Applied Thermodynamics for Marine Systems

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Lecture - 26

Modification of Brayton Cycle
Principle of Heat Transfer
Conduction through Simple Geometry

Good afternoon. We were discussing regarding the modifications which are generally done to the basic Brayton cycle. The first thing we discussed is that the exhaust which comes out of the turbine will still have enough thermal energy which can be utilized. If we do not utilize it, it is going to the ambient atmosphere. Not only we are wasting a potent source of energy but also we are creating some amount of thermal pollution. I have mentioned that with this thermal energy, the compressed air can be preheated and then the preheated air can be sent to the combustion chamber. This increases the efficiency of combustion and you have to burn less amount of fuel. I will quickly draw the block diagram.

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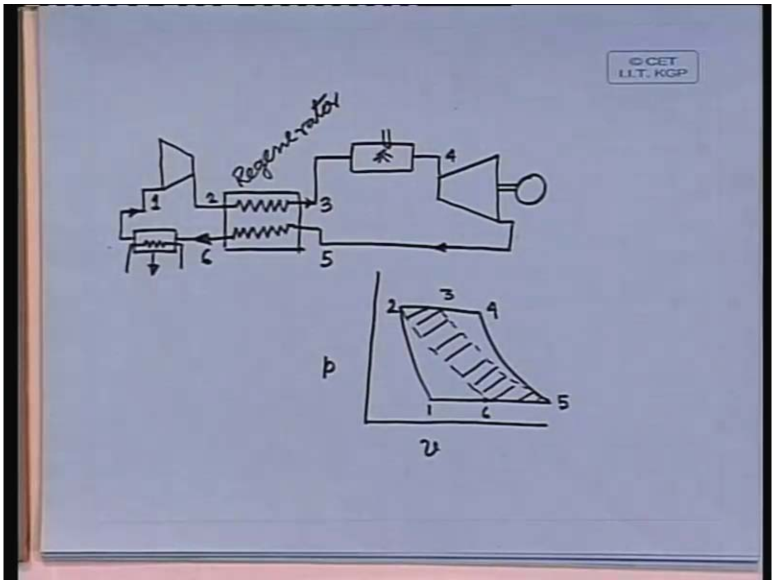

First, the compressor, then from the compressor the stream is being preheated, then after that, it goes to the combustion chamber where we have fuel spray. Then it is goes to the turbine which generates power. Then this one is the exhaust gas and ultimately through the regenerative heater, it goes to the ambient atmosphere and this is again ambient atmosphere. So, if we want to show a closed cycle, it will be something like this. After this, it again goes back. This is one unit where you are rejecting heat and after that, it is going to the compressor. If we show a closed unit, it will be like this. This is an idealized representation of the gas turbine cycle with regenerative heating. Let us put 1 here, then after the compressor it is 2, then this is 3, this is 4, this is 5, this is 6 and this is 1.

The PV diagram will be 1 to 2 compression, then 2 to 3, or 2 to 4 heat addition at constant pressure, then 4 to 5 expansion and 5 to 1 heat rejection at constant pressure. 1, 2 and then 2 to 4 I told you, 4 to 5 and 5 to 1. The only thing is that in between, we have got 2 to 3 and then we have got 5 to 6. So, this is internal. That is what I mentioned in my last class. But why I am repeating again? There is a reason. How to analyze this cycle? We will not go into a rigorous analysis but let me try to give some sort of hints as to what can be done for the analysis.

What can be done for the analysis? This heat exchanger now plays a very important role in the performance of the cycle. This is the regenerative heat exchanger or we can call it regenerator. This plays an important role in the performance of the whole cycle or entire cycle. For this heat exchanger, we can have some sort of effectiveness defined. We know what type of heat exchanger we are using. We can define some sort of effectiveness for this heat exchanger. Actually, heat exchanger effectiveness may be a slightly new term or probably you are familiar with it. Unfortunately, we do not have much time, so that we could have discussed it in detail while studying heat transfer, but probably …. not have that time.

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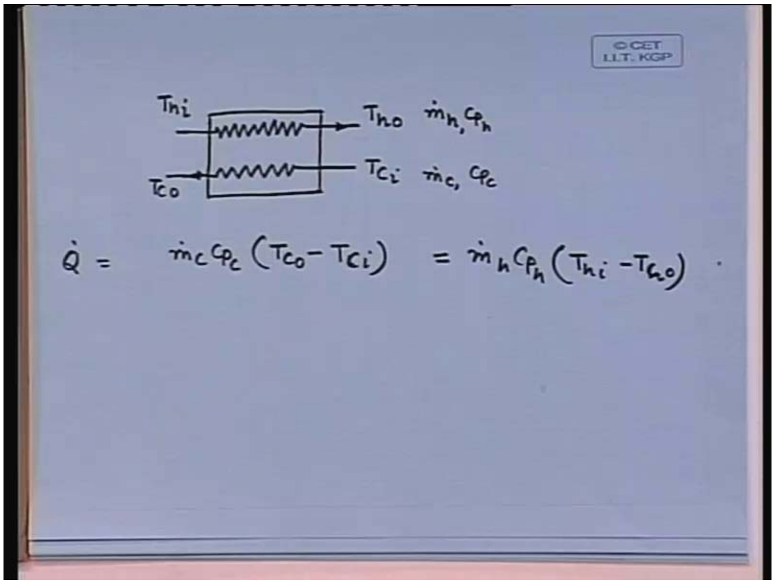

$$\dot{Q} = \dot{m}_c C_{p_c} (T_{co} - T_{ci}) = \dot{m}_h C_{p_h} (T_{hi} - T_{ho}) .$$

This is the schematic representation of a heat exchanger. This is one stream of fluid, this is another stream of fluid and they are exchanging heat. This heat exchanger - let us say this is hot gas; it is coming at T_{hi} inlet, going out at T_{ho} outlet. This is cold gas or cold fluid T_{ci} and $T_{co;}$ so inlet and outlet temperatures. One can, from this, determine the maximum possible amount of heat transfer. What is the maximum possible amount of heat transfer in this case? For that, one has to know what the medium is and what the property of the medium is. Let us say we can have for this m dot h is the mass flow rate of the hot fluid and C_{ph} is the specific heat of the fluid, then m dot c is the mass flow rate of the cold fluid and C_{pc} is the specific heat of the cold fluid. Then we can get what the heat transfer is by the hot fluid and what is the heat transfer for the cold fluid. They will be the same quantity but we will get two expressions. What we can get is Q dot is equal to m dot c C_{pc} into and, for this there will be temperature increase, so we can have T_{co} - T_{ci}, this is the rate of heat transfer and this will be identical to m dot h C_{ph} and T_{hi} minus $T_{ho.}$ This will be the heat exchange by the hot fluid.

Now, which temperature will be or which change in temperature will be higher? We cannot say that. Whether it is a cold medium or a hot medium, we can only say for the medium for which this product is small for that we will have the highest temperature differential. We will have the highest temperature differential for the medium which has the lowest product of specific heat and mass flow rate. So, this is actually heat capacity rate. The m dot C_p is the heat capacity rate. So the fluid which has the lowest heat capacity rate will have the highest temperature increase. What could be the highest temperature increase?

Let us say the cold fluid is having the lowest heat capacity rate. The highest temperature the cold fluid can reach is the inlet temperature of the hot fluid. It cannot have a higher temperature than the inlet temperature of the hot fluid because that will amount to violation of second law of thermodynamics. It can have at best a temperature which is equal to T_{hi} at the exit. That gives us the ideal or the best temperature differential which we can get. But actual temperature differential will be T_{co} minus $T_{ci.}$ Actual divided by the ideal will give us the effectiveness of the heat exchanger. So that is how we can have some sort of a merit or some sort of a performance criteria of the heat exchanger defined. Once we define that, then we know the performance criteria of the heat exchanger. If you do this, then you can predict the intermediate temperatures and then from there we can determine the cycle efficiency.

Basically one has to bring in some of the heat exchanger characteristics to analyze the cycle, when there is regenerator. This is relevant for heat transfer, which we are going to study. This is relevant for gas turbine so that is why I wanted to spend a little time. This is one of the ways by which we can increase the efficiency of the basic gas turbine cycle and without spending anything for the fuel. But the cycle becomes complex and we have to bring one extra equipment. So, the regenerator has to be brought in. Most of the gas turbine power plant particularly stationary gas turbine power plants should have a regenerator to extract some of the waste heat. This regenerator or the waste heat which is recovered from the regenerator can be used for other purposes like it can be used in a steam power plant also. But that is a different issue. Then what else can we do? We can do some other modification in the cycle.

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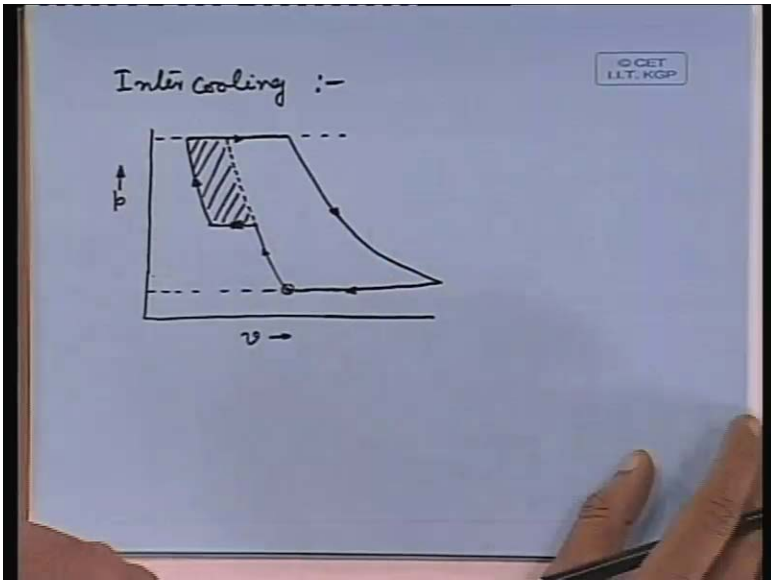

One of them is inter-cooling. We know that the compressor is a very crucial part of the gas turbine power plant. The compressor is a very crucial part of the gas turbine power plant because it eats away a lot of motive power developed by the turbine - that means the power which is developed by the turbine, a large part of it is utilized for running the compressor. Some how, if we can make the compression process more efficient or less energy extensive then we are gaining. We know that a compressor when we compress the gas its temperature increases and further compression becomes difficult. Instead of compressing the gas in one stage, if we can have multi-staging of the compressor and in between two different stages or two successive stages, if we have inter-cooling, then we can gain in terms of power. So how can it be done?

I am first drawing the cycle diagram. Let us say this is my pv diagram and I am working between two pressure limits. These are the pressure limits. I have to start from this point. This is the initial point but I will not go to the pressure at one stage. So what I will do is, I will initially compress and as I am compressing it along an isentropic path, the temperature of the gas is increasing and then I will do inter-cooling. If it is perfect inter-cooling then, after inter-cooling this temperature and this temperature, they are the same; but perfect inter-cooling is not possible. We will have some inter-cooling and then again, we will have the compression up to the final pressure. Then combustion, which is replaced by a constant pressure heat addition and then expansion and then constant pressure heat rejection. So this is our pv diagram for the entire process.

What we can show is that if we could have followed compression in the same stage, we could have gone along this path. But, here we have gone through this path and so this much extra amount of work we are gaining or saving. That is why by inter-cooling we are going to have some benefit. I have shown only one stage of inter-cooling that means compression by two stages. It can be done in more stages, but it becomes complex. So, more than two inter-cooling is generally not done. We can have some benefit by inter-cooling the gas after one stage of compression. I am not drawing the block diagram basically there will be two compressor; in between there will be an inter cooler which will be cooled where the gas will be cooled by a suitable media. Similarly, one can have the expansion also in different stages. Let us draw the diagram; then it will be understandable.

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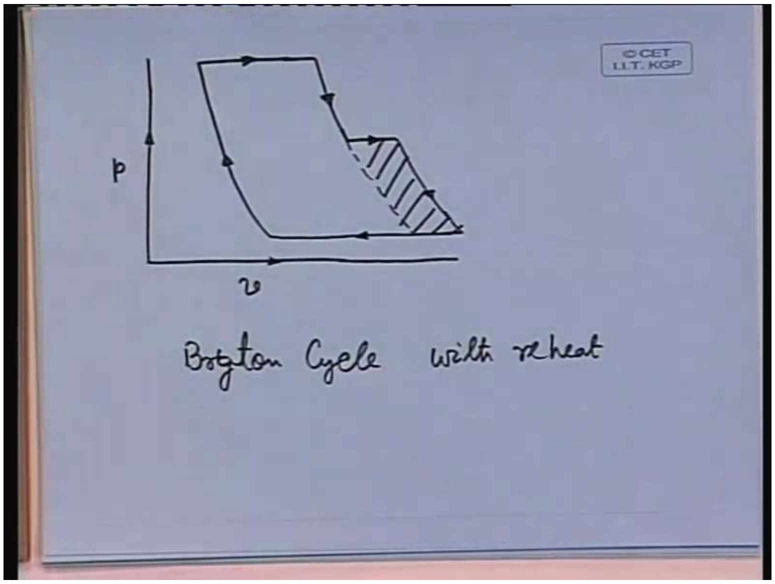

Let us draw without any inter-cooling. We have the pv diagram of the process. We are starting from here. We have gone to a very high pressure. At that pressure, we are having the combustion process which is replaced by an ideal constant pressure heat addition. Then we are having the expansion. After expanding the hot product of combustion to certain intermediate pressure, we will take it out of the turbine. Then we can reheat the gas. This is reheating. We can reheat the gas - that means it can be done like that. The combustion process we will have in two stages. Initially we will not burn all the fuel in the incoming air stream. Part of the fuel will be burnt in the incoming air stream. Then, the product of combustion which is still having lot of oxygen content that will be allowed to pass through a turbine where partially its pressure will be reduced and its temperature will fall. Again, in that exhaust, we will burn the rest of the fuel utilizing the available oxygen there and then it will be allowed to expand in a turbine which can be a low-pressure turbine. Second stage of the turbine.

Here we can see that this much amount of extra work we are gaining by reheating. So we can write Brayton cycle with reheat. So, what we have got? We can have inter-cooling, we can have reheat, and we can have regeneration. If all the three things are used together, then we will have some respectable figure as the efficiency of the cycle. One interesting thing - let me combine reheat and inter-cooling together. Then what will we get?

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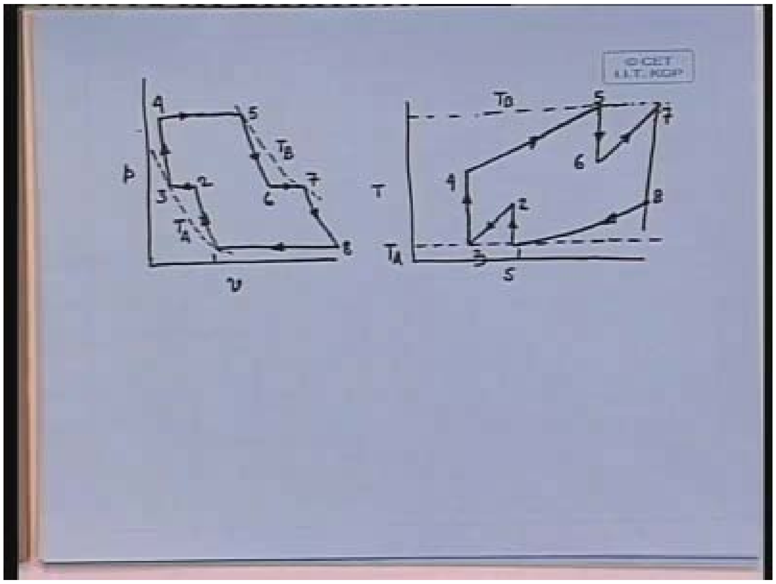

This is the pv diagram. We are starting from this point. We have gone through the inter-cooling process, then second stage of compression, then expansion, reheat, and then expansion in the second stage of the turbine. Let us say this is the isothermal line and let us say this is the isothermal line here. One is the isothermal line here and another is the isothermal line here. So after inter-cooling we are coming back to the same temperature with which we have started our cycle. Here also after reheating we are coming back to the same elevated temperature at which we have started our expansion. If we show this process on TS diagram, how does it look?

This is the TS plane. What we have done, let us give some name - that will be easier. 1 to 2 compression, then 2 to 3 inter-cooling, 3 to 4 compression in the second stage, 4 to 5 constant pressure heat addition, 5 to 6 first stage of expansion, 6 to 7 reheating, 7 to 8 is expansion in the second stage and 8 to 1 is constant pressure heat rejection. Now, if we want to have this in our TS diagram, let us try to have it. Let us say this is T_A, one isotherm and this is T_B another isotherm. Two isotherms I have shown. Initially I have started with point 1. I have gone for the isentropic expansion. Then I have gone for cooling. If I have gone for cooling and come back to the same temperature T_A, let us say our T_A is somewhere here. This is T_A. We will come back somewhere here. This is our inter-cooling process and then we will go to 1, this is 2 and this is point 3 after inter-cooling. Then we will go to 4. Again, isentropic compression and 4 to 5 is constant pressure heat addition. Let us say this is T_B, the maximum temperature. We will have isentropic expansion 4 to 5, 5 to 6, and then I will have the reheat process which is a constant pressure heat addition. So this and this curve will be parallel. Then we will have isentropic expansion. So somewhere here this is 6 to 7 and 7 to 8. Then I will have constant pressure heat rejection which is something like this. 1-2-3-4-5-6-7-8 and the representation of the pv diagram is also shown in the TS diagram. Why have I done all this? It is like this.

Let us say instead of one inter-cooling and one reheating, we have got very large number of inter-cooling stages and reheating stages. Then what will we get?

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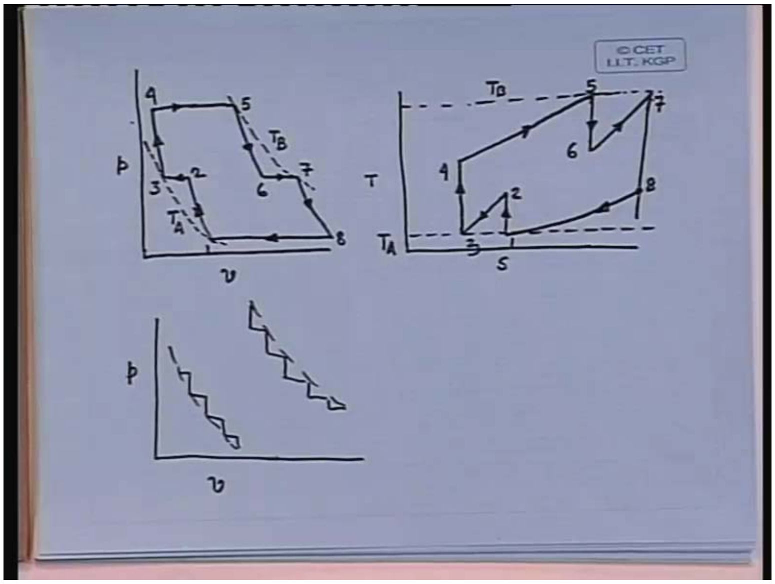

What we will get is like this. In pv diagram, this is one isotherm and this is our another isotherm. Around this isotherm, I will get reheating stages like this and around this isotherm I will get the inter-cooling stages like this. This process if I continue, then this zigzag curve here and this zigzag curve here will coincide with the isothermal processes or they will coincide with the isotherms. What will we have then? We will have two constant pressure processes and two isotherm lines. So, we will have two isothermal processes and two constant pressure processes. Then what is the advantage? We will see that the efficiency will increase if these two processes are made closer to the isothermal processes. Then we will see that our cycle efficiency is increasing. That is why even if we cannot approach the isothermal processes we are trying to approach it.

Even then, with finite number of inter-cooling and reheating, we will have better efficiency and that is what is done in gas turbine cycle. We will have inter-cooling as well as reheating. When you are having inter-cooling and reheating there are certain other advantages also. First is, you can go for higher pressure ratio and in a single piece of equipment, pressure fluctuation is not very high. So, if we want to compress the gas in one compressor in single stage, then definitely you have to go for a very large pressure ratio. A part of the compressor is subjected to small value of pressure, part of it is very high value of pressure means air is entering at a small pressure and then it is going out at a high pressure. The entire thing you have to make it so that it can withstand that high value of pressure whereas if you can divide it into three different stages, the compressor which is handling the highest pressure can be made of thicker section whereas you can have thinner section in other designs.

These are some advantages. Those are mechanical advantages but other than that we have got lot of thermodynamic advantages and that is why we want to have both inter-cooling and reheating. I think with this, we will end our discussions in this chapter that is air standard cycles. What we have done is we have done some sort of thermodynamic analysis for the cycles. With this, we can have the basic idea of the cycle performance. From all the cycles we can also determine what the efficiency is, how much work is being done and the variations possible with the cycle, so that their efficiency can be increased.

Now, the last topic which we like to deal is heat transfer. Again, with the available time, we cannot make much detailed discussion regarding heat transfer but I will try to cover the basics.

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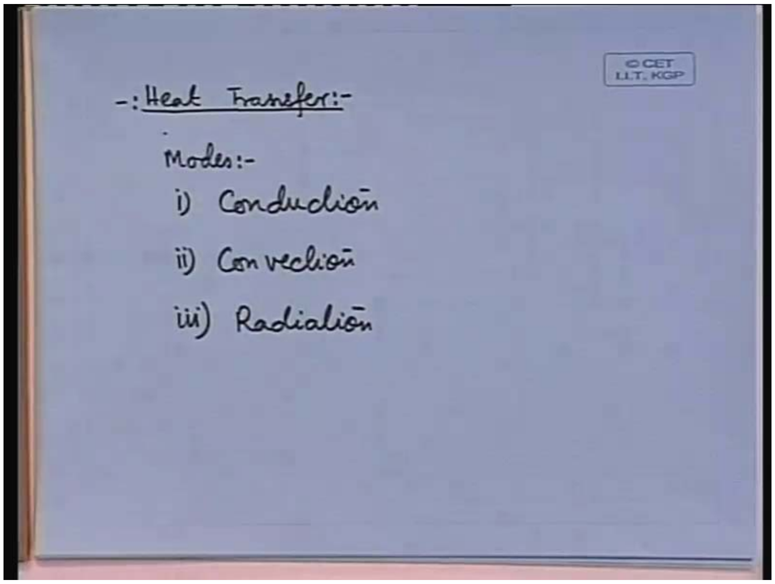

-: Heat Transfer :-

Modes:-

i) Conduction

ii) Convection

iii) Radiation

Heat transfer is a very important topic for number of engineering disciplines because we know that thermal energy is important as far as the power production and production of low temperature is concerned. We have seen that in number of cycles, we have got conversion between thermal energy into mechanical energy. Actually, at the end we want to have mechanical or electrical energy, but that can be produced or that can be converted from thermal energy.

For thermal energy whenever there is a temperature difference, we have got transfer of heat and that is why we should have some basic idea regarding the science of heat transfer. These are known things but even then, for continuity we should discuss. There are three modes of heat transfer. They are conduction, convection and radiation. We will discuss them one by one and some of the basic laws of conduction, convection and radiation, I like to give you and some applications where we need to use them.

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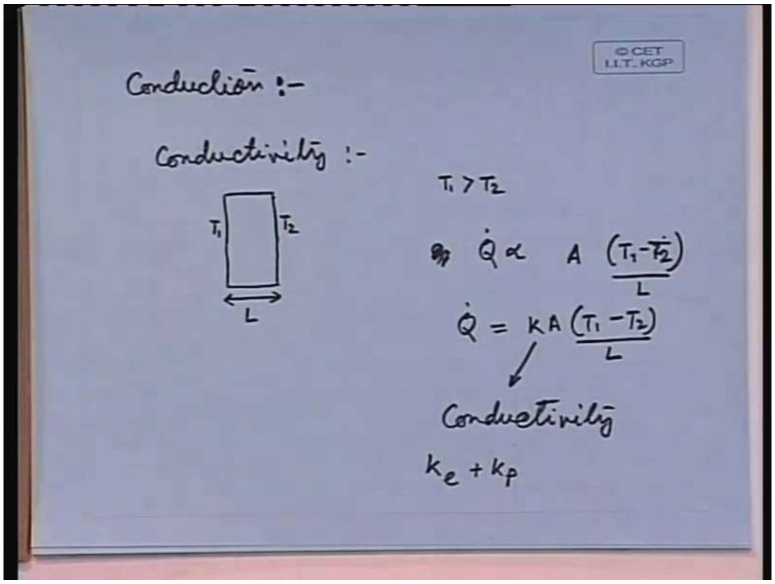

Conduction :-

Conductivity :-

$T_1 > T_2$

$$\dot{Q} \propto A \frac{(T_1 - T_2)}{L}$$

$$\dot{Q} = kA \frac{(T_1 - T_2)}{L}$$

Conductivity

$k_e + k_p$

If we think of conduction, conduction is the mode of heat transfer where the thermal energy is transported from one molecule to the neighboring molecule without much bodily movement of the molecule itself. It is like this - as the temperature increases, the molecule will have higher energy of vibration and this vibrating molecule will transfer energy from the neighboring molecule. In metal, the free electrons also take part in this type of energy exchange. Without going into the details of physics which we do not need to know in a very detailed manner, because we are analyzing the engineering aspect of heat transfer, we can see there are two modes of heat conduction. We have seen that mostly metals are very good conductors of heat; they are also very good conductors of electricity. There is some sort of a similarity between the conduction of thermal energy and conduction of electricity. In both the cases, free electrons play a very important part in transporting the energy.

Again, we have seen that some of the non-metals are also good conductors; some forms of carbon or graphite are also good conductors of heat. In that case, we do not have enough free electrons but the lattice vibration takes part in the transportation of thermal energy. Basically, one can broadly say that there are two phenomena which are important for the thermal conduction of heat. One is electron and another is lattice vibration or it is known as phonon. These two are mainly important for the conductive heat transfer and for the conduction heat transfer one can define a property which is known as conductivity. One can define a property – conductivity.

So conductivity is like this. We do a simple experiment where there is conduction along a particular direction. Let us say we have got an infinite wall, plane wall like this and we are maintaining one temperature T_1 here, one temperature T_2 here and let us say T_1 is greater than T_2 and we have got a thickness L. Then we will see Q, the rate of heat transfer, let us put it as capital Q, that will be proportional to T_1 minus T_2, that will be proportional to the area through which we are allowing the heat to pass and that will be inversely proportional to L. We need a constant of proportionality. For the time being, let us put this is equal to KA T_1 minus T_2 by L. We will see that the amount of heat transferred will be different for different type of materials when the wall materials are different. The constant of proportionality that is a function of the material or that is a material property. This property we call as conductivity. This is conductivity.

What I was discussing is that for conduction, two phenomena are important. One is free electron and another is phonon. Similarly, we can define that conductivity is due to conductivity of electron and due to the conductivity of phonon. One can write this is a summation of K_e plus K_p, actually it is pho, so K_p, and in case of metals, K electron is much higher. That is why it is interesting to know that if we know the resistivity or electrical conductivity of the metal, there are physical laws by which it can be correlated with the thermal conductivity of the metal. So there is some sort of a correspondence between the electrical conductivity and the thermal conductivity. Knowing this what we can do is like this.

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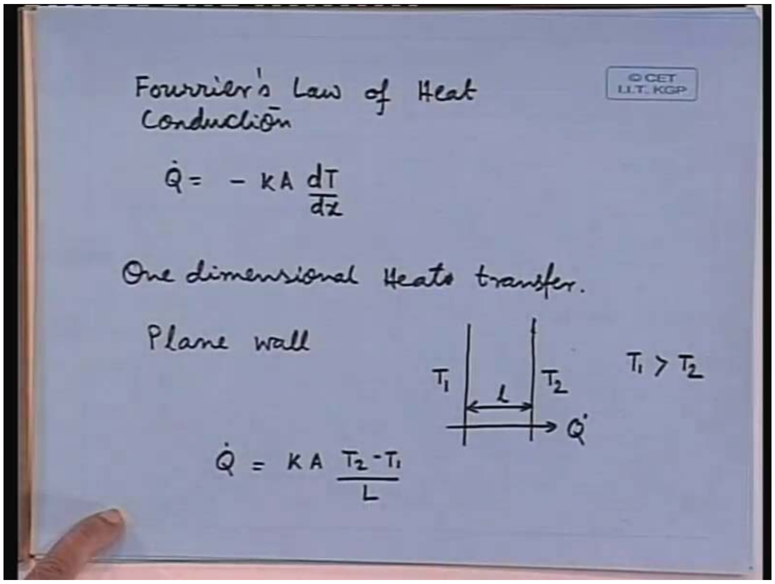

Fourrier's Law of Heat Conduction

$$\dot{Q} = -KA\frac{dT}{dz}$$

One dimensional Heats transfer.

Plane wall

$T_1 > T_2$

$$\dot{Q} = KA\frac{T_2 - T_1}{L}$$

The law which is the basics for analysis of conduction is known as Fourier's Law of Heat Conduction. This says, Q dot is equal to minus KA dT by dx. This is Fourier's law of heat conduction. K is the conductivity of the material, A is the area, T is the temperature and x is the direction. There is a negative sign indicating that heat transfer will be in a direction in which temperature decreases; that means from high temperature to low temperature we will have transfer of heat. This particular law one can apply for different cases and get the result by which one can determine the rate of heat transfer. We can take three different cases and this particular case what we are considering now it is heat transfer only along a particular direction, unidirectional or one-dimensional heat transfer. So, one can call it one-dimensional heat transfer.

Now we can take different cases. The plane wall, we have already discussed. So in the case of plane wall this is the situation. So what will we get? One of the wall, is having a temperature T_1 and another wall that is having a temperature T_2 and let us say in our convention T_1 is greater than T_2. Then heat flow will take place in this direction so this is Q dot and this is L, so Q dot is equal to KA T_2 minus T_1 divided by L. So it is like this. How did you get this equation? One can have it like this.

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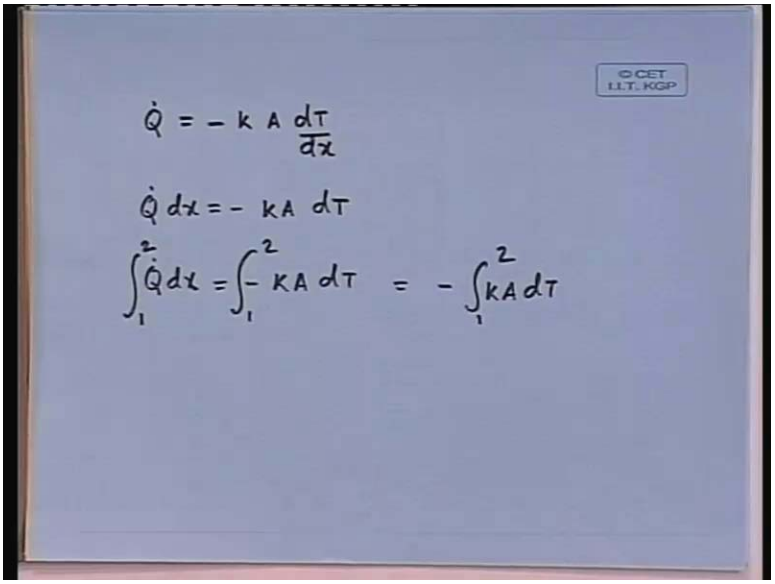

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$$\dot{Q} = -kA\frac{dT}{dx}$$

$$\dot{Q}\,dx = -kA\,dT$$

$$\int_1^2 \dot{Q}\,dx = \int_1^2 -KA\,dT = -\int_1^2 KA\,dT$$

Q dot basic equation is minus KA dT dx. So Q dot into dx is equal to minus KA dT. Now this integration can be done. One can integrate it; that means Q dot dx 1 to 2 and that is equal to minus KA dT 1 to 2 and that means minus KA dt 1 to 2. This integration can be done and the result which I have shown can be obtained if K is independent, thermal conductivity is independent and area is also constant between point 1 and 2. Then only this integration can be done. That is what we have done. In the plane wall, we have assumed the area is constant. Plane wall means area is constant and we have also assumed K to be a constant. If K is a function of temperature, then what we could we have done? Knowing the functional relationship between K and temperature, one has to do this integration. There are number of situations where K is not a constant. This property, thermal conductivity is generally a temperature dependent property and we have to determine this by integration. We cannot take K to be a constant if the temperature interval is high. In that case, you cannot take K to be a constant and then one has to go for the functional relationship between K and T. With this, let us go for circular cylinder or annulus.

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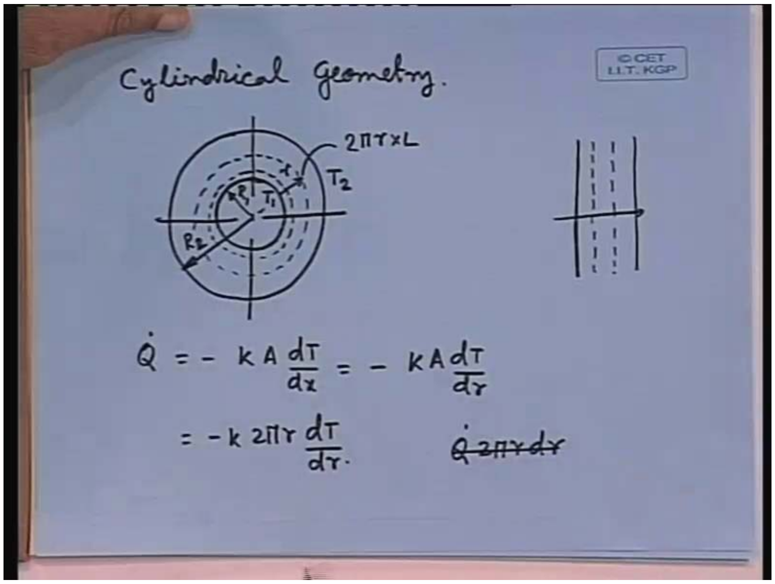

Cylindrical geometry.

$$\dot{Q} = -kA\frac{dT}{dx} = -kA\frac{dT}{dr}$$

$$= -k\,2\pi r\frac{dT}{dr}.$$

~~$\dot{Q}\,2\pi r\,dr$~~

Let us put it as Cylindrical geometry. Let us take the example that we have got an annulus. In the annulus, one wall of the annulus we are keeping at T_1 and another wall we are keeping at T_2. If T_1 is greater than T_2, then from the inside surface heat will be transferred to the outside surface. This is a very common example, like we have got a steam pipe and over it, we have got some insulation. So, from the outer surface of the pipe, the heat will be flowing or heat will be transferred to the outward direction. The outside temperature, where the temperature is low probably it is at atmospheric condition there will be flow of heat. In this case, also, we can use the same formula and we assume the cylinder to be infinitely long. So there is no end effect. We also assume that temperatures are uniform. Inside wall is at uniform temperature of T_1 and the outside wall is also at uniform temperature T_2. Material property is not changing in the annular direction. So in that case, the heat flow will be only along the radial direction. One can write Q dot is equal to minus KA dT by dx that is nothing but minus KA dT by dr along the radial direction. This is r and let us say this is R_1 and this is R_2.

We have got two radii R_1 and R_2 inner radius and outer radius. In this case, we can do the integration but area is not constant. Area is a function of radius because what is the area through which heat is flowing vis a vis, if I have the plane wall this is the area at the outer surface, this is the area somewhere in between, this is the area somewhere in between and heat is flowing in this direction. But in the case of the cylindrical geometry, we have outer surface area, some intermediate area and then some other intermediate area. We can see that the intermediate area is different from the inside surface and that is different from the outside surface. In fact as we go from the inside surface to the outside surface, the area of cross section through which it is flowing changes; area of cross section through which it is flowing changes.

A general expression of the area, let us say at radius r will be 2 pi r into L. Try to visualize. Along the L, we have got infinitely long cylinder. Otherwise, this analysis is not valid and there will be end effect. Sometimes we write 2 pi r assuming that L is equal to 1; so we are doing for only unit length of the cylinder. So heat transfer, whatever rate of heat transfer we will get if we multiply it with required length, we will get. Sometimes it is done like this. If we do this then we will have minus K into 2 pi r dT by dr.

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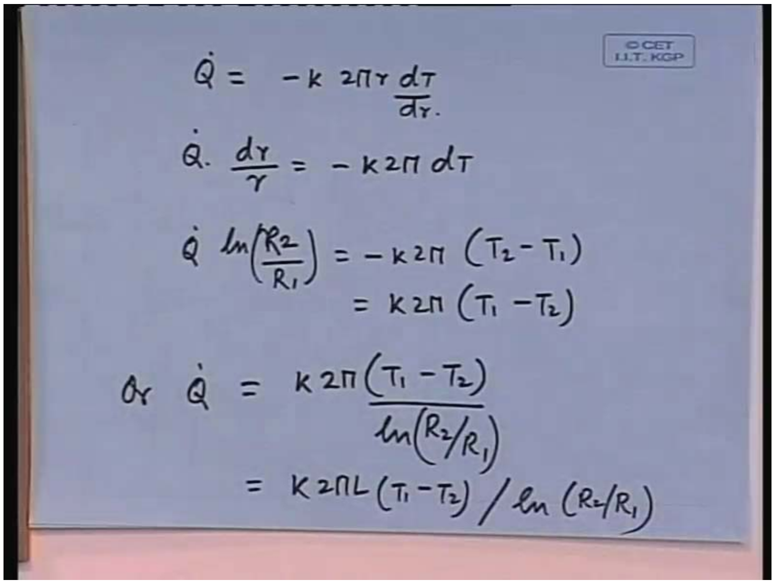

$$\dot{Q} = -k\, 2\pi r \frac{dT}{dr}.$$

$$\dot{Q}.\, \frac{dr}{r} = -k\, 2\pi\, dT$$

$$\dot{Q}\, \ln\left(\frac{R_2}{R_1}\right) = -k\, 2\pi\, (T_2 - T_1)$$

$$= k\, 2\pi\, (T_1 - T_2)$$

$$\text{Or } \dot{Q} = k\, 2\pi \frac{(T_1 - T_2)}{\ln(R_2/R_1)}$$

$$= k\, 2\pi L\, (T_1 - T_2) / \ln\,(R_2/R_1)$$

Let us go to the second page. Q dot is equal to minus K 2 pi r dT by dr. Q dot into dr by r is equal to minus K into 2 pi dT. If we do the integration, then we will have Q dot into ln R_2 by R_1 which is equal to minus K 2 pi T_2 minus T_1 that is equal to K 2 pi T_1 minus $T_{2.}$ Finally, Q dot is equal to K into 2 pi T_1 minus T_2 divided by ln R_2 by R_1 or one can write taking the length into consideration that is equal to K into 2 pi L T $_1$ minus T_2 divided by ln R_2 by $R_{1.}$ This is what we get in case of a circular cylinder, when thermal conductivity is constant.

Let us take a small break and then again we will resume from here.`