Seakeeping and Manoeuvring

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Lecture No. # 05

Uncoupled Heave, Pitch and Roll – I

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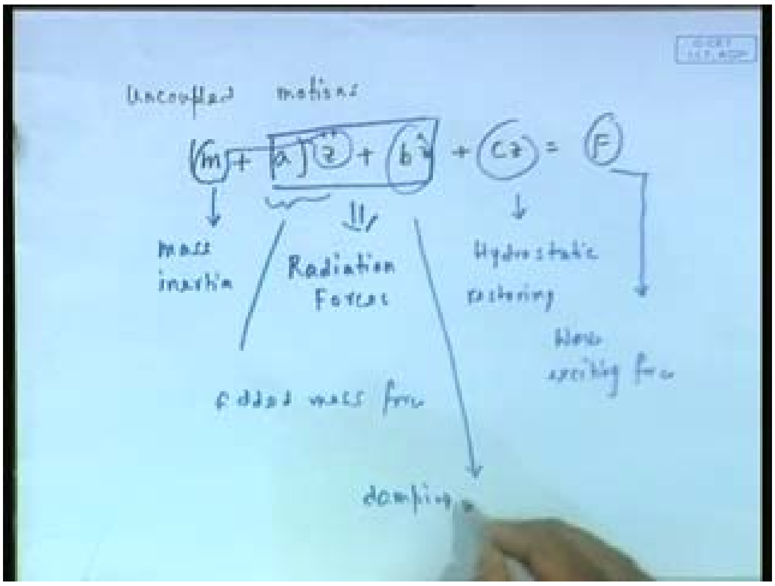

So today, we are going to start the lecture on uncoupled motions. See in the last class I derived or showed that equation of motion for an oscillating system for ship turns out to be of having a formed something like mass plus some part into well, I just put an simple this thing mode of motion, b into z dot plus c z equal to F, where what I said is that this part this was what was called as radiation forces.

This part is m z dot dot was mass inertia force, this was hydrostatic restoring forces and this was wave exciting forces and in specifically, this could be called added mass force and this could be called damping. But I think we will specify it to be radiation damping or potential damping force because it is arising out of only radiation damping as as we have derived earlier.

Say let us call it damping, because other wall also might look similar. Now having said that so this was single degree. If actually I consider a 6 degree of freedom that is a ship in waves, then every motion will affect every motion. And all this force, get coupled means, you will have some radiation force arising in the heave direction because of roll motion etcetera etcetera. But today we are going to talk about that to start with a simple heave equation of motion single heave equation of motion.

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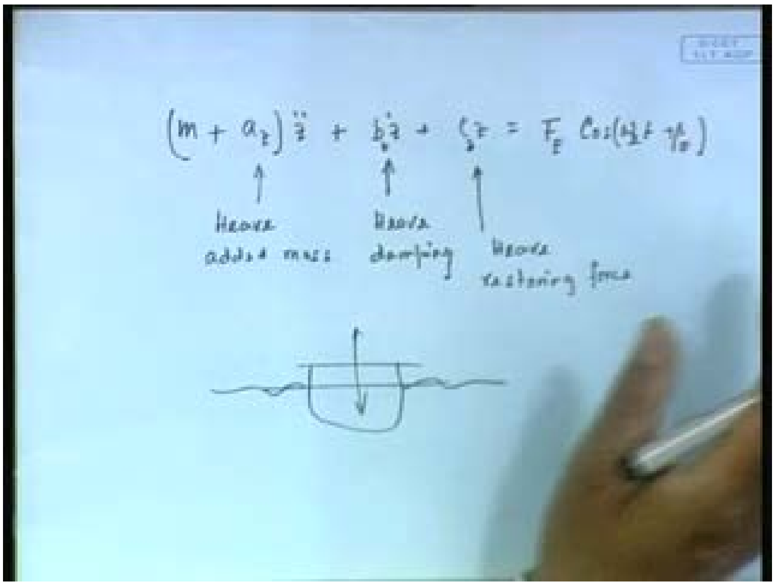

So now, that will look like the equation for that will have then m plus let me call this a z now. Now exciting force, we know the f. But I explained to you that in waves see everything is periodic with a periodicity of omega e. So therefore, this can be written as some amplitude of exciting force, let us say I call it this thing.

And I can write it as cos omega e t well we can also put a phase here but that depends on let me put a phase here also, let us say we can also put a phase here as beta. Because the phase you can also make it 0 depending on where you start from. Now this is a single degree equation of motion for heave, where I will call this to be heave added mass. We can put it also z here and this is heave damping and I also let me put it z also that is heave restoring force. Now before I discuss about these terms, we will first let us talk about the solution of this equation motion how does it look like.

You know, this is a equation of motion if the bodies undergoing only heave and nothing else. And we are assuming that as it goes heave up and down, that is body is undergoing this mode of motion. It is not exciting or causing any other mode of motion just uncoupled, you know we are making this assumption. Now you see this equation of motion, we will start now supposing we all know this it looks like and vibration equation. If I made this 0 and if I remove damping what I end up getting is what is known as undamped natural motion or equation for undamped free undamped motion. So, let us look at that so, if I just consider the inertia force and c z equal to 0 then this is what is my so, called undamped free motion right

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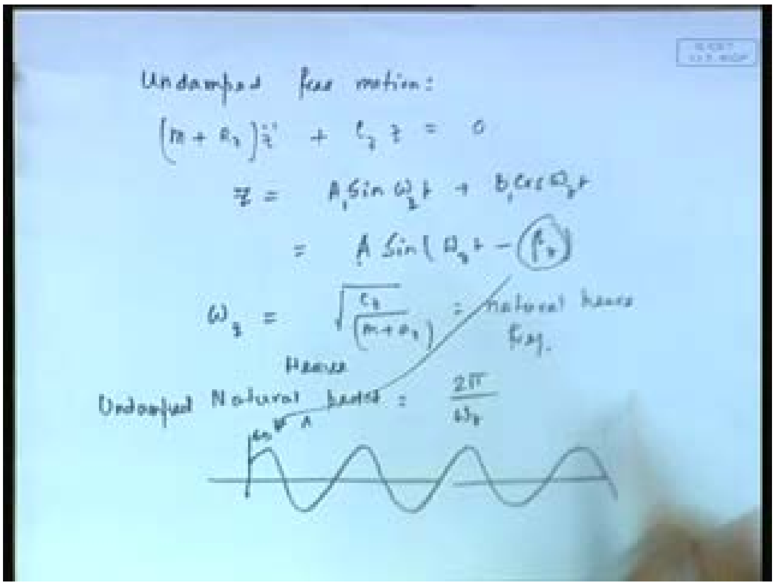

Now this, undamped free motion has a solution which will look like z equal to A sin omega z t plus b cos actually either you can call it, this way or I can call it let me call it A 1 b 1. So that, I can write it as a sin or cos omega z t minus beta z, as you knows it is the same way. Either you can write in as a 2 sinusoidal term or one sinusoidal term with a phase gap.

This is how the solution is and the most significant thing here is w z, this is given by square root of c z by m plus a z which is what is called natural period natural heave period sorry sorry natural heave frequency. So that, natural period is simply 2 pi is going to be 2 pi by natural heave period of course, I mean this implied here by omega z.

Now, natural period is very very very very important concept, this is why we have to insist on that afterwards. But this we know this is what is called well, we can also specify it is natural heave period for undamped motion natural free period, if you want to add the word undamped you can also add that. Because I have reduced the damping right I do not have damping here.

How does this solution looks like well it will look like a sign curve. The beta is going to be a measure of this that is a phase that is the phase, this beta measures in some sense this how much later with respect to well at t equal to 0. It is having a motion, essentially if I know A and if I know beta z, then this solution is fully complete which means that in this equation you of course, I am assuming we know m A z c z. So, here if I know well if I know that if I know omega z So, omega z is known the only unknown is A and beta z and of course, I can always solve it. If I had two initial condition but that is a trivial thing not very important.

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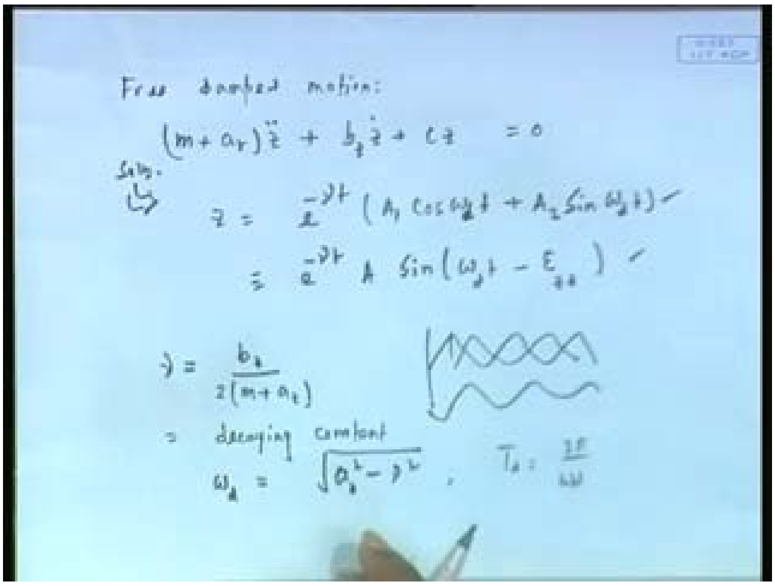

Now coming to the next level, that is my damped natural period, free damped motion. If I consider, that means here I have a damping but no forcing. So, here the equation will look like m plus a z double dot plus so, this as a solution this solution we all know this. So, we need not go through how mu t, we can call it d omega d this is d t plus A 2 sin omega d t or which is equivalently e power of minus d t I can call it this way. A sin omega d t minus eta, let us put something eta z d let me just call it some phase angle. See remember that any signal with a phase gap can also be written as sum sum of 2 sin.

Because you know sin a plus b is sin a cos b plus cos a sin b, if you actually apply you will end up getting this relation. This is very straight forward, I mean what I mean is that two sinusoidal curve is essentially one sin curve with a phase curve if you add this up if you add this and a cos curve. You will find out that summation is nothing but another sin curve starting with a phase curve. So, one can use either this expression or this expression I mean, so this is interchangeable. We need to know, that there are two unknowns in a signal A 1 A 2 or A and eta z 2 unknowns which ever form you write so this is known. In this case, this signal looks like that this part is a free vibration but there is a decay part. Now, this nu is given by b by b z by 2 m plus a z, this is called the decaying constant.

In fact this solutions that I wrote, we had made an assumption here that b z is small decaying constant is small. See if the decaying was very very high, then it could have just dived down a periodic motion but in a it is presumed. Water is a very less viscous water as such even if you take viscosity damping. Of course, here we are not considering even viscosity, then the damping is very low.

As a result b z is always very smaller b z by this this mu is very small so, it is always looking like that. So, this is a relevant solution for this as far as heave equation of motion is concerned. Of course, the other thing is w d this is given by, that this is a damp period frequency z square minus mu square root over. Now here or therefore, I have got T d equal to two pi by omega d at this point so, this is how this is the solution how does it look like.

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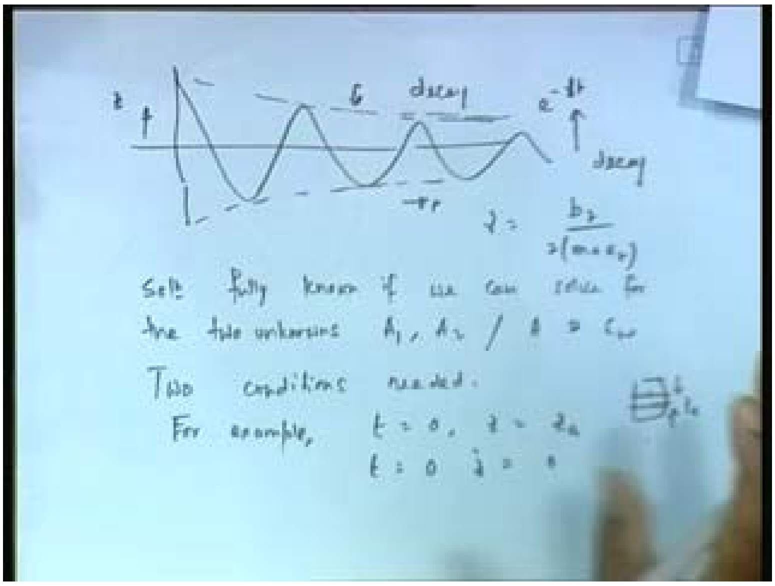

It was it will look like decaying, this is my z and this is my p and this decay this decay is e power of minus mu t depending on this mu, it was large as decay faster longer this part this is decay decay parameter right. What is this as we have found out this is actually b by b z by 2 I just wrote it in last slide.

So, this is a decaying parameter once again, if I want the solution very simple I have two unknowns because the solution that I looking back at that two unknowns. One was you can call it either this and this or this and this if I know this, this and this the solution is like this the solution is fully known. If I could determine this to unknowns I can determine this to unknown if I have two condition such as somebody says let us say that, I have got in initial condition.

Say, let me put it this way that is A 1 A 2 or A and eta z t as this one as I mentioned and for that you need two condition. So, we can put in form of initial condition. For example, I can say that for example, let me see I can say that at t equal to 0 my z equal to z a and also t equal to 0 my z dot equal to 0. That is very actually, very you know valid point what I mean is that it is something like you are pushing the body down. So, I know at t equal to 0 what is my see I pushed it down to an extent z a. So, I know what is my initial displacement that is what is at t 0 z is how much I pushed it down no imposed motion left my hand. So, at that time an initial velocity z dot is 0. So, I know this for example, so if I know this I can find out c 1 c 2

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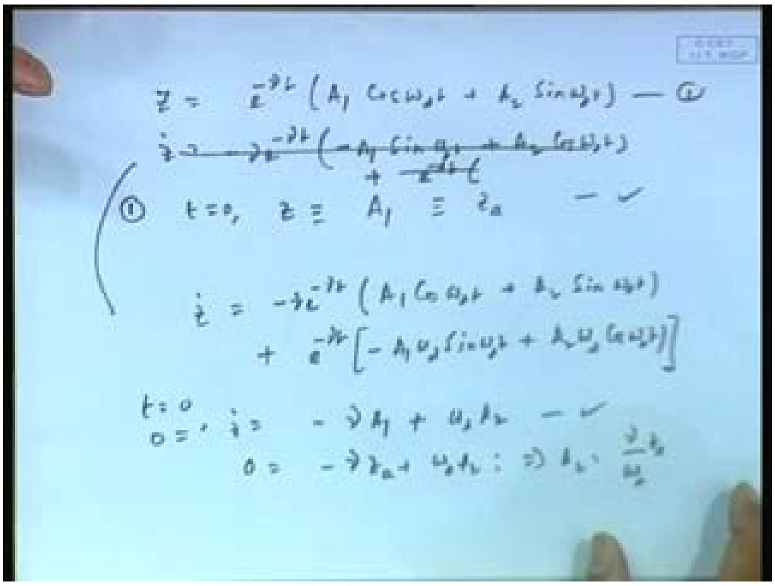

I just I give an example of that see, if I look back here again I will write it in this form. In c 1 c 2 form, let us say if I wrote z as again e power of minus nu t A 1 cos omega d t plus A 2 sin omega d t, I have z dot given by minus nu e power of minus nu t is becomes minus A 1 sin omega d t plus A 2.

So, I know that see at t equal to 0, z is equal to from this equation c 1, from one I will get z is equal to simply A 1 right. You see because at t 0, this is 0, this is also 1 so, z becomes A 1 cos you know 0 that is A 1. And which of course is equal to my z a, because this my initial condition z a becomes a.

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[FL] omega d will come right that is what you are saying thank you, see here your omega d will come right sorry sorry you are absolutely sorry absolutely, we are going to get actually you are right this was actually for this differentiation.

Then I am going to get one more term that is e power of minus mu t right right right sorry about that. We are going to get here, well well let me let actually deduct that wait, let me write it now you're you are absolutely right. So, this is I am sorry this is going to be first minus mu e power of minus mu t, then A 1 cos omega d t plus A 2 sin omega d t right plus e power of minus mu t. Here I m going to get this differentiation that is what you are saying, right that is minus, we are going to get A 1 omega d into sin omega d t right plus here, we are going to get A 2 omega d into cos omega d t that is right right you are absolutely through. So, in any case what therefore, is happening from this one. Obviously, at t 0 my z dot becomes this this goes off, then my z dot will become minus this A 1 right plus I will end up getting omega d into A 2.

So this becomes my one equation and this becomes one. So, from there this is actually equal my initial condition is 0, so I end up getting 0 equal to this. So my point, you say here is that I have found A 1 is z 1 and here I have got 0 equal to minus mu z a plus omega d A 2. So, this gives me A 2 equal to mu z a by omega d right.

Really speaking, this I just went through this, it is very trivial my full point of saying here was that we had two equations sorry one equation with two unknowns. So, to solve that I need two equations, so to get the two equations I need two conditions. It can be any two condition I can say any two condition. So that, I can get two independent equations, so this is once you know that two equations you can solve for it that is all. A really it is also not very important it just to prove the point, but what is most important for us, now is the the forced part the full solution.

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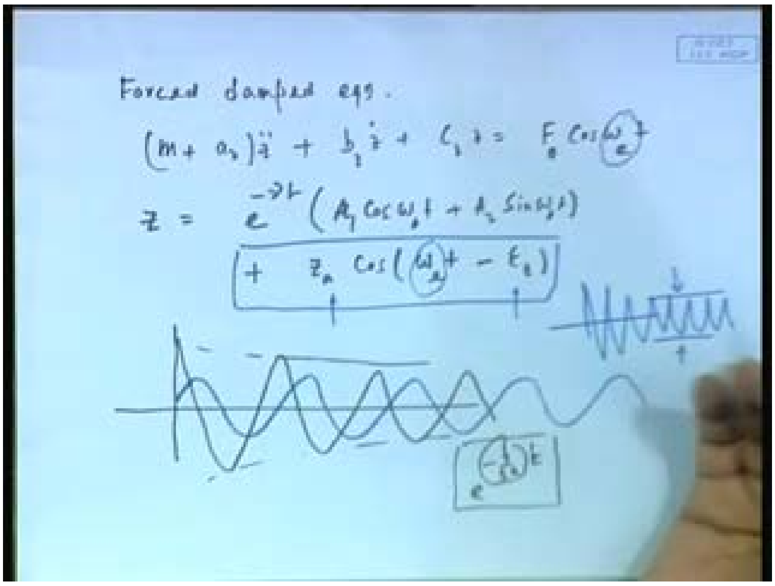

That is now you'll look at forced damped equation of motion the final one. So, here I have got let me call it here F, let us call it F 0 here, this e. Now, you know this has everybody knows this it has two solution, its solution can be written as a sum of the few so, called complimentary solution as the full solution. So, it essentially the solution can be written as z equal to e power of minus mu. One is solution for the homogeneous that is going to make it 0 and 1 is for the full power. So, this is something like or A 1 let me call it.

Now here will come an interesting part, plus there'll be sinusoidal solution forced solution e t minus say epsilon z. Now you see how does it look like the first one is this one. This one is going down like that and a second one is actually a forced solution of omega t. You are actually adding the two together solution now the question is that what is our interest.

Our interest is obviously in only in this part, why because you see given sufficient time our this thing will decay because of mu factor. So, essentially if I were looking at a steady step solution. That means, if I am holding for a long time what would happen the body will begin to heave only with this oscillation, forced oscillation because this one will decay down.

In fact, the full solution may look like, if you were to draw experiment something like that and ultimately it will become a straight line. So, this is what I am looking at this is what is my interest fully, so we are looking at only the steady part of the solution. Again here my unknowns are this and this and what you notice here is that this is oscillating at that exciting frequency omega e yes. No no no no not at all, all structures you see the question is this decay the we are not talk of q or see or the question. It is nothing to structures here it is that we are looking at a this is a decay factor minus v by A, when minus v by a would make e power omega t small if t, in fact in real life just 1 minute will become 0 actually.

Because see you are having e power of minus b by 2, A this factor whatever way it is with t if you give t 1 second, 2 second etcetera shortly it becomes 0 in practical. In fact, in reality it may become actually, you would not even notice it will be actually 0 in almost like 3, 4 periods 10 second 15 second or so. Remember it is nothing to do with structure it is connected to the decay part of it.

And we are only looking at the steady part of the solution, when we are saying forget about few more (( )) structures, even (( )) structure. When the wave comes I am looking at for 1 hour, 2 hour, 3 hour kind of solution not in 10 seconds that goes off. And second thing is that wave always existing, you cannot actually fix a time when it started you know, when you are looking at that.

See suppose you today go on boat well, it does not oscillating yesterday onwards not that it has started 0, cannot see and suddenly started. So, what you see is always a steady solution the decay part is just not there and simulation also shows. In fact, we will see later on by numbers b by 2 A and that how it is how fast it decays. Now, we will look at steady part solution and that part of solution of course, we know it.

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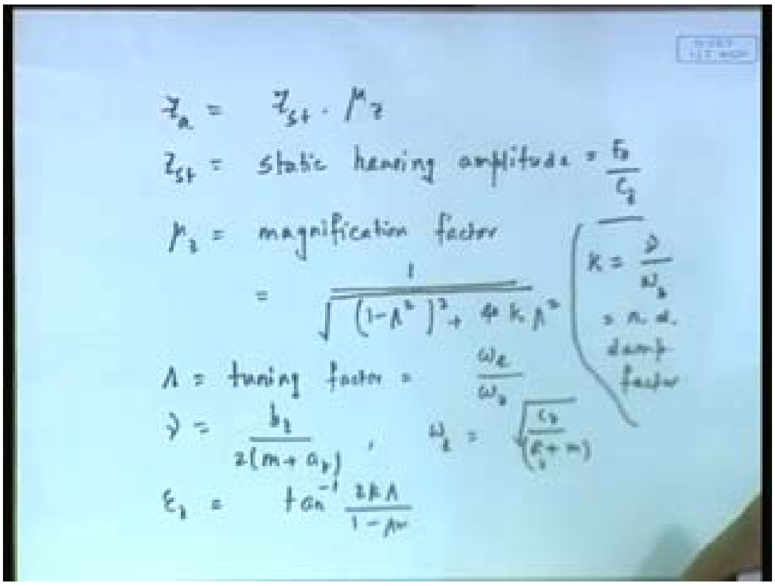

So it turns out that the solution for this steady part, again from vibration is given by like this z a is given as z static into this solution. When z static is known as static heave amplitude, well we are calling heave amplitude but what I am saying is that typical solution for a vibration forced vibration equation. This will be F 0 by c z is this a solution mu z this this all we know it, that is why I will not going to go through this in the the solution part.

What we what we need to know in this course is to tell the typicality of heave motion. You know like how does it heave, how much it does it etcetera, etcetera. Because this part anybody was studied, you know any vibration system mechanically you'll we will find out standard things and the other things is the epsilon right, what we wrote epsilon z. Now we had to write the kappa part and of course I had missed out here one. That is this this part this is given by it is called non dimensional damping factor. See what I mean here is that solution for this equation is fully known to everybody. It is you can write it this way and let us look at the diagram that is more revealing.

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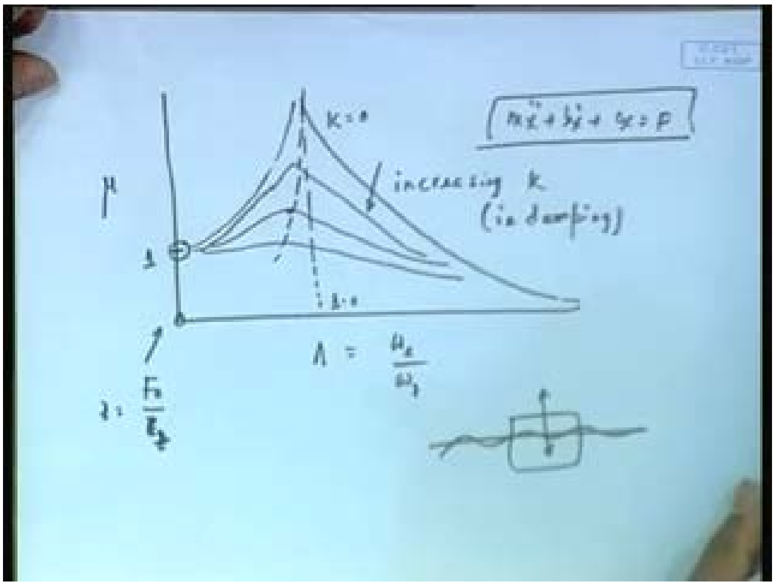

So, it will look something like this, if I were to look here, mu this is here, mu equal to omega e by this is always easier actually, what we are plotting is z verses omega. Now, you can write this in z was omega e, if you want to write instead of omega e. We are writing a non dimensional form of omega, that is omega e by omega z. And instead of writing z we are writing mu because z is mu into z s t, z s t is a constant so, it is a same thing. This is why it is easier this becomes something like that it goes to etcetera, where this is for damping 0 and this is increasing, that is damping.

This is actually one magnification factor, see the thing is that again let me put it this way. This is actually one undamp one and this is actually the damp one. See what is happening understand this all of you have seen this, what what does it show it shows that this equation that if I have the body here heave. Number one is that at low frequency, very low frequency my we need to see this very very important you know. At low frequency at the limit of omega tends to 0, my magnificent factor is 1 which means it simply goes up as F 0 by F 0 by what c at this low frequency. At this region first of all, if I had no damping, I would find out that at tuning factor omega a by omega z my magnification is theoretically infinity but if you give a slight damping it will come down to finite value.

As you keep giving more damping, it actually the peak ships towards this side lower side because this correspond to omega d the damping natural damping period. You go down this side, here it will eventually come down ascend particularly to 0. Now the important point of this we all know this you see this part is known. But we want to know is not the solution part say, importance of this solution with respect to heave motion. Remember this equation m x dot dot plus b x dot plus c x equal to F. It is actually this vibration equation, everybody have studied it is ubiquitously occurring in nature any oscillating system, this is you know phenomena string vibration, any vibrating system, even atom vibration whatever.

The question is not in this solution all I showed here is a typical solution which also all of you know very well. What is important is that the characteristic of that with respect to heave mode of motion that is very important in heave. See heave is not going to be like atom oscillation, it is not going to be like a string oscillation, what is the special like that characterizes the motion.

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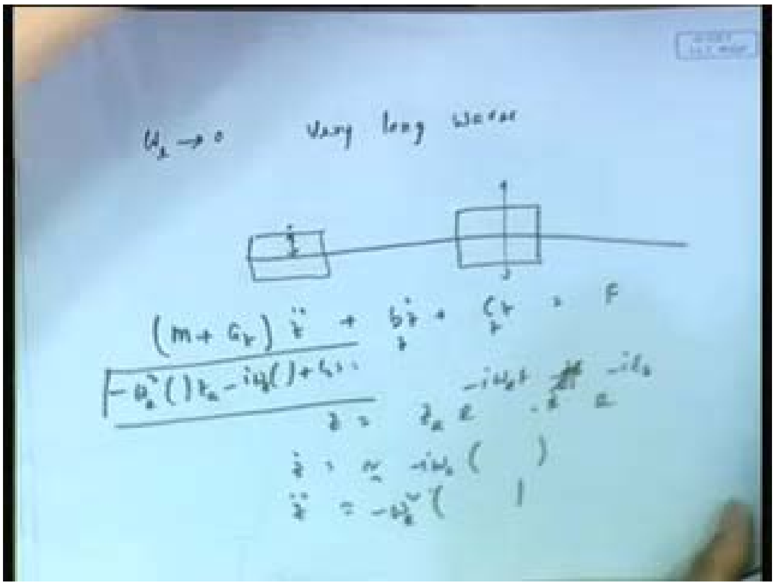

So, here I want to tell you this see look at this low end, what the low end means, now let me look another picture, then now omega e tends 2 0. What does it mean, it means very long waves infinite period. So, you have this very long wave wave body here, now we will again write down the equation of motion well. I just now you know z, you can write z as z a, where I will now this time I am going to write the sort of you know e power of minus I omega e t, let me write it this way.

But there is also e phi may be e power of minus I beta or no I what we call epsilon in the real e power of minus i. I used the term earlier, let us see the phase phase was used as epsilon phase. So, we will use as epsilon z there is not important really speaking but I mean I want to add this now. You see z dot is going to be proportional to minus I omega e into this thing and z double dot is proportional to omega square e into this.

What I want to say velocity has a proportionality of omega e multiplied by displacemen,t acceleration has proportionality omega e square. Now when omega e is very small omega e is very small what would happen obviously so no. So, this equation will look like minus omega e square into something z a, minus I omega e into something plus you can say c z into z equal to F, now it look something like that.

Let us pay attention on that see the first term the force, the added mass force is in proportion to omega e square. Second term damping is in proportion, of course forget the phase gaping proportion to omega e. Third one is not it is a constant. Now when omega e tends to 0, this will actually have no influence very less value this also will have very less value. So, my F z becomes simply F by c z that is what is happened.

Now, what is c z we will come to that but it means is that it is happening so slow. In fact, we will find that the phase, also phase gap become zero with respect to exciting force, this will simply go up and down with the water. Means if water goes up body goes the water comes on body comes down it will simply the wave with hydrostatic force.

So that means, every time if you stand here you are going to find your water surface a free boat, same the free boat is going to remain the same. You do not do not even feel that you are moving, because first of all it is moving very fast. So, that is what is happened in tsunami type of wave very long wave, you do not even know that there is wave coming. Because you have actually you have gone up and come down with the water.

So, as you stand you always look at the water, you you do not feel you are moving. First of all omega is small means t e is very large. So, it takes all the time just image it takes 10 minutes to go up 1 meter, so how would you feel it. So this is one extreme of this graph, you know this part and we can call this region to be hydrostatic motion. That is this side of motion is dominated by hydrostatic force, it is a hydrostatic force that dominate.

You can make all kind of mistake in a z, b z does not matter, do not even bother very low frequency the body is going to go up and down. So, I can make 100 percent a z in estimation does not matter I still get a good estimation. This is why the small boat in large waves the simply ride the wave, so this is what we call hydrostatic dominated motion. Now coming to this other side, this is this side now also we have see, now let us look at this side, high frequency side.

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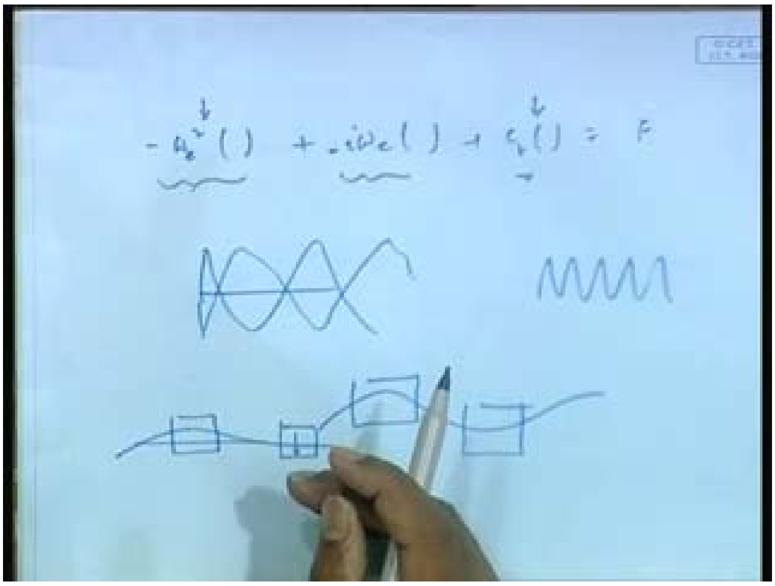

Now here again if I look at this equation, see it was omega e square into something that was my inertia force plus I omega, say it is minus here into something plus c z into something equal to force. Now you see when omega is very high, high frequency motion is very fast. What happens this becomes the much more dominating force this also becomes fairly dominating force this becomes compatibly less important. So, what happen is that it become inertia dominated and you’ll find out that, the force of this and force of this is opposite in sign. As a result there’s a phase gap, I did not put the phase gap.

But if you are to put a phase gap between them theoretically, there will be a change of ninety degree here the phase gap. That means, look at this earlier I had got c z into something is F, now I have got minus omega square minus something into z equal to f. Obviously, the z a in this case if it was like that in the other case it is going to be just opposite.

So, what is happening this is very important, in this case in the first case my boat was riding this. But now in the lower end my boat is out of phase with that that means, when the wave is rising, it is sinking it is more dangerous. You see that is what I wanted to tell you from physical point of view. That means, if I were to draw this here now in this case when the water is rising my boat is sinking.

Here when water is this thing my boat is rising. So, you see I end up getting an oscillation related to the water surface much more pronounce which is of course more important. Because what happens just imagine this case, the free surface is rising the way this crest but my ship is going down. So, there is a chance of water my my free body is reducing.

So inertia dominated motion typically have a responses which is out of phase with the forcing of the waves typically. Now the third thing that is important is the other part that is damping part. Now this this part now this part this small part here this values peak values depends purely on b or mu k value, k is nothing but damping. You will find out that if you get k 0 is infinity, if you get k with 0.001 it will come down to final value drastically

So, this is extremely sensitive on damping. So, this part inside part what we can call damping dominated motion but it is very small part only. Because really speaking estimating this amplitude requires a good estimate of damping b very good estimate of damping b and damping is always very difficult to estimate. But we can call it damping dominate and in fact that theoretically this is where there was a change of phase of you knows like that e z value, e z value actually undergoes a phase change in this case. So, this is the kind of modes of motion that we find.

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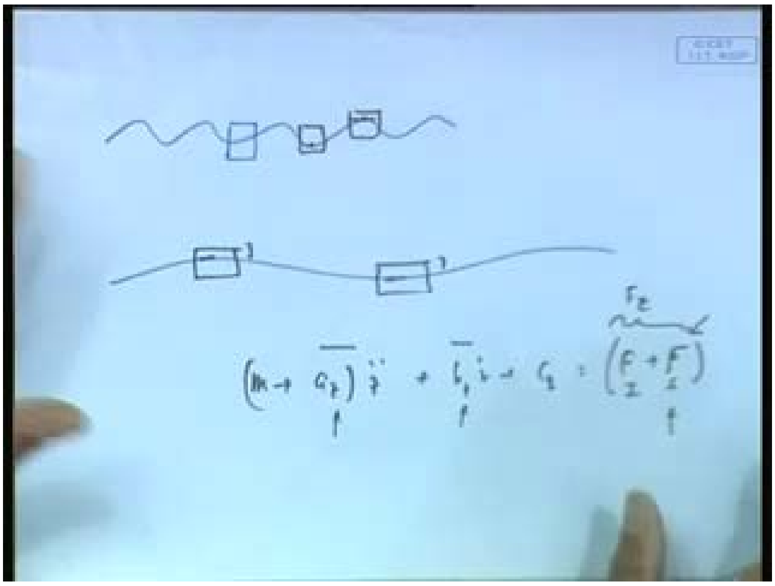

So, we will find out therefore, that you see three cases, one is high frequency wave there is a it is going like that it is in this case sinking more and here rising more. You know this gap is changing go to this long wave it is always riding the wave, it is always having the same. That means, this free boat part remains constant more or less constant so and it is very easy to study this motion. Because I can I do not have to actually determine a z, b z etcetera, again if I were to look at this equation of motion. In fact, what would have happened is that you know that even this part this has got two part. Actually this was if you know the inertia and I said that in the first class it is incident with and scattered with together combines and gives this F e, this F e in principal.

Now, at very low frequency the body is going to disturb the waves very less, so scattering also is very small. Because very long wave, the body simply goes up and down since it goes up and down, it does not you know like dissect or it is not going to disrupt the wave field. Even if the body was not there wave will looks exactly the same. So, this becomes very small as such because these are also proportionate to omega e actually.

So, you end up getting only this part very easy to find out, very easy to figure out, no problem at all. And let us say you made a you are doing a solution, you made a mistake in this estimate which is of course, is very usual to do. Because hydrodynamics is on this and on this, this, this and this. So, in static dominated force there is no hydrodynamics practically, you can make all mistake in hydrodynamics and you end up getting a good estimate.

In inertia dominated force which happens to be more critical in some sense because there is a out of phase. You unfortunately require to have a good estimate of this force and this force, and this force. Because it is the hydrodynamic forces that decide inertia dominated motion. And it is inertia dominated motion, where typically I have a tendency over free boat to change.

So so therefore, this is a situation where things that is critical I might demand on calculating a z, b z and F a s dynamics correctly is also become more important. See however, opposite that the critical scenario, I have less hydrodynamics, I could have afforded to have bad or you know or poor estimate of hydrodynamics. But this is a situation here that you cannot ignore hydrodynamics. Therefore it is the hydrodynamic forces that are more important for inertia dominated this thing and also for damping, if you look at that for that damping part. So, this part is extremely important for us to understand.

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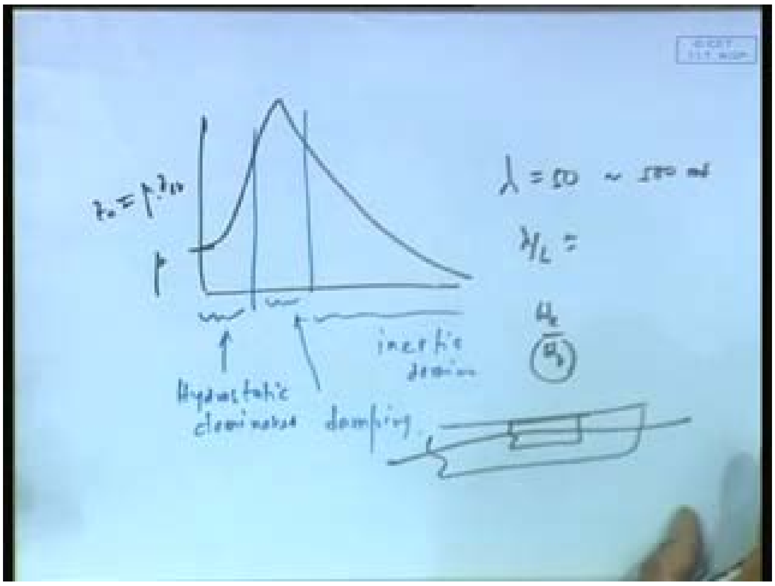

So, again if I were to look back this, I just draw it here again a typical this. So, this once again I call this z a equal to something right, you know like z a equal to mu into well z s t and I plotted only mu here. Whichever you plot mu becomes 1 means it is hydrostatic force etcetera. So, this edge lower frequency range is once again, I tell you hydrostatic dominated, this side inertia dominated, this is damping dominated but this can be narrowed down to a small part, this damping. Now you tell me we will try to tell from physical point of view, what is the importance now look at that. Typically, you know you'll find out that or we will find out afterwards from the point of view of wave length.

What is the wave length in open ocean, we have seen it may be something like may be 50 meter going to say 500 meter something like that. If you if you look at that it will be of this range in fact, lambda by L if L is ship length may range some value depending on the L of course. Now what happen is, if I take a boat small boat of 30 meter or 40 meter let us say 50 meter boat. Then all the waves are existing which appeared to be long with respect to the boat because lambda by small h is 1 and goes to 10.

So, relatively what would happen the boat that have a tendency to follow the wave. So, mostly if I have a small boat, then we are looking at always relatively high frequency motion, why I say relatively because again this response will remain remember omega e by omega z. So, omega z comes in picture, omega z will of course become low this I will come later on. But let us say if the shape is this and if the waves are much longer with respect to that then you are afraid to go up and down.

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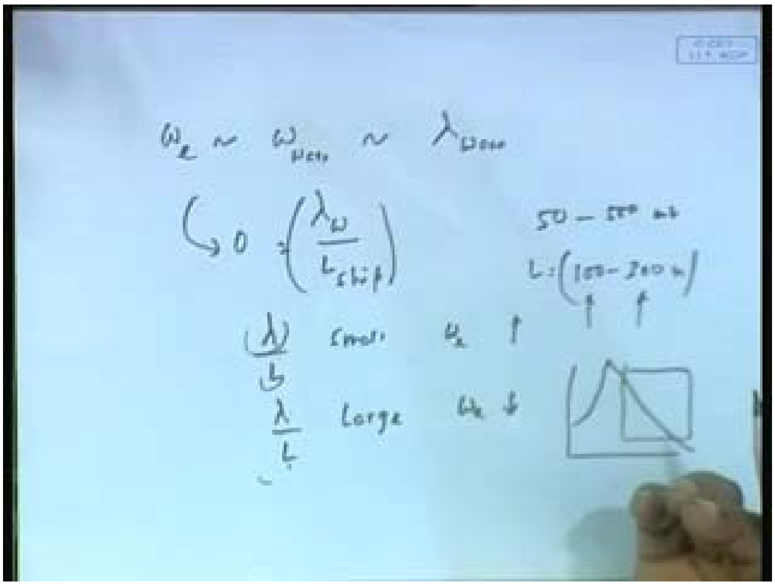

But if the shape is much longer with respect to this. Then obviously, what happens is that you have a situation, where you have got essentially a relatively high frequency wave. Now I can relate omega is lambda, remember omega e can be relate to omega a into omega g k lambda omega e, can be related to omega wave, can be relate to lambda wave. So, I have omega is lambda wave by L ship. So, I can support some of this make it in order of this, I can relate that in some sense it is order of wave length by ship length. Now what is happening obviously, when wave length by ship length, lambda by L is actually lambda is small, it is small means omega e is high. When lambda lambda is larger long wave means omega e is of course small, peroid is large.

You know that is what typically is the question of lambda by e, now you see for a given lambda if L is small then obviously, what will happen for a given lambda L is small, this value becomes large. That means, small shapes feel the waves to be large, large ships will fill the waves small. Now going by the open ocean 52 meter wave to 500 wave length obviously, what happen if a 50 meter one ship fills 500 long ship as 10 times its length. But a 500 meter long ship or a typical 200 meter long ship a ship typically we have got say 100 to say 300 meter. L ocean goings are something like that a typical ocean going ship is something like 100 meter, 200 meter, 300 meter. You know that a v L c may of this length, a typical small cargo ship also of this length.

So for this once obviously, what happen say 300 meter long ship, there are waves which is 200 meter that is going to be small. And therefore for actual practical shape, we are in this region for large ocean going ships, you see I cannot avoid studying hydrodynamics. I am trying to emphasize the importance of that. That if you look at large ocean going ships, then large ocean going ships would encounter waves where the encounter frequency is moderately high and therefore, inertia force dominates as an example you will find out

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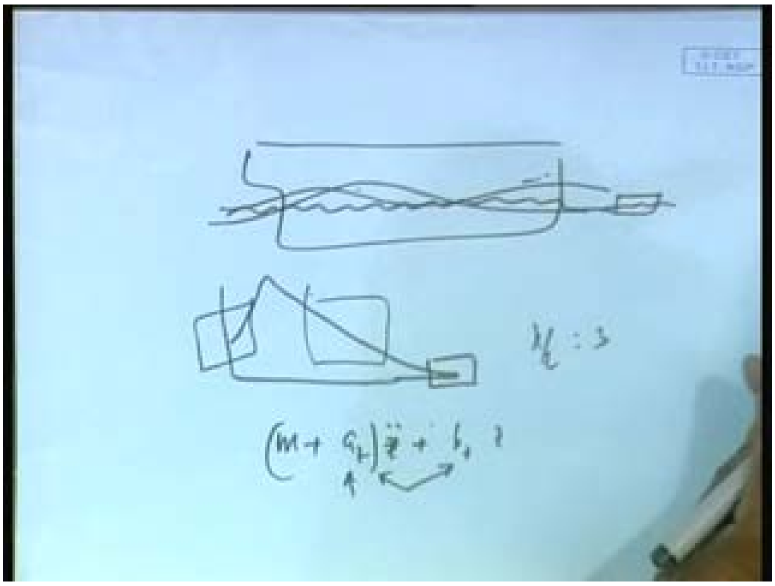

These example if you ever go to port you will find out there is a large ship. If you see there are some small ripples and I you used to always wonder personally myself, that in such cases the boat does not move up and down. You you think that and there is a small boat next to it, you'll find out that as if the boat is grounded. You know I used to feel wondering that this boat since grounded, you know these waves are flashing by and ship does not move a small boat here is slashing up and down. So that means, what in height this is, so small it is actually so high frequency the inertia. In fact, it is something in this region for this boat that it does not have any force to move up and down at all.

But slightly more and something like that this will now begin to up and down. And of course, in open ocean these things always happens and in this case these motions are this region. We are shifting to this region remember for 300 meter long ship to go to this region, I need 1 kilometer long like wave. That means, lambda by L to be 3, if L is 300 meter lambda is 900 meter but most lambdas in open ocean are 300, 400 meter itself. So, I end up getting lambda by L to be around 0.75, 1, 1.5 time

And in such cases I have inertia domination. So, my point of saying is that large ocean going waves, you cannot avoid studying dynamics. And in order to get dynamics, now I have this problem of having to estimate the dynamic part of this. This part I have to estimate to some moderately you know acceptable degree. In the other case I could ignore it completely I can even make it 0 also some very rough estimate. If I met 100 percent in that I might have made 5 percent estimate in of motion.

In the in the low frequency range but here if I make 100 percent error, I might make 50 percent error in my motion, that is very important. That is that tells us that add and mass damping and exciting force becomes an important parameter for us to study. You me just cannot avoid it, now just last last few minutes of this class.

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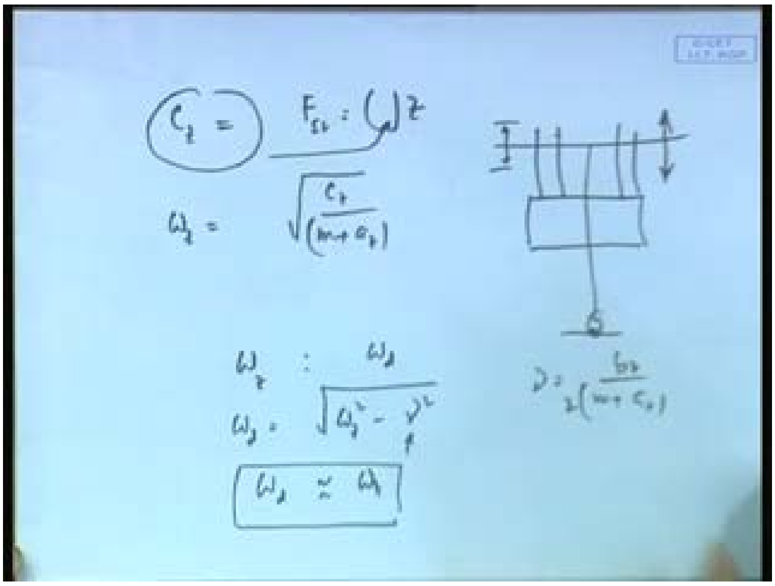

Let me talk about this natural period, c z well omega z equal to root over of c z by m plus a z to do that I first need to have an estimate of c z. How what is c z, I will talk about that but before that let me tell you the difference between omega z and omega d. See omega d is omega z square minus mu square mu is b by m plus a z 2. This mu is a very small number, typically for water it is not you know water is not thick oil, water is one of the least viscous thing. Of course, there is no viscosity also and this is actually damp radiation damping.

What is origin of that I will tell you later on in next classes when we discuss b and a. But what I want to say here is this is small. So this is small means, this square is even smaller. So, suppose this is point 1, this I 0.01 numerically then what happened omega d is approximately same as omega z. So in all practical purpose, when we study natural period and we say resonance as you know that, when omega e is omega z it is resonating what is why we call that is this peak peak value.

So, we can estimate the resonance simply taking omega d to be s omega z. So, our importance become omega z rather omega d, which is also very useful for me. Because if I had to do omega d, I had to get mu mu is to know b z, b z is more difficult to estimate. So therefore, let me talk about this is very important because if my omega e happens to be this, I am going to be very large heave motion right.

So, natural period becomes a very very very important point, especially for off shore structure and off shore structure. It is digging oil from somewhere always stationary, if it begins to heave very large you might stand here and not feel but this oil, this line is going to have this tension. So, you actually cannot allow or cannot have oil explosion going on if the heave exceeds some value say 3 meter or 2 meter or something other

So, you know if it is heaving very largely you actually end up having these problems. So heave becomes very important, when it will have large if the natural period meets the exciting period remembers. So, natural period becomes very important very very important from the point of view of study. Now for for this I need this now, what is c z we will now try to tell about that you see the c z.

And I will just I think that, we talk about this c z tomorrow's class because we do not have time but we understand here one thing is that c z is hydrostatic restoring force. So it purely comes from hydrostatic, there is no hydrodynamics having said that hydrostatic it is not exactly the boyancy force which is what we will try to find out it is the unbalanced hydrostatic force. And remember that we have always called hydrostatic force F s as something into z and this something is this c z.

So, we will derive that c z expression tomorrow. So, that we can estimate this value for practical structures and figure out which range they are. Then we will find out very ironically that, it turns out what typical shapes omega z is actually in the range in which oceans exist, waves exist. So, we will get back to that tomorrow