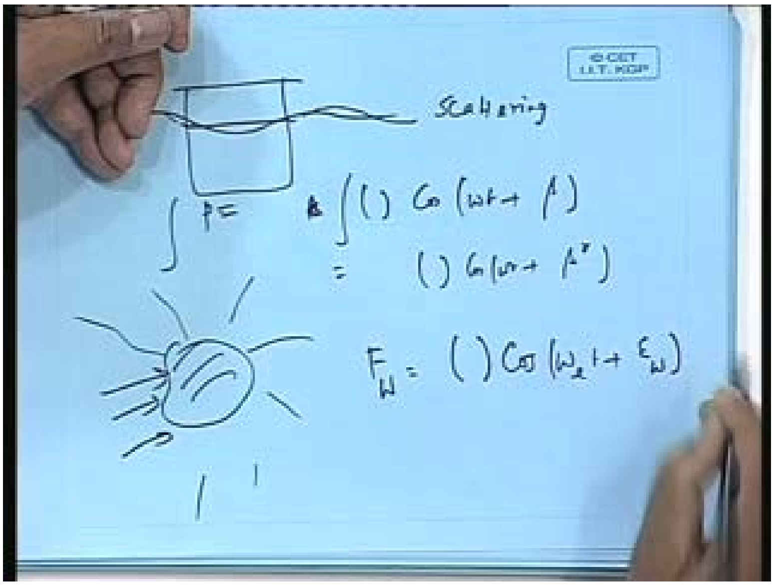

The point is not that, this is one part of the force. Now, now you can write the force always say remember that, this wave comes. Now, pressure is also a sign something some something into cos omega t plus some phase, see everything is kind of an cos omega t plus.

So, when we integrate the pressure, this will also it will be something into cos omega t plus some kind of phase, some may be another phase, what I am trying to say is that whenever, you integrate anything any sinusoidal function, every point is sine of same frequency, so obviously, when you add it will be same sine function.

So, the Froude Krylov force also will be sine function, but of course, reality is that we have got also the fact that this wave has come here hit, but it has got scattered, so you know it has got kind of reflected back, we do not call it reflection, we call it scattering, because reflection would be when it is like a light.

When a when you think of a light going to a mirror it gets reflected, but when you see a light falling on a particle, it gets scattered means it is getting disturb scatter, this is how our C.V.Raman said why sea sea is blue for example, light scattering. Same thing it is scattering, when now this object is something like say something like that, now waves come then is going to get scatter all all direction.

So, point is that you will see here, first I thought that there is no scattering. So, I could only know incident wave and I find the pressure this is Froude Krylov force, but in reality there will be also scattering. So, that reality is more difficult here, but ultimately if you add them up let us say we do not find it, you will find out this f wave to be some kind of a amplitude into a again a cos curve plus, let me say this into say wave.

You will be able to find out that it is some kind of constant, we will write that to be say f a some amplitude into this thing, this amplitude of course, you have to find out based on the Froude Krylov force plus scattering, that is a very complex subject, so I will not deal with how to find it out, how to find it out are pure hydrodynamics? Just like how to find added mass and damping. So, let my say that I can express that, I can find out the pressures then I can find out integration of pressure at the force.

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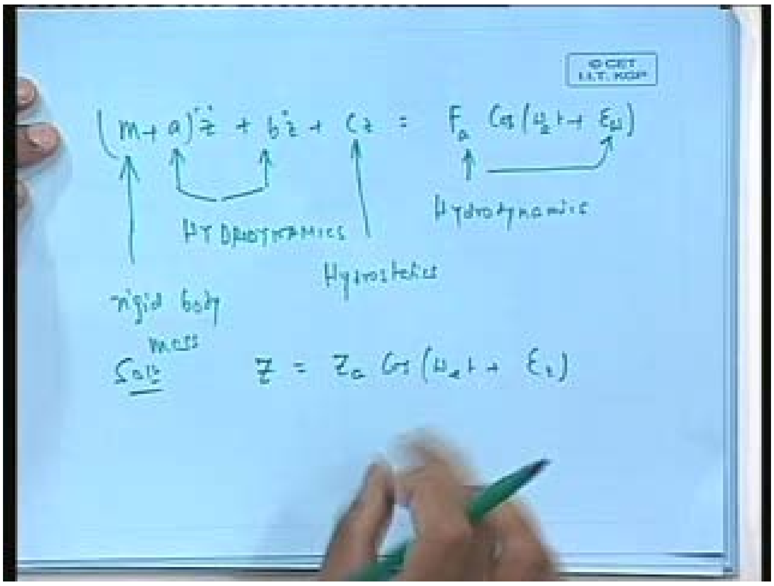

So, if I do that, I end up getting this equation what I have got earlier, that mass plus a z dot dot plus b z dot plus c z equal to here I have got f a into cos omega e t plus e, let me call it this.

You see what happen is that? This part and this part hydrodynamics, this part is also hydrodynamics. In fact, this and this in fact, you have to solve a complex hydrodynamic problem, if you want to actually estimate this the the wave force coming as well as the added mass damping.

This is what is this this is hydrostatics. This we have done buoyancy stability course and this of course, is rigid body mass only. So, what happen you can set up an equation it looks like that which has got actually mass into acceleration equal to hydrodynamic force, hydrodynamic force is you know rearranged, you end up getting an equation looks very very similar to a standard vibration equation mass spring dash pot.

Now, I will talk very briefly about how to solve with which you probably have done it, but solution of that, I want to tell is actually most trivial thing everybody knows how to solve it, what we do not know is how to get a b and f, because, that is where the hydrodynamics lie.

In any case we will not go into that because that is a complex hydrodynamic problem we can say a added mass b damping, this is exciting force wave exciting force we call that, the term is with a phase angle, this is something that is hydrodynamics, in fact entire research if you think of that or sophistication of calculation is hinges on how do you estimate this.

You do not make a mistake in c, because it is only hydrostatic calculation, rho g a w p you do not make a mistake in mass, because you know mass all the profit analyze in estimating that, which we are not going to talk.

I will now talk if you have this how done you solve it, what would look like, what would said look like? This is a very standard thing, the solution for that I can write actually here this solution for that here, solution will look like always z equal to z a cos omega e, we have t plus epsilon z, where see it will the solution for that will be always like this, because this is a sine function will lock that and z a and e z the phase, that is the amplitude and phase is well known.

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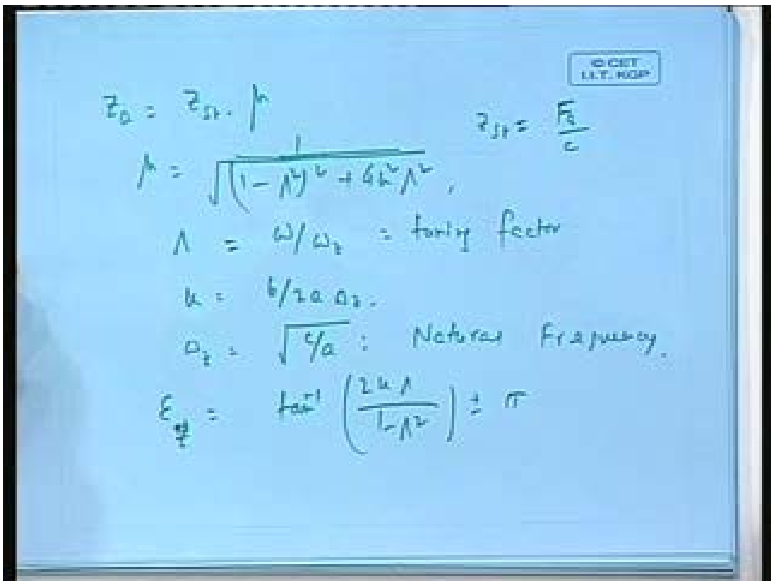

I will just write that that part for completion it turns out the z a is what we call is a static part into mu this is called tuning factor, mu is known as one by one minus, this is you would have done that in some other course, I am quite sure in this class itself, right you would have done in vibration course, this positive it because this is standard vibration.

So, i will just go every quickly on that this is called w by w z tuning factor, k is known as damping factor, 2 a omega z omega z is called c by a natural frequency, this all you know. And e w I we wrote know is tan inverse, I think it is e z tan inverse 2 k mu by 1 minus mu square plus minus pi like that.

What I mean is that, let me not do not worry about this this part of detail, because everybody knows this solution, I will simply basically show how it looks like, z s t into this or z s t should be I mentioned that is f by never mind this we know. Actually z s t is given by f a by c, that is called you know z s t if you look at that if you look at that supposing, there is no acceleration, there is no damping that means, it is moving, so slowly, that you can neglect that you can neglect that, you know because very slowly it is moving down.

So, the z dot that is very small because, omega you know it is proportional to omega square, if omega is small it is very small, so we neglect that, neglect that here only this then z becomes f a by c, z a becomes z is z equal to f a cos this thing by c, so that.

The amplitude

The amplitude become f a by c that is called static restoring force, this is what we have done. In fact, if you push it down by a weight how much it is parallel sink age that is what it is. And what dynamic does is that that part gets modified by a factor, what you call this mu factor and this this is called magnification factor, this mu is is called magnification factor.

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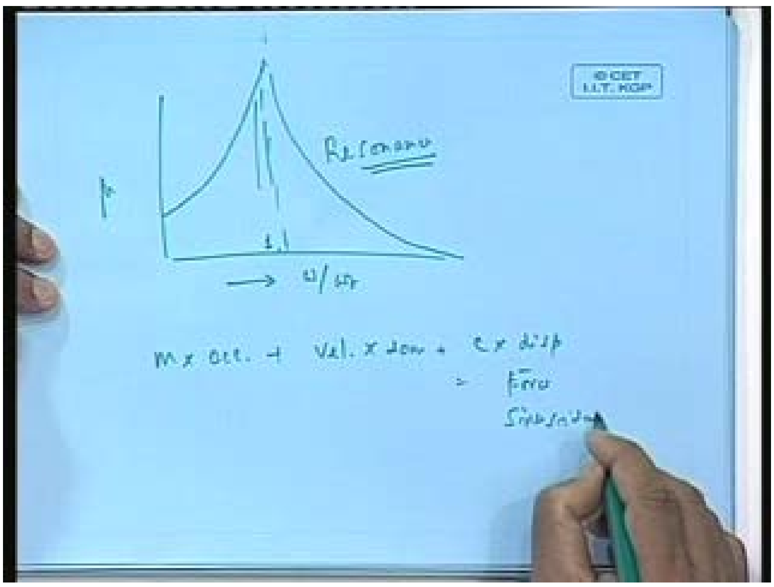

And, how much it magnified depends on a something called tuning factor, that is if there is a natural period you know comes down, this is all standard thing I mean now I will just draw this picture, that how it looks like typically you know that you have got this here, may be omega by omega z you can write or omega you can write is because 1.

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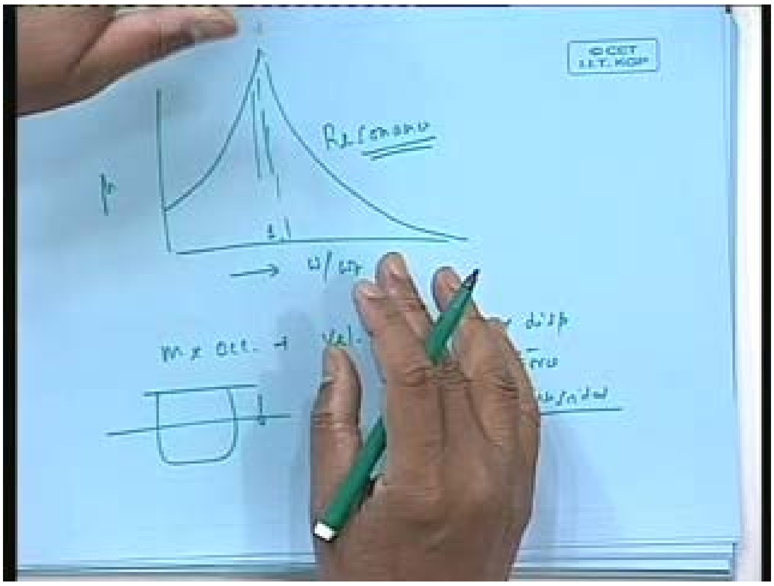

This is your magnificent factor mu, it goes something like this and you know goes to infinity and comes down like that. This is where it is equal to 1, everybody knows that this is what is called resonance, this is called resonance.

So, you see the characteristic is very simple, therefore ultimately what happens is that, even the ship equation is nothing but, mass into acceleration plus velocity into damping plus c into displacement equal to force, this is the equation, force is nothing but sinusoidal.

Therefore, there a solution is sinusoidal; therefore solution is covered by exactly same thing as what you see in any vibration book, mass spring dash pot. So, solution everybody knows that, there is a natural period if the natural period, if the forcing frequency is equal to natural period you have very large motion, you know what is called resonance motion that you know.

When when natural period and forcing period, see think of that, now what is the meaning of that suppose you take a body here, I pushed it down it will oscillate at some period, that is called natural period, everybody has I do here it has a vibration natural period.

Now, here somebody is forcing the wave comes and hits it at every 10 second. Now, supposing this is natural period is to move at 10 seconds and at exactly 10 seconds the force comes. So, it is got to go move very much up and down, this is what we call resonance.

This is why, you you say that the soldier should not march on a bridge, this is very standard physics problem, you know the bridge might have a natural period, now if the soldiers go on a rhythmic march and that that forcing you know tuck, tuck, tuck, if it matches no if it matches the frequency it can very have high vibration, so you have avoid that.

We will see that, this is a very high practical implication for ships, natural period has, but the solution is like this, so we we we can just go from that, here next thing because we need to go little faster.

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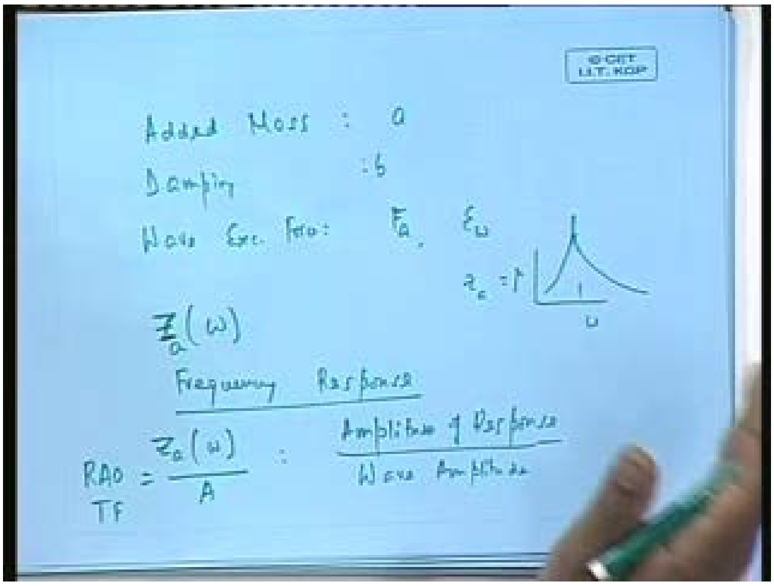

So, what I am trying to say is that, the solution for a typical regular wave motion, therefore it becomes sinusoidal. And therefore, you can solve for it provided, you know added mass a damping b and wave exciting force, both there are you know f a and e w I mean the amplitude, if you know that the solution is know everybody knows the solution.

Only thing is the characteristics of the solution as you know we call frequency characteristics, because this is a solution is the function of omega, z z a is a function of omega, because you know the amplitude, that you find out is a function of omega because, you see if it is you know as you have seen this response is like that, you know this is omega this is my magnification, which can be called to be the z response.

So, response amplitude of response is equal to you know this is what is called frequency response curve, this is actually a again this is very well known to us. Frequency response that means, supposing the forcing is that frequency omega, what is the amplitude, because, it is depending on the on the frequency, we we will see that when the frequency is very low it is one type, it is very high it is one type, and it is in between like when frequency become same as natural period z a becomes very high.

You see normally people do not call in terms of frequency response, you want to non dimensionalize it what you do is that, you say z a omega bar unit of wave amplitude that means, amplitude of response bar unit wave amplitude. This makes sense, because you see the waves are linear.

So, you want to find out how much the ship would drive, how many meters far meter of wave height, so suppose there is a 10 second wave, if the wave height was 1 meter it goes 2 meter, if it is height is 2 meter it goes 4 meter, this is why you say you better find out, what is the response per unit and this term is known as people call it RAO, you can also call it i will write these two later on.

See, again z a omega is the result, what is that? It is means what is the response amplitude for a given frequency, how much it is having for a given frequency. Now, the question that comes is here is that obviously, that will depend on the wave amplitude incident wave of is say 10 meter high.

Let us say, I take a 100 meter long wave same 100 meter long wave, that is if I take hundred meter long wave I fixing my omega. Because, you see length and you know a l omega t are all related to each other. So, if I take an hundred meter long wave means, I am taking a certain omega, but the same 100 meter long wave may be of height or amplitude 1 meter or 2 meter or 3 meter or 4 meter.

Now, the question is that, this theory is called linear, so if it is 1 meter it goes 0.5 meter heave then 2 meter should be 1 meter heave, so that means, the heave that z a value per meter of amplitude remains constant, because it is a linear theory, so called. This is why, instead of calling z a, we must call z a by A, that becomes more makes more sense.

So, this part this z a omega by A you can call it by h omega or number of terms, this is known as I say response it is the the common term that, we will use or you will hear is it is called response amplitude operator, people will call the word by RAO or R A O, you know.

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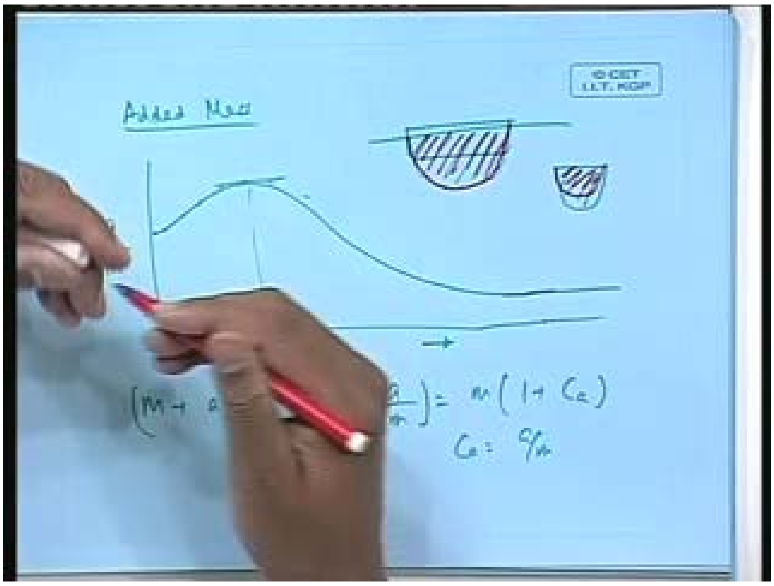

Some other people like in, if you are look at this is very standard frequency response in an a electrical engineering any other engineering, they use the word transfer function some some people, both are same thing I just want to tell you that.

We use the word in ship people RAO always you know you will see that, what is the RAO? It is the very common term, you cannot not know it, but if you would use the RAO term to an electrical friend, he will say what that is? Then you say it is this is transfer function he will tell, because they call it transfer function. So, I just want to tell you that, this is what it is.

Now, this is an interesting part that, I want to tell in a in a short while is how does this, now we have found out that it depends on, I bring it back here depends on mostly added mass damping wave exciting force. I will discuss this little bit, because added mass and damping therefore, it becomes crucial.

Suppose, you have a ship, if you have an added mass and damping, this is not you you found it out already. Suppose, you you find it out, then you can very easily go to that problem and solve for any, what is the response, so if you have this this and this with you. Initially and and hydrodynamic problem is to find out this, then you can find the solution, we are not going to talk about the solution we what we will talk is, how does they look like? You know, how a look like you knows b look like, etcetera, etcetera. Just have an have an idea you see.

Typically you know, I will how see added mass, if you look added mass, normally added mass is a mass people write this as added mass is like that, non dimensional factor you can call it to be added mass coefficient, because what is the added mass per mass, you see that way you know if my ship mass is 20000 ton and added mass is 30000 ton, I know that added mass is one and half times mass.

So, it is better to use that factor, then at you know you see m plus a, you call it m into 1 plus a by m you call it m into 1 plus c added mass, when c added mass equal to a by m this makes more sense, because this numbers are easier.

So, you define added mass normally as added mass coefficient, in other words how many times of mass is added mass. See, that makes more sense, because you know your ship may be having 21937.25 tons of weight. Now, an added mass is 32918, you do not like that kind of number, you rather will tell added mass is actually 1.35 times. It is much easier at at you also this is also gives you a feel, because even the small body or big body it is a proportion you know. So, you know that it is so much more.

So, what I am trying to say how it looks like, typically if they depend on frequency and they will actually look, you know like in a heave case it will turn out that, it may look something like that. It has a frequency dependence, it is you have to find out at every frequency and you know like plot this, it normally tends to an what you call a constant at at a low frequency when you go very low, it has some value at higher frequency or it might go up, it has some kind of a dependence. At some frequency it can be actually high and it depending on the ship type it can be very high.

Some rule is like that, if you take a body here a ship body a beam here, a added mass is approximately equal to mass of this semi circular water. If you take a hull, this green line is my hull, then a cross section then added mass is approximately equal to the mass of this water.

So, you can see that if you have a very small like a barge, you know you, you take a ship like this, cross section then added mass is going to be less than the mass, because added mass is going to be water of this mass. Typically, you know in a rough rough figure, see this I will explain better.

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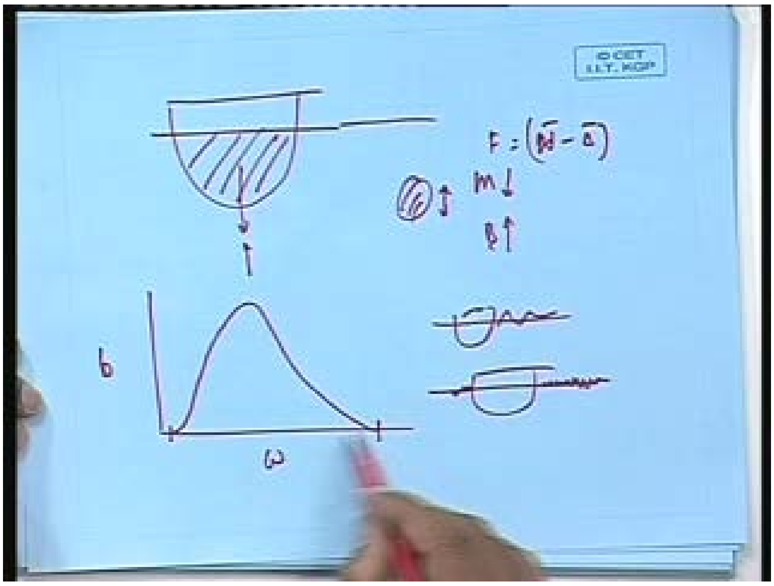

See, it is like this, you have a plate here, you are oscillating it what happened? Every point of water gets pushed down, so large amount of water will get pushed down, so it turns out that the added mass that, you find out is approximately same as mass of a semicircle of that projected area, mass of this. Now, you take a ship here more or less that has a feeling, suppose I am looking at a cross section of a ship see this is my ship here.

Sir this is mass of the water f that semicircle

Mass of the mass of the water of the semicircle, so I have a cross section, so I have show the cross section a barge a, barge will look of course, thin and like that, this is that you know tip, I mean b is much b by t is very high. So, you have the the mass of this water, remember mass of the ship is nothing but, mass of this cross section because nothing but, displacement is volume.

So, here the mass of the ship is area under the green line and added mass is area under the red line. So, in this case you find added mass is actually comparable and more than the mass of the hull. Now, if you take the other extreme of is a navy if we get, which is like this you know thin line, then the mass of the ship is under this line, but mass of added mass may be this line.

So, in any case you can see for a typical ship in heave case added mass is a large number, because it is almost comparable to mass. If it is a small actually if it is a small barge like that, you know like cross section very narrow barge added mass can be three times of mass as much as three times of mass.

And, smallest case normally all almost all ships no ship is as, so thin and narrow you see normally B by T is at based 1.5, so it is a in heave in heave ranges between may be 0.8 of, I mean mass to as much as three times of mass depending on the frequency. Now, you say where is the frequency coming, you see if you take a body and you oscillate very fast, then there is a kind of acceleration in mass if you do very slowly, you will not find, so much mass being pushed out that is why, again it depends.

Sir this added mass will be effective mass.

Effect, you can say mass like added mass may may call it virtual mass, yes you can call it actually they actually mass plus added mass can be called effective or virtual mass. You know, if you add add see m plus a see when you have a mass m accelerating, then mass into acceleration is force, but in water when you do it it feels that it is a greater mass and that total mass is called virtual mass or you can call it effective mass whatever that extra part is added mass.

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Yes and what what I mean is that that what is the value of a e, depending on the frequency and the shape if suppose you have 10000 ton barge, it may feel like 30000 ton being accelerated you see it can be two as much as that. So, it it is a huge number it is not a small number, this is what I wanted to tell.

That's a normal when you take a mass.

You dip in water

Yes.

When you weight it it is less than what it it actually was.

Buoyancy

No no that is not, no that is no connection with that you see.

Sir once we are talking about the effective mass.

No that no no see this no see this.

Look, this is a mass here you say that when you are in water the mass is less, actually mass is feels like 0, because the entire mass is balanced by buoyancy, that is why it feels 0, but that does not not mean when you accelerate that there is no force there.

You take a body deeply water here accelerate it, it will still have that force see what is happening is that, actually there is a mass force there downward, there is a buoyancy force there upward, because what we feel net in our hand is net force of f equal to m minus b w minus b, you feel it is feeling small you never feel w, you actually always feel w minus b.

See, whenever you are down in your holding a body in an under water, you are not able to isolate m, you are what you feel is total f this is why, but the body feels that pressure. So, the force will exist on the body always, you know that is that is the answer to that.

Having something in your hand under water moving it slowly bring nothing,, but you try to move it quickly.

That is another thing, that is a frequency dependence that when you do very slowly the amount of push is very less, so added mass become very small, but if you do in between, actually another theory is there, if you do very fast very fast, it will be also small, you see that is another another thing if you do very fast it turns out that the waves are very less, if there are some theories depending on the frequency.

Now, say that other thing that, I have to tell you is the damping, the damping is more interesting that is b, b as a function of omega that the wave creation, you know that that when you do that this wave get created.

It turns how the damping is always like that, very high frequency waves not damping, very low frequency waves not damping, why? You see, damping is connected to wave creation, now you will see that, if you take a body takes all the time to go up and down, there will be no waves.

So, there is no damping force, no waves created no energy, you know if you take and take all your time to go up and down. If you do very fast, what will happen you will find that it makes repels, just repels, small repels, you know like very small repels, they do not have much energy, because energy is proportional to also length somewhat.

Therefore, both ends it is 0, but in between there is not 0, this is actually a prove because in between you know it does create waves, so therefore, it must be 0 here 0 here, but in between something else.

This is actually one of the examples to show that, added mass and damping are depending on frequency, so you cannot find out for a ship added mass is 10, what frequency. You see when you say added mass is you know, this ship has added mass of you know 10000 ton does not make sense.

Generally, speaking mostly in vibration you will say that, because in vibration you assume it is a high frequency value. In vibration if you assume it is high frequency, but in a ship motion you cannot say added mass is so and so, you have to say I will ask you what frequency, so it is frequency dependent phenomenon, it depends on frequency.

This is why you cannot find a and b just like that, you have to find it for very you know over the range of frequencies, that is very important part of this added mass and damping. Any how, now exciting force is more complicated, so I am not going talk about exciting force right now, but what is important here to talk is one part called this response, this part is very very important, I want to just show you from here.

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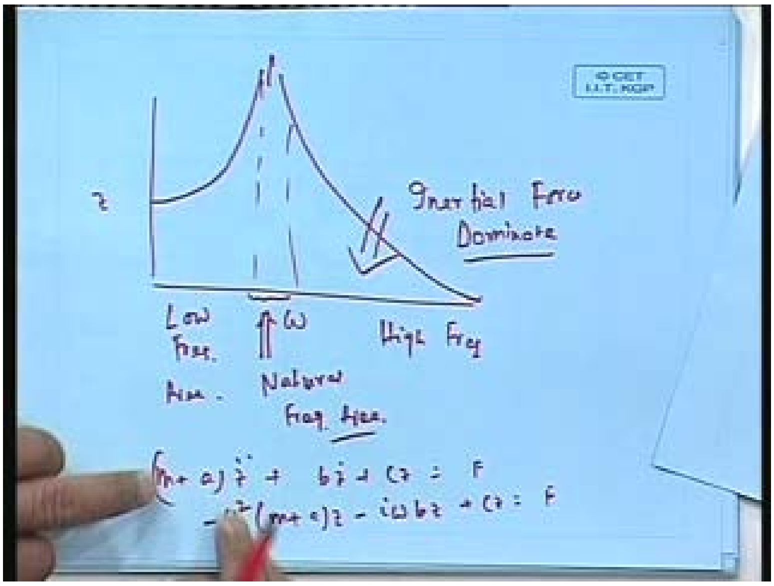

Now, typical response curve, if you see this is very interesting part see omega of versus the response z how it looks like. Now, we have seen that it actually looks something like it goes like that and comes down like that.

You know, this goes to almost infinity, now one can break it in three parts, actually one can say that this is in part near resonance, this part is near resonance, this part this central part, I will say the low frequency area this this can be low frequency, this is high frequency and this is the natural frequency area.

Why I say this? There is a there is a reason you know you take this equation m plus a z dot dot plus b z dot plus c z dot equal to f. Now, z dot dot is equal to proportional to omega square, so I can write this to be something like omega square m plus a z, because you you can find out z dot dot is omega square z with a minus sign or something, a minus I can write it here I omega.

I mean let us forget it, b z plus c equal to f, what I mean is that added mass posses proportional to omega square, damping is proportional to omega and of course, this is proportional to just like that, there is no omega there, why I say that? You see, even omega is equal to very high then obviously, this is the very large number, this is also large number compare to this, so this becomes the dominating force.

So, here at this region it is inertial force dominating that means, in here you you have to find out added mass damping carefully, because this force this force are more important, you can have large error in this, that does not matter.

You see, in high frequency phenomenon becomes like that, low frequency phenomenon if you go here, then omega is very small, so omega square is a very small, so you can neglect that, you can neglect that. In fact, it becomes only this in fact, it will always go to z by A will always become one low frequency long wave by ships simply goes up and down with a wave, just up and down with a wave in a very long wave that is a common phenomenon.

Why it is so? Because you find out that, this is small you can neglect that, this is small you can neglect that, so it is like c z equal to f, so f equal to f by c that is all, and you can one can prove that, that become actually equal to 1 z becomes equal to you know no magnification factor mu equal to 1.

So, this becomes dominated by what is called hydrostatic force? So, this area static force dominate, this here inertial force dominate what dominates here? This is the natural frequency area, where that omega omega z omega by omega z equal to tending to 1 at this area, omega z depends purely on a b by or c, I mean you know like natural period at this.

In fact, it depends on root over of what you call c by m, natural period forget it, this part gets dominant purely if you see by damping. If there is very low damping, actually theoretically there is no damping this will be infinity, but in ships you have damping, so it becomes like that, you know small.

So, this is a part, where damping comes in and I will just tell you a example, a next time with respect to roll why you put and all kind of staff. Because, if you did not put it would have roll like crazy, you see you know like when theoretically anything, theoretically if there is no damping it goes to infinity, I will come to that may be in the next part of the lecture.

See, now this is at the typical how it looks like a frequency domain response, how it looks like? It heave this is very important to know, I am talking only of heave just to give an example, very long wave the ship just goes up and down hydrostatic force is dominate do not worry about an hydrodynamics.

Make all mistakes, forget added mass damping you just take any number, you will be still close by go to the other end that means, suppose there are ships, you know say lengthwise there is a ship of 100 meter long? If you take waves three times or more then it will be this side, if you take waves of 100 meter or may be 60, 80, 100, 150, then your somewhere at this side.

You cannot afford to make mistake, so much in a and b added mass damping. It becomes a dominate force there and of course, damping is the most difficult part to estimate, so what we do normally is that, we only find out what is natural frequency and try to ensure that you avoid natural frequency, I will come natural frequency is so crucial in our design and motion that I will discuss that little more at length in the next part of the lecture you know. So, now, I will stop this part of the lecture.

Preview of next lecture

Lecture no. # 28

Ship motion in irregular waves - I

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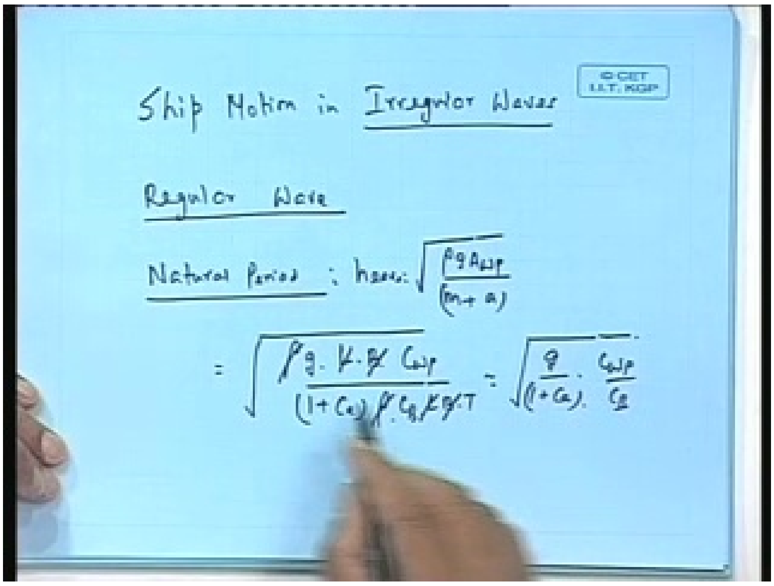

See, now a natural sequel to motion in regular wave would be motion in irregular wave. So, although I am going to call this Ship Motion in Irregular Waves, I will still pickup little bit on the regular waves part and continue.

See, again motion in irregular wave obviously, should follow study of motion in regular waves. So in fact, we have to continue little bit on motion in regular waves, because some of these parts are very crucial in order to study irregular wave part.

So, I will still continue some small discussion on regular wave motion in regular wave, what I will do, see let us talk about some simple thing of natural period, why it is important? Now, natural period is turns out to be equal to for in heave equal to rho g a, I mean just writing the formula a w p by mass plus added mass, this is the formula.

Now, if you take a typical ship in fact, we can work it out a various waves, but if you take a typical ship value, let me work it out here rho g A w p is actually L into B into water plane area coefficient. Mass plus added mass can be written as 1 plus C a into mass, mass is rho into C B into L B into T root over of that.

See, I am just breaking it down just for benefit C B into length breadth draft into rho gives you mass into 1 plus added mass coefficient give you this, this is rho into g A w is L B since if you so. If you do that cut this if you cut that then rho gets off then, it becomes proportional to g by 1 plus C a g by 1 plus C a C w p by C B.

I think now there T there, g C w p T will be there, now if you work it out for a typical ship you know, you will find that this is almost like a one, see this will be almost like a 1, now T let us say C a is actually also 1, so it is 2.

Let us say, it is you know 1, so it is 2, so 9 10 by that is a 10 by you know it is 10. So, you can say, if g if you take g to be 10, 9.8, you know approximate 10, 1 plus C a to b 2, this it is becomes 5, so it is 5 by T root over of 5 by T you take any T.

Can be broken to many sine curves, that point remains how to break it down and there are now absolutely standard mathematics flourier analysis etcetera, to do that, this is irregular wave input.

Now, I have to have an output, this is my waves, this is my irregular motion, say motion. This also now, this also look something like that, so this can also be broken down like this. This can also be broken down like this.

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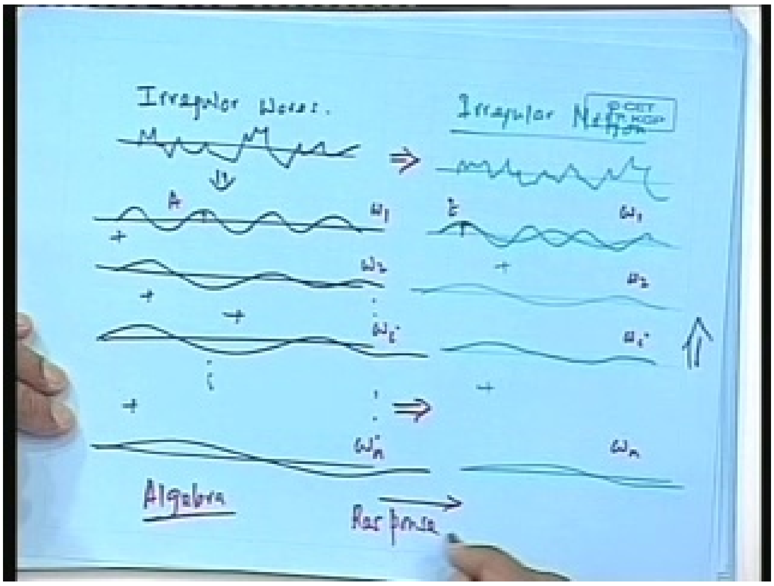

Now, you see oh sorry I step broken it down in the same frequency. Now, let us say that this frequency is we have broken it down to a frequency omega 1, this omega 2 like that this is omega say i like that, this is a omega n number of frequencies.

Here also, I can break it down to this same one, omega n now what happens? This gives me this; this input gives me this output. Now, I say that this input is nothing but, this plus, this plus, this plus, this, which I have done by spectrum (Refers Slide Time: 54:12).

Now, what I do is that, I find out that if this was my regular wave, what would be my output? That is, if this was my see say you know A what is my z here that means, this by this for that frequency, because that is what I have done.

See to get this from this was what I so far talked, that is how it behaves in a regular wave that means, supposing this is my regular wave or amplitude a 5 meter what would be my heave. So, going from here to here this way is finding out response then a regular wave.

So, what I did I break it down to this sine components for each one I find out, if this was a wave then what would be my regular, the if this is regular wave, what it my regular heave motion, so I break it down to this then add them up to go there.

So, you see the algebraic, this is the picture that you must I mean keep keep in your mind, this is broken to this piece. So, this way then next step, I go from here to here then next, I go from there to upwards.

So, you see I have this break it down to these pieces, what is this breaking down? This is basically this breaking down is algebra pure, you know what is this going from here to here this is actually response in regular waves. A regular wave given what is the output, what is this to this, another algebra just adding them up. So, you see getting here to here is not so tough. It is actually in your mind very simple, because if I can break it down to these pieces, which I have done in a spectrum.

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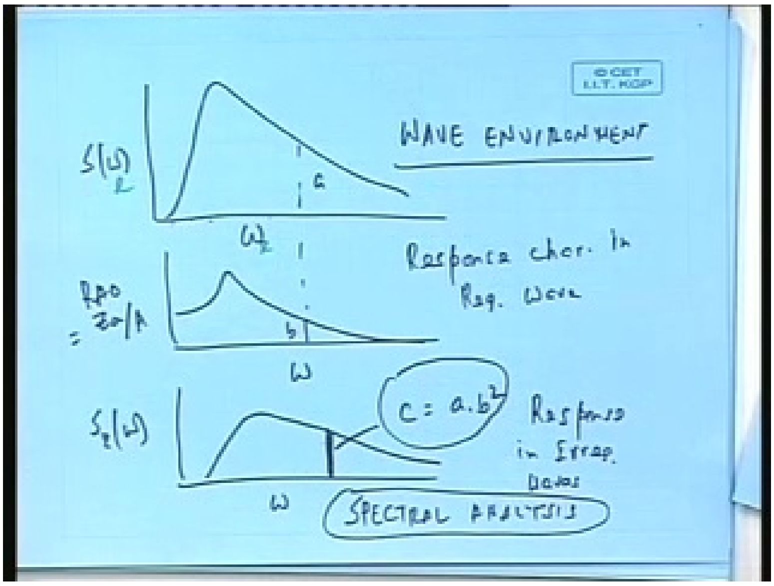

Fixed reference point, because you have taken the record at a fixed point, but ship is moving. So, the all those irregular wave what the ships sees each one, see this one is broken down to number of waves. But, the ship that this same wave would feel to the ship to be something else, because this omega will feels to be, this omega I will feel at omega e i this omega I will be feel as omega e i.

Because, supposing this is composed of let me give an example, a 10 second wave plus a 12 second wave plus a 14 second wave. For the 10 second wave, when the ship is moving at 20 knots we will feel like an 8 second wave.

That 12 second wave will feel like something else, so first of all although I said that, this has to be transformed initially to how the spectrum would be if you are on the ship or as the ship would feel it. I will actually go to that next class, but I will just tell you why it is important? See, again there is a ship here, it is stationary here there is an observation point here, the ship is stationary and if you observe the wave looks like that.

But, if you now move, then same observation point is changing, so if you are moving now, then this same thing will get squeezed, so this is in kind of omega domain this is in omega e domain. So, first of all if this looks like something like this, this should look little more spread, because it will be stretched omega, these each omega is suppose, to changed to omega e i that means, I have to first change the wave measured that is spectrum given, which is for a stationary point to an observer sitting on the ship.

So, there is a transformation there, before I can start this procedure, that I mentioned last you know a before see here, I mentioned this to this, I have one step, I have to first change this to omega e first I have to change that to omega e.

So, first from this to omega e, that I will discuss next class, but that is also algebra, but you have to know that you know, because after all we are depending on how you move this itself changes and when this changes this combination will call the change.

So, this also as an effect on how you actually can tune or you knows like, just last thing is that supposing a ship sea state force, there is a peak wave coming at 10 second. Now, if you are going at a different speed that 10 will appear 8 second and suppose your resonance is 8 seconds, then you are going to be bad badly hit, change the speed you make it 9 second it will not is so bad.

So, we will discuss that in next class the the response part at more length, so today we will end it here, Thank you.