Applied Thermodynamics for Marine Systems

Prof. P.K. Das

Department of Mechanical Engineering

Indian Institute of Technology, Kharagpur

Lecture - 27

Introduction to Convective Heat Transfer Forced and Free Convection

Let us continue with our discussion of heat transfer. We have seen plain wall, then we have seen cylindrical geometry. Basically, we have seen heat transfer in an annulus. Then we can think of another application, where we have got a spherical geometry.

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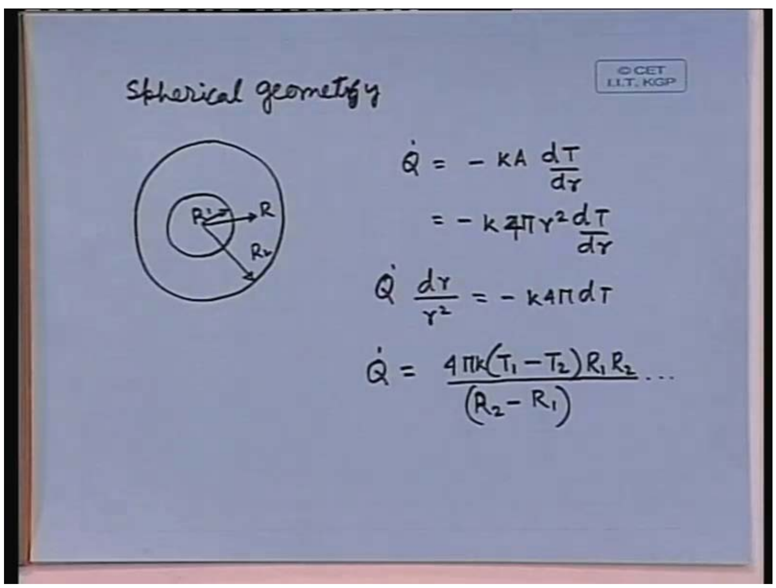

Spherical geometry: Here also, we assume that the thermal conductivity remains constant and there is symmetry as far as heat transfer is concerned. That is why heat transfer will take place only along the radial direction. One can think of spherical cells. It is very difficult to have a three dimensional diagram. So, basically what I am drawing is a spherical cell, section taken. This is the outer cell and this is the inner cell (Refer Slide Time: 02:07). This is R_1, this is R_2 and any intermediate radius, R. We will have Q dot is equal to minus KA dT by dr, in this case also. Here, actually I have written capital R, but here it is small r; but this is the same thing. This area is the spherical cell area at any intermediate radius R and we can write it as minus K 4 pi r square into dT by dr. That is the surface area of a spherical cell. Then, we will have Q dot into dr by r square, that is equal to minus k into 4 pi dT. What will we have after integration? Here, logarithmic R will not come. I can write down the final expression. Q dot will be equal to 4 pi will remain and then it will be T_1 minus T_2, where T_1 is the temperature at the inner radius, T_2 is the temperature at the outer radius and we are assuming that T_1 is greater than T_2. What will be the R term? K we have forgotten, so we to put k. Then, we will have R_2 minus R_1. Then at the top, we will have R_1R_2. So, 1 by R_1 minus 1 by R_2; this will come from integration.

Let us not bother too much regarding this, because this we can do in any moment from the basic laws of integration. This is how we will get the heat transfer for plain wall, for cylindrical wall, and for spherical geometry. One can try different other things and one can write the generalized expression of conduction equation.

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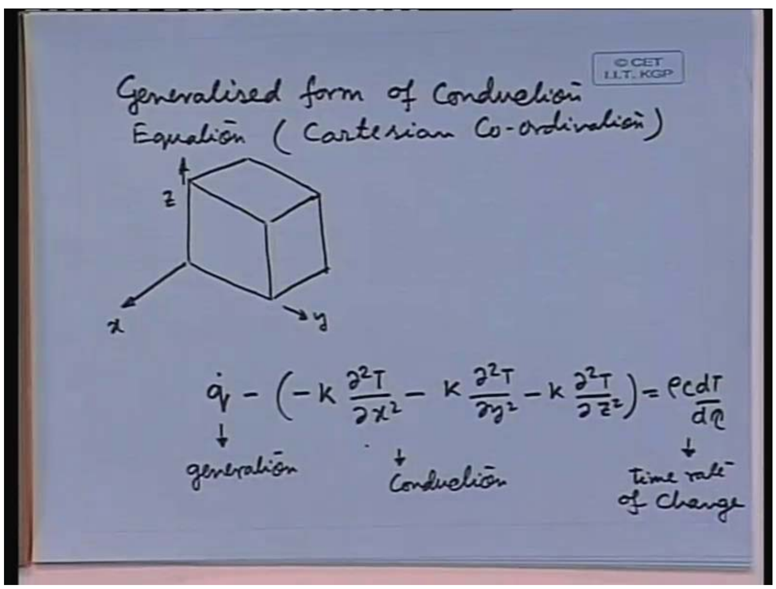

This is generalized form of conduction equation in Cartesian co-ordinate, because in cylindrical polar co-ordinate and spherical polar co-ordinate we will get different expressions. Actually, this equation one can derive easily, but I will not do that. Let us say, we have got some sort of a control volume which is a three dimensional control volume. Then, we can have accordingly (Refer Slide Time: 05:54) this is x, this is y and this is z. So, x, y, z are the three coordinates we can have. If we apply Fourier's law of conduction and make some sort of energy balance, assuming that there is only conductive heat transfer, then we can write down like this as q dot minus of minus k d square T by dx square minus k d square T by dy square minus k d square T by dz square is equal to rho c dT by d tau. I will tell you what I have done.

I am not going for the rigorous derivation of this equation. I have considered the time rate of change of the temperature within this volume and I have considered heat transfer along any direction of the Cartesian co-ordinate. Along with that I have considered that in this volume, at each and every point, heat is being generated. q dot is actually volumetric generation. Sometimes, people put it either as q_v or q triple prime, because in a volume, this is the rate of generation of heat. This is conduction and this is time rate of change of energy (Refer Slide Time: 09:04). For steady state, this term will become 0.

Let us try from this one. What we have done is, if we simplify this equation, we will have q by k plus d square T by dx square plus d square T by dy square plus d square T by dz square, which is equal to rho c by k dT by d tau.

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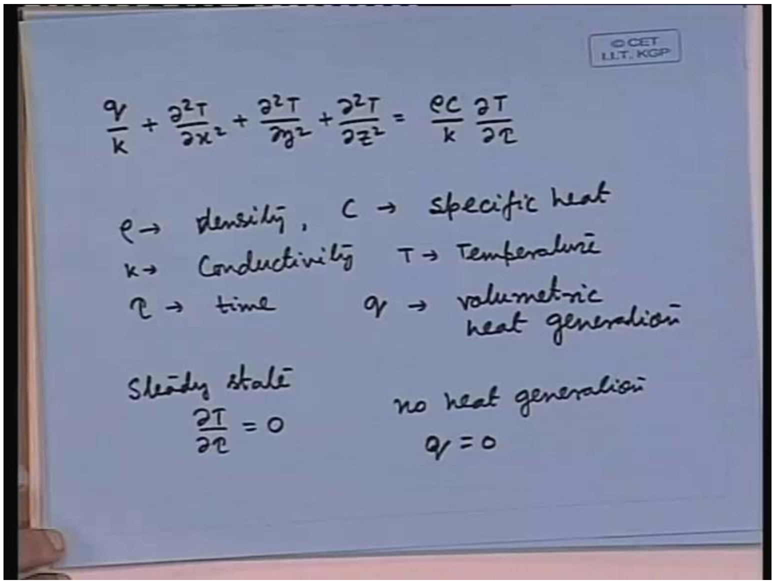

I hope, you know all these quantities. Here, rho is the density of the material, C or C_p is the specific heat, k is conductivity, T is temperature, tau is time and then q is volumetric heat generation. Let us see whether from this generalized equation what we have done earlier, whether we can get it or not. In steady state, we will get dT by d tau is equal to 0. Then no heat generation and so, q is equal to 0. Then, if we have only in one direction, d square T by dx square is equal to zero; d square T by dx square will be equal to zero, if it is one dimensional heat transfer. Then, we can integrate it. So, dT by dx will be constant; so, no heat generation, q is equal to 0 and here when we have derived this equation, please look into this particular expression. Here, this is valid for constant conductivity; not only constant conductivity, the material is also homogeneous and isotropic. That means conductivity is not changing from point to point; it is not changing for different temperatures. If we have such a type of material that means it is homogeneous, it is isotropic and we do not have any heat generation, we are considering steady state, moreover, heat transfer is one dimensional, then in that case, we are having d square T by dx square is equal to 0.

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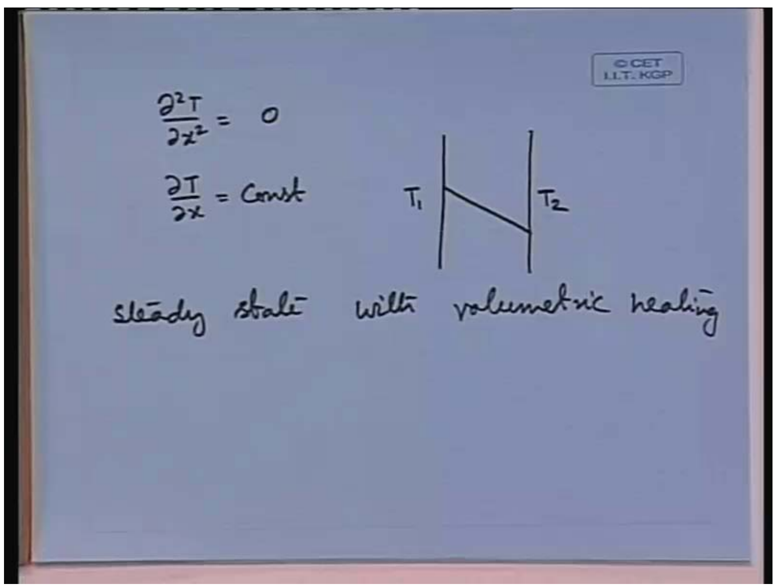

Here, dT by dx is equal to constant. There are two walls; one is kept at T_1 and another is kept at T_2. The temperature profile will be something like this; dT by dx is equal to constant and from here, we can get what is the rate of heat flow. So, the generalized equation is also valid for these special cases which we have derived. If you look into this equation, this is not fully generalized. Here, one can again make it more generalized by taking different conductivity in different directions. That is also possible; one can take k_x. In that case, it will be not k into dT by dx square but d dx of k dT by dx. We are not going into those details, because that is not within the scope of this particular course, but one can make it fully generalized. The derivation of this equation is also not very difficult; one has to only consider Fourier's law of conduction along different directions.

Another important aspect one can get is like this. Let us say that we are considering a steady state case, but there is heat generation; steady state with volumetric heating. One situation could be you are analyzing the heat transfer of a slab or cylinder, where electric heating is there. The material itself is a conducting material and through this we are passing current. Then we have got Joule heating in the conductor. So, it is volumetric heating. In that case, what will be our equation? My equation will then be q by k plus d square T by dx square, again one dimensional conduction let us say, is equal to 0. Then, we will have a non-linear temperature profile. These are only mathematics. There is not much of extra physics involved in it. The physics is the same Fourier's laws of heat conduction. But after that, one can have different mathematical expressions depending on different situations. This is what we like to do or we like to discuss in conduction heat transfer. Then, we go to convective heat transfer.

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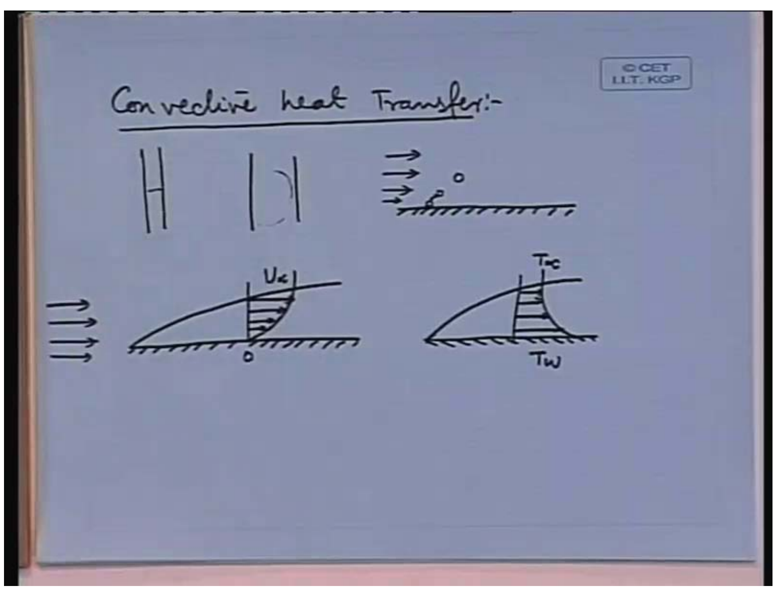

In the convective heat transfer, there will be physical motion of the bulk medium. In the conduction there is no physical motion of the bulk medium, but in case of convection, there is physical motion of bulk medium. So, it will take place in a fluid. What is happening? For energy transport, as there is bulk motion, the hotter fluid is coming from its own place to the colder region of the bulk of the fluid and the colder fluid is moving from its own place to the hotter region or to the high temperature region. There is some sort of inter-mixing. But if we look at the molecular level, there also heat transfer is taking place by conduction. That means one fluid molecule to another fluid molecule, which is just adjacent to it heat transfer takes place by conduction. But what is the difference with pure conduction is that in conduction all the fluid, all the molecules of the material are at its mean position. Here, the molecules are having motion.

In conduction, one molecule can give energy only to its neighboring molecule, because its mean position is fixed. However, in case of convection, it can give energy to its neighboring molecule, but at the same time it can change its place, it can change its position. Initially, at time T, what was its neighboring molecule? At time t plus delta T, its neighboring molecule will also change. In convection, we will have a higher rate of heat transfer. What I mean to say is like this. Suppose there is a fluid medium and if we consider heat transfer only by conduction, we will have some rate of heat transfer, but if there is convection along with it, we will have a higher rate of heat transfer. The example is we know that we produce hollow bricks. Because we have air in between that and as air is a bad conductor of heat, this will create some insulating effect. This is true as far as this gap is small. When the gap will be large, then by convection, air can also create some good amount of heat transfer. One has to remember that in convection, the mode of heat transfer between two adjacent molecules is conduction, but over and above, there is a motion of molecules due to the bulk motion of the media.

Basically if we see the interplay between conduction and convection, it is like this (Refer Slide Time: 20:46). Let us say this is a surface and on the surface there is flow of fluid. Adjacent to the wall, we have zero velocity due to no slip condition. Here, the heat transfer is purely by conduction. After that, the fluid molecule which is getting heated up will have some motion. The next molecule will transfer the heat by conduction and then the other molecule which is away from the wall will have higher motion, higher velocity. So, it will have the capability of transferring the heat to a larger distance. So, we will have a greater transportation of thermal energy from the wall to the bulk of the fluid.

In case of fluid flow, as we have discussed already, adjacent to the wall we will have a strong radiant of velocity, which is known as boundary layer. Let us say, this is a flat plate and we have got fluid flow like this. So, we have boundary layer like this (Refer Slide Time: 22:12). This is the boundary layer. Similarly, this is called velocity boundary layer. Similarly, adjacent to the wall, we will also have a boundary layer where there will be change in temperature and that is called thermal boundary layer.

Let us say that the wall is hot and the adjacent fluid is cold. So, we will have a boundary layer like this. The temperature will change like this. The highest temperature at the wall which is T_{wall}, here it is 0 and this will go up to the fluid temperature which is $T_{infinity}$. It will go up to the velocity of the fluid which is $U_{infinity}$. Here, the velocity will change from 0 to $U_{infinity}$. Here, it will change from T_w to $T_{infinity}$. When there is a fluid flowing past a wall whose temperature is different from the fluid temperature, we will have two boundary layers. One is a velocity boundary layer and another is a thermal boundary layer or temperature boundary layer. These two boundary layers may be extended up to different levels. They may not coincide with each other; in most of the cases, they may not coincide. Either the velocity boundary layer will be thinner or the temperature boundary layer will be thinner.

Let us try to understand the convective heat transfer and then we will discuss certain rules for determining convective heat transfer.

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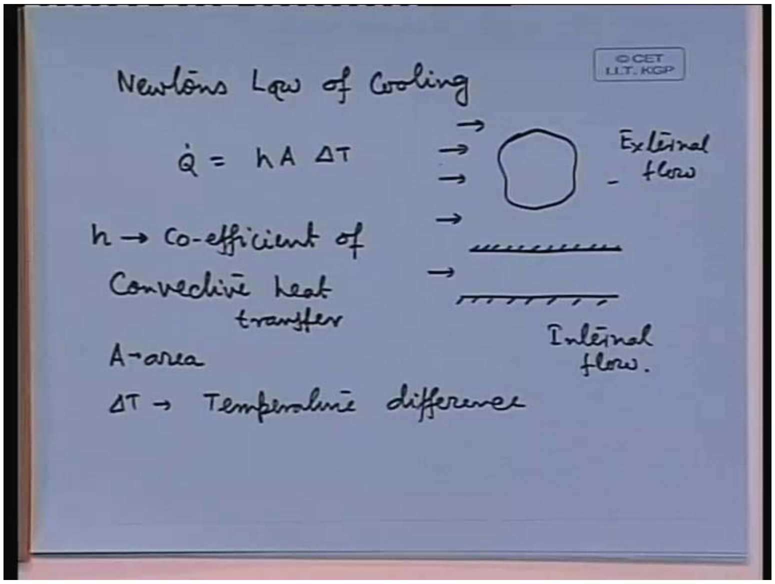

In this regard, the law which was given by Newton long ago is still the guiding law for convective heat transfer. That is known as Newton's law of cooling. In engineering practice and in number of scientific practices, when there is a heat transfer between a solid body and a fluid, we are interested in that phenomenon. In that case, Newton's law of cooling says that Q dot is equal to h A delta T. Delta T could be like this. Let us say, there could be different things and suppose that this is one phenomenon where this is a body and this is the fluid which is flowing through it or the phenomenon could be something like this. This is a conduit of any arbitrary shape and fluid is flowing through this. This is one situation and this is another situation. This is known as external flow and this is known as internal flow. In all these cases, Newton's law of cooling is applicable.

Only thing is that one has to define these quantities properly. Let me define it. Q dot is the rate of heat transfer as we have already mentioned, h is the co-efficient of convective heat transfer, A is the area. Here A is the surface area and here, A is not the cross-sectional area, it is the perimetric area, the inner perimetric area; that is also a surface area. Because heat transfer in or is? convection in these cases, this is a surface phenomenon. Basically, it is external surface area, this is internal surface area, though, from fluid mechanics point of view, there are differences between these two phenomena. I am going to explain that. So, h is co-efficient of convective heat transfer, then A is the area and delta T is the temperature difference. This delta T has to be defined properly, or rather it is like this. A is more or less unambiguous. A is the physical surface area that you will get. But h and delta T are to some extent related to each other. The way you will define delta T, you will get a particular value of h, which means that when there is external flow over a flat plate, what we can have?

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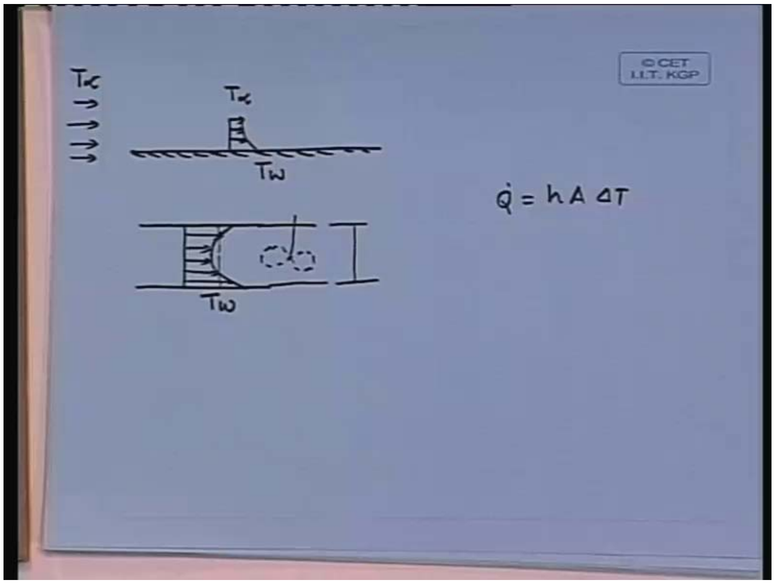

Let us say that the flat plate is kept at a constant temperature T_{wall} and we have got fluid flow. Fluid is initially approaching the plate with a temperature of $T_{infinity}$. There will be thermal boundary layer and ultimately at the edge of the thermal boundary layer, we will have $T_{infinity}$. This is T_{wall} and this is $T_{infinity}$. Heat transfer is basically between the temperature differences T_{wall} to $T_{infinity}$. Our Newton's law of cooling where Q is equal to h A delta T, in this case, delta T will be T_w minus $T_{infinity}$. But, let us say, we have got flow inside the pipe wall. What is happening? Here, the temperature profile is something like this. At the wall, we have got high temperature and at the center we have got a lower temperature; if we are passing it through the tube that can happen?

Let us say, the tube is electrically heated and through this, we are passing the liquid or gas or it is a boiler tube where it is getting radiant heat and the fluid which is water, is being passed through the tube. In this case, what could be our delta T? For simplicity sake, let us take T_w is constant. What is the fluid temperature? Basically, some difference will be there between the surface temperature and the fluid temperature. What could be the fluid temperature? What type of mean temperature we will take? Again, I am telling that h is dependent and how we define the temperature. One temperature is surface temperature, but how we define the fluid temperature? In this case, what is taken is that we take the bulk mean temperature. There is a temperature profile like this. Let us assume some sort of imaginary process.

Let us say, (Refer Slide Time: 31:33) we have got some imaginary stirrer here, we stir this. If we stir this, well mix this, then what will we get? We will get a constant temperature. Let us say, the constant temperature I show like this. This is the constant temperature we will get. This mean temperature we will call as bulk mean temperature or mixed mean temperature or sometimes it is called mixing …. temperature. We take that temperature and then, we define h. If we could have taken some other temperature, we could have got some other value for h. Let me now proceed with h.

This equation we could understand that this is Newton's law of cooling. This is very useful information, because suppose I want to calculate what the rate of heat transfer is, I have to use this information or use this equation. In this equation, the most crucial thing which we should know is h. Convective heat transfer gives us the means by which we can determine h. So, convective heat transfer is all about the determination of h and this point should be made clear. In convective heat transfer, we discussed different methods; time and again, I am emphasizing the same thing. In convective heat transfer, we discussed different situations so that we can determine h.

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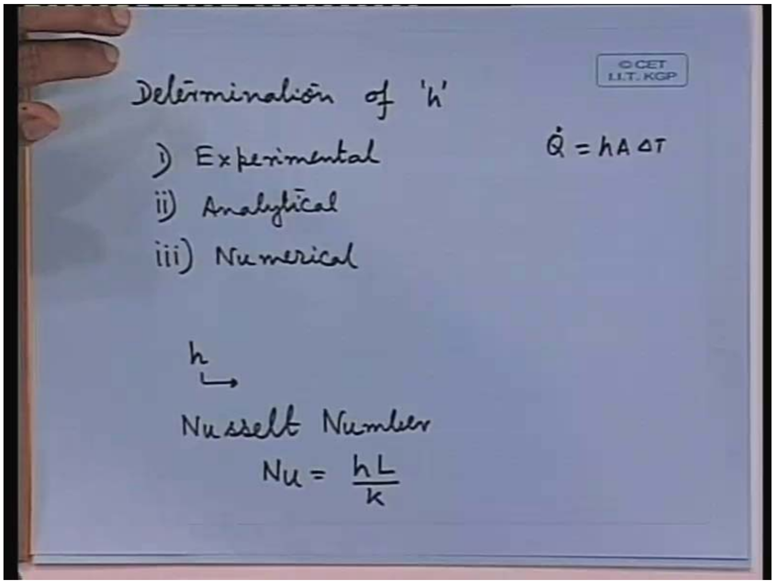

How do we determine h? The thing is that mainly there are three methods of determination of h. One is experimental; experimentally one can determine h. Second one is analytical and the third one is numerical. Actually, these last two methods are inter-related. When we try to determine h, what we do? One is experimental, you measure. I have written this law. Q dot is equal to h A delta T. A is known from the geometry, Q dot, the input heat transfer or the rate of heat transfer, you should have some method of determining. Let us say, you are going for electrical heating, you know it or you are heating by condensing steam, you know the incoming steam temperature or the condensing steam is at saturation temperature. So, you know its latent heat, you measure the conduction and then you know the heat transfer rate. So, Q dot should be known by some method, A, generally is known from the geometry and delta T is the temperature measurement that you have to make. That is how you can determine. This is the experimental technique.

In analytical and numerical, only the method of solution is different. Theoretically, we are doing this. What we have to do? In both the cases, first, we have to determine the temperature profile in the fluid. For determining the temperature profile, we have to consider three conservation equations. One is conservation of mass, another is conservation of momentum and the third one is conservation of energy. The three conservation equations I have to write down. Sometimes, if the velocity profile is known or velocity information is known, only conservation of energy I have to write down. From there, I will get the temperature profile. One can get the temperature profile by analytical method. In most of the cases, analytical method is not possible and so we go for numerical technique. This is the main procedure. Though the details of the procedure are not suitable for this particular course, we do the theoretical analysis like this. All the three conservation equations are solved to determine the temperature profile. The solution technique could be analytical or numerical.

Generally, we get analytical solutions only in limited cases. Let us say, we have got laminar flow. In simple geometry, we can get analytical solution. But if the geometry is complex, if we are going for turbulent flow, for most of the cases, we have to take the help of numerical technique. From there, we determine h. I will not discuss these techniques, but let me discuss some other things which are important in connection with convective heat transfer. h, convective heat transfer, it is customary to express this convective heat transfer in a non-dimensional form. The non-dimensional number which gives the co-efficient of convective heat transfer is known as Nusselt number. Nusselt number is denoted by N_u, which is equal to h L by K. Here, we can see that dimensionally it is matching. Let us quickly do that.

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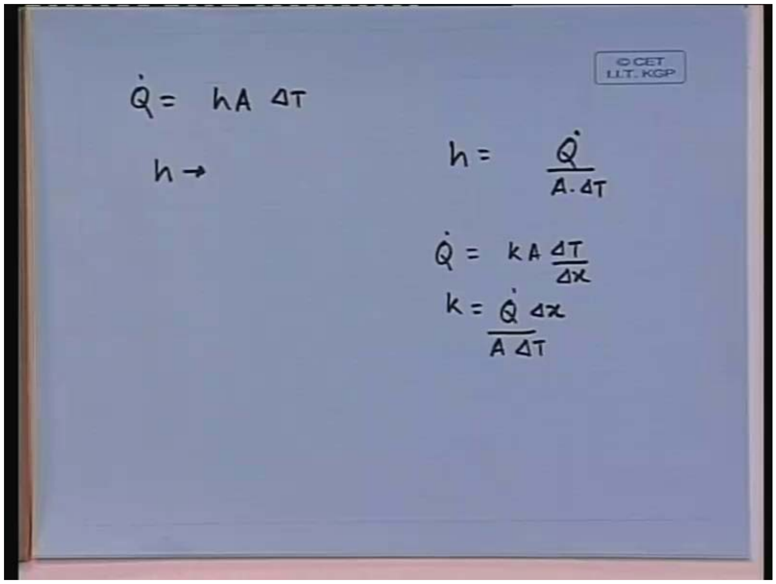

Q dot is equal to h A delta T. What could be the dimension of h? Let us determine the dimension of h. We are defining a non-dimensional number which relates this heat transfer co-efficient and that non-dimensional number is your Nusselt number. In the unit of h, we are getting per meter square, rate of heat transfer per meter square. h is equal to Q dot by A into delta T. Again, if we write conduction equation, we can write Q dot is equal to k A delta T by delta x. k is equal to Q dot into delta x by A into delta T. That is what we can write. If I take a ratio, then you will see that h L by K is becoming a non-dimensional number and that becomes the Nusselt number. The heat transfer co-efficient is expressed in terms of Nusselt number and there are either analytical expression for Nusselt number or there are correlations for Nusselt number. The correlation which we get for Nusselt number will depend on a number of parameters.

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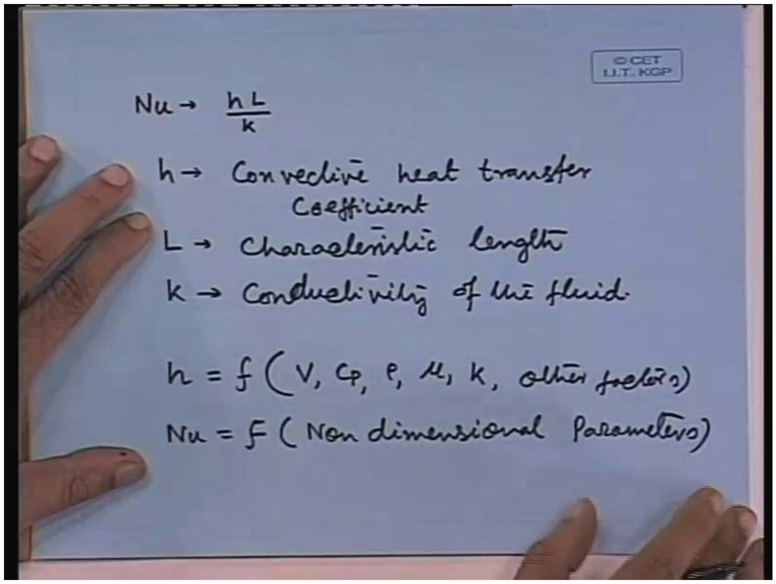

This Nusselt number is nothing but h L by k, where h is the convective heat transfer co-efficient, L is characteristic length and k is the conductivity of the fluid. h L by k which will be different for different situations; Nusselt number will be different for different situations. Basically h will depend on a number of parameters. What are the parameters on which h is dependent? h will be dependent on the velocity of fluid v, on the specific heat of the fluid, on the density of the fluid, on the viscosity of the fluid and on the thermal conductivity of the fluid. The temperature difference will not come directly, but indirectly it will come, because all these properties are temperature dependent.

Again there are some other factors, which I cannot just write by symbols, like whether it is internal flow or external flow, what type of geometry it is and what type of surface characteristics are there? There are other factors. Similarly, we can have Nusselt number as a function of F, of several non-dimensional parameters. As Nusselt number is a non-dimensional parameter, we can express Nusselt number as a function of several non-dimensional parameters. Let us see, just to have an idea, what are the different situations? h or Nusselt number or convective heat transfer co-efficient, depends on the type of convection. Generally, there are two types of convection. One is forced convection and another is natural or free convection.

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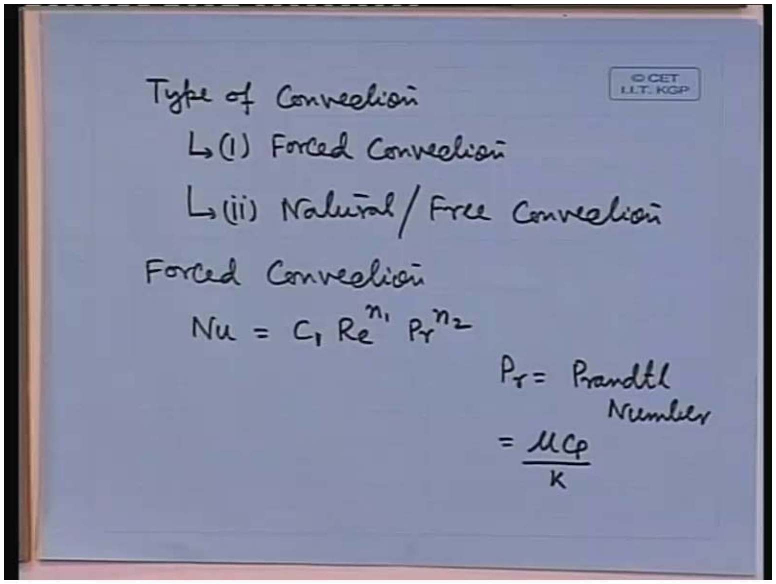

As the name suggests, in the forced convection, we have got flow field which is generated by some external agency or by some fluid mover. We can have either a pump or a compressor and we have got the fluid motion generated by some moving body or it is generated externally. Whereas, natural or free convection is mainly due to the difference of density and some sort of body force field should be present. The gravitational field is one of such body force field by which we can have natural convection.

Generally, when we mention natural convection, we refer to the situation where there is a body force field and due to density difference, we have some sort of convection current, but there is no external moving agency for the fluid and there is no other external agency which is responsible for creating the velocity field. One can have different types of convections; instead of the gravitational field, suppose we have got electrical field, we can have similar type of phenomenon. Let us not discuss those issues. Basically here in forced convection, the fluid motion is created by some external agency and in natural convection it is due to the density gradient and density change that is due to temperature. So, the temperature difference itself creates some sort of motion. There are two types of convections: forced convection and natural convection.

Again, two types of situations I have discussed. One is internal and another is external. In all these cases, I will have different types of relationships for Nusselt number. If it is forced convection, I will have Nusselt number is equal to some sort of a constant $C_{1,}$ then Reynolds number, R_e to the power of n_1 and Prandtl number to the power n_2. This type of relationship I will get. I am not giving these numbers, because these numbers will vary from situation to situation. This constant, then there is one number here and then there is another number here. For forced convection, I will get this type of a relationship. We are familiar with Reynolds number. What is Prandtl number? Prandtl number is P_r, which is equal to mu C_p by K. Here, mu is the viscosity, C_p is the specific heat and k is the conductivity. This is what we will get in case of forced convection.

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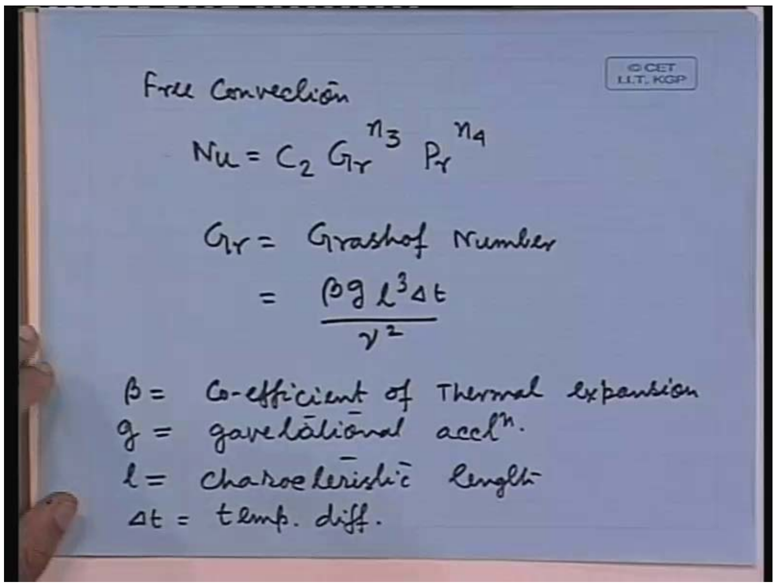

In case of free convection we will get, Nusselt number is equal to some constant again, let us say C_2, Grashof number, which is a non-dimensional number, to the power n_3 and Prandtl number to the power n_4. Here, G_r is equal to Grashof number and it is given by beta g l cube delta t by nu square. Beta is co-efficient of thermal expansion, g is the gravitational acceleration, l is the characteristic length, delta t is the temperature difference, nu is co-efficient of thermal expansion and nu is equal to mu by rho. Actually, nu is called kinematic viscosity.

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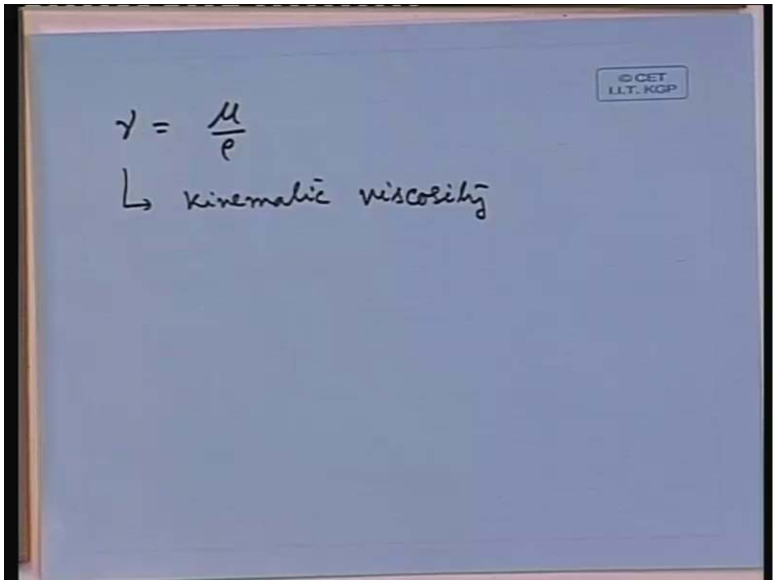

From situation to situation, these C_2, n_3, n_4 will vary. There is no point giving these numbers. But this is how we have got correlations and using these correlations, one can determine Nusselt number. Then, knowing the characteristic length and conductivity of the fluid, one can determine h. Then using Newton's law of cooling, one can determine the rate of heat transfer. Finally, with this, we can conclude our discussion on convection. We will not discuss much about radiation, but little bit we will do in tomorrow's class.