Seakeeping and Manoeuvring
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Module No. # 01
Lecture No. # 36
PMM Tests – II

Good morning, some of you can go this side now [FL] [FL].

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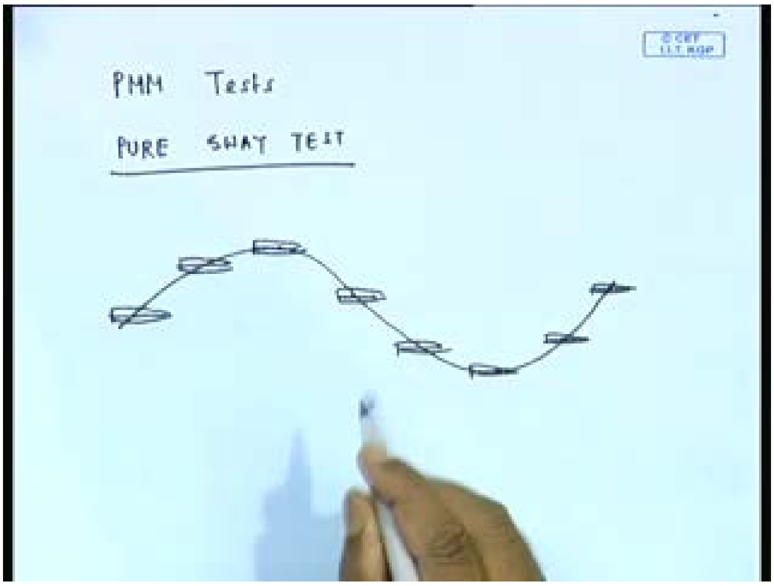

See, so today well I am going to talk about, we are discussing PMM Test right. So, yesterday we talked about pure sway test all right, these are all part of dynamic tests. Now, I will repeat little bit of this Y in, Y out part, it seems there is little confusion.

Let us look at this pure sway test, what has happen here see is that, the up the body has been made to sway, oscillate (No audio from 01:16 to 01:31) like that. So, the body has been made to oscillate in the pure sway mode, now the confusion that maybe there is with respect to Y in, Y out and the forces. So, I will just clarify that before going to the next one of pure yaw test. Now, let me look at it little more carefully, we will look at it here. Let me take this off this this one pages have come here; see I will just look at the maths part of it.

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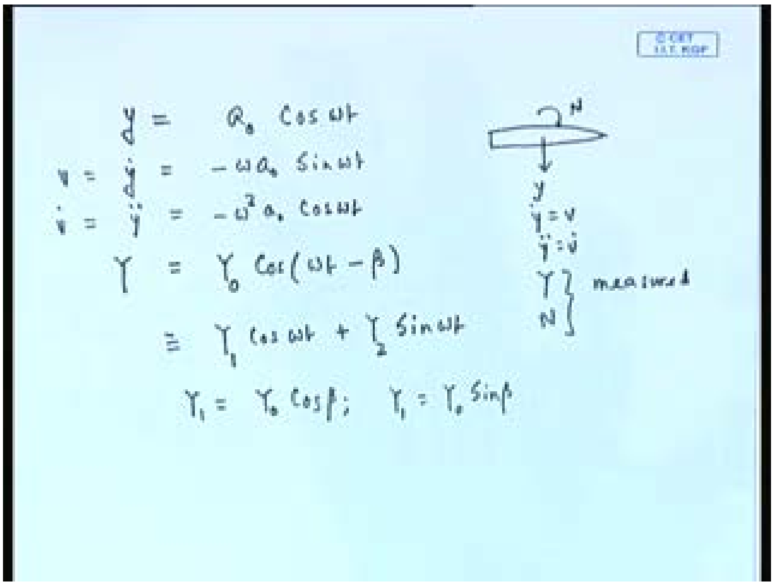

Now, I had y given by a 0 cos omega t. So, v which is given by y dot will be given by minus omega a 0 sin omega t, v dot was given by minus omega square a 0 cos omega t.

Now, I found out the force Y force, I measured the Y force alright that is what I measured, that means, if I have this body here, remember this is my direction y, y dot equal to v, y dot equal to v dot, and Y force. And of course, also N, N would be N and Y are similar, this measured right. Now, this Y that we have measured will be sinusoidal in some form of sinusoidal.

So, let me call this to be, this will look like something like Y, let us say cos minus beta or I can call it to be say Y 1 (No audio from 3:39 to 3:49). I can call it this way, because any this force can be like that, you can always check that know, this is Y 0 omega t cos beta plus Y 0 sin omega t sin beta that means, in fact, you can say that Y 1 is nothing but, Y 0 cos cos beta and Y 1 is going to be minus sin, this is ok.

So, this is what we are calling, actually we are calling this this one, Y 1 as Y in, in force, because it is in phase with this with a minus sin of course, if I want to put give it a mode Y. See, here now look at now we keep this and I just draw this the the sinusoidal oscillation when you will see.

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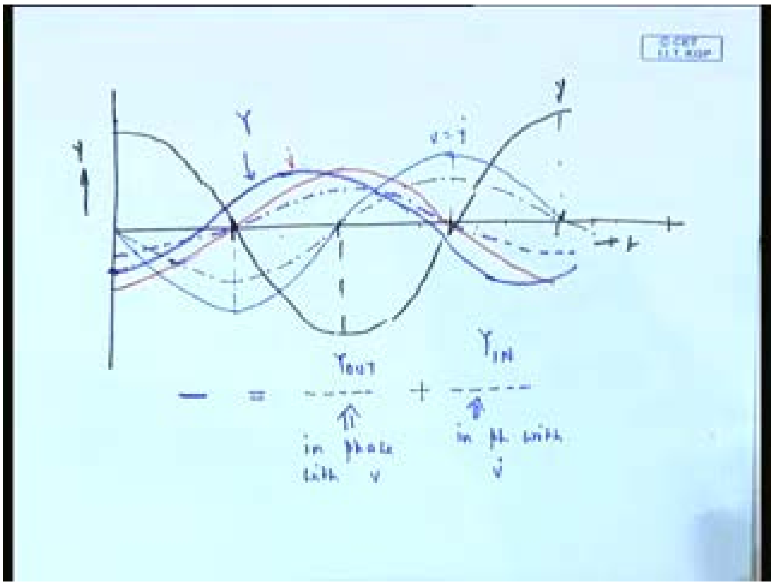

First of all Y, Y is the cos curve. So, if I were to draw here, Y is a cos curve, now let let me put it here for simplicity this side Y, it is easier for all of us to understand. So, is a cos curve, means it goes like that, let me put some 1, 2, 3, 4 (No audio from 5:05 to 5:24), this is my y, what is y dot, it is a minus sin curve minus sin curve (Refer Slide Time: 4:48). So, sin would have been this, so minus sin would have been something like that. Let us take another pen (No audio from 05:44 to 06:00), this is my see here, v equal to y dot minus sin curve with an amplitude omega, forget the amplitude part, now we are looking at the phase part; what is v dot, minus cos curve? (Refer Slide Time: 5:54)

So, v dot is going to be, let put minus cos curve means opposite of that. So, it is going to be (Refer Slide Time: 6:23) (No audio from 6:23 to 6:31), this we agree, there is no doubt on that, the question comes with force part. Now, I have got a measured a force, so I have no choice in the force part. So, what has happen, the force might look, now here comes the question of colour, which colour to use, let us use the blue colour. Let us see the force, force has come to be wait (No audio from 7:00 to 7:23) something like that, it has the phase gap, this this my force (Refer Slide Time: 7:00).

Now, this I can write as I showed you, one as a part of a cos curve, one as a part of a sin curve. That means, this this blue line, this dark blue line can be taken to be part of two, one is essentially you can say in phase with that, that is this blue line is part of this one and a part of (No audio from 8:05 to 8:24) (Refer Slide Time: 7:39). See, here now I will tell you this, this blue line (No audio from 8:30 to 8:43) (Refer Slide Time :8:30), once again you see this this this is my measurement of Y force, that is Y cos something with a phase gap which is nothing but Y 1 cos omega t, Y 2 sin omega t, etcetera with a minus sign.

So, that means this blue line can be written as if it is this small blue line, actually this small blue line is starts from here in fact, they should start from here, whatever because this two were 0, it should have started from here, let us never mind that this blue line is some of this dotted line plus this line, that means I can break this into two parts.

Now, you see if you look at this now dotted blue line, the dotted blue line is in phase with the v dot line which is out of phase with the displacement. So, in other words, the one that is in phase with the acceleration that part, that is the dotted where is the acceleration, acceleration is here v dot this one, the dotted blue line, the dotted blue line here is actually in phase with v dot right, see v v dot is the red line.

Red line is the v dot line, acceleration line, red line is acceleration line and dotted blue line is in phase with the acceleration line, whereas the dotted black line is in phase with the this light blue line which is velocity line. So, that means this is, so I am calling this to be Y in and this to be Y out, this is what is being called. In other words, what is happening is that you are calling this side the cos part, this part is a question of calling you did not call in and out, you can call Y 1, Y 2 does not matter. So, now once again, this is you understand this part, see here. I have a displacement, I have a velocity, I have n acceleration; if I were to do once more this in only a simpler one with just with velocity acceleration, let me put that in a better way of if the velocity acceleration.

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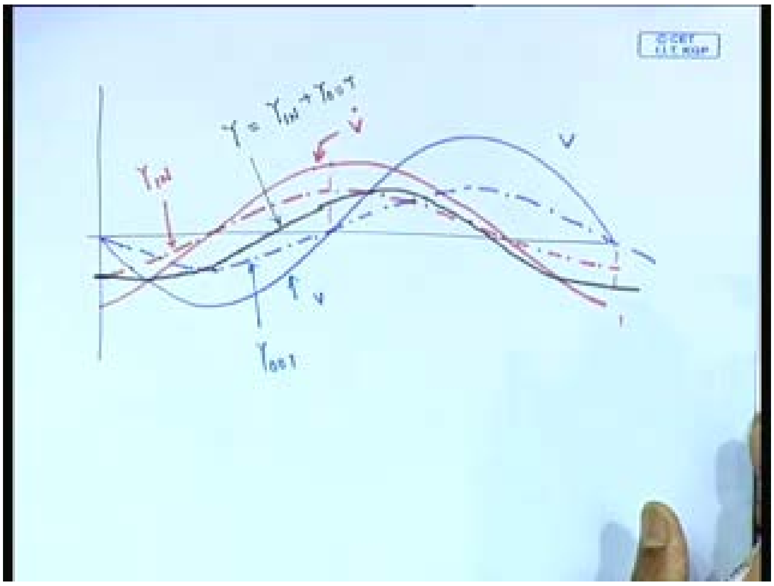

What would happen is that, I just take the velocity acceleration here, see here velocity was this line, see my velocity became this line, this is v agreed, my acceleration is the red line (Refer Slide Time: 11:50), that is my force my force is some line here, now this particular force I I actually let me show the breaking part and then I can show this.

The the force turns out to be equal to (No audio from 12:18 to 12:28) last (No audio from 12:29 to 12:42) or in in if you sum this up (Refer Slide Time: 12:17), then of course the force value is here, let me put it this way, here the force value is supposed to be this two line from here. So, the force value may look something like that, see this is the now now now, again we will go very carefully on this part, see here this v dot this v, black one is Y force, black one is equal to Y in plus Y out.

What is Y in? (No audio from 13:37 to 13:43). So, what is happening, once again the this black line is the force that I measured, but this black line is this one, Y 0 cos omega t minus beta, but this as you know you can write this as Y 1 cos omega t plus Y 2 sin omega t. That means, this black line is a cos omega t curve and this is actually, we are the this one and a sin omega t curve.

See, this is a cos omega t curve, and this is a sin omega t curve, this sum this two dotted line sum give me the black line or conversely the black line is what I have got can be written broken down this two. So, I am only calling one of them as in phase, one of them as out of phase, that is it. That means, one component which is actually, see whether you call Y 1, Y 2, Y in, Y out does not make a difference.

The main point is that there is the part of the force that is what we can call in phase with acceleration or displacement, and the other one is out of phase the acceleration or in phase velocity, that is all. So, this what we have calling this way, we will actually see this later on, I thought I will to the towards the end show the entire thing in a little more equation form to see how this thing work out.

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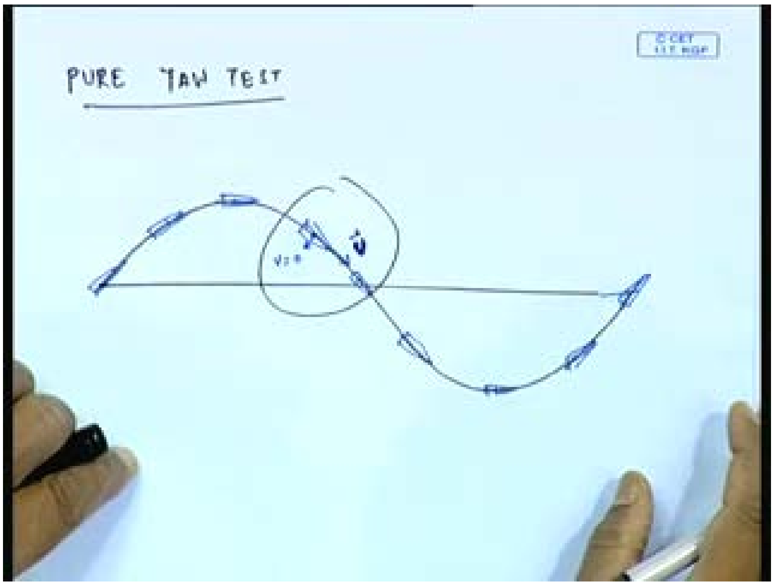

But let us now go out to my, the the other point today, what we are talking of pure yaw test. In this case, what we need to do, I want to make sure the model moves always tangential to this (No audio from 15:54 to 16:09), I will just draw this. So, I what I need, remember I need to make sure that the model moves like that, because that way what would happen if you look any point here.

See here, I have got here v. So, this side, know sorry that is v let me call it big U only or U bar, this side v is 0. That means, this only rotating, so there is no v, but there is only rotation because after all you are just rotating in this way, that is the idea, how do I achieve that, question number 1, how do I achieve that, now let me just blow up one part. Let let me take this part and blow it up and see, what is the way of doing this, so let us look at this part, just one part.

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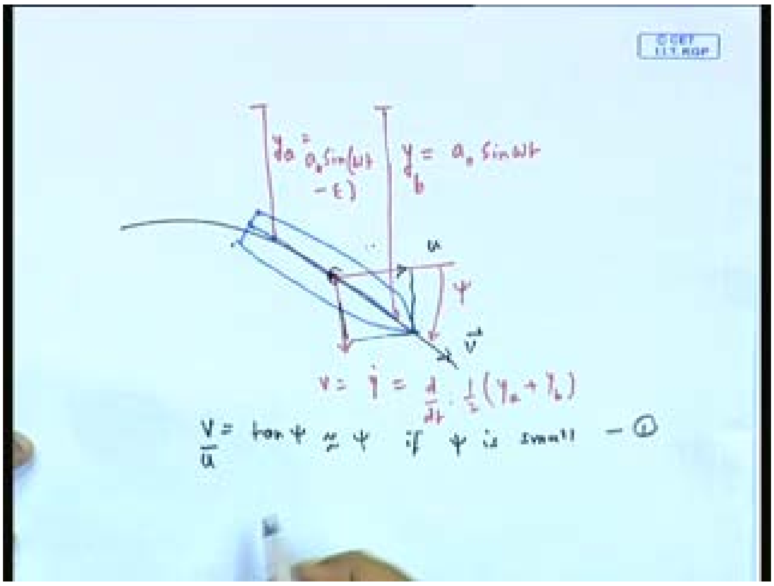

So, where I did is this way, model is here (No audio from 17:15 to 17:22), now you see here (No audio from 17:24 to 17:35), so I have here y, yesterday we call it, let me call it bow or forward, say this is a 0, then y a sin omega t minus epsilon.

There is a phase gap you have to keep (No audio from 18:17 to 18:44). See, here we will we will go carefully, see what is happening, I have to make sure that my velocity vector v is tangential to the path line right, that is a that is a requirement, look at this here. I have to always orient a body that is the velocity vector must be tangential to the path line that is the requirement.

So, all I it is a very simple, I will be actually asking you to derive this formula, but I will tell you the the clue by which it should be done, the question is that, see what is happening here, now what is the velocity here, this velocity here is given by v as y dot d by d t of half Y a plus Y b agreed, then my v, what is the psi.

Now, you now remember, this is v and this is u, actually this psi is always small angle. So, u and v equal to close by although this diagram shows very exaggerated one. Now, what is happening, see here v is equal to or v by u equal to let me call it here tan psi, of course it can be also become equal to psi if psi is small, which is what we will be taking, this you agree, because this is here, this is here take the velocity vector here. Obviously, v by u if you draw this diagram here, this is v, this is u this u. So, this psi we are calling this to be psi. So, that relation how is good that is one, all right now the other question is that v is equal to, now now here v is this, v is actually d by d t of y dot that is d by d t of half of d by d t of Y a dot plus Y b dot. So, it will involve omega epsilon here.

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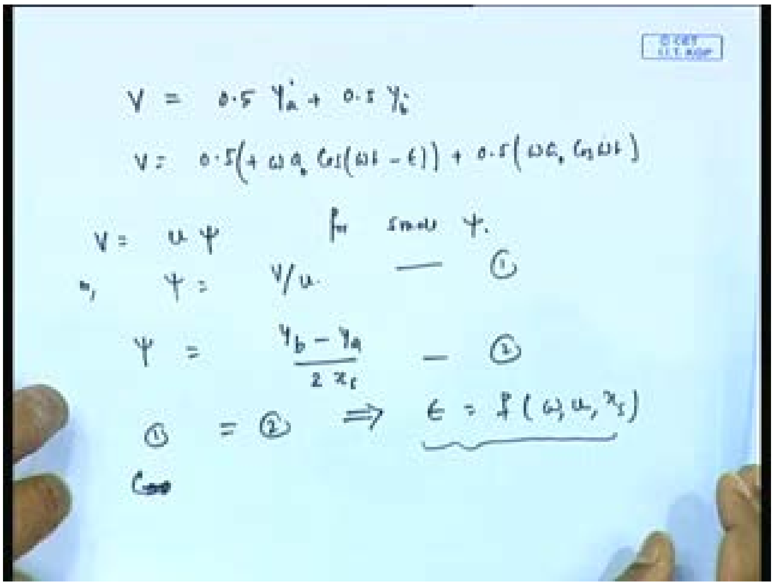

In other words, what is happening here that if I were to let me go to the other one, see v is 0.5 Y a dot plus 0.5 Y b dot. So, this if I were to do Y a dot Y b dot see, Y a dot is what minus a 0 cos omega t minus epsilon minus omega a 0 cos omega t minus.

That means 0.5, I am just writing this just for the sake of writing, no plus, it is plus omega a 0 cos omega t minus well this is b actually epsilon plus 0.5 essentially like that. So, in other words now, I what will I have v equal to u tan psi. So, v for psi small or rather we will write that way psi equal to v by u, this is 1, this is the relation 1 which is what I am writing, but there is a relation 2 there.

What is psi? Psi is given by Y b minus Y a divided by this angle, see psi is also Y b minus Y a divided by this distance, because this small any how this distance is small, we are taking this psi to be small, if you want you can always do this by you know like this is this way sin psi cos psi, this by this is cos after all right. So, the, but we are taking small psi.

So, what is happening for small psi once again, the psi is also Y b minus Y a by 2 x x that is 2. So, when 1 equal to 2 will give you epsilon as function of omega u omega u essentially omega and x s, this equation I will want you to solve. But if you solve it, the answer that will come out to be which is what I am just writing here, by solving you will end up getting that or I write it next line, next page.

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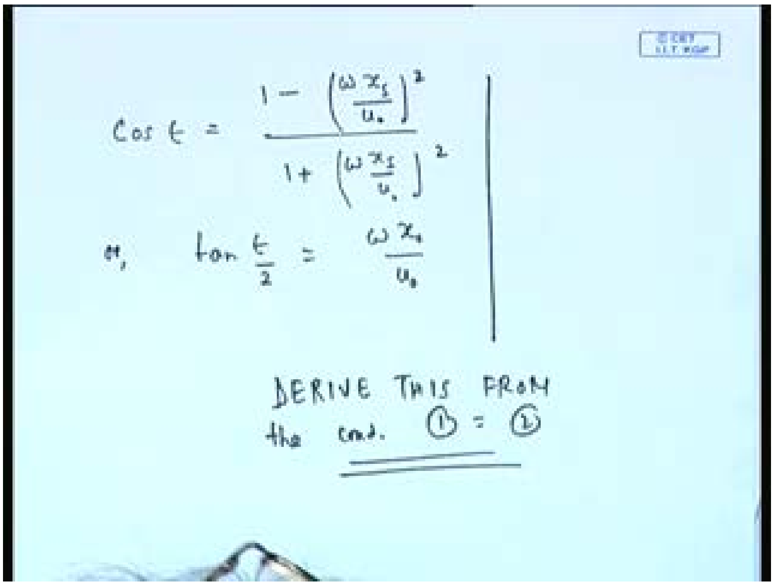

You end up getting that (No audio from 23:53 to 24:23), you can call it u 0, I am calling this to be actually we are calling it to be u 0 (Refer Slide Time: 23:53). See, once again let me tell you this, what you are doing, see this is an assignment to you. So, that is why I am trying to tell you, this will be an assignment to derive that expression, what we are trying to find out is that, what is the condition or necessary that is what must be the value of the epsilon.

So, that the path line (()) or the velocity vector v is tangential to the path line, this is the requirement, v must be tangential to the path line, remember here I have a path line here, say I have a path line here, my velocity vector must be always tangential to this, this is my requirement. So, the question that I have asked myself is what is the epsilon I should keep, what is the signal I should keep, what is the phase gap I should keep, see here you can see from here the phase gap, what I am doing is that I I want to make this phase in such a way that is always goes like that.

See, this phase this two line I am opposing in such a way that is always tangential to that, this will only achieved by keeping value certain value of epsilon, how do I derive it that is what the one I was trying to tell, the condition is very simple that this epsilon angle is this minus that by l, this minus this by this length is epsilon for small value of epsilon.

Similarly, now that is one, also epsilon is given by v by u right, because tan epsilon is v by u tan is epsilon, because angle is small, But what is v, v is u dot y dot, y dot is half Y a plus Y b d by d t of that. So, here I have got epsilon coming, there also I have got epsilon coming. That means, this equation that I am trying to tell, in this I have got epsilon coming, in this also I have got epsilon coming, because this has epsilon here or in time of cos.

So, I equate this with that, then I will have I will have an expression with the only unknown being epsilon. So, I will know what is the value of epsilon, that means the question I have ask is what is the value of epsilon for which 2 is equal to 1, this you solve it, and if you solve it the answer that comes out that is what I I have said it here is given as cos inverse of this or cos epsilon as this.

This (No audio from 27:04 to 27:18), this is a I am leaving that to you for the class to derive you know because it is a simple, it is a basically in algebra. So, when you carry out the algebra, you will be able to find it out. Now, what happen, actually this is very simple case you know, people work it out this algebra or much complex case of an arbitrary body of Y a Y b; in other words, what is this Y a arbitrary value of Y a Y b, supposing I give a 1 here, a 2 here, this distance is also not excess, etcetera. In in that case also, you can find out a general expression, essentially all I am looking is that, what is the expression so that this becomes tangential. Obviously, it will depend on u, it will depend the frequency of oscillation and it will depend on the distance, this two distance. So, here it does the result comes out to be this, now having said that what happen, the now we come back to this expression again, so here we end up getting this.

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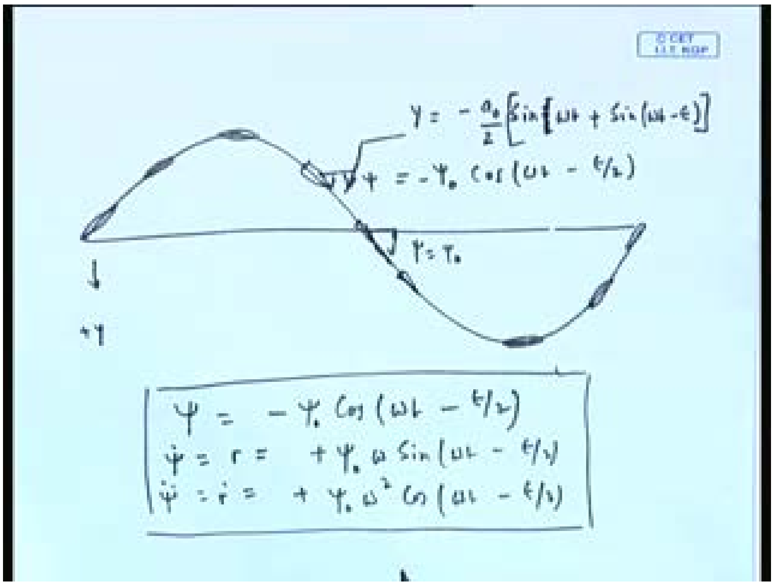

Therefore, if that epsilon is there, I will just draw it again, bodies (No audio from 28:30 to 28:41), etcetera, so here I end up, getting here this path line turns out to be y in this diagram, I am just showing because this is my plus y sorry and this is my u of course, this is the maximum value psi, but generally speaking psi just the heading angle psi with the minus sign here.

You will find out that this expression will come out, psi will obviously become a psi, see having done the expression I can now find out psi, psi is nothing but y 2 minus y 1 by 2 x, 2 x comes in and if you do that it will turn out that obviously, it will be a sinusoidal function. In this case, it will turn out to be minus y 0 cos omega t minus epsilon by 2, in other words, if I keep an if I keep a phase gap epsilon given by the expression this.

Then it turns out which is also what you should derive that, first of all the fact that epsilon must be so and so, in order to make sure my path line and velocity vector tangential or velocity vector is tangential to path line, that will give you by this expression. And if you put this expression, you will end up getting psi to be given by this, now this is very simple, now what has happen, see I have got a heading angle psi given by a sinusoidal function. So, what I end up getting this equal to minus psi 0, now this part is very simple.

Then, psi dot equal to r equal to plus psi 0 omega (No audio from 31:13 to 31:29), exactly same as what we have done before. That means, I have a heading angle given by a curve, cos curve the heading angle if I were to plot heading angle, rotational velocity sin curve, rotational acceleration cos curve just like what we have done before.

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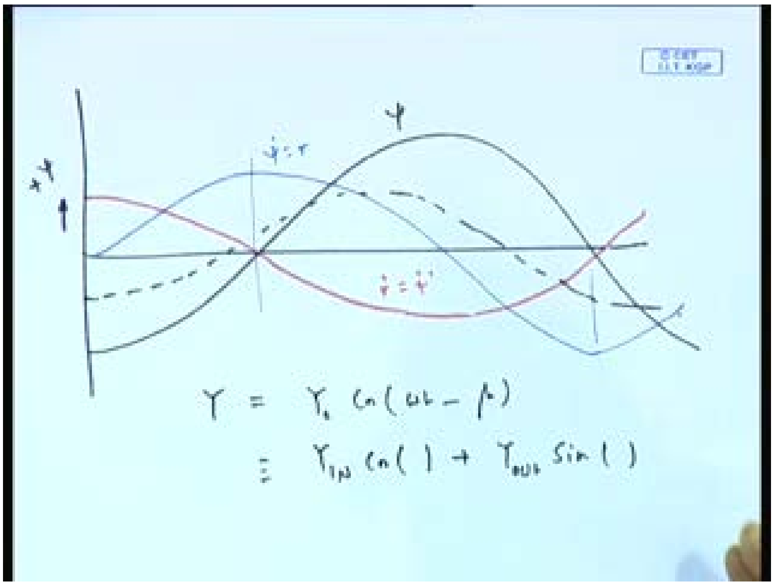

In other words, if I were to draw the psi curve and all, let me put it here again psi is a minus cos curve. So, if I see this side plus or rather I call this you know plus here, this time this side plus psi I put it. So, minus cos curve will look like this is psi curve with a with a gap here. I am just putting this epsilon 0 just you know choosing the value from somewhere; r is going to be, therefore plus sin curve, r is plus sin curve that is this is plus side.

See, here r dot equal to plus cross curve, and similarly you are going to get the force, exactly same way. Now, I do not have to go further beyond that, you are going to measure a force, some some force will come, something like that. Again you have this plus and minus part, so you are going to find out here again Y force which will be Y 0 cos maybe minus say beta.

Here, we write other way round, see cos was this thing you have Y in. So, the procedure becomes same, I do not have to actually go through a produce anymore, the procedure is exactly same, you are measuring the Y force, N moment. In case 1, I had only y that is only v, no r and v was a sinusoidal curve; now I have only r, no v, r is the sinusoidal curve.

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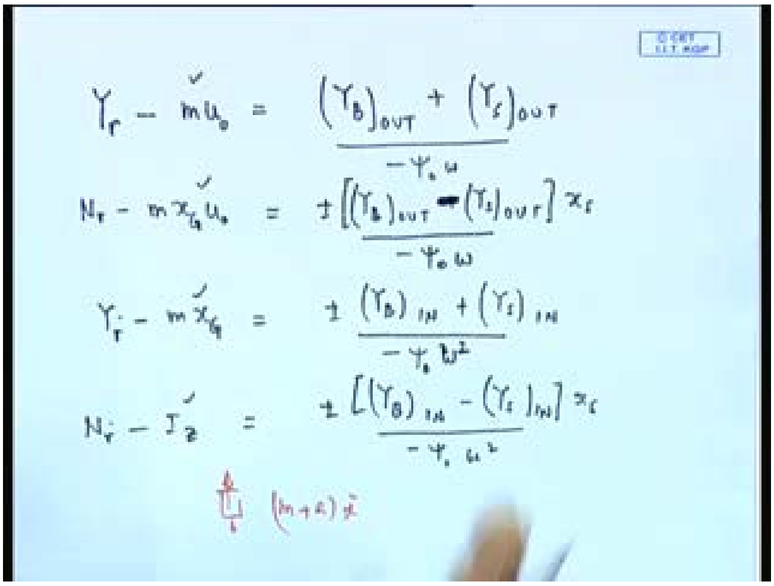

So, the you know the end of story in a sense, because it is exactly same. So, of course, here when you want to use the equations, then it will turn out, therefore the this thing, Y r minus m u 0 will turn out to be Y b. If you let Y b out and Y a, there is bow (()) out. I will tell you why, this there is no, see this terms come because of the, see when you I will tell you about this term, let me write it down this (No audio from 34:40 to 35:10). We are writing plus, minus, etcetera, just so that you make the appropriate.

This will be actually minus probably, this is this will be minus here, sorry and N r dot minus I (No audio from 35:56 to 36:08) (Refer Slide Time: 35:56), there is not in this will be again mistake here, this is out, this is in, here this in (No audio from 36:20 to 36:29). See, the thing is that this terms are coming, see this part is ok, you know the problem you may you may the the question is very simple, once again same procedure this, for this value, this is the Y b out, N b out, etcetera, what we have done before. The only thing that, you might doubt where this expressions come, this side expression come that is because, remember whenever I measure a force, I always measure the forces which includes the mass inertia.

For example, if took a body and I oscillated that in water, I will end up getting mass plus added mass into acceleration force, I will not find out in the added mass acceleration. See, if I took a body for example, in water in a, I accelerate that, my equipment will measure mass plus added mass into acceleration that force, not only added mass and the acceleration force. So, the measurement of Y b out, etcetera includes this parts that is see m u 0 into velocity m x G u 0 into velocity m x G into acceleration I z into acceleration.

Say for example, this remember is I z into acceleration, this is my inertia force. So, that is why this term has to be taken into consideration always, because remember in other words what you should do, if you do not want it take Y b out, total measure minus this part will give you the Y b out for hydrodynamics only is the same thing. I could take this minus this much and then divide by that part again, this is the same thing.

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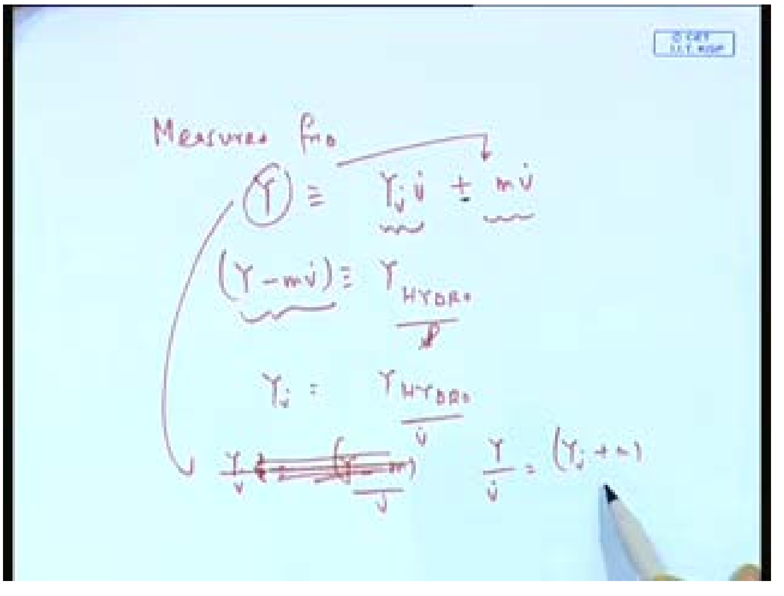

In other words what I am saying is that, just think of this the measurement, the measured force say Y will include always say you know like, I just give an example Y v v dot plus m v dot, this part is also included. So, if you do not, if you want only this much what you could do, you you can do m v dot and define this to be Y hydrodynamics and then take Y hydro divided by v dot to get Y v dot. Same thing we, I I am just try to explain to you, what what I am saying is that the measurement forces will always includes this, as a result if I want to find out hydrodynamic physics in a body inertia force, hydrodynamic force, measured force is always includes body inertia forces.

So, if I want to determine hydrodynamic forces, first of all I should take this off from here and then divide by v dot to find out the derivative that means, this would be my hydrodynamic force. And therefore, Y v dot is Y hydro by v dot that is what we have done, these are nothing but the v dot here. So, m this inertia force that is this into thus part, inertia force have to be taken out, or you can write this way whichever ways you can.

In other words I mean, here I can write basically I can write this v dot here, say in this expression I can write v dot to be Y minus m divided by v dot, what we have written here sorry not v dot Y v, y no Y v dot equal to y minus no sorry no no, I made a mistake what I can do is here see here, this Y by v dot equal to Y v dot plus m, this is what we are getting know, this expression.

See, here Y Y v dot is Y v dot plus m, this is exactly what we have done, this minus this by this exactly that is what we are getting here this expression. So, it is the same thing, I just wanted to tell you this part. So, this is how we are doing my, our you know like this rotating PMM technique, etcetera. I just want to tell you here quickly, I do not know if I have time, maybe I should quickly go through that, there is a nicer way of doing this entire thing in a more maths way to show all this parts you know, I will very quickly try to go through this, it is going to be all kind of equation.

See, here what I want to do, we have shown this separately saying that I can do pure sway pure y pure sway pure yaw by taking this phase, etcetera, etcetera. But, you know the entire thing can come out in a equation form, if I play with the equation of motion with this.

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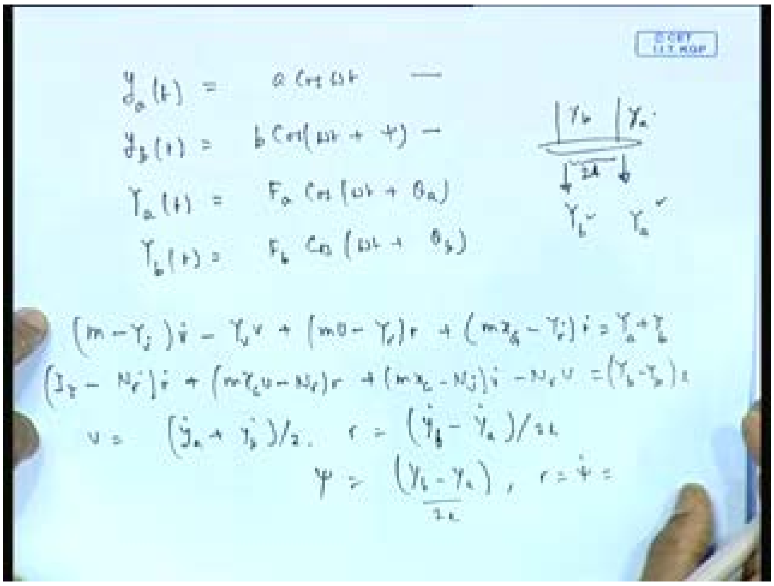

For example, see here, I have this two strucks, Y a up struck was given by omega t, let us see, this abstract motion (()) struck motion is given as b, I am just writing as general expression, b cos omega t plus psi some phase gap there.

I am measuring forces of it is coming out to be say f a cos omega t plus theta a forward you will turn out to be F b (Refer Slide Time: 41:41). Now, what I said here, this is an input data again this start Y a, Y b, measure forces Y a Y b, this is measurement of forces, this is the input data I am giving which I what I what I determine, I have given input a b psi or input data, I have given this what I measured F a, theta a, theta b, F a f b theta a, theta that is what I measured right.

This is exactly what I have done in a equation, now if the equation of motion is like that linear equation of motion was given by Y v dot v dot minus Y v v plus m, I just writing very quickly because time, you can always write it out, this is my external force, this net force is Y a Y b that is what I measure.

Similarly, here I z minus (No audio from 42:58 to 43:17) into l, we are calling this to be l (Refer Slide Time 42:58). This is net moment, this I am calling this to be l the distance, now v equal to y a dot plus y v dot by 2 and r equal to y v dot minus y a dot by 2 l. I will leave it to you to work it out, you see this this part very simple pi equal to y b minus y a by 2 l. So, r dot is r equal to psi dot, remember psi equal to y b minus y a by 2, this is actually know, this 2 l comes in, because this is actually 2 l here sorry this into this this, this is 2 l, this is 2 l.

This is the moment is y b minus y y v into l, because this is y b l y a l, sothis is this by 2 l. So, r equal to psi dot r equal to psi dot that gives you this expression, we we we now now what happen, if you make the distance as a equal to b that y a equal to y, the basically a equal to b, if you make this this same.

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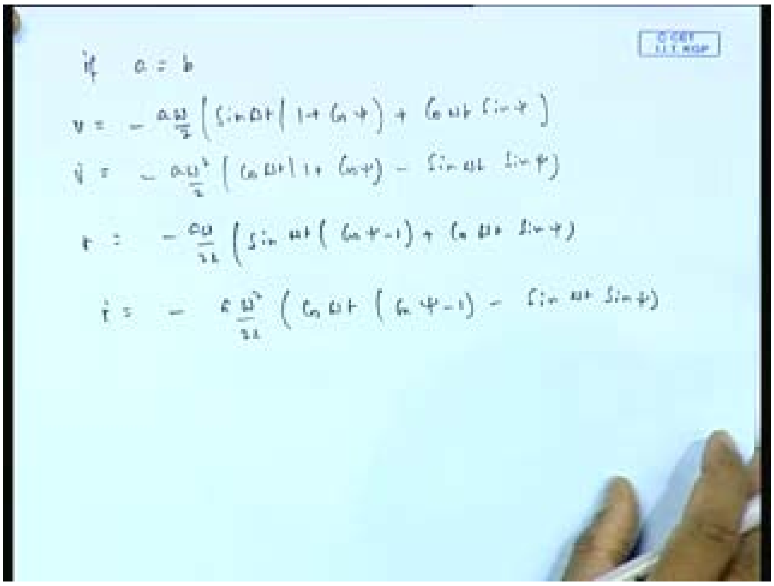

If remember, a equal to b basically means a and b this amplitude of this two motions are same, then why, what we end up getting is like that. I want to just show my very quickly. you can work it out yourself (No audio from 45:12 to 45:48) (Refer Slide Time :45:12), and r dot transfer to be minus a omega square by 2 l cos psi minus 1 minus sin omega t sin psi.

Now, here basically if you equate from v v dot this thing sin and cos term both sides, then we will what what what is meant by that, see we know this v remember v is given by this plus that by 2, r is this by that by 2. So, v if I put this expression, there will be sin and cos term, because this is cos a cos b sin a and sin b, similarly r a of sin b cos term.

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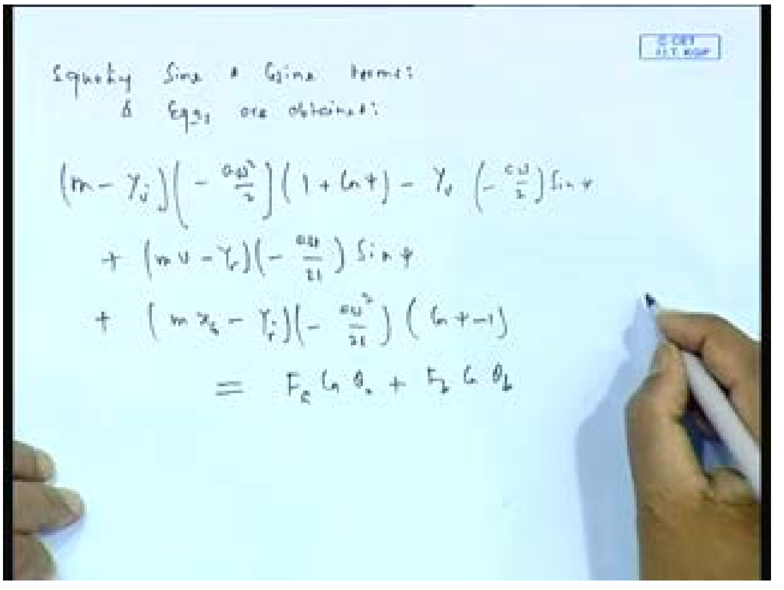

So, now what is happening, this side has sin and cos, this side when when I equate the sin and cos terms then I end up getting four equations, why why I am telling you is that, let let us first equate this four terms (No audio from 46:46 to 47:06), maybe I will ask you to do the entire, I will just show you one equation, but you can do the rest part ((No audio from 47:12 to 47:57). This will become equal to F a cos theta a plus F b cos theta b at what what we are doing.

I will tell you one again now that see here see here, I know this v v dot, r r dot, this is these are having sin and cos terms right, I put this expression here on this, what will happen this side, I have sin and cos terms, here Y a plus Y b, Y a plus Y b also has sin and cos terms, because cos omega t plus theta, etcetera, so now this side and that side.

So, I equate sin with sin, cos with cos, if you do that you end up getting actually four expression here that you basically end up getting four expression, you end up getting four independent equation, this is one if the one that I showed you just now, this is one, like that you will get four equation, the interesting point is that this four equation after you get, I will not go through the four equation, but you do this four equation.

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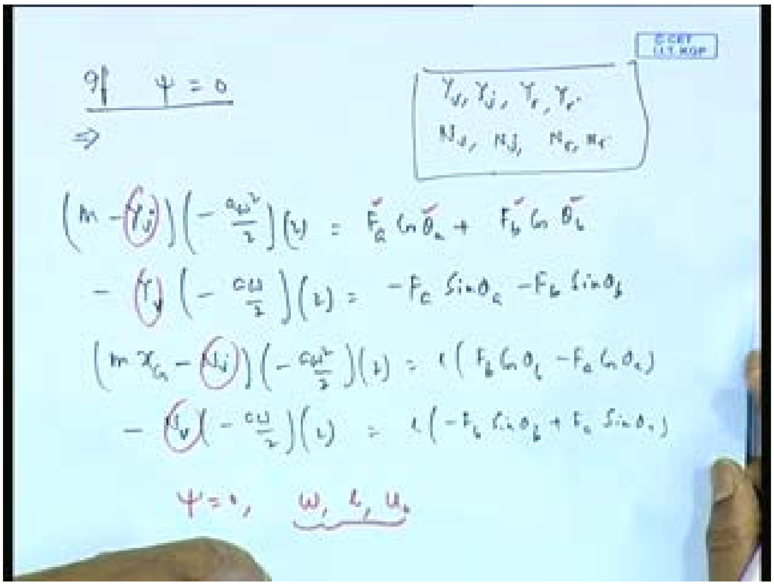

Then you will see that if you put psi equal to 0, remarkably you get four equations, actually here you are going to get four equations with unknown of Y v, Y v dot, Y r, Y r dot, N v, N v dot, N r, N r dot, you will end up getting like this four equations with eight unknowns, those unknowns are the one this eight unknowns. But now, the most interesting part comes out, this is I will leave it to you for exercise, now you make psi equal to 0, you will end up getting four reduced equation with only this four unknowns.

For example, you will end up getting, I will just give you one example, Y v dot (No audio from 50:07 to 50:55) and last one (No audio from 50:57 to 51:10). See, here you will get this four equations with the unknown this four, this one, this one, this one, this one (Refer Slide Time: 51:14). So, what is happening, you see here the interesting point therefore, so when I make psi equal to 0 that I of course I can always make psi equal to 0, because here you see this psi, if I make psi equal to 0 here, then I will end up getting four equations with four unknowns.

Therefore, for when I do the experiments psi equal to 0 that becomes the pure sway test, you have the four equations, I can solve it. I am measuring F a, F b, theta a, theta b, four unknown I am measuring, remember. So, I have measured this, I have inputted basically what did I input, I input psi of course, I this case psi was 0, but I have omega, I have l, I have u 0, this is given values.

So, for a given value this, this, this with psi 0, I end up getting, I measured F a and theta a, four unknowns and the four equations I can solve for the four unknowns, same way same way if you make psi equal to 180 degree, you will end up getting just or you know the other four equations, where I will have only Y r dot, etcetera, etcetera.

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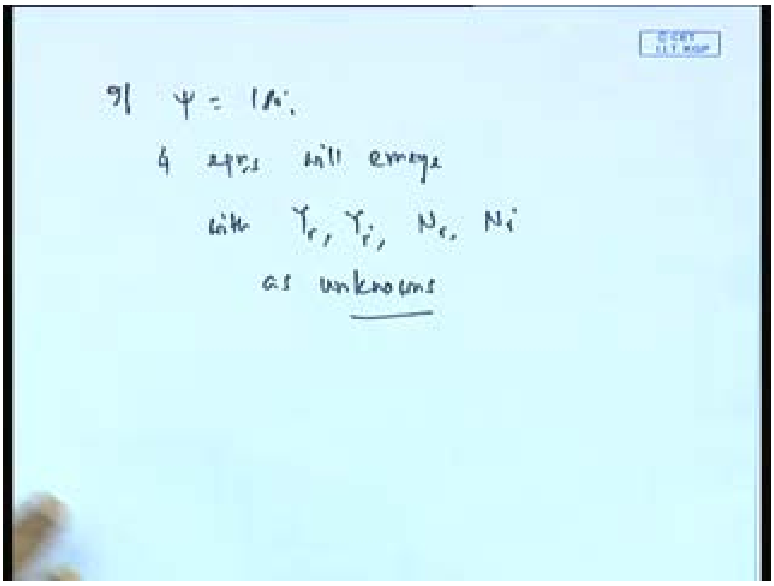


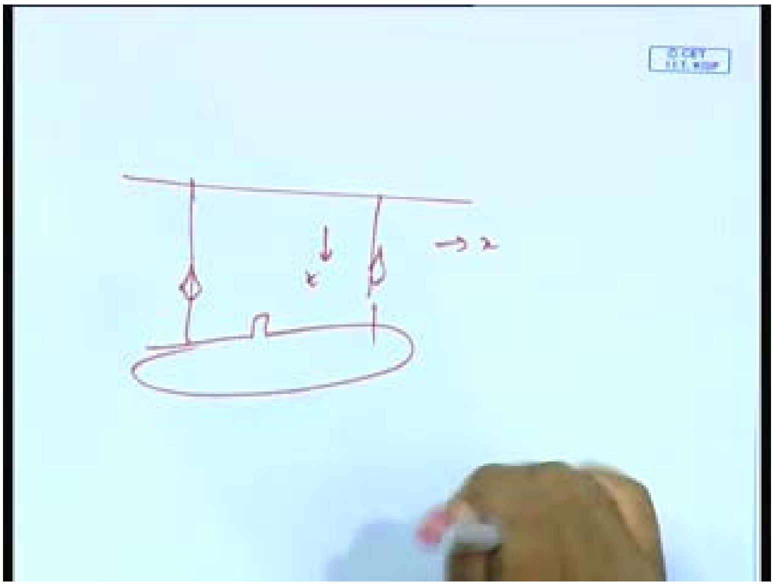
That means, similar way if I make psi equal to 180 degree, so if four equations will will emerge, (No audio from 52:49 to 53:02) you can solve it. So, what is happening, therefore see why I wanted to show you, this tells me an interesting way, supposing I do not keep psi equal to even 0, see I can you know the why I am showing that you know do not make it pure sway, you do not make it pure v, I can make psi equal to 45 degree, I get four equations with eight unknowns.

I take i psi equal to another angle, say 90, say 135 degree or whatever, I get another four equation with eight unknowns. So, I will get total eight equations with eight unknowns, I can solve for it. So, the what, why I am saying you know, therefore team and as a technique can be generalized, you need not gave necessarily psi equal to 0 and 180 or pure sway or pure yaw, although that is what is done.

I can also have sway or mixed, make two experiments and separate it out, you see the the hints are tantalizing. In other words, if you have the capacity to go through the maths (()), you will find out that you can determine the non-linear coefficients by simply taking this playing around, that is why I want to show you this.

If you go by equation of motion and you can generalize it in a much more complex way, although conventionally you would normally do pure sway and pure yaw, and this is what we do. I will just want to tell you at the very end that for submarines, what happens, you also want to know you know like if you look at my this thing, this is my horizontal plane motions, but in submarine you also want to know vertical plate.

(Refer Slide time: 54:40)



So, what they do in a submarine, the planar motion mechanism actually is vertical submarine is modulus (()) like that, here I can oscillate it this way.

So, I can oscillate it vertical plane, this is my z, this is my x, but the same one what I should do is that, suppose the body is kept this way, this this is very interesting, the submerge body I keep it this way, I oscillate. But if I want y Y v and all, all I do is this way and oscillate it; I just rotate the model 90 degree and oscillate. So, I can get horizontal motion, because in deep water, I can simply make this as horizontal plane by making the body like that.

So, you see (()) PMM what is called for submarine, where it is called submerged, I can actually oscillate the vertical mode, but this vertical direction can be made the x direction, y direction by simply turning the model upside down. So, you can get both Y v, etcetera and also Z w, etcetera, that is what is called vertical planner motion mechanism applied for submerged bodies, whereas for surface bodies, we use horizontal motion mechanism, because my free surface has to be there, it has to be half submerged, you cannot deeply submerge it.

So, this is about planner motion mechanism, I will end it here and we will see from next class, we will probably go to control surface design and control rudder part. So, this, but as I said this derivation of this phase angle that I mentioned earlier, kindly do that as an exercise to see that this expression comes out, gives you a good feel. And if you have the tenacity, you can generalize that for any value of a and b, Y a, Y b, etcetera you know unsymmetrical part. Then you will find here, you have a much more complex expression, nevertheless an expression is possible, with that I will end it here.