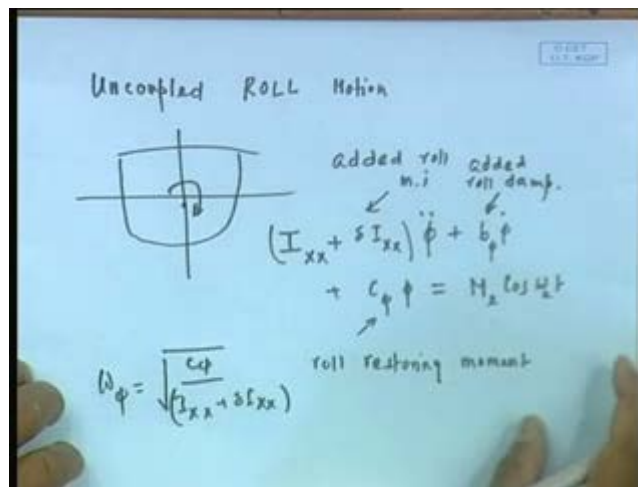


Seakeeping and Manoeuvring
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Lecture No. # 09
Uncoupled Heave, Pitch and Roll - V

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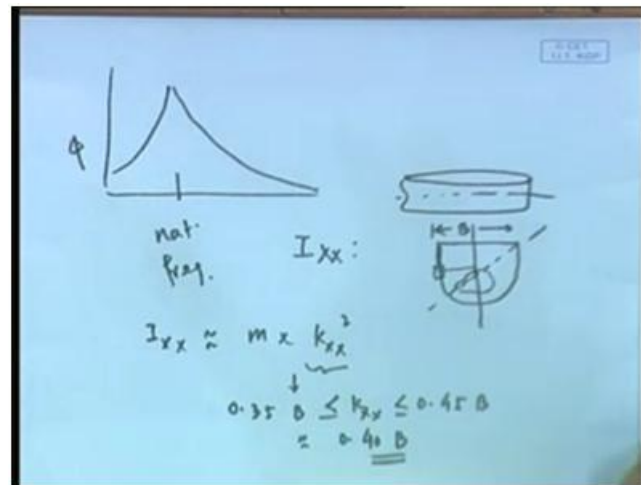


So today, we are going to talk about uncoupled roll motion, the last part or last lecture on this uncoupled motion; as you know that roll we are defining it roll is essentially dynamic (()), again the form of the equation looks exactly same, we end up having I, but here it is I xx rigid body plus delta xx phi dot dot plus you can say b phi phi dot plus c phi phi equal to M well, we can say just the same equation of motion.

Once again all single degree equation of motions have the similar look. So, we have here added one moment of inertia in roll mode, we have damping in roll mode, we have restoring force in roll mode. This is if I were to write added roll moment of inertia; this is roll damping; this is roll restoring moment.

So, form is exactly same there is just nothing that we need to worry about; obviously, it will have its natural frequency omega phi once again given by square root of c phi by I xx plus delta xx, which is very important the equation motion solution looks exactly same as phi will look going like that coming like that etcetera this is the natural.

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So, there is nothing to it; **the** as per as this part is concerned, the solution there is nothing to it we all have studied that same as in roll, same as in peaks, same as in heave; what we need to of course, talk about relative magnitude that how do I estimate these values and what is the kind of natural period at what range it lies etcetera. So, we will start with this I_{xx} you see. Now, we have this ship and x axis is this one if I had to draw a section x axis is going through this is my B let us say. So, I_{xx} of course, is mass into K_{xx} square what would be a good guess for K_{xx} that is the question I want to ask you.

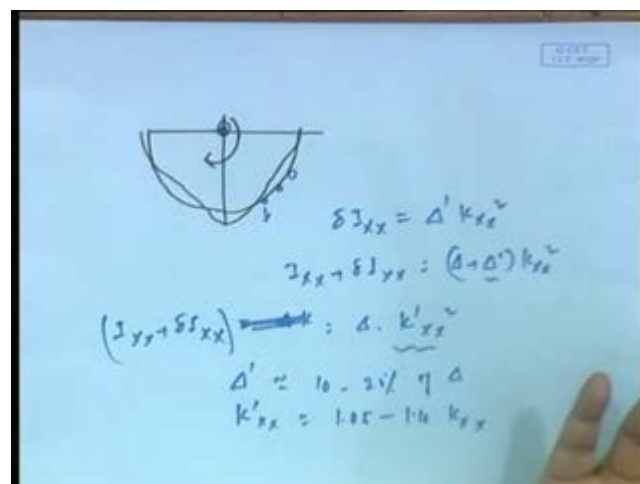
Remember K_{xx} is a about this axis, now in case of pitch we took K_{yy} as around 0.25 l or 0.3 l, but here of course, it is a function of B, because K_{xx} is the mass of this all this masses into the distance square. Now, what happen; obviously, in a ship you have the shell plate. Now supposing it was only the shell plate, I would have had my radius of gyration located at that point itself, but of course, I have also inside many things. So, what I mean to say therefore, you can guess that it is going to lie some here towards this side, it is not going to be 0.25 B or you know this is being B here, this half B it is not going to be two point B its going to be more than that it is going to be hanging towards more like 0.5 B. So, a good estimate that the turn out to be approximately 0.35 B; you can take it nothing is given as 0.40 B. Please understand that, we are in a engineering course, we require to make good judgments.

In case of a longitudinal mass moment of inertia, I took my k_{yy} to be approximately at 0.25 or 0.3, because mass was distributed more or less along the length per section of course, it is slightly less as you go forward. But in this case it is like an hollow cylinder;

if it was a hollow cylinder, I would have had my radius of (C) at 0.25 at 0.5 B. But of course, it has got many weights inside here and all that. So, it will pull down, but it will not pull down, so much as a result you will see these numbers they are more towards outside 40 percent of B approximately. So, that is a good estimate that is what I want to tell you.

Supposing, I want to make that well, you cannot say exactly what there are empirical formulas if there is time, I will mention the empirical formulas, but otherwise it is in that range, if you want to make an estimate. Remember once again this I_{xx} is mass moment of inertia it is not to be connected to I_t that you use in you know like gn meta centric height study.

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Now, look at this added moment of inertia. Now this is another thing that you pay attention we I want you to pay attention. Suppose, if it was a cylinder and I rotating it about this point tell me what would be the moment of inertia for this added moment of inertia. We have discussed that and that is why I am asking you suppose there was a fluid particle here when the body rotates does it accelerated the answer is no, because we are talking of non viscous flow as a result if it is semicircle and you have got you know rotation point exactly here you have practically now well you have 0 theoretically 0 added moment of inertia.

So, now it is not semicircle really, but it is something else you still expect the added moment of inertia to be extremely small, because naturally it do not disturb see if I push it down in heave mode or in a pitch mode also gets pushed down it gets much more

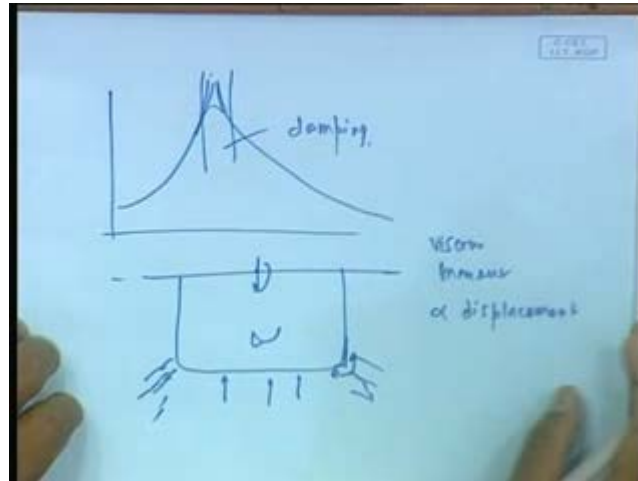
pushed down, but in roll mode you really expect very less water to be disturbed its only logical to think.

So; obviously, my this term ΔI_{xx} which I can write see I can write it two ways I can write it to be the Δk_{xx} square where I can think as if you know in if I write it that way then my I_{xx} plus Δs_{xx} they become Δ plus Δk_{xx} square I can write it this way or I can write ΔI_{xx} as Δ into K well I can write other way round **sorry** I_{xx} plus this thing as Δ into K dash xx square, thinking that there is a augmentation of this or augmentation of mass it is a both way the same thing, it is a question of you know like describing this as a mass into x square.

Now, what happens is that it turns out to be the Δs_{xx} therefore, if you want to estimate it is a small number it turns out to be as if Δ dash is something like 10 to 20 percent of Δ or if you do it K dash xx it becomes like 1.05 to 1.10 times of k_{xx} . Because when you are squaring it remember K_{xx} square approximately. So, the effect of these added moment of inertia in roll is very small 10 20 percent. So, even if you ignored it not much of you know like difference occurs in your estimation what I am therefore, telling added moment of inertia in roll is not a very significant number, which is obvious from the picture come in heave added mass was it can be 2 to 3 times the mass, in pitch mode it was same almost same or more than the rigid body moment of inertia, but in here it is only 10 20 percent of rigid body moment of inertia.

So, see the difference it is order of magnitude lower than corresponding added moment of inertia in pitch mode. So, even if you have ignored it you will not be too much of from a practical estimate that is an important point.

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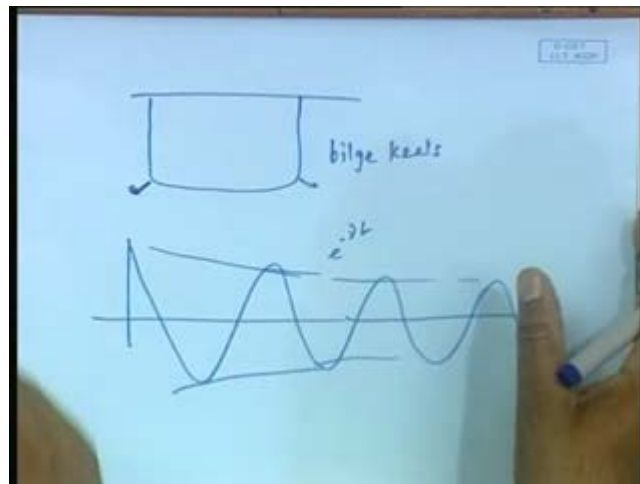
Now, we will have to talk about damping. Damping becomes extremely important as we know if I were to look now this picture you see this zone dominated by damping. But it part slight this damping also will bring it down, now the thing is that here if I look at this picture once again. Well added mass and damping I kept on telling added moment of inertia and damping are two parts of the same coin. So, if this disturbing very less the fluid; obviously, the damping is also going to be very less that is my potential inviscid damping.

So, if damping is extremely less; obviously, this is going to be very high. In fact, damping is very less you know in generally speaking. So, damping is a very, very important concept for ship. Now, what happen here comes this typical question you see, in all modes of motion I presume that water is non viscous. I use the damping also based on non viscous theory, you may question why it is not viscous. So, the question well there is always a question the answer was that well viscosity has very less influence on the forces because most of the cases the forces are normal forces as we pushed down fluid is trying to push up normal force it is not really going along it. So, the viscous effect is very low excepting for roll mode sub motion.

In roll mode sub motion what happen when I turn it here, you see this corner it is rotating now a fluid particle cannot just go around it theoretically also, because it cannot go around a corner. So, what will happen it will separate out why does it separate, because it cannot stick to it on the separation is only a because of viscosity without viscous without being viscous flow cannot separate out you if I have a sharp corner if I rotate it I oscillate it from this corner there are flow separation.

Therefore, this is the only mode of motion, where viscous force becomes important, how does viscous flow act viscous force is always proportion to displacement and therefore, it is considered to be damping force any viscous force you will write down any viscous force or moment in this case is proportional to displacement.

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So, what happen roll is the only mode of motion where viscosity plays a very crucial role not only that on top of that what we will be doing or people do it is that you deliberately introduce here that bilge keels let us, why do you do that because you want to actually introduce or increase damping why we do that. So, that my resonant peak this peak comes down significantly. Remember I will show you the roll modes otherwise damping radiation damping is very low if you were to run a computer program you will find this roll this peak going to be very high giving a roll of 50 degree, 80 degree, 90 degree like that, because this becomes very high practically, because compared to heave and pitch my radiation in viscid damping roll damping is very, very low.

So, but; however, there is some roll damping for viscous even that is. So, because roll is the only mode you will find out that if I were to start a body rolling it will keep on going for a long time very long time, you know like if I just displace it say 10 degree and leave my hand, it will keep on going rolling for a very long time, the decay is very small that shows you know this decay is $e^{-\gamma t}$ we have seen it very small damping.

So, we will come back to this damping issue little later, but let me first talk about the natural period and then we will come back to the damping issue. Because roll for roll once again I want to tell you we are talking about heave pitch and roll in heave we have

discussed all about the nature of the single degree freedom equation and its solution same solution exists for pitch and roll.

Now, then when I come to roll I should only speak about what are the characteristic of roll modes of motion compared to the other well one of the important thing is damping very low damping which gives you very a large peak roll. So, peak period is very important we very small viscous damping. Now, for prediction I need to know a viscous damping we deliberate introduce a bilge keels typically for a ship to introduce additional viscous damping. So, that this kind of gives you a viscous damping to lower down lower the period etcetera. So, we will get back to this viscous damping part, because roll is the only part of motion where viscosity is important you cannot ignore it.

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The image shows handwritten equations on a whiteboard:

$$c_{\phi} \cdot \phi = \frac{\rho g \nabla \cdot G M_T \phi}{(W) (G z)}$$

$$c_{\phi} = (\rho g \nabla G M_T)$$

$$T_{\phi} = 2\pi \sqrt{\frac{m \cdot k_{xx}^2}{c_{\phi}}} \approx 2\pi \sqrt{\frac{(m+m') L_{xx}^2}{\phi}}$$

$$m+m' = \frac{\rho \cdot \nabla}{g}$$

Below the last equation, there is a handwritten note: "1.1 to 1.2" with a horizontal line underneath.

Now, I wanted to get back to that after I speak about the natural period. Let us see about natural period or c_{ϕ} . Now see c_{ϕ} by ϕ is my restoring moment. Now I know that if I were have a small angle displacement then my this thing is $\rho g \nabla$ which is of course, w into $G z$ which is $G M T$ into ϕ this is nothing, but $G z$ we all agree with that $w G z$ is my restoring moment z is $G M T$ into ϕ . So, therefore, this part is my c_{ϕ} that is c_{ϕ} . Now, let us look at natural frequency and let us put directly period. So, period is of course, 2π root over of i plus δ i . So, that I can write this to be mass into K dash xx square or other way round at 1.1 mass into K_x square let me write or rather let me write it that way only let me write it this or it be easier to do **sorry** we work this out in steps.

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$$T_\phi = 2\pi \sqrt{\frac{f \cdot \rho \cdot K_{xx}^2}{g \cdot GM_T}}$$

$$= 2\pi \sqrt{\frac{F K_{xx}^2}{g \cdot GM_T}} \approx 2\pi \cdot K_{xx} \sqrt{\frac{1.1 \cdot \rho}{g \cdot GM_T}}$$

$B \approx 40 \text{ m}$
 $K_{xx} = 0.4 \cdot B \approx 16$
 $GM_T = 2 \text{ to } 3 \text{ m} \approx 17-18 \text{ sec}$
 $T_\phi \approx 1-10 \text{ sec}$

So, I go back to this. So, T_ϕ 2π ; so mass plus m dash I let me put 1.1 time this thing you know some fact let say put some factor may be let us see some F into mass is ρ into v this is actually around 1.1 1.2 otherwise said. Look at this what I am saying is doing is once again this part m plus m dash I am writing this as f into ρ into this is my m and this f is actually around 1.1. Mass plus see m plus m dash K_{xx} square we have talked about that as if mass has been augmented the augmentation is only about 10 percent 20 percent. So, I can write this as f into mass is ρ into v f may be around 1.1 1.2 it is easier to do that way divided by here I have got ρ g v into $G M T$ **sorry** here I have got K_{xx} square.

So, this turns out to be 2π see no here there is this gets off. So, I have got basically I say K_{xx} square into F by g into $G M T$ let me write this as 2π K_{xx} square root of let us take this F as 1.1. So, outside may be 1.1 or rather let me put this 1.1 around 1.1 or so. This is the counter values we get. So, now let us, you may have being knowing about this, but I want to work out the numbers for a typical ship, so that we are understanding, what is happening let us say, take a guess let us take a ship of the approximately give me a number of typical ship say you 200 ship or so 40 meter 40 the big ship we will say 40 meter you are saying.

So, this will be approximately a large tank or a something like **yeah** say some 250 meter long ship fine then K_{xx} is may be approximately how much say 0.4 times v that is going to be 16 or. So, say 0.4 B 16 how much you will take $G M T$ here no **no no** see whether you I by v or not remember I by v will give you B m t , but you want $G M T$ meta centric height typically may be 2 meters or. So, for this kind of ship 2 3 meter will be there.

Let us say 2 or 3 meter 2 to 3 meters say. So, I want to figure this out how much how much it comes out to be there 2π into about 16 into root over of say 1.1 I write say 10 into say 2 how much this comes out to be can you tell me, well 1 by 20 20 is approximately 4 and half 1 by 4 means, 4 how much this comes out to be actually 2 may be too small it may be 3 for such a large ship, because you are looking at a very large ship remember 40 meter breath ship. So, three would be probably better I mean, I would think. So, smaller ship will be much lesser, but if you work it out may be I let us put it 3 31 by 30 square root is about 5 and a half or. So, so if use 5 and half or 3 6, 6 into about 18 second or. So, you are getting 17 18 second whatever in this case you are getting normally, but this is for a very large vessel these also explains why large what is are more comfortable

If you are to take a much smaller boat you will find out this will become smaller, but this will also become much smaller. And you might end up getting actually roll period for typically small boats may be again about 8 10 second for small vessels. What I want to see here is this thing T_ϕ is proportional to these of course, all of you know it K_x by root $G M T$ this everybody knows or I can say now proportional to root over $G M T$ this is the typical case for a ship for stability I want $G M T$ to be high, but if I have $G M T$ high my T_ϕ becomes low.

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$$T_\phi \propto \frac{K_{\phi}}{\sqrt{GM_T}} \propto \frac{1}{\sqrt{GM_T}}$$

$$T_0 = 15 \text{ s or more}$$

$$GM_T \uparrow \Rightarrow T_\phi \downarrow : \text{STIFF SHIP}$$

$$GM_T \downarrow \Rightarrow T_\phi \uparrow : \text{SOFT SHIP}$$

Now, for small boat specially, it turns out that if the T_ϕ is actually 8 second or. So, you are not comfortable people are comfortable if T_ϕ is around 15 second or more typically a ship which has high $G M T$ gives you low ϕ this is known as stiff ship, but I

think you would have known that in your design calculation and the other way round would be a soft ship.

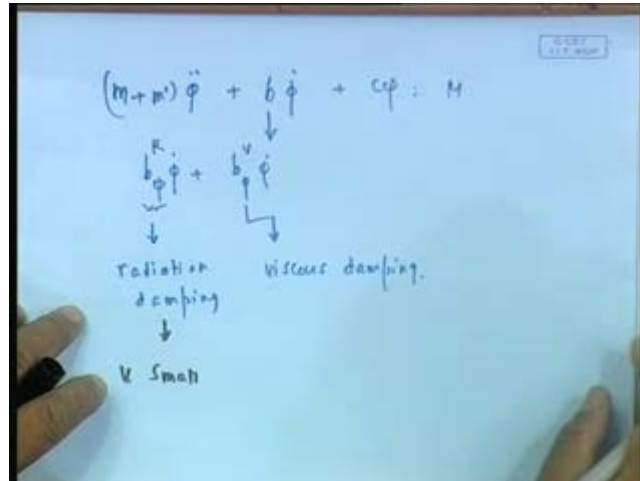
Now, the problem that occurs this is one of those you know like balancing thing that you need to do. See you take a naval ship in a naval ship really the comfort is very less important or even if you take a cargo let us say the cargo ship also it is only the you know ((C)) going in such a cases comfort being less important you might afford to have higher G_m because 0, because you do not mind lower T_{ϕ} , but passenger ship where the stability becomes an even more important issue you have a problem of deciding $G M T$, because actually you would want to have larger $G M T$.

However you end up getting larger $G M T$ giving you a lower T_{ϕ} which people do not want to go on, because you do not want to go on a ship which is lower than sixteen second or. So, typically it is not comfortable because rolling is much more than pitching. As a result you have to have a balance between the two. So, when you doing a design you actually need to balance this two specially, for a passenger craft, you do not want it too small too large $G M T$, but you cannot just keep on increasing $G M T$ simply, because you think that it is more safe.

So, increasing $G M T$ you save T , but beyond a point you should not increase it, that is a main point I am trying to tell you right now you know you cannot increase $G M T$ arbitrarily beyond a point, because if you do that you end up getting much, much uncomfortable ship; this is a typical problem that you have for in a design in one of those kind of design you know like you will say balancing at that one is to do.

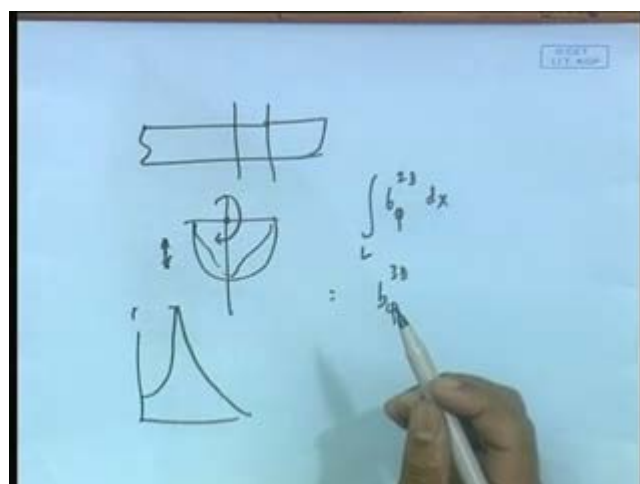
Let us talk little more about this damping part now one getting back to the damping part, what we say, these are this damping. If I were to write down my equation of motion again, let me write down again one, I want to say this way see this damping b_{ϕ} say m now this part.

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So, I cannot two part one is $b \phi \dot{\phi}$ well you can say $b \dot{\phi}$ one is say b let me call it $r \phi \dot{\phi} + v \phi \dot{\phi}$. So, this is my radiation damping, potential damping, inviscid damping, and this is my viscous damping, this is much more important as I said because I said that this is very small this also is small actually relatively, but at least compared to that it is not. So, small now see the question is that because damping is very very important as for as roll motion is concerned, because if you see if I did ignored it as I said if you do a prediction, you will end up finding that a resentment period, I have a roll which is something like you know non feasible 95 degree 100 degree like that, you may end of getting, because your damping is very low radiation damping suppose you are adding a computer program you know suppose, I will tell you about this $b \phi$ estimate earlier let me say that you know just next. This is my vicious damping let me say I ignored viscous damping one of the way I am estimating radiation damping would be again a stiff theory.

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So, have remembered that I have done this ship in various strips; so each strip I have actually various equations and various modes of calculations; there are theoretical solutions about added mass in this direction. Similarly, added mass in this is also available as I said earlier theoretically you know if you take a semi circle added mass in heave is equal to the displacement of this in deep water and rotational inertia is 0. So, there if you make it by some transformation those information is available based on the similar kind of graph and charts so, I can always integrate that sectional added a you know like 2 d added this thing and along the length I can get my 3 B radiation damping for roll motion.

Now, if I do that I end up getting a number and if I use that number in this equation and solve it I will end up getting a resonance peak, which will become very high, it will priory go to you know high number why it is, because the roll potential or radiation or inviscid damping coefficient is extremely low almost 0. The; however, we know the reduced mass so. So, therefore, this damping becomes very important now this damping estimating this damping is very difficult, because it is viscous phenomena if I want to estimate this damping I have to solve a Navier-stokes equation a viscous flow equation remember not a potential flow equation. So, very difficult so what is a standard would be to determine this damping of course, overall damping because you know you cannot separate out by experiment. So, you do an experiment to find out this it is a very common thing to do that, but what experiment.

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Handwritten slide content:

$$(m+m')\ddot{\phi} + b\dot{\phi} + c\phi = H$$

The term $b\dot{\phi}$ is broken down into:

- $\frac{R}{V}\dot{\phi}$ (Radiation damping) → V Small
- $\frac{V}{L}\dot{\phi}$ (viscous damping)

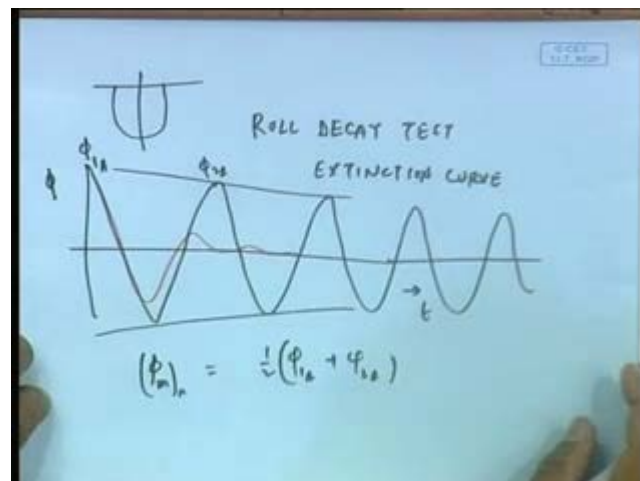
The term $c\phi$ is labeled as potential damping: $c = f(\omega)$

A diagram on the right shows a wavy line representing a wave with a downward arrow pointing to a box, likely representing a ship's roll motion.

Now, go back to this our previous lecture what we did we call that when we did this part of damping that is, the radiation damping that is, the potential damping we found it the function of omega remember that is, why the force oscillation test was done it depend on at what frequency you are oscillating. However here what happen this damping turns out to be fairly insensitive of omega, the potential damping once again you must get that in your mind see radiation damping is as I oscillate it the wave it makes see if I take this body, if I oscillate it makes this waves. The energy that has gone dissipated in the waves is radiation damping. In fact, you will be very easily find out that if I took a body and pushed it there is some waves, but if I just rolled it any this thing the wave that creates is very low you cannot even see it, which essentially would imply that the energy that has gone away by this in viscid you know like theory making waves is very low; that means, this part of damping is very low.

Of course, this part of damping theoretical is dependent on frequency, but our interest is practical we want to know, what is the total damping for the ship, for which it turns out this is more important than this part is not. So, frequency dependent therefore, I can now do a very simple. So, called decay test which is what is typically done.

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What is a decay test is very simple, you take a body or shifted to some angle leave your hand what you will find that the roll is becoming keep on going for a long time, for a very long term area you can do a test now. Why you could not do this for heave and pitch added mass, because this test would give you the heave damping and added mass only for that frequency natural period. So, called damp natural period, but here I could

do that, because damping that I am looking for I am thinking it is not function of frequency.

So, this is a test very simple and you know like. In fact, people do in lab most hydrodynamic lab this test is always done, this is what is called roll decay test, this is my extinction curve you just oscillate to some point leave your hand it will keep on decaying and. In fact, you will find out that if I were to superpose here a typical you know like heave you will see heave would become like that in two oscillation will become like that if you just push the body down you will see two oscillation go like that that actually shows that heave damping is much more even from potential point of view, but roll damping is even with a viscosity very low, and this is I repeatedly saying this, because this is very important, because without this damping if you have to predict motion you would not get a good prediction at the resonance period.

So, this now how do I find this damping out actually I will not go through a theory, but supposing I measure this peaks here. So, I can find out say theta 1, theta 2 etcetera I can find out the decay part let me just write it down from here. See if I were find out see here theta 1a theta 2a etcetera. Then I can find out theta mean per oscillation n nth oscillation that will be something like half of say half of say phi 1a plus phi 2a they of course, decaying it that is the mean period its decaying it again oscillation period m. It turns out that if I were to plot that here, what is my phi mean, amplitude of phi mean against n it actually decays it.

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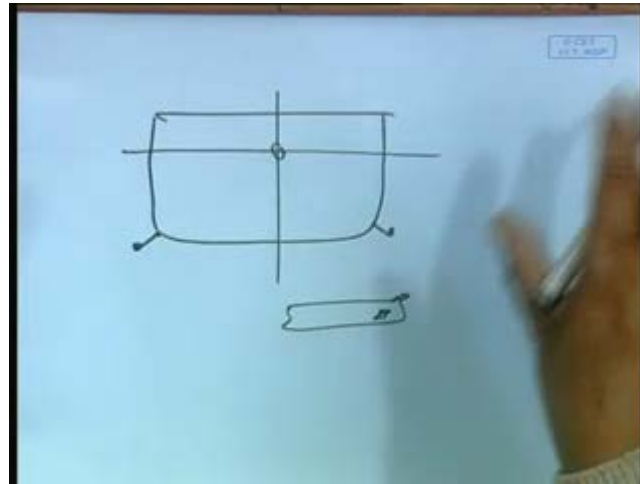
The image shows a whiteboard with handwritten mathematical derivations. At the top left, there is a graph with a vertical axis labeled $-\frac{\partial \phi_m}{\partial n}$ and a horizontal axis labeled ϕ_m . A straight line with a positive slope is drawn. To the right of the graph, the equation $-\frac{\partial \phi_m}{\partial n} = k \phi_m$ is written. Below this, the equation $-\frac{\partial \phi}{\partial n} = \frac{\pi^2 b_p}{T_p \cdot \delta G H_T} \phi_m$ is written, with the fraction $\frac{\pi^2 b_p}{T_p \cdot \delta G H_T}$ circled. An arrow points from this circled term to the k in the equation above. Below the circled term, the equation $\Rightarrow b_p = \frac{k \cdot T_p \cdot \delta G H_T}{\pi^2}$ is written.

In fact, we this will be decaying, but what we do is if I want to we plot $d\phi$ by dn it basically will be decaying minus, because in other words what happen it turns out that $d\phi$ mean amplitude by dn is equal to k into ϕ n we can fit a graph like that, actually here we can put ϕ m here if we put this one, and it turns out from basically from energy balance. See here, what is happening is that because of damping it amplitude comes down. So, we can have a energy conservation principle about work done by the damping force and work done by the restoring force etcetera it turns out that by doing this balance again why not going to this the derivation you will see that $d\phi$ by dn turns out to be π square b is this thing T ϕ into δ into G M T into ϕ m .

So, from there since I know G M T δ etcetera and I know this thing this is a actually k this k I found out. So, we see this part that is this my damping this is my $d\phi$. So, this part is nothing, but this k . So, therefore, $d\phi$ can be found out as T k into T ϕ δ into G M T by π square or what I want to say here is without going to this that an extinction test is a very important test that is always done, the test is very simple you probably would be doing in a lab also, simply keep it rolled leave your hand it will keep on decaying and you get a very nice curve you will see that you get a very nice curve for many oscillation it will keep on oscillating.

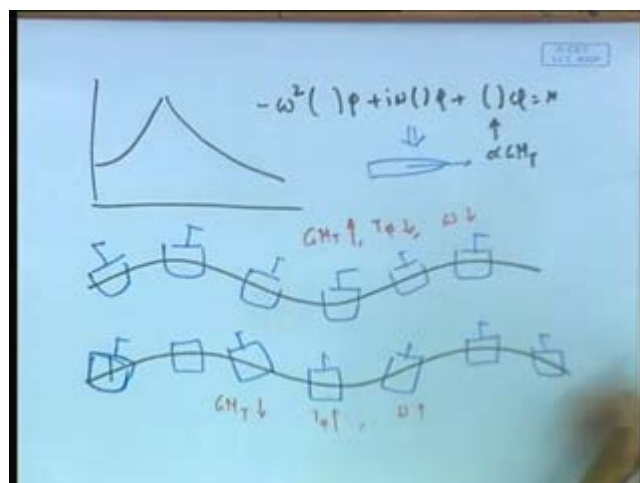
You just want to know how much it is decaying and this decay; obviously, as you know is e power of minus ν t basically, you can also relate this decay to be e power of minus ν t this ν is actually b by $2a$ you have seen this earlier all. So, from there easily we can find out from the decay the damping that the b part now that the various now theories of relations to get the b which is one of which I showed there are other ways of getting the graph also. So, the question is you do find the damping here and the damping is extremely important and it is this reason why almost all ships will have a bilge keels. You see that you always put a bilge keels at the middle where do you put the bilge keels that is the another thing you put exactly the bilge.

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In fact, for typical ship we are calling this to be bilge area remember this to be bilge area and you always put it here why, because you get the maximum leverage. You see when it is rolling here it gives you maximum the damping effect, because of this damping of course, in roll it turned out to be a much more complex thing, because it turns that it also depends on speed it also depends on many other phenomena etcetera. In nowadays in this, this is one way of what is known as passive device to control roll passive, because you do not do anything there ships coming up here in the ship you put some kind of fin outside a fin, which will actually control the fin to reduce pitch as well as roll etcetera they are all becoming part of it because you do not want large motion people do not want to be on a ship.

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Now, I wanted to tell you about another part of this characteristic of roll motion once again we have to go back to this, and you see here this part of roll I want let us take a

wave now at low frequency what happens, I have a static roll flag is here, flag is here like that. Always remember once again I am looking at a beam way, because you expect; obviously, largest role to be at beam way means, the wave is coming from you know the ship is here and waves are coming from this side you expect largest roll. So, in a beam wave let say typically at low frequency that is the wave is very large then, the ship is going to be in this way, because it is just like in a pitch what we have seen its follow the, but the problem happens in the other end this end we end up getting an opposite phase, where I cannot cross properly for this.

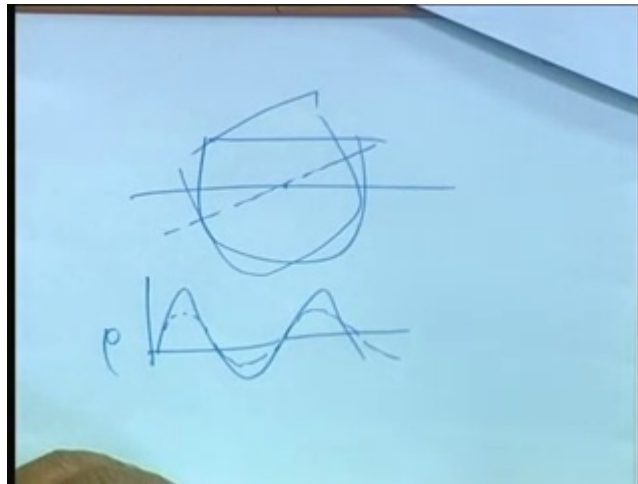
We can clearly see why it is. So, much more dangerous right very you can call it hydrostatic roll now normally what happens once again its interesting part we see, this will happen not only low frequency also, you can say at high meta centric height if $G M$ is high this see an important distinction see. In pitch mode of motion and heave mode of motion we only talk of frequency, because waves are much longer, but in here typically restoring force should dominate. Now, restoring force will dominate suppose restoring force dominates you see once again the equation remember the equation of motion will look like ω^2 into added masses into ϕ plus or plus or minus $i \omega$ into damping into ϕ plus restoring moment into ϕ equal to m .

Now, if this is nothing, but proportional to $G M T$ if this is very high even for a some ω is very high; obviously, this will dominate. Remember that in a ship $G M L$ you do not it is already high, but do not change it by factor of two etcetera. So, we have already seen that, but $G M T$ can become half a meter can become 2 meter or 1 meter $G M T$ have proportion wise $G M T$ have a chance of changing more widely, because it is a small number by itself 50 centimeter to 1 meter is factor of two whereas, a $G M L$ which is 150 meter will not become 300 meter it might become 180 meter you understand that. So, as a result what happens $G M T$ has a more influence. So, here what happens when my $G M T$ is high my vessel is safe you have this kind of rolling you can say this will happen at high $G M T$ low $d \phi$ in other words this is actually if I were to write this way this one is $G M T$ is high $T \phi$ is low and of course, you can say ω is also low.

This one is on the other hand $G M T$ is low $d \phi$ is; obviously, this two are connected and ω can also be slightly high. In fact, I am just discarding the ω part, but I am saying this measured from point of view of $G M T$. So, what happen is that if $G M T$ was high there's a tendency for the vessel to align itself to low wave which of course, is

safer you what is this you can study this purely by hydrostatics, because it hydrostatic rolling in other words if I were to look at that basically what is happened there is a restoring it is like I have healed it there is a restoring moment that has made it back parallel to this line.

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See here, if I were the vessel was initially like that my something like this you know let me also show this here something like my vessel was like that initially waterline is like that my water line has become like this waterline become like that my vessel is also turn to like that right. It will just try to, because it is very slow it is trying to, because the restoring moment is very high it will there is no space it will try to go up that way. But so; that means, this would have what will happen it very high you know restoring moment very low natural very quickly it will turn $T \phi$ roll means it can it is the ability to turn very fast the restoring moment is too large uncomfortable, but safe this one on the other hand when I have got $G M T$ low $T \phi$ is high end of having this.

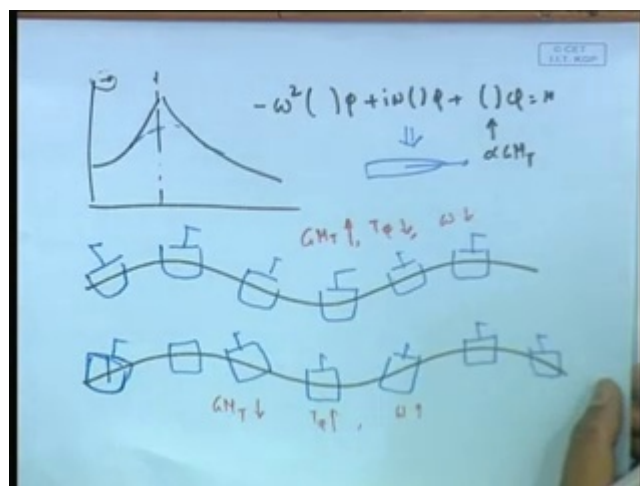
Now, you see the problem that that we have, you want to have your design ship more or less in this range, because this is most comfortable for a ship. So, you want to have a more comfortable ship. So, therefore, you want to be an operating this range. So, we do not want $G M T$ to be too high, but the $G M T$ is not very high then there is a chance of its getting into the waves. So, there is a risk factor more. So, therefore, from roll design point of view you have to actually have a wave proper you know like thinking you cannot make a stiff ship, but if you want to make a soft ship then you have to be careful regarding that it does not become dangerous.

So, therefore, what people do you actually introduce damping what is happening by damping, what happens the amplitude is reduced normally and slightly the period, but amplitude is reduced this is actually brought down this is a reason why you have this damping devices coming up in roll instead of rolling 15 degree you make it roll say 7 degree.

Remember period does not change much, because damping b is as a small number. So, ω with damp damping and ω without damping does not differ very much, what differs is the amplitude you would have rolled something like this. Now ϕ would have been something like this, you actually bring it down to this essentially and that is the reason. Why you have so many kind of rolls stabilizing devices you know, which will in future lecture we will talk active, passive device you know nowadays control like all kind of fins you put are controlling it essential, because you of course, one thing is that is that if amplitude is less your acceleration is going to be less you also will have a you know a more comfort it will be more comfortable. So, this is about this static rolling.

Now, let me just talk about some empirical formula that we have I thought, I will spend this other time on this finding out this added mass and roll heave moment of inertia having said this 0.4 0.45, since we have little time we purely thought of doing this, let me think about the phase gap part, we let me just spend little more time on this.

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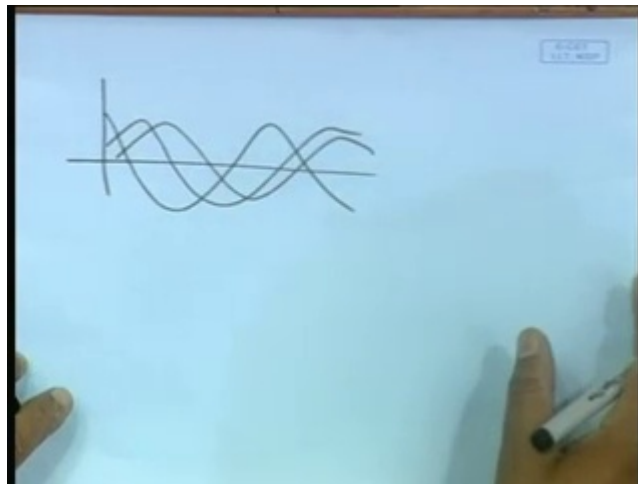


See here one thing we are finding out that all of them, if you look at a phase that you know all this theoretically at this region the phase is basically, out of phase with waves

here it goes an **sorry** in this in phase with waves at this point it actually goes 180 degree change and here it becomes out of phase with waves.

Now, a good point of we have seen in heave and pitch is that in this low frequency the starting point differs, in heave the phase is with the crest as crest goes up and up down it goes up and down, but in pitch it is with the slope here also it is with the slope at this, but slope in the other plane Remember if the waves are coming straight forward really speaking you have very low roll you expect roll to be much more when it is coming beam wise. So, it is slope from beam wise. So, in a sense there is some difference in the phases between the free signals and that is how you want it, because if you do not want it to have that same phase when everything is occurring very high, but we should realize and that tells me that if I were to draw pictures if heave was like that you want peaks something like that you want roll something like that, and there will be there will be phases between the between the various signals.

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Of course, you cannot say that when it will be so, but phase always is an important parameter to be found out doing a solution, but without phase we really cannot you know like say the history of the motion and all we are talking is uncoupled motion when we will talk about couple motion we will find out that there is more complications. Let me just spent this rest of the time by telling about this some empirical formula, which may be you will find useful for determining the moment of inertia added moment of inertia moment of inertia for ship.

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$$I_{xx} + \delta J_{xx} \approx M \cdot \underline{\underline{K'_{xx}}}$$

ordinary merchant ship:

$$\left(\frac{K'_{xx}}{B}\right)^2 = f \left[C_B \cdot C_U + 1.10 C_U (1 - C_B) \left(\frac{H_e - 2.2T}{T}\right) + \frac{H_e^2}{B^2} \right]$$

C_B : Block coeff.
 C_U : upper dk area coeff = $\frac{A}{L \cdot B}$
 H_e : effective depth of ship superstructure
 $= D + A/LBP$

So, I have this I_{xx} plus δJ_{xx} it is written as here mass into K'_{xx} dash square and there are some formulas given for this that what I wanted to tell I wanted a thinner pen all of them are same (No audio 46:30 to 46:19) see this is only to tell you some idea regarding you know several empirical formula that has been developed that (No audio 47:00 to 47:19) H_e square, so some formula where why I am saying this, because you know I will expect and we will we can do some problems where these are all part of ship parameters for your some typical ship you can estimate these then you can actually determine the natural period from $G M T$ etcetera. Upper deck area coefficient, this area by length into breadth deck area coefficient H_e is effective depth of ship superstructure depth that is given by D plus A by $L B P$ this just for your reference only D of course, is small than breadth that.

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D : moulded breadth
 A : projected lateral area of superstructure
 \approx dk above main dk
 L_{pp} : Length Bet. Perp
 T : draft
 B : beam
 f = constant
 $= 0.125$: passenger, cargo
 $= 0.13$: tanker
 \vdots

Let me just write it down then we just discuss it projected (No audio 49:11 to 49:41) 1 b p is L DP T is draft and f is a constant (No audio 50:01 to 50:29) that see what I just wrote here you know like you do not be. So, worried about this in a sense it is just to tell you that there are some empirical formulas available based on ships for initial you know initially people have done it to estimate this, but I tell you I want you to just see this is not a it is just in case you want to do a estimate having said that you can always take this K x by B 2.4 for first estimate.

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The image shows handwritten formulas for the periods of heave, pitch, and roll. The formulas are as follows:

$$T_{\text{Heave}} = 2\pi \sqrt{\frac{k_z \rho V}{(\rho) A_{\text{wp}}}} = 2\pi \sqrt{\frac{k_z}{\rho g A_{\text{wp}}}}$$

$$T_{\text{Pitch}} = 2\pi \sqrt{\frac{k_{\theta} k_{\text{yy}} \rho V}{\rho g \cdot GM_L}} = 2\pi k_{\text{yy}} \sqrt{\frac{k_{\theta}}{\rho g \cdot GM_L}}$$

$$T_{\text{Roll}} = \dots = 2\pi k_{\text{yy}} \sqrt{\frac{k_{\theta}}{\rho g \cdot GM_L}}$$

Below these formulas, there is a boxed note: $8-14 \text{ sec}$.

Now, let me just sum up, because today I want to complete this single degree of freedom that now this natural paired which I always consider most important see, what we end up getting let see here T of heave, we write this three formula down, and we want to do this time they are quickly for a barge all the three to know, how they are for a typical ship the one example we took earlier was for a very large tanker for which you are undergoing 18 second, but you know that has take some smaller 100 meter barge or so.

So, this was 2pi here you see we will take this factor k into added mass factor in heave let us say into it was K z into mass right, mass was rho g v into rho g A wp no naughtt rho g v rho v **sorry** let me write it down 2pi square root of this is added mass factor K z into rho is cancelled out volume by g into A wp this was a formula when K z was maybe one or two something like that K z was added mass factor how many times of the mass is this rho gets cancelled out. So, g this thing now T of pitch 2pi here again I will write from this K factor you know like K let us say theta factor into remember we had here K

yy square into mass, this is my I divided by here we had rho g v into G ML I have got rho g v into G ML.

So, again this I will this cancels out this comes out to be $2\pi k y$ I can take it out square root of and T of roll similarly, it was $2\pi K_{xx} K$ of this thing into $g e M L M T$ so here what you notice is that now we can make this estimate for this if you want I think we will not have time here. Now, see this part understand this look at this added mass part this may be 1 2 or. So, important you cannot **sorry** not 1, 2 2, 3, because it is already 1 plus you know like 1, 2, 3, 2, 3, 2 and a half 3 etcetera cannot be ignored absolutely you have to take some value 2 or so.

Similar is this, but this is very close to one that is 0.1 see here it is K_{yy} and $G M L$ this is K_{xx} and $G M T K_{yy}$ is a function of a length typically $G M L$ is also function of length. So, there is a length function there and more or less it is of some order 10 12 15 second this is the function of b this also the function of b.

So, again this is also of similar order. So, what it turns out is again we have we are seen earlier this two also of the similar order. So, if you look at order all of them are ranging between say 8 to 14 second or so I mean, something like that it is all within all of the three of them is within the everyday occurring waves this is the problem see, whether it is roll whether it is pitch, whether it is heave all of them lie somewhere, where I have the everyday waves coming.

It is not like offshore structure, where I am deliberately designing to be far of even roll. So, having said that our aim would be this two point of view of law of motion this law of motion is, but more importantly the natural pitch to be make slightly higher, because of comfort point of view, because mostly if you go to any vessel it is roll is much more, because the amount of rolling is much more for the simple reason that the damping is very low, you end up having a roll of say 6 degree 5 degree etcetera pitch is not never.

So, this is where I will end this now the our talk on this single degree freedom equation of motion, but this point is extremely important and we have to always remember this, but typical ship you cannot avoid it offshore structure we have avoided it and this two are you know like comparable for offshore structure. So, with that, we will end it for today's class.