

**Performance of Marine Vehicles At Sea**  
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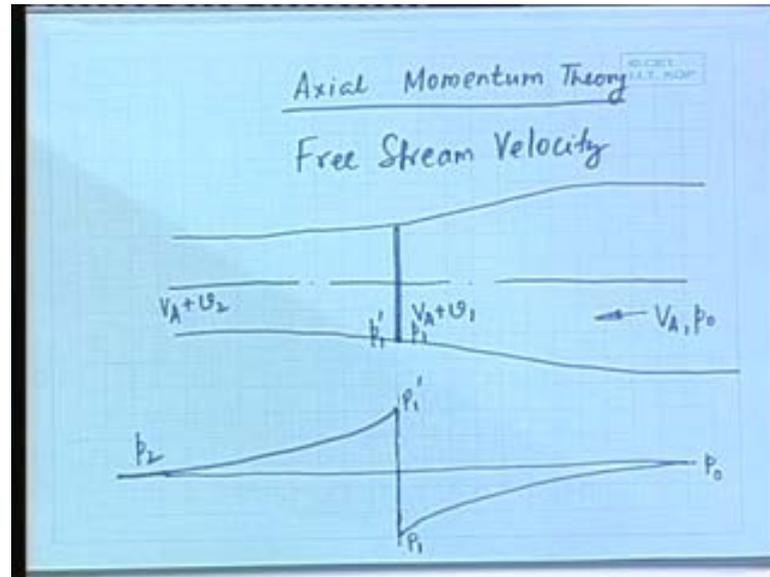
**Lecture No. # 18**  
**Propeller Theories Part - I**

Good afternoon. Today we will talk about Propeller theories, that is, how a Propeller works in water. There will be two lectures in this and the intent of these two lectures is to understand the basic principles of Propeller action rather than going into details and trying to design Propellers based on the theories. In case you are required to design a Propeller using these theories, you have to go deep into the subject and study further, so that you can use it for design purposes.

Now, we will try to understand the basic principles of how a Propeller works in water and how it generates thrust, that is, a forward force which propels the ship forward. Beginning, in the beginning of nineteenth century, propellers came into being used in ships - screw Propellers. They are conventionally called screw propellers because the principle of Propeller action is like that of a screw.

The theory of Propellers action at that time was not understood at all, but it seem to work. Today we understand Propeller theories, and therefore, Propeller action in water in a much better perspective, but I must also add that perhaps our knowledge of Propellers action in a fluid medium is still incomplete. One of the earliest theories of Propeller action was proposed by Rankin and also by R E Froude somewhere around the beginning of nineteenth century. Surprisingly, even that, even though that theory was very simplistic in nature, the conclusions drawn from that theory still hold good and this has shown the way to go move into higher theories later on.

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So, the first theory that was proposed by Rankin and R E Froude was called the so called axial momentum Theory. Can you see? Axial momentum theory where the propeller was considered as an actuator disc instead of consisting of blades; the propellers was considered as if it was a circular disc which was rotating in water. This action of the actual actuator disc was suppose to increase the pressure filed in the fluid across the propellers disc, and therefore, generate a thrust. What was not explained in this theory is how does the propeller change the pressure field. It was just assumed that there will be a change of the pressure field across the propellers disc.

Now, just imagine that there is a propeller, a disc moving in water like this, and if the Propeller moved forward, then we could assume as if the Propeller is standing still and water is moving backward such water, if the propeller was just moving forward like this and we considered it to stand still, then we could think that the water is flowing in the opposite direction in equal magnitude, such velocity was called the Free Stream Velocity.

That is, Free Stream Velocity of water is equal and opposite to the forward speed of the Propeller. If there was a ship, it would have been equal to the forward speed of ship, but in the opposite direction. So, therefore, whenever we are talking of water velocity, we will inherently mean the water flowing past in the opposite direction and the propeller is standing still.

Now, this disc is rotating without any forward motion of its own; it is rotating and water is flowing past it. When that is happening, we would assume that the pressure changes from one side to the other across the propeller disc. It is not explained how this pressure change takes place. This is something like a black box a propeller which changes the pressure from one field to another by its action of the rotation of the propeller disc. Now, far forward of the propeller, if the water is flowing past this way, far forward of the Propeller, the pressure is say  $P_0$ ; there is no change in pressure; also far aft of the propeller, pressure would come back to normal again say  $P_0$ .

But across the propeller disc, there is a change of pressure; that means, as if the pressure increase slowly to a large value or decrease slowly to a large value just before the propeller disc and then it increased across the propeller disc. There was increase in pressure and it came down gradually to  $P_0$  again. Now, if I draw this diagram, how would it look? Let us say this is where my Propeller disc is working. There is a water velocity; we will call it  $V_A$  which is the Free Stream Velocity. Water is flowing past this propeller disc at a speed  $V_A$  that is the Free Stream Velocity.

Now, what we are saying is, if this is the position of the propeller disc, how will the pressure change. Pressure we said here would be  $P_0$  let us say at far forward of the propeller and it is dropping to a value, negative value just before the disc where it has a increase in pressure, certain increase in pressure due to propeller action, and then, this increase in pressure will reduce further to the same value  $P_0$ . We can call this pressure  $P_1$  and this pressure  $P_1$  dash, that is, before the propeller disc, the pressure was  $P_1$ , and the after the propeller disc, the pressure is  $P_1$  dash. Is that clear?

Now, what happens to velocity? We know when there is a change of pressure, there has to be a change in velocity. Now, since there is a increase in pressure here, there would be a decrease in velocity initially, and then, due to this increase, this increase in pressure, there will be a decrease, but you see this, **this**, pressure is dropping here; that means velocity is increasing. There is something that happens here and then again pressure is dropping. So, velocity will increase again. Is that clear? Also we know that fluid will observe the continuity principle, that is, fluid cannot be created or destroyed and the flow will be continuous.

So, therefore, there cannot be an abrupt change in velocity across the disc. Do you understand me? There cannot be an abrupt change in velocity. There will be a gradual change in velocity, no doubt, but there cannot be an abrupt change in velocity. Now, you see here, if the velocity was  $V_A$  here, what would be the velocity at the disc? See this pressure, it is reducing. Therefore, velocity will go on increasing till the disc; also beyond the disc, there will be a slight increase in velocity because drop in pressure, and across the disc, velocity will be constant. There will be no change in, abrupt change in velocity here.

Whatever change is there, it will be gradual. So, generally there will be increase in velocity as if the Propeller disc is pulling the water from forward and pushing it aft. Am I clear? Yes. So, what will happen? If we assume as if the water is in a cylinder as a mass of water is moving like this, this velocity is increasing; mass has to be conserved; that means, as if the water is getting constricted, you understood? As if the water is getting constricted and going forward.

So, if I draw that out, it would be like this. As if the water here was coming and it gets constricted at the Propeller disc and slight constriction even after that and that is how the velocity will be. Is not it? So, we can say, if we say the pressure was  $P_0$  and velocity was  $V_A$ , then at the propeller disc, velocity will be slightly more than the speed of advance, and here, again it will be also slightly more than speed of advance. Let us call this  $V_2$ . The incremental velocity  $V_1$  is at the propeller disc and the incremental velocity  $V_2$  is at the far end, far aft. Is that clear? And pressure, we can call it  $P_2$  here,  $P_0$  here and  $P_1$  to  $P_1$  dash across the propeller disc.

Sir

Yeah

Well, we explain that. We really do not know how the propeller works. This theory does not tell how the pressure increase takes place. We have only assumed that there is a abrupt change in pressure because of the propeller action. This abrupt change in pressure will affect velocity, no doubt, but generally because of constriction of the slip stream, there will be a increase in velocity which will continue across this pressure field will affect  $V_1$  and  $V_2$ , but there will be increase in velocity all the same. Is that clear?

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$$P_0 = P_2$$

$$\text{Mass of fluid } m = \rho A_0 (V_A + U_1)$$

$$\text{Thrust} = m [(V_A + U_2) - V_A]$$

$$= \rho A_0 (V_A + U_1) U_2$$

$$\text{Power} = T (V_A + U_1)$$

$$= \frac{1}{2} m [(V_A + U_2)^2 - V_A^2]$$

$$= \frac{1}{2} \rho A_0 (V_A + U_1) [U_2^2 + 2U_2 V_A]$$

$U_2 [U_2 + 2V_A]$

Now, I have already told you that the pressures for a far forward and far aft will be same as, as, the static pressure. So, what we have here -  $P_0$  equal to  $P_2$ . Now, what are the assumption we have made so far or what we are going to make as we along to find the efficiency of a Propeller based on this principle? We have made some very drastic assumptions. One assumptions we have already mentioned that there is a change in pressure and we do not know how it is.

We have assumed that the velocity field is constant across the disc, that is, at any point on the disc along the entire circle, the velocity here is  $V_A$  plus  $V_1$ . That is constant if there is no variation of velocity across this. This we have assumed. We have assumed or we will assume now that there is no viscosity. So, the flow is ideal, and further more in this diagram, whatever I have told you so far is the change in velocity only in the axial direction.

Though the disc is rotating, we will assume right now the all changes in velocity are only in the axial direction as shown in this diagram. These are very drastic assumptions, but we have to start from somewhere. So, we make these assumptions as it was done by Rankin and R E Froude. Now, what is the mass of fluid passing through the propeller disc? Let us call it  $m = \rho A_0 (V_A + V_1)$ . Is that correct? What is thrust? Rate of change of momentum though mass into  $V_A$  plus  $V_2$ , please see the diagram, minus, no,

$V A$  only  $V A$  plus  $V^2$  here and  $V A$  here. So, the rate of change of momentum in the entire fluid will give us the thrust force generated.

So, how much does it come to,  $\rho A_0 V A$  plus  $V^2$ . Am I right? Now, power delivered by the propeller. What is the power delivered by the propeller is same as rate of change of momentum, rate of change of circular momentum, rotational momentum. Do we have any rotational momentum? We do not have rotation of the fluid. So, we cannot use that. Am I right? We have not got a rotational energy put. One of our assumptions is that fluid is not rotating.

So, we will say power is work done by the thrust. So, what is the thrust, that is,  $T$  into, what is the work done? Across the disc  $V A$  plus  $V^2$ . So, this is the work done by the thrust. This is also equal to change in kinetic energy, rate of change of kinetic energy that will be half  $m V A$  plus  $V^2$  whole square minus  $V A$  square, and we can write half  $m$  we write  $\rho A_0 V A$  plus  $V^2$  into, **into**, how much?  $V A$  square will cancel. So,  $V^2$  square plus  $2 u^2$  into  $V A$ , which you can write as  $2 V^2$  plus  $V A$  by  $2$  or  $u^2$  into  $1$  plus  $V A$ ,  $V^2$  into  $1$  plus  $2 V A$ ; it is wrong. So, it will become  $V^2$  into  $V^2$  plus  $2 V A$ . So, now, these two will be equal, is not it? Work done by the thrust and the kinetic energy change, rate of change of kinetic energy, they are same; they represent power.

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Handwritten mathematical derivation on a whiteboard:

$$T(V_A + u_1) = \frac{1}{2} \rho A_0 (V_A + u_1) u_2 (u_2 + 2V_A)$$

$$= \rho A_0 (V_A + u_1) u_2 (V_A + u_1)$$

$$\frac{1}{2} [u_2 + 2V_A] = V_A + u_1$$

$$\boxed{u_1 = u_2 / 2}$$

$$C_{TL} = \frac{T}{\frac{1}{2} \rho A_0 V_A^2}, \quad A_0 = \text{Disc Area} = \pi r^2$$

Thrust Loading Coefficient

So, if you put that, what do we get?  $T$  into  $V_A$  plus  $V_1$  is equal to half  $\rho A_0 V_A$  plus  $V_1$  into  $V_2$  into  $V_2$  plus  $2 V_A$ . What is  $T$ ? We have got  $T$  or this is equal to  $I$  can write the  $T$  part  $T$  into  $V$  again. What is  $t$ ?  $\rho A_0 V_A$  plus  $V_1$  into  $V_2$  into  $V_A$  plus  $V_1$ . Now, see equate it, this cancels, this cancels,  $\rho$  cancels,  $V_2$  cancels. So, you have half  $V_2$  plus  $2 V_A$  is equal to  $V_A$  plus  $V_1$ . Now, you can see this also cancels. So, what do you get?

(O)

No

This is what you get. This is a very interesting observation. What does it mean? The increment of the velocity at the disc is half the total increment in velocity between far fields. Do you understand? Is the total increment in velocity is  $V_2$ , then the velocity increase of the disc is half of that. Do you get it?

Yes sir

Now, this thrust, we have got the thrust  $T$ . Now, if we put it in non dimensional, form the thrust so called thrust co efficient. This is called thrust loading coefficient –  $CT_L$ .  $T$  divided by half  $\rho A_0 V_A$  square. What is  $A_0$  here? Disc area and is proportional to radius square or diameter square whatever.

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Propeller Efficiency  $\eta = \frac{\pi \cdot V_A}{P(V_A + U_1)}$   
 $= \frac{1}{1 + U_1/V_A} = \frac{1}{1+a}$   
 where  $a = U_1/V_A$   
 $a$ : axial flow factor  
 $\eta = \frac{1}{1+a} = \frac{2}{1 + \sqrt{1 + CT_L}}$   
 ①  $a > 0 \rightarrow \eta < 1$   
 ② For higher  $\eta$ , increase Propeller dia.

Let us remember this. Now, let us find out Propeller efficiency from whatever we have done. What is the efficiency of a propeller of any device? Output divided by input, and what is output power? Output power is, this is the output power. This is not the power we calculated from water. This is the power that overall the power that is being used, that is, thrust in a speed Free Stream Velocity of  $V_A$ .

So, the useful power, if we had a ship, the ship would have taken this thrust power  $T$  into  $V_A$ , and the input power on the other hand is what we have calculated. Do you get it? That is,  $T$  into, what have we calculated? We have calculated the input power know  $V_1$  plus  $V_1$ . This is the power of the, this is the thrust power generated by the, is inherent in water and this is the power we are using. This is the useful power; this is the input power.

Now, what is this?  $1$  divided by  $1$  plus  $V_1$  by  $V_A$ . This we can write as  $1$  plus  $a$  where  $a$  is equal to  $V_1$  by  $V_A$ . Now, this  $a$  is called the axial flow factor, that is, the ratio of incremental velocity of water at the disc to the Free Stream Velocity. Do you understand? So, we can also represent efficiency as  $1$  plus  $1$  divided by  $1$  plus  $a$  which is also equal to  $2$  divided by  $1$  plus  $1$  plus  $C_T$ .  $C_T$  we just defined. We have defined the thrust coefficient  $T$  divided by half  $\rho a V^2$ .

Now, you can derive this  $a$  is  $V_1$  by  $V_A$ . We have got  $V_1$  and thrust equation we have given. So, if you use all those values that are given, you can show that efficiency can also be represented in terms of thrust loading coefficient. What are the inferences we are getting from this representation of efficiency? We can derive very interesting observation from here. Whenever we are talking of designing a propeller, we are interested in having a highly efficient propeller. Is not it? We want to increase efficiency.

So, we must have a limit to what efficiency we can have from the Propeller, and you can see from here that  $a$  is ratio of incremental velocity to Free Stream Velocity, which is always positive is less than  $1$ , but it is positive; that means, the velocity actually increases across the disc. This  $V_1$  is always positive; that means that is an increment of axial velocity. There is no decrease, because of this type of pressure distribution, there is always an increase in velocity. Therefore, this  $a$  that we have written is always greater than  $0$ , greater than  $0$ . What this means? Efficiency is always less than  $1$ . It is not equal to  $1$  nor greater than  $1$  as per this simple axial momentum theory, understood?

The second conclusion we can draw from here is also very interesting. How do we increase the efficiency? We have to increase the axial flow velocity, incremental velocity. We have to reduce this. If you reduce, this a reduces and this increases. Is not it? But that is very difficult. We cannot do anything with the disc is working. There is some incremental velocity. How can we do? How can you control that? Let us look at this equation.

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The whiteboard shows the following equations:

$$T(V_A + U_1) = \frac{1}{2} \rho A_0 (V_A + U_1) U_2 (U_2 + 2V_A)$$

$$= \rho A_0 (V_A + U_1) U_2 (V_A + U_1)$$

$$\frac{1}{2} [U_2 + 2V_A] = V_A + U_1$$

$$U_1 = U_2 / 2$$

Below these equations, the Thrust Loading Coefficient is defined as:

$$C_{TL} = \frac{T}{\frac{1}{2} \rho A_0 V_A^2}, \quad A_0 = \text{Disc Area} \propto D^2$$

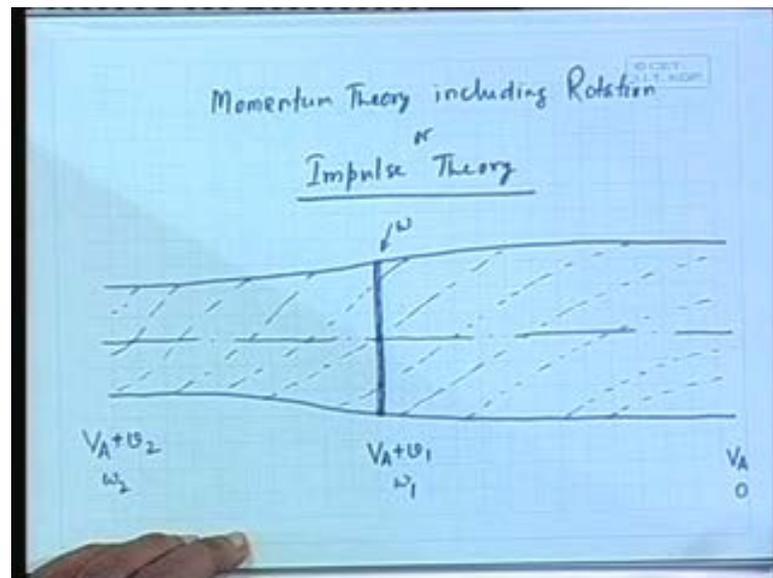
The text "Thrust Loading Coefficient" is written below the equation.

Here, to increase efficiency, what we have to do? This is the only variable we have with us. Should we increase or decrease? Decrease; that means, if we look at the coefficient, this quantity has to be decreased. So, to decrease this, we have to reduce thrust. Now of course, we cannot reduce thrust because thrust is required to give a particular speed to the ship; our thrust required is constant. We cannot reduce that. If we want a particular speed, we cannot reduce thrust. The Propeller to design is for this thrust if we reduce this thrust designing.

Look at the bottom  $V_A$  is constant speed. So, the only thing we have to play with is  $A_0$ . If we want to reduce thrust loading coefficient, the only option we have is increase  $A_0$ . What is  $A_0$ ? We have written here is proportional to or diameter square; that means, the bigger the propeller, the higher will be your efficiency. Do you get? So, that means this is conclusion one and the conclusion two is for higher efficiency increase propeller dia.

Now, these are the two major conclusions that we get from this simple axial momentum theory. These two conclusions still hold and they are valued. But as you might have noted we have made very drastic assumptions for this theory, is not it? and one of the main assumptions we have made is that there is no rotational increment of velocity; that means, as if the water is only having straight line velocity, but if we have a disc, of course the water will also rotate, is not it? You cannot avoid that.

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So, a modified theory was proposed later after R E Froude, and again, sometime in nineteenth century only which was called as momentum theory including rotation or it was also known by another name Impulse Theory, which had all the assumptions of the original momentum theory except this one. That it also suggested there, **there**, has to be an increment in the rotational velocity of fluid.

The axial velocity will change as has been suggested earlier, that is, slow change will be there. There will be a change in pressure and then velocity will change for the downstream. So, we have a  $V_1$  incremental velocity at the propeller disc and  $V_2$  incremental velocity far away from propeller disc. In addition to that, there has to be a change in rotational velocity. Now, far forward of the propeller, there is no rotation of fluid. There would be a rotational velocity imparted on the fluid because of the rotation of the disc. That rotation if it was  $\omega$ , then we can say at the disc, there can be additional fluid velocity, rotational velocity on the fluid as  $\omega_1$ , and far field, there

will be a rotational fluid velocity  $\omega_2$  would like to see what is the relationship between  $\omega_1$  and  $\omega_2$  and how it effects efficiency.

So, axial momentum theory deals with that. Now, to understand the flow with rotation, I can only make a simplified diagram. This is the propeller disc and you have a flow field here  $V_A$  Free Stream velocity; axial velocity increases by an amount  $V_1$ , and here, the axial moment velocity increases by an amount  $V_2$ . The angular velocity here is 0; here it is  $\omega_1$  and here it is  $\omega_2$ . The disc is rotating at a speed  $\omega$ .

The disc does not have a forward velocity. Disc velocity, disc is not having a axial velocity, but it is having a rotational velocity which is  $\omega$ . Now, we have seen that there will be a contraction of the slip stream across the disc, and this is the slip stream which is contracted the disc and goes far field. The rotation if we just think like this that the water is also rotating as it moves forward. So, if I represent it, how will it look? Like that as if water is also having a tangential velocity not only an axial velocity, but also a tangential velocity. Again, we must understand that how this tangential velocity is created or how the pressure change takes place is not explained in this theory, but however, it also gives us very interesting conclusions. Therefore, we should be seeing this.

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$\omega$ : rotational speed of disc  
 $dm = \rho dA_0 (V_A + U_1)$   
 $dT = dm [(V_A + U_2) - V_A]$   
 $= \rho dA_0 (V_A + U_1) U_2$   
 $dQ = dm \cdot \omega^2 [U_2 - 0]$   
 $= \rho dA_0 (V_A + U_1) \omega^2 U_2$   
 Workdone<sup>W</sup> elemental Thrust  
 = change in translational K.E.  
 $dT (V_A + U_1) = \frac{1}{2} dm [(V_A + U_2)^2 - V_A^2]$   
 $= \frac{1}{2} \rho dA_0 (V_A + U_1) (U_2^2 + 2V_A U_2)$

So, there is a slow increase in rotational velocity which we have already explained -  $\omega$  is the rotation, rotational speed of the disc  $\omega$ , and  $\omega_1$  is the incremental rotational velocity at the disc itself, fluid, of the fluid at the disc itself and  $\omega_2$  is incremental rotational velocity of the fluid at far aft. So, now, here to appreciate thrust etcetera, we will assume an elemental area rather than the whole disc area because the rotational velocity will change as per radius rather the moments etcetera will change as per the radius.

You have to understand now that we will now have a rotational momentum and rate of rotational change of moment, rate of change of rotational momentum etcetera which will also contribute to power, which we did not have in the previous case, and that moment from the axis will depend on the radius. So, we will now consider not the full disc area, but we will consider an elemental area  $dA$ . So, the mass flowing through the elementary, elemental area  $dA$  will be  $\rho dA (V_1 + V_2)$

You remember the assumption we made that across the disc velocity, axial velocity is constant everywhere. We made that assumptions still valued. So, for an elemental mass of fluid passing through an elemental area will be  $\rho dA (V_1 + V_2)$  across the disc. Is that correct? And what is the thrust, elemental thrust due to this area -  $dF = \rho dA (V_2 - V_1)$  rate of change of momentum. So, you have  $\rho dA (V_1 + V_2)$  into  $(V_2 - V_1)$ . Is that correct?

And what is the torque of the fluid now since there is a rotation? Rate of change of rotational momentum which we can write as, you tell me if you understand this or not.  $\omega_2 r$  is the speed at that radius;  $\omega_2$  is rotational speed. So, when you convert it to tangential speed, it becomes  $\omega_2 r$ . So, the momentum will be  $\omega_2 r^2$ . Rate of change of momentum from here to here is  $dF r = \rho dA (V_2 - V_1) r$ . So,  $dM$  you write  $\rho dA (V_1 + V_2)$  into  $\omega_2 r^2$ ;  $r$  being the distance of the elemental area from the axis of the propeller. Is that clear?

Now, work done, we have already seen work done by elemental thrust. This we have done in the previous case. We have said this is equal to change in kinetic energy. Is not it? We write here change in translational kinetic energy, not the rotational kinetic energy. Rotational kinetic will be equal to the work done by the torque. You understand there are two sets of velocities - one is the transition translational; other this is rotational.

So, the work done by elemental thrust, thrust is a linear force. So, work done by the elemental thrust will be equal to rho change of energy due to change of kinetic energy due to axial velocity. So, if we put that, what do we get?  $dT V_A$  plus  $V_1$  is the work done at the disc is equal to half  $d m V_A$  plus  $V_2$  square minus  $V_A$  square, is not it? Can we simplify? This we can write half rho  $D A 0 V_A$  plus  $V_1$  into  $V_2$  square plus  $2 V_A V_2$ . Now, if we put as we did before, if we equate is the same equation actually, what we did last time and this equation is same and we can show that  $V_A$  plus  $V_1$  is equal to  $V_A$  plus  $V_2$  by 2 or  $V_1$  is equal to  $V_2$  by 2, that is, even when you are imposing a rotational velocity, we see that far field increment in axial velocity is twice the axial velocity increase at the disc. Is that clear?

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The image shows a whiteboard with the following handwritten equations:

$$V_A + U_1 = V_A + U_2 / 2$$

$$\text{or } U_1 = U_2 / 2$$

$$dQ \cdot \omega_1 = \frac{1}{2} dm \cdot \omega_2^2 [U_2^2 - 0]$$

$$= \frac{1}{2} dQ \omega_2^2$$

$$\text{or } \omega_1 = \omega_2 / 2$$

$$\eta = \frac{dT \cdot V_A}{dQ \cdot \omega} = \frac{(\omega - \omega_1) V_A}{1 + U_1 / V_A}$$

$$= \frac{1 - a'}{1 - a}$$

Now, you do this for the torque. What is the work done by the torque? Work done by the torque will be  $dQ$  into  $\omega_1$  at the disc. The fluid is having a torque  $dQ$  elemental torque and rotating at a speed  $\omega_1$ . So, this is the work done by the torque is equal to rate of change of kinetic energy, rotational kinetic energy. So, that is half  $d m r$  square into  $\omega_2$  square minus 0. So, if we put the values, we find half  $dQ$  into  $\omega_2$ . You remember  $dQ$ , we had derived  $dQ$  as this quantity.

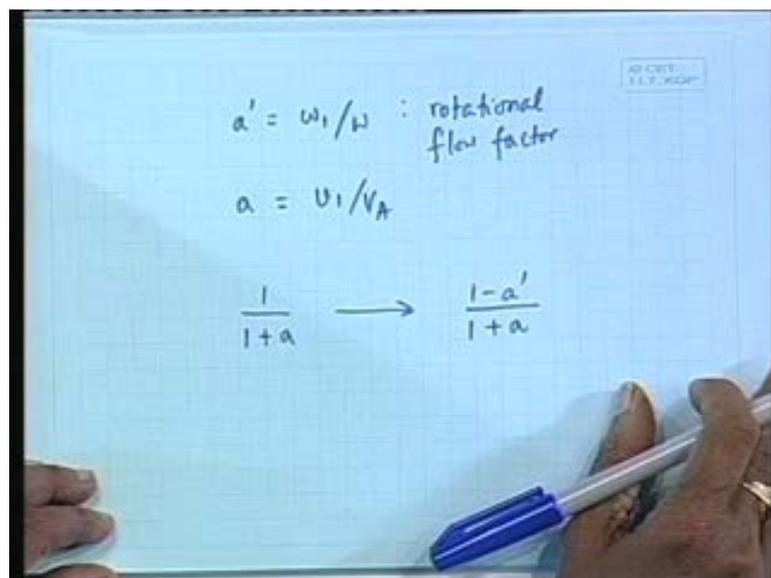
I have not gone through. We can easily work it out it will come to half  $dQ$  into  $\omega_2$  or this gives us the very interesting observation that, by this theory also we find that incremental rotational velocity far field is twice the rotational velocity of fluid at the disc

itself, is that clear? So, the law holds for both axial velocity and rotational velocity, that is, at the disc, the increment in both axial velocity and rotational velocity is equal to half of the increment in the axial velocity and rotational velocity far field, is that clear?

Yes sir

Now what is efficiency? Now, the Propeller efficiency will be equal to output by input which we can write as  $d T$  into  $V A$  divided by  $d Q$  into  $\omega$  which also we can simplify from the previous derivations we have already done as  $\omega_1$  minus  $\omega_2$  into  $V A$  divided by  $1 + V_1$  by  $V A$  and this can be shown to be  $1 - a'$  divided by  $1 + a$ , where  $a'$  is equal to  $\omega_1$  by  $\omega_2$  which is called the rotational flow factor.

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As already we know is the axial flow factor which is equal to  $V_1$  by  $V_A$ . So, thus you see the efficiency of a propeller when we include the rotational velocity changes from simple  $1$  divided by  $1 + a$  to  $1 - a'$  divided by  $1 + a$ .

Yeah, ok.

So, the previous step also this should be  $1 + a$ . So, this efficiency changes by due to the rotational velocity. So, now, you can see the propeller efficiency depends on the

incremental velocity of the due to axial velocity and also incremental rotational velocity. We have already seen that axial velocity increase will depend on the diameter of the propeller. The rotational velocity on the other hand will depend on the rpm of the propeller. So, the efficiency of the Propeller will depend on two factors, that is, the diameter and the rpm as shown in this equation. Thank you. We will stop here and we will start with more advanced theories in the next hour.

(Refer Slide Time: 45:30)

The image shows a whiteboard with the following handwritten equations:

$$V_A + u_1 = V_A + u_2/2$$

$$\text{or } u_1 = u_2/2$$

$$dQ \cdot \omega_1 = \frac{1}{2} dm r^2 [\omega_2^2 - 0]$$

$$= \frac{1}{2} dQ \cdot \omega_2$$

$$\text{or } \omega_1 = \omega_2/2$$

$$\eta = \frac{dT \cdot V_A}{\frac{1}{2} dQ \cdot \omega_2} = \frac{dT \cdot V_A}{dQ \cdot \omega}$$

$$= \frac{1 - \omega_1/\omega}{1 - u_1/V_A} = \frac{1 - a'}{1 - a}$$

We have already seen increase of diameter will increase this increase efficiency. What about the top? What will happen to omega? Should it be more or less? More increase efficiency. If it is more, numerator will increase.

Numerator will

Increase

Increase

You are right, but there is a small hitch here, that is, the, this sign. The actually a dash is less than 0, that is, if the disc is rotating this way, you, you, have to understand that water is flowing the other way. You got it just like axial flow ship is going this way flow is this way. If we assume, the ship to be stationary. Similarly if the disc is rotating in one way, then the flow is in the other way. There is a decrease in this flow because of the this

thing. There is an incremental;  $w$  and  $w_1$  are there.  $w$  is the flow of the disc.  $w_1$  is the increment from 0.

So,  $w_1$  is positive, but  $w_1$  is less than  $w$ . We will see that in the next theory; it is explained better, but at the present moment,  $w$  is the Velocity of the disc, and what is  $w_1$ ? Please look at this diagram, this diagram. We have got the  $w$  as the disc velocity  $\omega$ , and we have said from 0, the water Velocity increases to  $\omega_1$  and far down is  $\omega_2$ . What is the relationship between  $\omega$  and  $\omega_1$ ? Can the water have an incremental velocity equal to the disc velocity? The answer is no.

It is less than the disc velocity. So, what happens here? If you assume that, what happens to a dash?

a dash.

Sorry

Greater than

So, this will turn positive, this will turn positive. There is a positive effect of the rotational flow, and if we want to increase it, we have to increase, we have to decrease  $w$ . Do you understand? How do you decrease  $w$ ? That means reduce  $r P$ , that is, a propeller efficiency increases. We have seen before that if the diameter increases and propeller efficiency also increases if we reduce the rpm for the same thrust. If it is possible to reduce rpm, then propeller efficiency will increase.

This is still the case in propeller designs as you will appreciate those of you who know about propellers. As the ship size increases on loading on the propeller increases, we go for slower and slower engines, that is, for very large power, the Propeller rpm is of the order 70 to 80, whereas as the power reduces that we come for smaller and smaller ships, the rpm increases. You understood?

So, if we increase the diameter, propeller efficiency increases; if we reduce the rpm, propeller efficiency also increases. So, our aim should be designing propeller for increasing diameter and decreasing rpm so that we can get the required thrust. You cannot say that same propeller run at slower rpm and I will get more efficient behavior,

no, because you are not generating the thrust required. Are you understanding what I am saying? We will stop here and next lecture we will see the more recent theories. Thank you.