

Seakeeping and Manoeuvring
Prof. Dr. Debabrata Sen
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture No. # 33
Non-linear equations and Model Tests

(Refer Slide Time: 00:31)

The image shows a whiteboard with handwritten equations. At the top right, there is a small logo that says 'SCEY IIT KGP'. The equations are as follows:

$$m(\dot{v} + ru + x_G \dot{v}) = Y \checkmark$$

$$I_{yy} \dot{r} + m x_G (\dot{v} + ru) = N \checkmark$$

$$Y = Y_{HULL} + Y_{RUDDER}$$

$$N = N_{HULL} + N_{RUDDER}$$

$$\left. \begin{matrix} Y \\ N \end{matrix} \right\} = f(\dot{v}, \dot{r}, v, r, \delta)$$

Ok, see in the last class we were discussing about non-linear equation of motion. What I was trying to tell is that, if we were writing the equation of motion as, say Y equation of motion as... etcetera. **The see excuse me**. The main issue was in these forces, so what we said is that Y is essentially Y hull, if I were just taking this and N was like that.

Now, Y and N essentially we assume that; no, this hull hull forces let us say, or if I take both of them $v \dot{r} + r \dot{v} + v r \delta$, this is what we were saying.

(Refer Slide Time: 01:58)

$$\begin{aligned}
 Y_{HULL}(\dot{v}, \dot{r}, v, r) &= \underbrace{Y_v \dot{v} + Y_r \dot{r}}_{\text{added mass}} + \underbrace{Y_v v + Y_r r}_{\text{damping part}} \leftarrow \text{linear} \\
 &= [Y_v \ddot{v} + Y_r \ddot{r} + X_u \dot{u} r] \leftarrow \text{added mass/inertia} \\
 &+ Y_v v + Y_{v|v} v|v \quad (+ Y_{vv} v^2) \\
 &+ Y_r r + Y_{r|r} r|r \quad (+ Y_{rr} r^2) \\
 &+ Y_{vr} v r \quad (+ Y_{vrr} v r r + Y_{rvr} r v r)
 \end{aligned}$$

Now, let us take one by one and what we did. Say Y_{HULL} , we said this is a function of \dot{v} , \dot{r} , v , r , just when I take 2 d equation. So, this was expanded as $Y_v \dot{v} + Y_r \dot{r} + Y_v v + Y_r r$. Now, in a linear equation we are writing up to this much, but what is happen is that in a non-linear, this is for linear, and this part is the added mass, and this is the damping part.

What said in the last class is essentially, that see this added mass part is fully known from equation, and it is only the damping part that we keep expanding. So, essentially in a non-linear equation what happen, I have added mass part that comes out to be, in fact I can show you this will remain, but I end up getting an extra part that is actually, given as what you call $X_u \dot{u} r$.

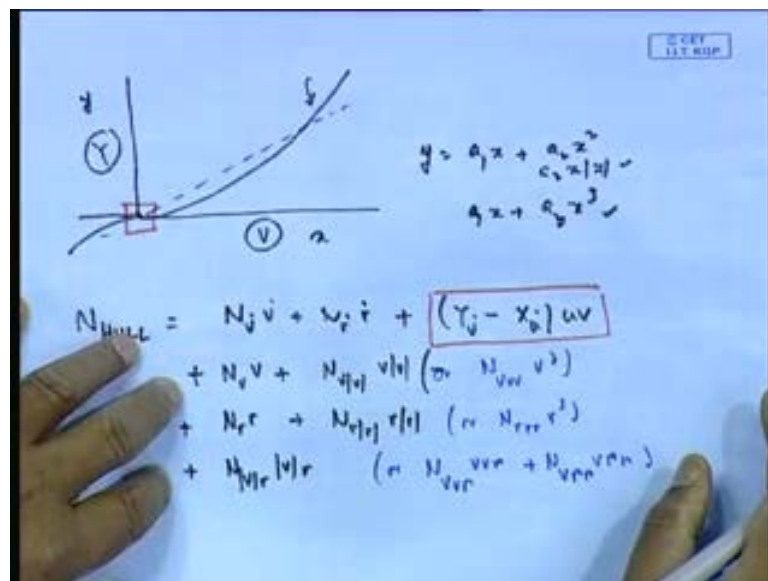
See, this part comes out, this additional part, this expression. I showed that in last class that the entire expression for inertia force is fully known, so it becomes like that. See, the problem this is the inertia part, or added mass part. Problem was with respect to this part, which depends on v and r . So, here what is happening is that, it turns out that this force can be, or some people will replace this, I will write.

Then let me write it down, then I will tell you what. See, what is happening now, this is the very important point. See, this part of the force is fully known to us, that is the added mass part, and it assumes only a linear part, linear part and there is an additional part that comes in like this, and in fact this can be neglected because $X_u \dot{u}$ is very small, X_u

dot this term is extremely small, so it can be neglected this part. But, the most important point is to recognize is that, when I meant by non-linear is that Y against v, it is not a straight line. Basically, it turns out that because it may not be a straight line, I need to have an additional part either this way or this way, depending on how I model.

Now, I there are two models, normally people will take, if I take only this. See, after all what I meant is here, again I will like to show you, there is lot of repetition, but let me also show this.

(Refer Slide Time: 05:53)



So, if this value goes like that what is happening is that, if I were to use only Y v v, what it means? It means, I am presuming that this Y is varying like a straight line right, but actually I find out that Y may not be varying as a straight line, it may be varying in a curve.

So, what do I do? I have to model this curve, because I need to find out Y as a function of curve. So, this graph can be fitted if I call this for example, Y versus X in a graph, then Y can be fitted as a 1, because it goes to 0 0 plus a 2 x square, that is good enough to give you this curve, some curvature.

Or of course, **now the or of course**, we can use by a x plus a 2 x cube. So, here that is what I mentioned here, Y it is used this see basically you need a non-linear term, an additional higher order term, one term to make this straight line curve. Now, which one

we should take, a cubic obviously, a second order polynomial is good enough the way we do in a in any, like ship hull, like description.

But, if I use a $2 \times \text{square}$, the problem that remains is that, I want to make sure that the Y value and x value are of same direction. That means, this Y force and this v, if I were to write this way. If v becomes negative, then Y must become negative, that means this graph must basically go like that, and that is not possible by this, because this particular part would make sure that when x is negative also, this value is positive. So, this graph will actually show something like this up, going up like that.

So, you have to then take this as a $2 \times x$, which is what is call it will be actually, the symmetry relation will be mentioned like a cos curve, not a cos curve, like a sin curve The sin will go like that, it'll be like a sin curve, that is that negative side sin minus theta should be minus sin theta, etcetera. Or of course, you can take 2, some people may have bit trouble with this mod, so you may take x cube.

But the main point is of course is that, what the most important point of non-linearity means that, either you take or this or this means actually, what you are trying to find out is capture the value of Y against v which is like this, that is the main thing main thing is that I did not assume with a straight line, because now I want to see the behavior of the body, not just for small value of v if this value of v was very small, value of v was just within this range then of course, I could have presumed v to be straight, Y to be straight then against v.

But now, my interest is to go beyond that, and when I am doing an experiment I want to find out exactly, how Y varies with v. That is why, the question of model comes in, so I have this. Now, this is a question of modeling, either you take this or you take this plus this, or you can even take 3, it is up to you. Normally, taking three may not mean much, some people can also take 3.

So therefore, the question of how in reality, what is non-linear coefficient? There is no fix thing, it depends on how much you want to take. See, first you have to model with certain coefficient basically, I have a curve, I have to therefore, fit a curve with some assumed polynomial, that is my model and then my next job would be that, what are the coefficients for the polynomial? Like what is my a_1 a_2 or a_1 if I call it a 3, or a $1 a_2 a_3$ depending on, which one I choose.

So, first I have to assume a model, then I have to find out the coefficient for the model. Remember, these are coupled, there is nothing like non-linear derivatives, it depends on which non-linear coefficient you have used to model the force, and that one are the one you have to determine. Now, you see here this is the, I just show you possible model, this is like, that this is like that, and when I take v and r for example, when I take combination of v r , I can have either v r one term of course, here actually mod part comes in, that one has to take one of the thing take in mod again, in order to avoid that you can take this way v v r , v v r plus v r r , v r r .

This is called a cubic, see here linear and cubic term, linear and cubic term or linear and quadric term, or some people may take linear, cubic, quadric term. So basically, these things are showing v and r couple. Now this is of course, v r , but now so you understand that part. So, this part is what we are talking of non-linearity, so ultimately what happen, supposing I took this coefficient, what happen?

See, this model for hull forces, what it turned out that I have to find out by experiment unknowns. What are the unknowns? This is one unknown coefficient, this is one coefficient. Now, I come to this later on, because what happen this u r can be coupled with this r apart, and I will show this little later on in experimentally. Basically, what happen, whatever is varying with r , see this Y r plus x dot u this becomes one term, because here I end up getting, see this and this combine becomes Y r plus x u dot u into r .

And I can call this to be ν Y r r , I can absorb this within this basically. Anyhow but, as far as coefficient is concerned, the other coefficient is this one, then either this one or this one, and this, this or this, this or this and this. These are the terms I have to find out at in the Y Y coefficient. Now, I like to write this N part little bit, because N part is an important part that comes in specially, with respect to one part of the moment, this N part. If I write N , again I have got N v dot, v dot plus N r dot, r dot, but there is a very important term that comes out, this is, or I should write here as usual

Now, there is a point of my writing this one, I am just writing this still, etcetera. See, here one most important thing that is that bears like repetition or emphasis is like this. See, if you look at this, these are exactly same, the only difference that comes in this case is one term basically, this one, added mass term. This value, here this additional part

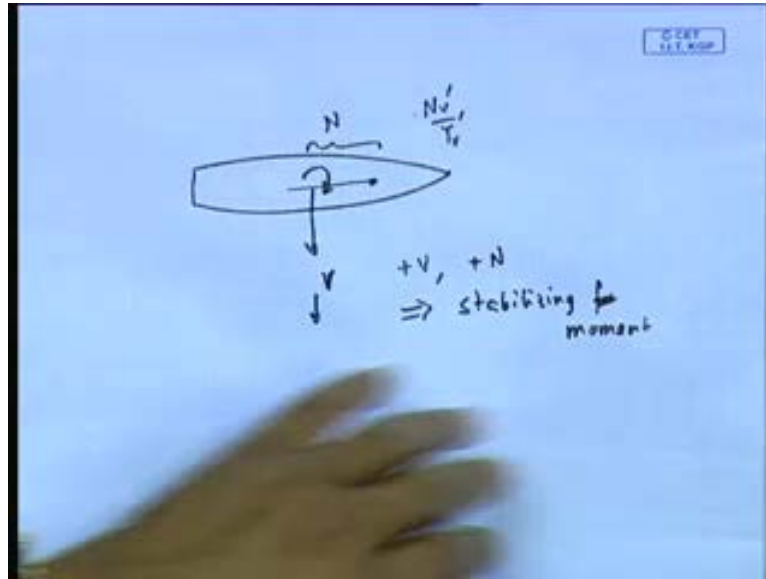
came this value $x u \dot{u}$. Now, $x u \dot{u}$ is surge added mass, added mass in the forward direction, ship is here, added mass in this direction, very small, you can even neglect it. But, this one $Y v \dot{u}$ minus $x u \dot{u}$ is very small, but $Y v \dot{u}$ is added mass in this direction, because $Y v$, very large value.

So, this value becomes a very large value this $Y v \dot{u}$ one thing, and $Y v \dot{u}$ is always negative, always a large negative value. Because, $Y v \dot{u}$ is minus of surge sway added mass, so this is a negative value. Now, you see this term is combines with this term, because this and this are v , depending on v . Suppose, I have got a certain v , say I have got some plus v , then I am going to get a negative large force here and depending on value of $N v$ positive or negative, I'll get a force here.

Now remember, we have mention earlier if you recall, if $N v$ was a positive force, was positive number, it was always trying to stabilize, because that we were saying in order to make ship more stable, $N v$ should be made more and more positive, less and less negative. Suppose, $N v$ was large negative, try to make it less negative means, when $N v$ moves towards positive side means, turn is important, it is trying to make the ship stable, that basically means if I add is for example, a skeg for example, a feather at the end, then $N v$ becomes towards positive.

So, $N v$ go make, that means this combine term, they are against v , what it means is that. See, suppose I get a positive v then I should try to get a positive N moment, so I let me explain this here.

(Refer Slide Time: 15:43)



So, if I get, say this v here, so if I get a positive v , if I go this side, this is my basically positive N . If I get more moment this side, it will try to stabilize the, why because, remember that we have always said that, the center of point here, this point p is given by this distance N v by Y v , and as it is moving backwards, it becomes more and more stabilizing.

Now, N v is always negative, so this was taken as negative, so it becomes more and more positive, it will go on stabilizing essentially, what it means is that obviously, if N was suppose to be positive with v positive, or less negative that means, it is trying to turn this side, I end up getting a positive moment, stabilizing. But, here the problem is coming here, this value, this particular term is always negative for a positive v , because Y v is very large, Y v dot is very large number.

So, this value always represented a stabilizing moment, and this is known as a munk moment. In fact, this is something that is very important in our marine object, and I can tell you one interesting thing, that whenever people try to, like take the dynamics of a aircraft structure and missiles for example, and try to use that for marine bodies, they never understand that added mass forces are important.

Because, in aircraft added mass forces are very small, because the acceleration of fluid forces are extremely small, so there you can neglect it. But, in marine structure this becomes extremely important, and because you can see that Y v dot is like almost like a

mass, $Y \dot{v}$ proportional to the mass of the vehicle, let say the 20000 ton, there will be around 20000 ton into $u \dot{v}$, this is you can imagine the nature of the force, this is a force that is trying to destabilize it, trying to make it go away.

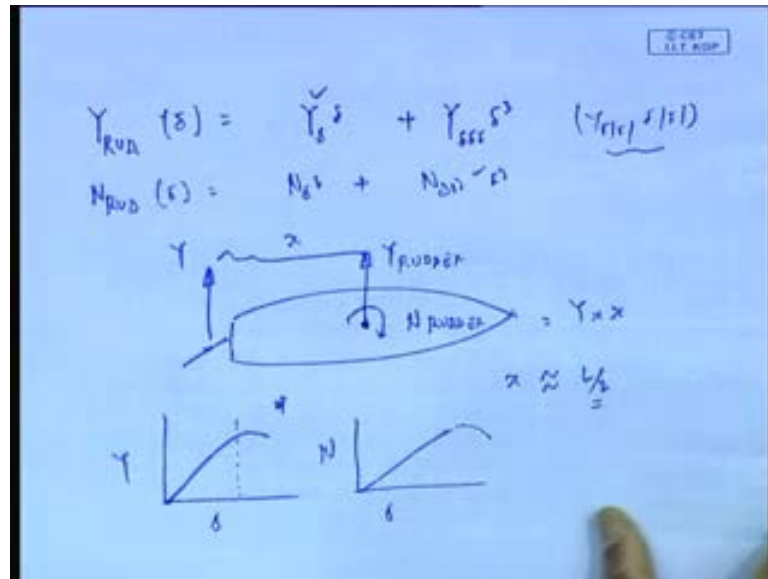
So, this force is extremely important to be understood and of course, in our equation of motion, we can combine this two, once again this 2 part and write it at $N \dot{v} \dot{v}$, because why I am saying is that, this is therefore, a coefficient multiplied by velocity gives me the net force. Here also is something into v , something into v . So, that means the total thing into v and I can call that to be the equivalent $N \dot{v}$ coefficient basically, this $N \dot{v}$ comes out from the real fluid, this is coming from the non viscous or ideal fluid effect, acceleration effect, and this two combine is basically, as far as nature is concerned you are not able to find out this and that separately.

So, what is happening, you end up getting expression like this, this is non-linear. Now, again before I got to this experiment, I have to tell you that, see here what did you find out? Once again I find out, I need to determine added mass coefficient, which is of course, only linear one. Because there is no, and this as I mentioned lass class exact expression is known, there is no non-linearity there, as far as this excepting this fact.

Here I need to find out this and this, or this, this and this or this. It can be also an, some **some** people have modeled this, this and this, all of the 3 as I told you essentially, this is by trying to tell you, how you model this graph, this graph can also be like that, whichever way. So, it is a question of how you model this graph, so this also it can be anything. So, that means I have this linear coefficient, these are known as non-linear coefficient, see $v \dot{v}$ or $v \dot{v} \dot{v}$, $v \dot{r}$ or this thing, this is known as coupled coefficients, also non-linear, $v \dot{r}$ or $v \dot{v} \dot{r}$, $N \dot{v}$, $v \dot{r} \dot{r}$, so I end up getting these coefficients. So, this is one part.

Now, this only with respect to the hull forces, but there is also coupling with respect to the rudder forces. Let me talk about the rudder for a minute before I go, see here the rudder force, let me talk about this before I go to know the captive test.

(Refer Slide Time: 20:31)



So, let me look at Y rudder, this is a function of delta rudder angle, so this you can write it as $Y_{\delta\delta\delta}$, there is only linear term plus, now this linear term only, but sometime you can also write basically, $Y_{\delta\delta\delta}$, δ , δ , δ , δ cube. This is the common model, instead of taking δ bar δ , because of the symmetry once again, that if you give minus delta, you used to the same value, same thing.

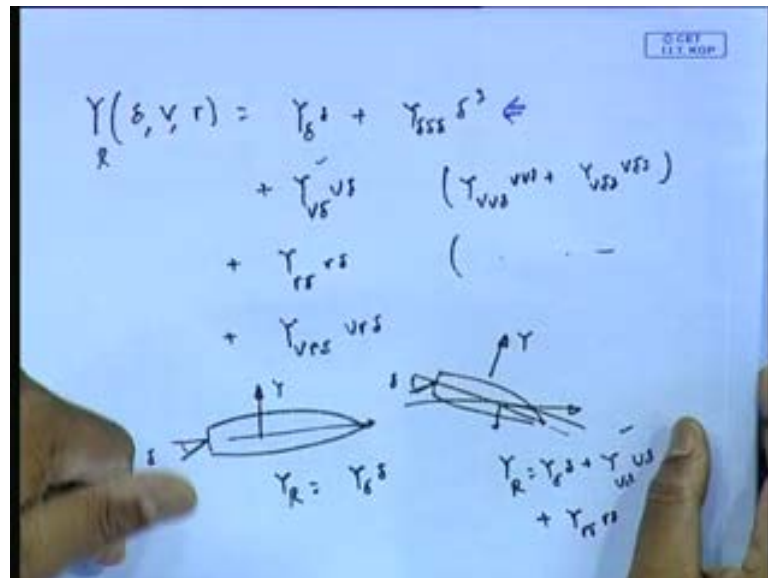
You can actually, take here $Y_{\delta\delta\delta}$, δ , δ , δ also normally, people do not take it as I told you essentially, what again you are trying to look at is Y versus delta, the variation. Now, but the same thing with N rudder ok. Remember, these two are normally taken together there is a reason for that, you see here, if I take here a rudder here, so this gives a force here. This force becomes Y rudder here, and if I multiply with this distance x then N rudder equal to Y into x approximately.

Approximately so, and when depending on the point of center to this point, and this is approximately L by 2 **approximately L by 2** because this will be around L, half the L and this very short distance. So this distance x is. So, what I am like therefore, telling that if I take this two, I must take this two, do not take this two and only this one, because essentially Y and N are somewhat connected. See if I were to take a modeling of Y force with this two, a non-linear variation then N also must be so.

That means if my Y versus delta is taken to be like that, you should not take N versus delta to be straight line, you also should take similar variation. That makes sense,

because N is nothing, but Y into x. This is only the linear, non-linear part in fact, mostly we do not even take this, because you'll know that you operate this, I mention last at to an angle it may be straight line, this is a hydro foil section like in aircraft wing, but symmetric, so there is a c L there lead coefficient, which is similar to Y.

(Refer Slide Time: 23:13)



But, now I want to tell one more thing, Y versus delta, we did that, but there are more complication may come. Now, Y rudder can also not only, it can also depend on this, so in that case I can have not only Y delta, delta plus, Y delta, delta, delta, delta cube. I can also have Y v delta v delta or of course, Y can have normally what happen is that, you do this or you take this way.

You can have also Y r delta, r delta and equivalently term. You can also have Y v r delta, v r delta. So my point therefore, you see each two, what is the meaning of that? See that, imagine this, what is my, what is the physical meaning of that, I want to tell you this. See here, what I am saying there is a ship here and I give this Y force, certain delta angle and it is giving me certain force, but now supposing the ship is actually turning with a v, with a velocity here and of course, that means.

If this point I give it here, so this delta I give, so this force that you end up getting here, this Y force normally to this body may not be same as this Y, because in this case the body was going on a straight line, but in this case it is having a small v. So, this is what will happen that there therefore, I will have, see here the force Y rudder may be if I take

Y delta, delta, but here Y rudder may be Y delta, delta plus $Y v$ delta v delta. Here of course, the v is small 0, what I am trying to say is that, see the force here and force there may be different, why because here I am going on a straight line, so flow comes in certain direction, I have got a Y force.

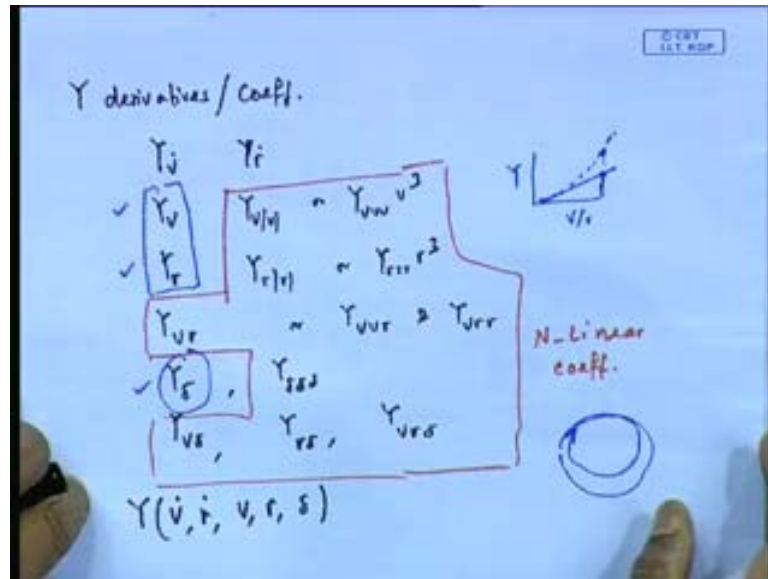
But, here same angle, delta angle I am given, say 20 degree rudder, but the ship is already on a turn, and it has got certain v and also certain r . So therefore, $Y r$ delta also, this force is different. Now, there can be a force arising a correction on this, because of the fact that the Y force now apply on a hull, which is already having some v . So therefore, Y is a function of v and delta. So therefore, rate of change of Y against v and δ square Y , were δv , $\delta \delta$ into $v \delta$, so this term comes in.

So, this is the physical meaning of that question of modeling, as I said these are normally small, so that it is a question of modeler, should I take this? Up to you, if you want to take a non-linear model, or should I take this? You have to decide. A modeler has to decide and of course, certain like experiences come in which people are now a days there are certain standard model being taken, where normally cubic is taken and these terms are taken up to one or sometime actually, what is happening there is a competition, either you take this or this normally.

It turns out, there are dominant feelings. One is the European $h s v$ a model, one is a so called American model. Some people take only this and this, some people take, instead of this, this, this and this, but most people will take a coupled term. Now, I can say that these are the non coupled term, because it is just telling me why varying with delta, these are what is called couple derivatives $v \delta$, etcetera. These are called couple derivatives, this is a question of modeling.

Now, if I understood up to this point, then we can now kind of list down what are the kind of derivative that we might end up having to find out, what are the derivatives that goes in a typical non-linear equation of motion. So, this is what we have said here, only Y the same thing will apply to N . So, let me now show that suppose I want to list derivative, Y derivative say, and then also I am only looking at Y and N .

(Refer Slide Time: 27:26)



So, Y derivative say, or coefficient you can call derivatives. So, we have Y_v , Y_r , then I may have Y_{vv} , Y_{vvv} or Y_{vvv} cube, then I may have Y_{rr} , Y_{rrr} or Y_{rrr} cube, I may have Y_{vr} or Y_{rv} and Y_{vr} , I can have Y_s . See, here ΔY , Δ , Δ , I can have $Y_v \Delta$ or I am just writing those ones $r \Delta$ and say $Y_v r \Delta$, one, one couple term I took, you can also have more.

So, you see now why I mention this see, because when I do experiment, I would at the same shot be able to find out all, because in a experiment you remember, I am going to be basically measuring Y versus some parameter, say v or r whatever. Since, I am going to measure that question of fitting and then getting all, there is nothing like I will find just only one, that is why I mentioned a mention then.

Because, when you do experiment you of course, will find out all, nature will not try to tell you how you model it, nature will give you exactly how the force varies for a given input parameter. What is my output for an input? That is what that means my Y for a v , that is what my measurement will tell me. We will of course, talk how we can isolate a measure that is different, but see you end up getting therefore, if I were to note down the non-linear derivatives, then this part this, this side is non-linear derivatives or coefficients.

This is linear, these are non-linear coefficients. Same thing will happen to n in fact, one can keep progressively more complicated, because I only take Y as a function of v and

delta, but if you take also roll motion then two more parameter come phi and p, roll angle as well as roll motion. So, there will be much more derivative that will come, that depends on the.

See now, I will come to this next part, the captive model test. Obviously, one of the biggest job therefore; is that, well I know the derivatives of course, I know only this linear ones, that is basically I know this and the corresponding n as far as my directional stability with control fix is concerned. So, this is very important, this is very important, this is very important. Normally, you will find out that these values can be estimated fairly well from theory also, but these values are much more important and we have seen that also. This, this and the corresponding N_v , N_r , these are the one that decides the sticks free stability. Stick free mean, control fix stabilities, stick fixed stability.

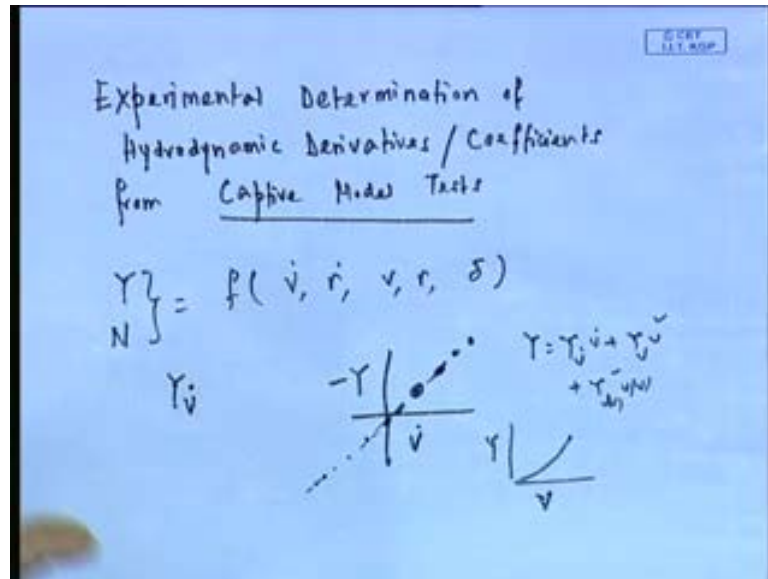
Also we have seen therefore, turning circle this, this, this and the corresponding N values are the one that tells me the turning circle radius to the first approximation, the turning circle radius, drift angle estimate, now you want to make an estimate. Normally, you will find out that if I wish an only linear derivatives, what I get if I take non-linear derivatives would be tight at turn, that means if linear gives me this kind of turn, non-linear derivative will give me tighter turn, normally. So, which means that, I am a requirement, that my ship must turn within 5 ship length and all this stuff.

So, if I could estimate them based on the linear derivatives, I will be more or less sure that the ship would meet the requirement, because non-linear derivatives always tend to make it smaller tighter turn, faster turn. That also make sense because, normally forces go like that, because that means in a linear you are predicting my Y force as this much, but actually the force is much larger. So, naturally that normally say.

So, we said this, now comes the question of funding this coefficient experimentally, that is what we are talking experimentally. Remember, I need to find this coefficient experimentally, this and the N, and as I said if there are other. What are the test we, and how we can go about it? That is one important part. Now, once again if you see that see here again if I write that Y was a function of, what is this? $v \dot{r} \dot{v} r \delta$ I am just taking this. So, when I if I want to find Y against v, what I should do?

I must make sure that, I have only v to the model and nothing else, because if I had other thing else, what I would measure would be Y arising from all the factor.

(Refer Slide Time: 33:03)



Let me again, this is the very important concept that I need to experimental. Let me write it down or I say sometime I use the word coefficient. See, everybody agrees that there is no denial that you need to determine the hydrodynamic derivatives, because they are the one that characterizes the manoeuvring behavior of the hull, that there is no doubt right, we have seen earlier also.

So, the next question of course, is that. How do you determine that? Experimentally, you cannot go build the ship and then measure it, you might do that, but that would have been after the design. The word captive model test, that resistant test that you do, etcetera, means that you restrain the model in some motion, it is not free running model that we are just making, you really cannot do it, because you have no control. Resistance test, what you do you hold it, allow only to heave and pitch may be, and then tow it captive model test these are captive model test.

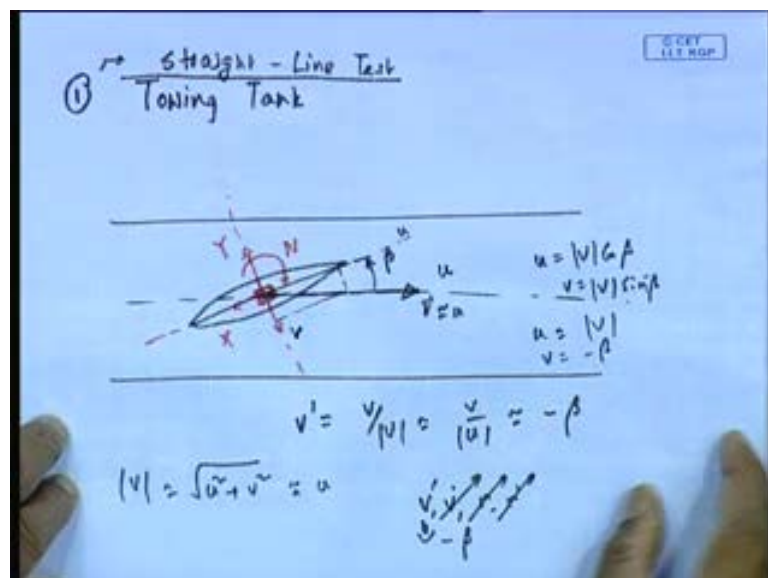
Now, here is comes the question, let us say Y and N I'll write this way, they are function of various parameter, I am just writing up to this much. Now, let us say I want to find out say Y v Y say v dot. what does it mean? I must measure Y force arising, what is this? Y force arising because of v dot, so this say minus Y if I do I had to measure these values, then I draw a graph, then take slope right, I need to measure this. I need to measure in experimentally, what is my Y force coming because of v dot or what is my this coming, because of that parameter, whichever parameter.

What about the other parameters? They should be 0, not straight line, they should be 0. So, this is the fundamental requirement that I have to ensure, that when I want to measure Y against one parameter, others are all 0. Otherwise, what would happen, the force I measure would have come because of also that parameter see for example, this both $v \cdot v$ and $v \cdot r$ there, then what the measure I value would be $v \cdot v$ plus $Y \cdot v$ may be plus $Y \cdot v \cdot v \cdot v$. So, you see, how do I isolate them? What I measure will be coming for all of 3, or if this r there also complication will come.

So, what I have to do, this is the important point you must understand that, if I want to measure Y or N against one of the derivatives against one of them, which means I am looking at variation of that, against that motion only. If it is Y versus v , only about v I can of course, measure this two both because I will have a graph like that. What I have to make sure that other parameter. See, if I want Y versus v , then $v \cdot r \cdot r \cdot 0$, $\Delta 0$ is not a problem as far as experiment is concerned. How do I do that?

So, this is where the challenge or designing the test comes. You see, as you go progressively higher, resistance is a simplest test, lets tow it, then sea keeping, then manoeuvring test, you keep going to more complexity.

(Refer Slide Time: 37:02)



So, I tell you the first test, which is the simplest and first test that can be done in a towing tank, Actually, I should say, **I should not test this in** first test is, we call straight line test, this can be perform in a towing tank. What I meant is that, the first test, that the simplest

is what is known as a straight line test. That first we will discuss, which can be performed in a normal towing tank and the test goes like that.

I, have this tank here, I have this central line of the tank here, all I do is keep the model incline, tow it there at u equal to u one you can call, and measure with dynamo meter here, whatever Y , and whatever. You see here, what we are doing with an angle of attack say, now here with angle of attack is taken from here. See, in a resistance test what you do? You have a model that is towed and measure x force, that is what you are doing right. What you do in a resistance test in towing tank? You have a model, you tow it along as a straight line, what you do? Measure x force with a dynamo meter.

Here what we are going to do is that, this first test you do the same thing, but you now tow it with an angle of attack β , and you measure with a dynamo meter Y and n , you can also measure x like this x , but more importantly Y and N for manoeuvring this way. You have to orient, that was that is a question of instrumentation, you orient the dynamo meter this way and measure along Y . Remember Y is actually, normal Y is actually, transverse axis to the ship, that is this is longitudinal axis, x axis, this is Y axis, the ship, the transverse axis to the body fix Y axis.

So, you measure Y and n , what did you achieve by doing that? Remember, what is β . Now, here comes the important question. See, v dash is v by v , which is equal to v by u , because if this small angle is there, this is equal to actually, minus β that we have said before. See obviously, what is happening is that, if you draw this thing here then, because the β the way you are defining is that you are measuring from vector to the x axis, so this becomes, see this is minus β for plus β .

Remember, in this diagram v is this side with this plus, and you are measuring β from the velocity that this is actually, v here which is close to u actually, this is u here. What is happening if there is a small β ? You can easily find out that u is equal to $v \cos \beta$, and v is equal to $v \sin \beta$. If β equal to small, then u becomes equal to v and v becomes equal to β , minus β , there is a minus sign here.

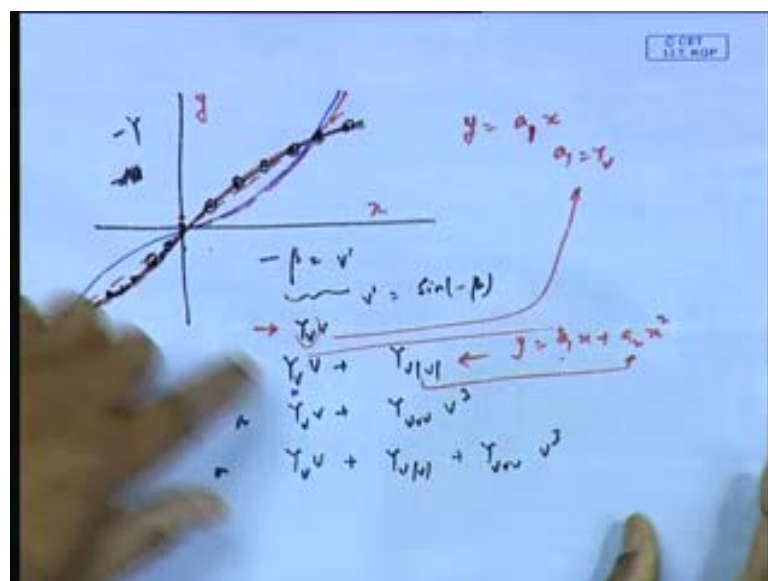
Therefore, v dash which is actually, v by this big v , which is v by u is equal to minus β , that is a non dimensional value, understand that. See, u is $v \cos \beta$, v being the velocity vector. Remember that again, I tell you this way v is equal to u^2 plus v

square root over is equal to u , because v is small. When β is small, v is small, this two first order it is like this. It is that just like saying, $\sin \theta \approx \theta$, $\cos \theta \approx 1$, if θ is small. You can also see u is $v \cos \theta$ β is small it will be v , and small v is capital v into $\sin \beta$, $\sin \beta \approx \beta$, that is β for small.

So, if you do not like, you can also put large number, does not matter. But, just for showing suppose you do not like this approximation, but you will know exactly what is the v . The point is of course, that you are here able to, you see what else is there in this, what is r ? 0, because I am not rotating it, right. When I translating it this way, what is my r ? 0, what is my \dot{r} ? 0, what is my \dot{v} ? 0, because I do not have any acceleration. So, what I achieve? I have only given v and could measure Y and n because of that v , you understand that no. In this case I could only impose a velocity, transverse velocity v u of course, will be always there forwards speed, because u is ubiquitously present e there is a mean forward speed always.

So, here what I did my speed v \dot{v} , r \dot{r} , this is 0, this is 0, this is 0. I could only have this which of course, is $-\beta$. So, not only that depending on what β it tow, I could change different v . So, what I do if I where to tow it at β equal to 1 degree, 2 degree, 3 degree, 4 degree, which means I am having different value of e dash. So, essentially I am doing this test here when I could measure for β , different β Y force and n moment.

(Refer Slide Time: 43:38)



In other words, what I am achieving in this test very interestingly is that, I have minus beta, which is equal to $v \text{ dash}$, and why I put it minus Y here, because typically this otherwise, this way the graph does not come minus n may be or n may be does not matter which way the n is. Why I put minus Y , because what is happening, then I do a test beta equal to 1 degree, I get a value here for Y value, forget this now. Why I put minus of course, because you know typically for plus theta dash you will get negative value, that is why I put minus. Why minus actually, it is nothing I can put plus Y and then the point will come here, because normally what would happen obviously, if I give this kind of angle here, then I will measure the force coming this side and I am calling by axis, this is plus.

See, when you do a measurement, you will get the instrument, the recording say 20 Newton, But obviously, in this our subject the coordinate system is very important, so you have to make sure this side the force is, if the force is this side I am calling negative, because my axis system is positive. So, I got let say the instrument showed me force this side 20 Newton, that means my measurement Y minus 20 Newton. I means this is very simple, and what is the beta? I have given this side angle, which is minus one degree, which is plus $v \text{ dash}$, so I have done that.

So, that is why I just want to show here, it does not matter. If you make it plus the point would have come here, that is all. So, then I do another one go here, I do another one go here. So obviously, I do a few of them, in fact I can go, in fact I need not even write this, I can write actual $v \text{ dash}$ has measured with respect to sin of minus beta, if I want large angle I can also write this way $v \text{ dash}$ as minus sin beta, I mean because we are doing large angle.

So, what would happen, you end up getting these points here of course, you have to repeat that exactly, on the other side, turn it this side with a minus value, because you have to always do symmetric, then you end up getting this value. So, you end up getting a graph well obviously, you have not got this point remember, you end up getting a graph like that. Now, you tell me from this what all coefficients we can find out.

Obviously, I can find out $Y v$ see, now what is happening $Y v v$ plus depending on what model you do, this or $Y v v$ plus $Y v v v v$ cube or I can even do a combine model what you call, condemn. See, now the question is like that, I have this value, or let me put also

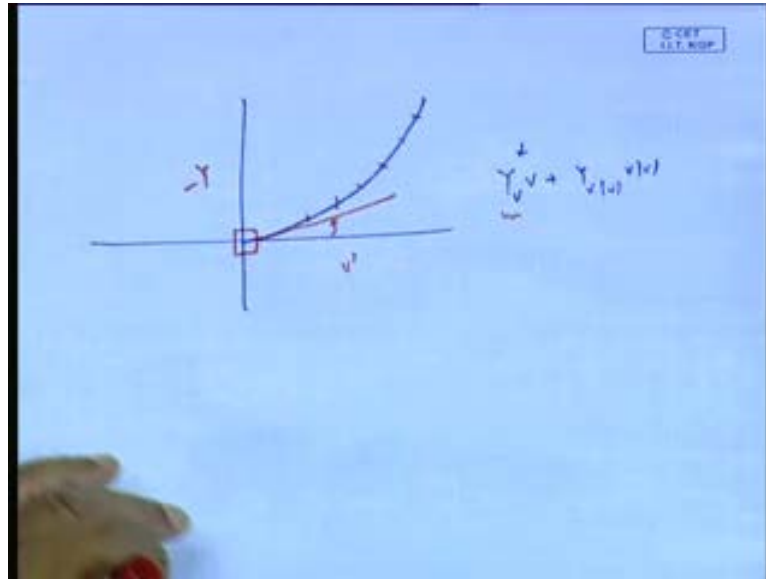
linear value only. So, this is pure linear, suppose my model was pure linear and I say that I have only linear model, I want to use this in conjunction with linear equation of motion.

What I will do? I will fit a this square fit curve. Let me call this Y , let me call this x , Y equal to a $0 x$ or a $1 x$, you can also have a 0 actually, if there is an offset there. Some models will have an offset, there is an residual value at v equal to 0 . It may happen because, of there is a design problem, it was not symmetric like exactly port star board of symmetric, as you go there is a slight force possibly, it can happen. I am just not going to those details now. So, if you do that you fit this graph a one is this simple, this equation.

Now, I want to model this and my model is this, and I want to find out. What you do? You model for thus Y equal to a 1 fit plus a $2 x$ square for whatever, like $x x$, so you have now fitted a graph like that, find out the coefficient and this is my, $Y v$ is this and $Y v$ is this. If I want to now fit this one, I can find. What I am trying to say that you have actually, got Y versus v .

Now, depending on, so it is not straight line suppose, it is all straight line also. Then depend the model you choose, you take that curve and fit that curve, this square curve simple as that. That is why I wanted to have this non-linear thing, because remember they are couple. When you do the experiment, there is nothing like linear, non-linear. Experiment is not bothered what model you took, experiment simply gave you what is Y force against v , it is up to you to model and you would have modeled with Y , normally $Y y v$ actually shows not this, in fact it mostly shows like this. Real life it shows actually, like this, it becomes larger with larger this thing like typically, I am just saying for typical this thing, it may look like that.

(Refer Slide Time: 49:19)



So, obviously in here you use that, now you see that the interesting points are many. Suppose, I use this, **now if I what is my let let me** I'll just want to go for this little bit here, let us say graph was like that, now if I use values like that say, in that remember what is my Y_v . Y_v is actually, slope, this slope. So, if I took only the linear coefficient in a measured fit and use this for finding out my stability, I may find out that ship is unstable with a sea negative, but actually the ship may be stable, because actually you are trying to tell the force is my actual, the force is more.

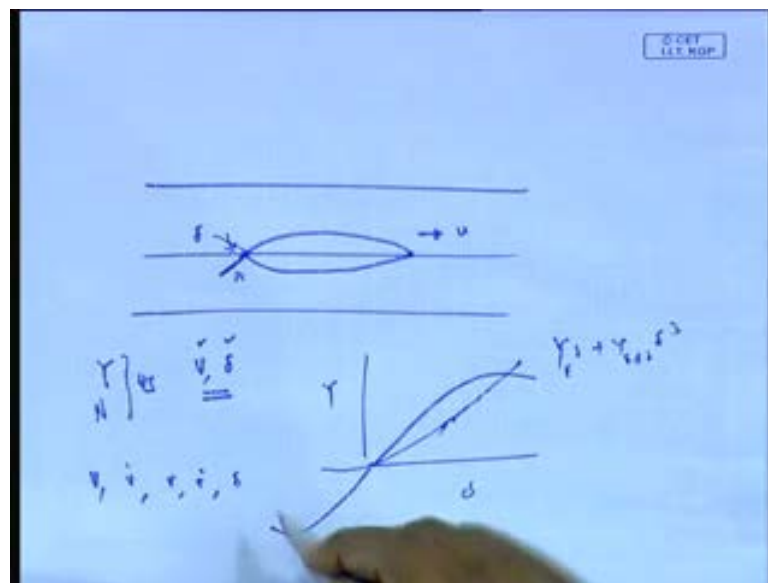
So, you see these things are very important for you to understand that linear derivative gives you only a behavior at this point, but unstable ship, direction unstable ship with coefficient, like with a control not working, does not mean it is going to be unstable, it is exactly same as angle of loll, somewhat similar analogous to that ship is negative g_m , does not stay at 0, unstable, but it'll go two degree and stay, becomes stable at 2 degree.

So, you must remember, this is way the question of non-linearity comes in, because actual Y versus v dash is like this. Now, we have just said that, now I will just want to end this test at this point, I want to tell you. So, if I do a straight line test, tell me what derivatives in this list I can find out. Where is that list? I bring that list and then I try to tell you. So, I have straight line test done, which is very easily doable in a towing tank, I do not need a facility for that, but I do need an instrument, a dynamo meter that can measure Y and r .

So, if I want to do it today in our towing tank, I may not be able to do in our towing tank, because I do not have a dynamo meter that measures Y and n . You see, I have a dynamo meter which measure x only. But, I need a, so called 3 component dynamo meter means, x Y and n , it should be able to measure. If I have that I can do this test, but having done the test among this coefficient what all I can find out, only this line or rather I will tell you, I am able to find out among the y 's only this line and of course, N equivalent N , $N v$ $N v v$, $N v v$ cube right. Why of this and also corresponding m .

But, now I want to ask you this question you tell me this anything else you see here, that can be found from this test. Remember, the test set up you know, this is the set up of the test, anything else we can do from here. I can do this, how I do this? Not only that, I can also do this, how I do this? I will now tow the model with $v = 0$, but with a δ .

(Refer Slide Time: 52:29)



Then I measure Y versus δ , I can do that. Remember, my v is 0, I can do Y versus δ , then I can measure this, then again I fit the graph. It is Y δ , δ plus Y δ , δ , δ , δ cube depending on what graph, basically it may not be like this way, it may be like that more.

You have to do with both sides. So, I am able to Y or n versus v , and δ . This two can be varied keeping the other constant. That means, I can among this, I can only get linear sway derivatives, because Y versus v r , v is sway motion, $Y v v$ not linear, linear and non-linear. Let me call the other way round, sway velocity derivatives. This is sway

velocity, if you call v , \dot{v} , r , \dot{r} and δ , sway velocity derivative I can find out. Linear and non-linear, that linearity does not come in picture here, I can find out rudder derivatives. Now after the break I can tell you how, I can also find out $Y \dot{v}$ or if you want $Y \dot{v}$, $Y \dot{v} \delta$, the cross couple terms.

It is possible, because I have varied this two, so I can always vary this two independently, I will show tell that later on. So, essentially of so many coefficient I can only find out few by this test, this is test number one. So, any how I will this class will formally be over now, then next class I will tell you how $v \delta$ can be found out, and then we go to the next level of test, test type two, where the rotational derivatives are found out. So, with that I will end.