Seakeeping and Manoeuvring
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Lecture No. #32
Non-linear Equations of Motion

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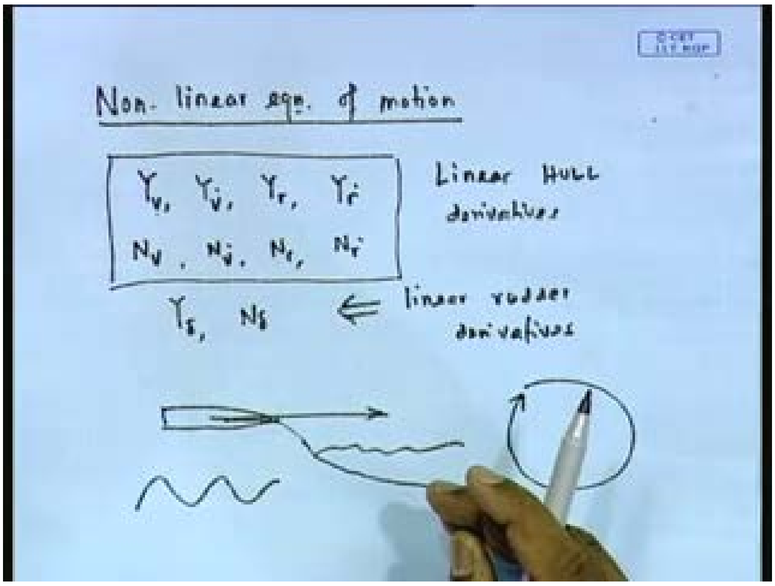

See, today I am going to talk about non-linear, see earlier we have spoken about, linear equation of motion, there were linear hydrodynamic derivatives, and we talked about the stability of the vessel without controls working, all of this was lie were like studying very similar to initial stability or small angle you know hydrostatic stability that when theta is very small.

You know the condition like something meta-centric height more than zero, but the point is that today I want to talk about little more general form of equation of motion, there is a reason for that see we have quantities like this linear quantities then of course, we have rudder quantities this were the linear hull derivatives, these are linear rudder derivatives.

Of course, in which we of course said all the divisions that this is linear velocity v; this is added mass etcetera, and we have discussed it many times what we found out at the beginning we found out that the stability criteria depends on this quantities that is one, secondly we spoke about turning circle, zigzag and some definitive manuals and of course, when we have turning circle manoeuvring also we talked about only turning circle predicted based on linear derivatives.

So, obviously as a consequence of this now my next job is how do I find out this derivatives, because I need to find it out know, when I am doing a design, I need to find out how much they are, when we want to find it out most experiments will at the same time also find out what is known as non-linear derivatives, now the question is what is non-linear derivatives?

So, when you do an experiment you always end up doing all together this is why I thought that first I will speak about non-linear equation of motion brief one class; before I go to how to determine the derivatives that is the purpose of my this talk why non-linear derivatives? Remember that earlier what we did we were thinking the ship was going on a straight line, what we did we wanted to find out if I give a small part of perturbation very small does it go on a this thing or does it gone a straight line. So, it was against initial small perturbation that was one but, when we did stimulation of zigzag etcetera, though we had turning circle, we had zigzag manoeuver in there of course, we did not talk of derivatives we only talked about actual experiment.

But you know when you do turning circle at 35 degree rudder the turn rate may not be small, and if the turn rate may not be small then the various assumptions based on with this I made this equation of motion may not be valid. So, I may require a non-linear equation of motion large amplitude just like if you want to study, what is my heeling moment at 30 degree you cannot use g m you use g z similar way you may not able to use only linear equation.

So, what is the concept of non-linear equation that is what I want to talk today and this comes in because non-linearity may be involved, when I make large trajectory stimulation that is when I make sharper terms, when I make quick zigzag etcetera force may not be linear, so this is what will today discuss now you see here.

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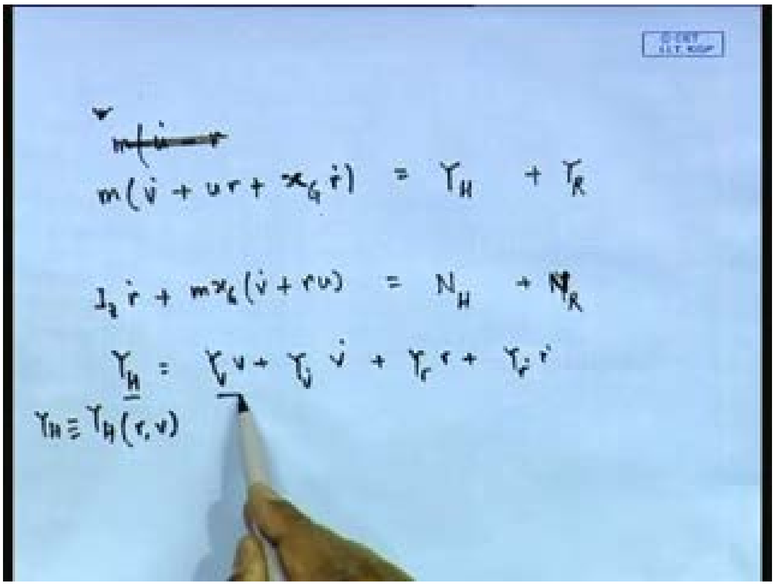

What was our equation of motion, it look like I just look at Y at N my equation of motion look like m u dot minus r v, this is not I will just write it again m because we will write only the Y equation, I just write these two and then we will see; this was my equation of motion remember just on the basic Newton's equation of motion mass into acceleration is force, what we did remember now earlier we had only this as hull force, this is hull force but, if I had the rudder, I also should have rudder force, rudder force N.

Then what we did we said this hull force Y H, because we said that I am only looking at very small values I end it up getting this Y H as Y v v, plus Y v dot, v dot, plus Y r r, plus Y r dot, r dot because Y H was taken to be a function of r and v. Since it was a function of r and v what we said is that we said by Taylor expansion it is like that up to linear order.

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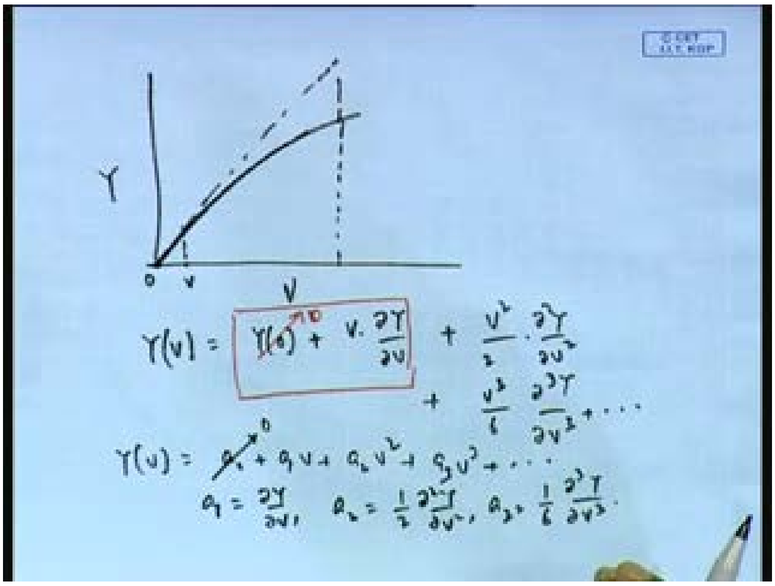

Now, let us take this case let say Y H against V what we did we said like this that I have here V, I have here Y I have this varying something; I am only looking at value at a small value V compare to this which is 0, so I said Y at V equal to Y at 0 plus V into dy by dv, plus v square by 2 d square Y by dv Taylor expansion is like that etcetera, this is the Taylor expansion what I did I only took up to this much.

Which means I said this is a straight line and of course, this was zero, so it becomes this d divided by dv but, now remember suppose V is no longer than zero and let say this graph is not this way the graph is actually steeping down, I have my V here if I only use this I will end up getting this estimate. So, what I need is to use higher order derivatives now, if I have to use up to this much; I will have found out V divided by DV plus V square by 2 dy by dv square if I took up to this much it will be one six.

Now, this is one way of looking at that but I can also say by a curve fit; I can also say that Y verses v is a 0, plus a 1 V, plus a 2 V square, plus a 3 V cube etcetera. I can fit a polynomial on this graph of course, this is zero because I know that at V 0, Y is 0. So, what happen if I compare this with this what do I find out? I found a 1 is essentially DY by DV, a 2 is essentially half of d square Y by dv square a 3 is 1 by 6 of d cube Y by dv cube.

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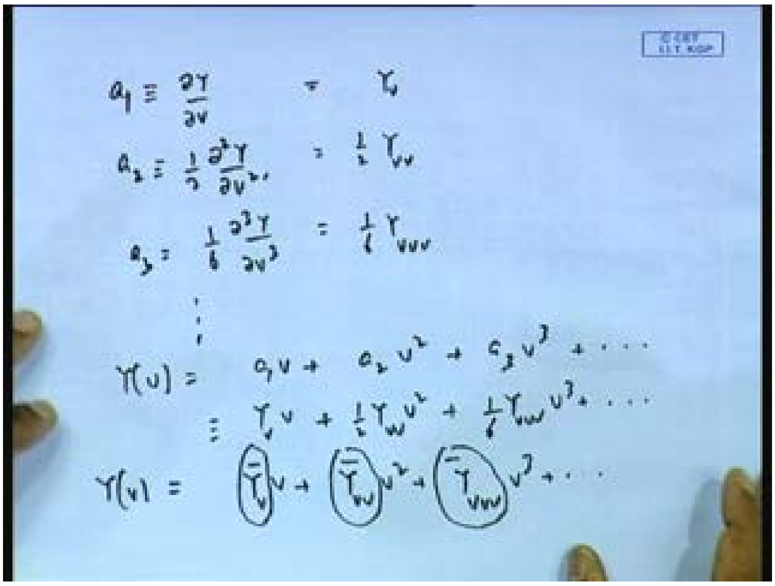

Now, comes the I will just rewrite this again in this one that means I find out that a 1 is equivalent to dy by dv, a 2 is equivalent to half d square Y by dv square, a 3 is 1 by 6 d cube Y by dv cube etcetera. So, if I have to call this Y by dy by dv as Y v then this is like Y v if I have to call d square Y by dv square as Y v then it is 1 by 2 Y vv and this is 1 by 6 Y vvv.

So, what in this nomenclature what happen my Y v has become a 1 v, plus a 2 v square, plus a 3 v cube, plus which is equal to Y v v, plus half Y vv v square, plus 1 by 6 Y vvv v cube plus, so that you can see that therefore, a 2 is supposed to be half Y vv, a 1 is etcetera this is the coefficient I can call this derivative but, what happen it is a question of nomenclature I can also write.

Now, this is very important for us as a convention I can write this to be as if, I will write this as Y v v, plus Y vv v square plus say I will call this with bar, so that my Y bar vvv is basically d cube Y by dv cube into 1 by 6 the question is that this thing see are just the coefficient; they are just basically a coefficient a 1, a 2, a 3 the nomenclature in manoeuvring is that the term by which you multiply say v cube you always write this coefficient as Y with a suffix vvv v cube that is the nomenclature use.

So, what happen when I represent Y as this way that is when my Y was not linear but, this way then I end up getting an expression like this, so what we are saying effectively is that I am making an assumption in this Y, that they are going to be a force against some parameter and therefore, they can have a non-linear variation up to this much, see what I am trying to do that I want to find out Y against V and I am saying that yes this is not a straight line, you know the analogy is theta verses G Z this is like that this straight line is what G M theta.

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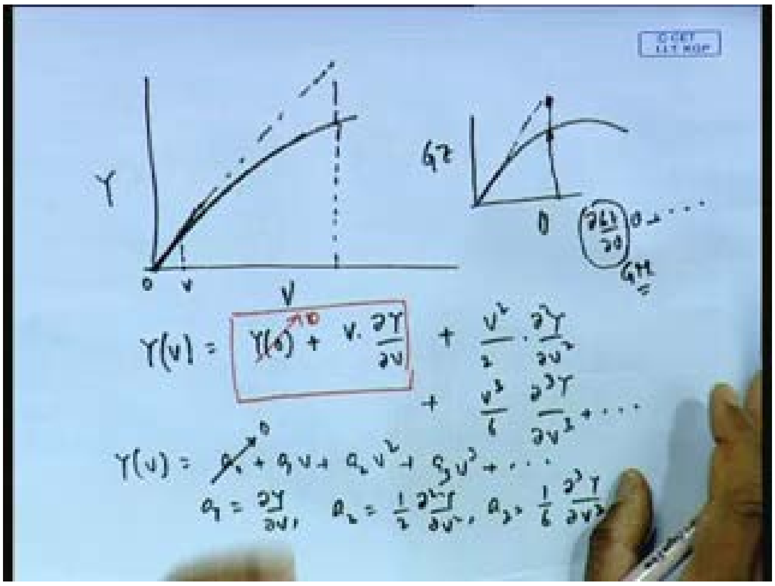

So, if I would have to make a straight line; I would have call G Z what I would have call Z G Z by d theta into theta plus this part is nothing but G M but if I want here I need this, I do not need this; if I have to use G M theta I would have up getting this as G Z. So, the analogy is exactly the same because, at larger value of the force there can be possible non-linear variation it may actually be like this, I need to use higher order coefficients.

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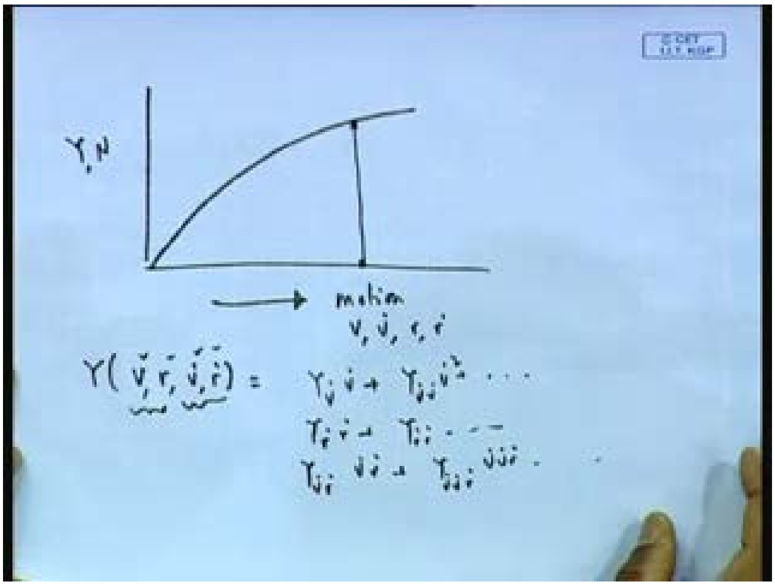

So, it effectively comes to the fact that I have here once again a motion it can be v, v dot, r, r dot whatever. Here I can have Y N whatever and this can be arbitrary like that therefore, I will have to express this, because what I am looking for is that given for v v dot r what is my the Y value that is why I am looking at.

So, for that essentially supposing Y is the function of v and r also v dot and r dot let us see, so what happen I will have here multiple parameter I can say it is v dot, v dot plus Y v dot, v dot square plus etcetera then Y r dot, r dot, plus Y r dot, r dot etcetera plus I will have v dot r dot, v dot r dot, plus Y v dot, v dot, r dot may be v dot, v dot, r dot etcetera there is no end I can have infinite number of coefficients as a general expression.

Now, what I showed here is a added mass that is only with respect to these two, then I have to have with respect to these two and the combination also, generally I can have a large expression, so if I have to go in a general sense then I have a let us take an example of just well in here also remember that this part variation of Y against acceleration is added masses, variation of Y against this is damping forces, now if so happens that obviously in a general sense we can have infinite number of terms but, shall we have it is there any physics behind it that is first question that comes in.

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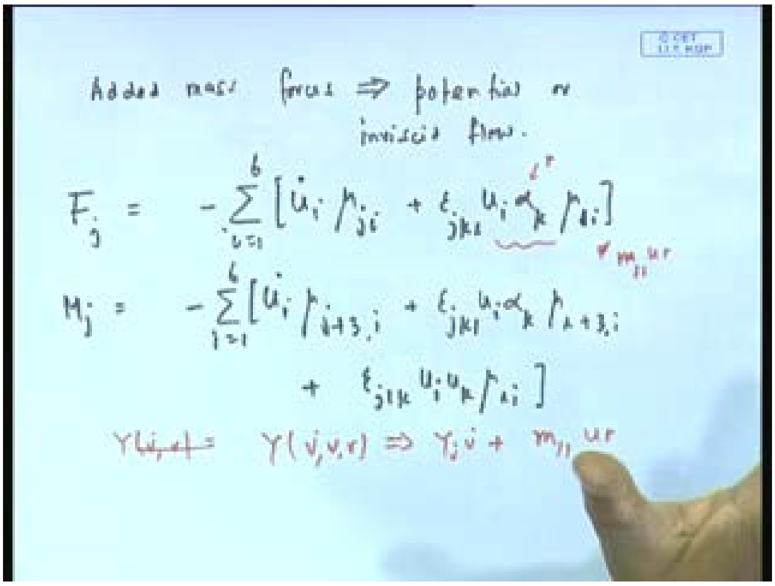

Now, it turns out that from potential flow theory if you use exact theory you know see added masses that is this arrives can be determined from potential flow and there is this part has been very well you know like studied general expressions for fluid forces on an accelerating body is known, this is basically a part of marine hydrodynamic force. So, we will not derive that obviously because it is a long expression but, the expression comes out to be something I just want to write this, so that you know F j becomes u i dot mu j i this is mu l I something like that M j is there is a similar expression this is j it is this is l, this is actually I do not expect you to remember all this but, I just want to show you just for knowing this j k l, l k l.

I see here I tell you we need not really too much worry about that but, there are some general expression known j is 1, 2, 3 direction 1, 2, 3 m 1, 2, 3 means moment in 1, 2, 3 the general expression mu j i is basically are called added masses say mu 1 1, mu 1 2 etcetera, u i 1, 2, 3 are the linear velocities alpha k 1, 2, 3 that is k is 1, 2, 3 are essentially the rotational velocities which is also equal to u 3, 4, 5 fives basically u 4, 5, 6 is alpha 1, 2, 3 is equal to rotational velocities epsilon j k l is known as Einstein's it is a convention if j k l are in a cyclic order that is 1, 2, 3 or you know 1, 2, 3, 3, 2, 1, 3, 1, 2 then this is supposed to be 1 plus 1 for anything else it becomes 0 and if it is anti cyclic that is 1, 3 ,2 etcetera then it becomes minus 1.

So, this is a general expression that has been derived by the hydro dynamicist to find out added masses why I am saying that because you will find out here actually from this the general expression for an accelerating body's added masses for example, suppose there is no acceleration body is going on steady translation then this becomes zero, alpha k also becomes zero, then you will be able to find out that F j becomes zero, this is the classical d'alembert's paradox.

If on the other hand there is an acceleration you will find out that there will be some velocity value for example, in our case we have let us say we want to find out f 2 as a function of certain parameter, if you place all of them you will be able to find out exactly what happen f 2 is my wide force you know 1, 2, 3, f 2 Y force and if I have to say I have no acceleration only velocity then this goes to zero etcetera.

So, you can find it out the my purpose of saying is that having done this one would find out that the exact expression for added masses that is the velocities depending on the u dot, v dot etcetera and known fully known you will be finding out that f 2 will become v dot into mu 2 2, mu 2 2 is my Y v dot minus mu 2 2 that is are by definition. So, this expressions are known why I am saying because the fact that I write here this one I do not have to make a general expression for this variation against the acceleration, variation against acceleration is well defined well known one does not mean to model it is known from theory.

So, as far as the variation of Y or N against the acceleration is concerned earlier what I did I took it this as Y v dot v dot, plus Y r dot r dot I took it bit here I am trying to show that I should take all non-linear terms but, I do not need to do that because these expressions are well defined in literature. So, the acceleration forces or if I have to go back to this hull force part here, if I have to define this again Y H as Y H due to acceleration for inertia, that is for v dot and r dot plus Y H due to damping that is v and r.

The first part inertial part is well known for that I do not have to take an arbitrary expansion that is the point I am trying to tell, the debate that comes, the entire debate that comes in modeling actually arises purely because of its damping term that is Y against v and r how do you model them? Now, I will come to this part how do I model them? Y against v to r.

So, what happen there can be number of models there let us see this part is here, Y against v this is why the debate comes see why against v dot acceleration forces are well known and I will write this expression afterwards, what happen to the acceleration part eventually right now, let us not you know like look at that we will find out that there will be some extra term coming; when we do this expansion of Y force but, in fact I can show this from here, what would happen you know that if you look at this term just for one second let us look at this first term.

I am looking at a case when I have got v and r, now in this you see there is a term v dot into mu 2 2 but, here I have j k l can be 1, 2, 3 when 1, 2, 3 j is 1, 2, 3 then there will be a term u alpha k and this thing this is not zero there will be term that will come this will be actually become r, there will be term equal to r that is this term, one term will arise which will be looking like it will turn out to be m 1 1 u r and additional tem will come m 1 1 u r.

That means my you see what happen my Y at v dot and of course, with r all that see my Y at v dot and of course, I am having v and r this arises to me v dot, v dot plus an extra term comes m 1 1 u r because of r, so this is arising because of added mass inertia forces. So, what I am trying to say inertia force part is well defined.

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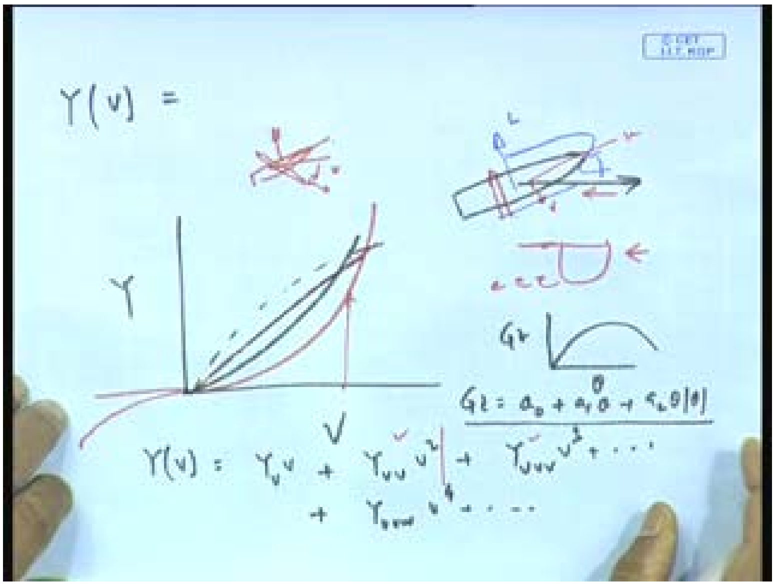

We know it exactly I do not have to expand this any further to go to any other terms it is the Y v part that is damping that is not known and damping is the one that it turns out if we plot v verses Y, it is where the debate comes why? Because number one the origin of the force if you want looks at that it may arise because of real fluid effect, velocity effect etcetera in fact, why it happen is that if you have a ship here and going like that flow tends to come so this act as a lifting surface number one.

So, acts as a lifting surface it works which can be of course, defined with potential float heel this is acting as if it is a lifting surface it gives the Y but, there is also flow coming this way because you know there is a small v there; so if I look at the cross section if I look at the cross section here flow comes this way as the flow separates out this is my flow separates out and this will give me some force.

So, there can one also a force arising because of real fluid effect I think understand the physics once again that, I have an ship with angle of attack going like that which means what I have here u, I have here v what does it mean there is a flow coming this side small v but, there is also flow coming this side if I stand here; I will appear to be the flow is coming this side. So, it is the body is acting the body is going like that, the body is going like this flow is coming this side, so it will act as a lifting surface of low aspect ratio whatever that will discuss afterwards that is one phenomenon which can be of course, defined by the means of angle of attack c l alpha and all but, there is also a flow coming this side small v which you try to separate out give me another force that will be real fluid force. So, as a result we end up getting two nature of forces, so this need not be like that this can be like this, this also can be like that we do not know how it is it can be non-linear variation.

So, this is why this part which is connected to lifting phenomena and not purely potential phenomena you have doubt that is why I have to model it, so I can model this now Y v as Y v v, plus Y v v v square plus Y vvv v cube like that.

Now, here comes the question you know if I use this what is happening normally again I have to tell you this, you look at this G Z curve how do you model this against theta you will see that G Z you are modeling a 0, plus a 1, theta plus a 2, theta bar theta etcetera maybe I should not oblique to brought that, so the point is here that normally I want to have some curve there is no point of going up to very high order, I can keep going you know like Y vvv v 4 like that normally what happen there is no necessity for us to go to a very high order to tell me that this is actually little away from straight line.

Because you know, even if I suppose I will take another graph here or may be this only even if the graph is like this, even if higher I can fit actually even a second order polynomial after all it is only one curvature; it is not going like that. So, essentially up to this is good enough because this is going to capture me this if I can fit a curve this is exactly what we keep doing in for example, in your geometry of ship hull we use simson’s rule because simson’s rule presumes first rule 141 that the graph is a second order polynomial, which is not a straight line and second order polynomial this is good enough to fit me locally one curvature.

So therefore, I basically want to make sure that it is not a straight line but, it is like that I want to model it before that if I have my v, I will capture this value and not this value by mistake you must understand this very clearly this is very fundamental point of the non-linear equations they are serves no purpose to take very large number of terms, all you need is a linear term plus one more term that will tell me the curvature.

Now, which one I should take? Should I take this and take this or up to should I take only this? Now, here is the point you see here suppose our ship is like this is my plus v there is a force here now, suppose is this way minus v how much is the force if my value is minus v my force is suppose to be negative but, if I have to use vv v square remember even if v is negative this will be may positive value, so you see what is happening here.

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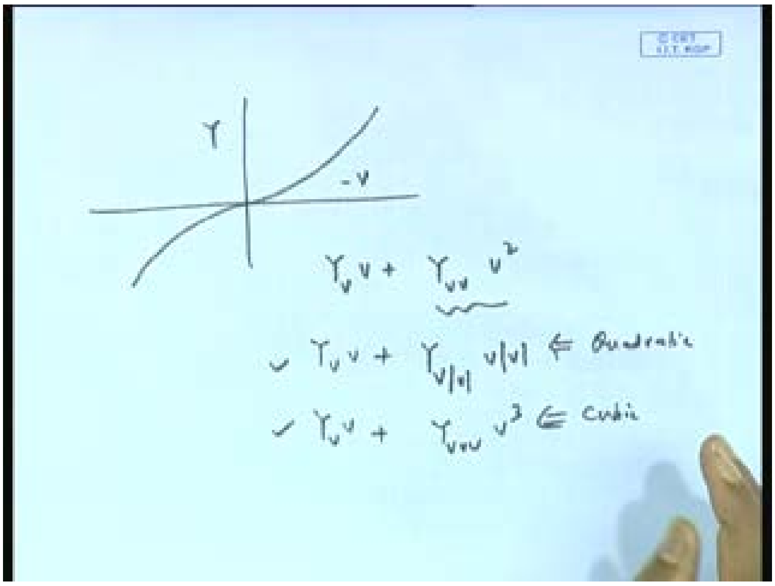

That my graph is like this and it is suppose to go like that Y verses v when I using actually this is not correct minus v because Y v is negative, so with or rather I should have put the other you know let do we like that, see it is like this no. So, see if I use Y v v plus Y vv v square remember this term become positive or negative whether v is positive or negative this becomes negative, so this will not make me a symmetry.

Then what I should do I should use this as Y vvv v what is called sign corrected quadratic term that means this value is going to have positive if v is positive, negative if v is negative. So, I should take therefore, a model of this or to avoid this confusion I should simply take vvv v cube because this one of course, is same so you see that the convention becomes either you use this or you use this that will capture to me up to this much but, the main point is I want to find out Y verses v and this variation which is not a straight line.

Why I am telling this you know when we I do experiment what I will do I will actually measure Y against v and I will get this graph, so because I got the graph, I would, I could easily fit a polynomial and find out the way I model I can find out both this and this or this and this depending on what I if I model by 3, I can find out 3 also that is why I wanted to talk this non-linear equation before, I talk how to determine this coefficient. So, this is very important so we have a one model this is what is known as quadratic model this is what is known as cubic model.

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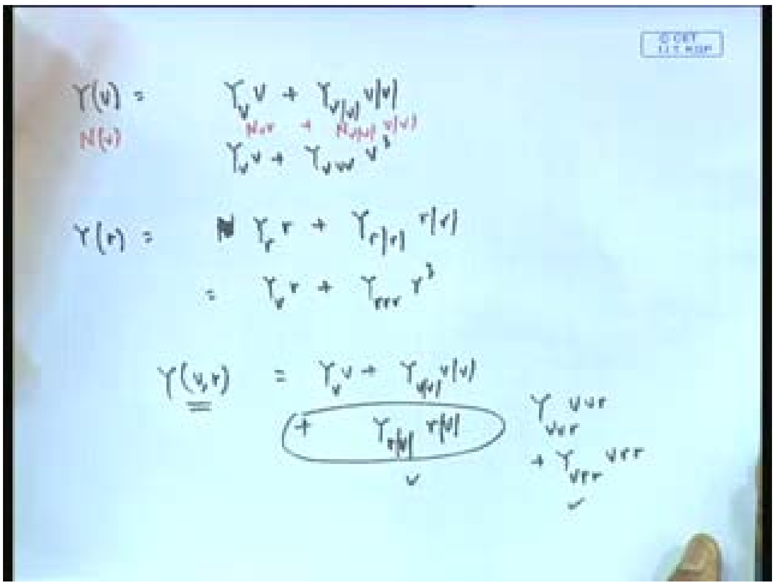

So, what happen Y verses v is normally therefore, the two choices I will tell you them Y verses v then I may have Y v v plus, Y v v actually v v is same as v square anyhow we can write it that way or Y v v plus, Y vvv v cube.

Now, you take off Y verses r N Y r plus Y r r, r r or Y r r plus Y r r r cube similar to that end we will happen you know like, you know like if you write here N v then you have just to make it n all places you know, if I have to make it N v here then all I have to do is N v v plus N v v v etcetera, etcetera. So, you see this is one of the model where of course, I am trying to find out v verses Y y against v what about the couple term v and r does depend on do Y depend on value of r also what it means is that physically my Y v r, Y depends on both v and r right. Now, this is Y v v plus Y vvv v, but what about the couple for term v and r that means what about when I have got certain r what is my Y v, so this is going to become then r v r v.

This is one model or I can make it or we can here I have to write this because of the same reason or to avoid that, I can write this part another model will be v v r, v v r plus Y v r r v r r this will take the your sign correction, so you see I can have either this or this when I want to take v and r.

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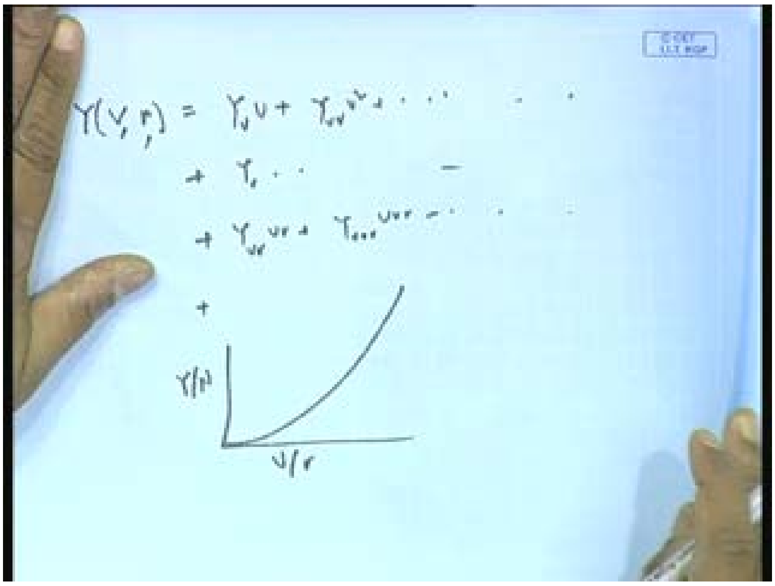

So, what would happen I have different models different way of basically like going in general of course, in general let me write once more again if I have to write Y as v and r then, I end up getting keep on going then, Y r keep on going then, I have got Y vr vr plus

Y vv r vv r, so there is no end of it I can keep on going and of course, if I have to put, I will see that about the delta part will come to that later on again.

So, I can actually have a whole lot of expression, large expression there, so when we talk of non-linear equation essentially I am looking at this non-linearity essentially I am looking at this variation which may be not linear Y or N verses v or r fact that it is not a straight line basic I want to model that so I only take up to some term, so there are if you see against v as I mentioned here against v I may take up to this or up to this against r I can take this but, against v and r I can take only one term r v r v or the two terms like this, this is what is the convention normally but, if you want to take more model you can also take more but, this is the convention.

Now, comes the other question of delta, what about delta? See I talk of Y v and r only v and r but, you will find out that I can also make it more complicated, I will show that this complication after I go to the delta part you see for example.

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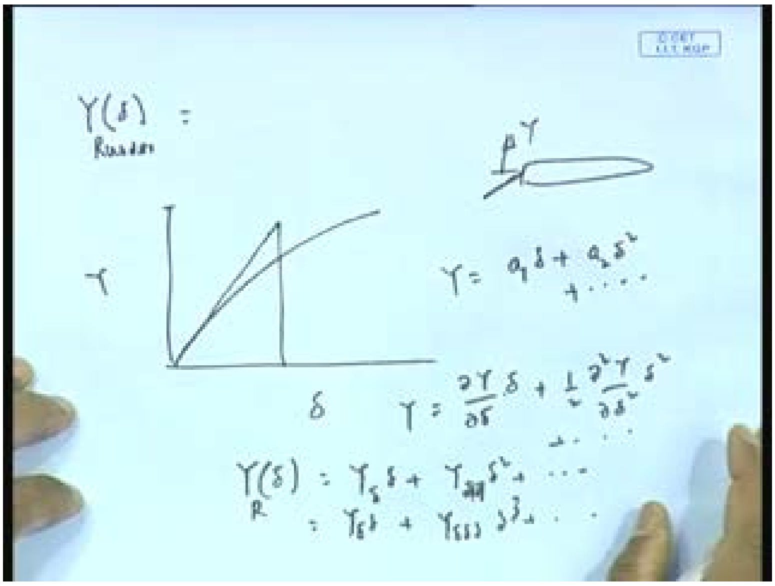

Now, rudder Y at delta that is Y rudder, what we are doing? Delta verses force you know I give the rudder; I want to find this Y force let suppose the graph is like that what is this? This is going to be Y force is going to be again a 1 delta, plus a 2 delta square, plus etcetera if see this would have been d Y by d delta into delta linear term but, I have 1 by 2 d square Y by d delta square into delta square goes like that.

Therefore, I can model Y also delta rudder force as Y delta, delta plus Y delta delta, delta square plus like that and again the same thing will come delta positive negative, I have to have quadrant correction. Remember delta is also like if it is on this side the force come this way and it in other side come this way, so again here the same thing will come so you can model this way or you can model etcetera that means Y against delta may look like that.

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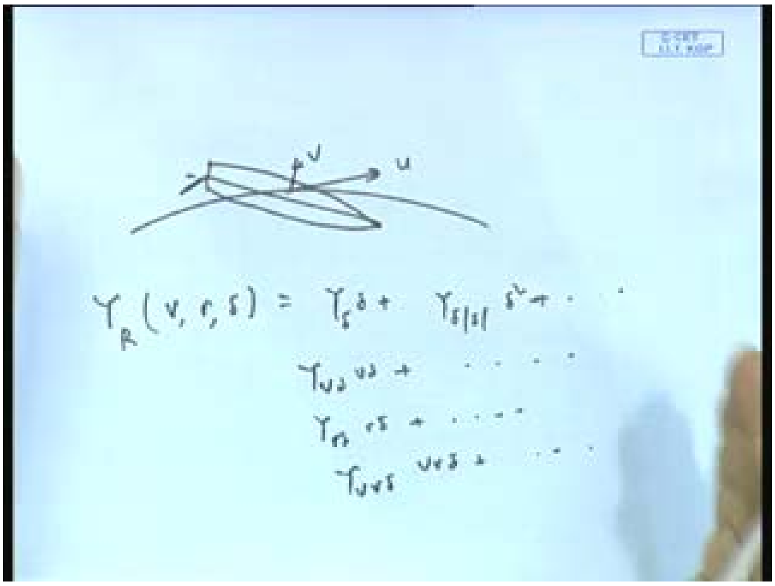

Similar, will be n against delta that is the rudder forces but, there are more twit remember, I am putting the rudder when I am having certain v that means; I am going turning like that with a v. So, I have a u here, I have a v here at that time I am putting this rudder, so my Y rudder force is a function of v, as well as r, as well as delta because after all I am applying the rudder in my simulation when I have already some vary of non zero v non zero r.

So, if I want to expand that then again this becomes Y delta delta, plus see Y delta delta delta square plus but, then I have Y v delta, v delta plus v and delta terms then I have r delta, r delta plus; then I can have v r delta, v r delta plus. So, I can have all kind of terms couple terms so called that means what does it means? This means that this means d Y d cube Y by dv dr d delta means for a non zero v, non zero delta what is my force that is small change in that force.

So, you know we can end up getting whole lot of such kind of terms there is no end of this term and we actually have to come to certain finite conclusion about how much I should use, there is no end of it from mathematical sense but, I will try to tell you that there can be some end from physical sense, I will just come to this we just show some example later on with a Y delta force for example,

So, this is how it is now let us talk of a more complex equation, now when the ship is turning we have seen the heeling we have seen that last time, when the ship turns it is also having a heeling phi now suppose my ship is turning here.

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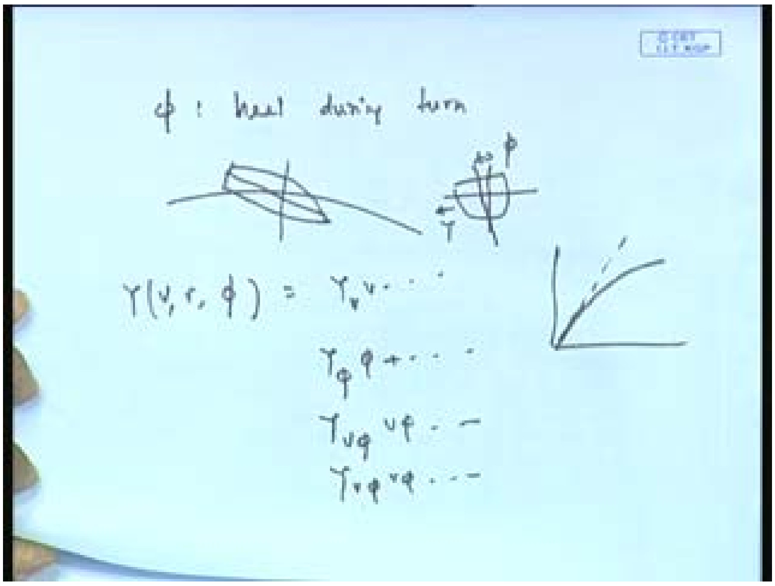

So, if you say cross section it is heeling outward like that, so when its heels my Y force that comes swill depend on the angle of here also suppose, you have two degree heeling there will be some effect of that; so then Y becomes function of v r also phi. So, then I can have this we have seen this v but, v v etcetera then also I can have phi phi etcetera then I can have also have phi phi etcetera then also I can have then I have v phi v phi etcetera then I have got r phi r phi etcetera.

So, what I am saying you add one more term then you have got all the couple this, this this, this, this, this all the three, so you know that that the equation therefore, can be expanded more and more therefore, what happen I will just write down what is the norm normally is taken but, there is no end of it.

So, when we talk of non-linear equation one of the purpose is you want to make sure that the non-linear variations are adequately represented not straight line because straight line is not going to give you correct result. It is very similar to trying to find out any idea under the ship’s curve where you want to use 141 role because that scatters for the curve not a trapezoidal role because it makes a straight line.

So, it is exactly same as that but there is no point of using 10th order polynomial to fit for example, you now the area under the beach, you just use a 2nd order polynomial because that scatters for this. So, similar thing is what is being done in non-linear equations as I said I will sum it up again the added mass forces are well defined, so you do not have to expand that the travel or the ambiguity or the debate arises because of the damping forces there also one finds out that it is up to quadratic or one levels up then comes the question of couple terms you know the couple terms are always small, there is no point of going to v v v r r phi etcetera, etcetera you can only go for one couple having said that people have done many experiments to find out what is strong, what is weak, should I take it, should I not take it. In real life is there a coupling actually of course, and come of its certain equations of motion some physical ground.

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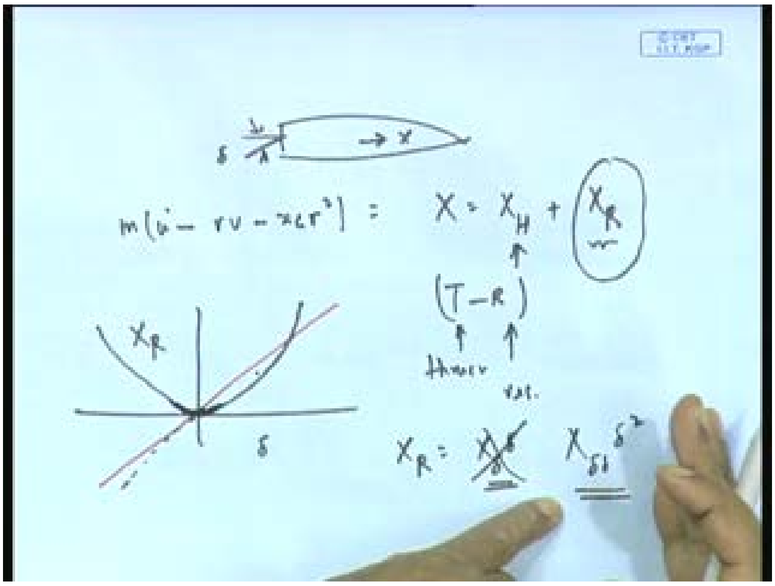

Let us, look at this X force because I need to now know the X force against rudder angle because, when I put the rudder angle delta remember my X force changes means rudder gives me resistance because I put that there is the resistance coming there is drag coming on because of rudder.

See the other part of resistance have been taken in the X direction of motion I will need to put see X equation of motion whatever I have that, you know like m u dot minus for example, rv minus x g into r square this was X force this X hull, this X hull was what was X hull remember thrust minus resistance primarily this is thrust, then I must have X of rudder because rudder is also going to give me an additional resistance here comes my question I am talking I am looking at this X verses rudder.

Now, tell me this X rudder verses delta how shall it looks like you? See here suppose as a force here, so the force goes like that what about this minus side, when I give minus delta what is my rudder force is it like this? Or it is it like this? It is like this therefore, the graph is u therefore, here the slope should be actually zero now, if this is zero then can I model this by seeing X R equal to X delta into delta.

X delta into delta tells me the graph is a straight line like that because delta is negative it becomes this side, so what happen see here, I need to model this from physical ground not this but, delta, delta, delta square I do not have a mod also here just this because that tells me whether delta is positive or negative I have X positive also the slope is zero at this point.

So, you see this is how the physics come in understanding comes into know which term you should keep, which term you should not keep similarly, one cannot similarly, one can work out which term should be kept, which term should not be kept what are term etcetera, etcetera and end up getting the expression now the general expression for example, for a four degree normally we have talked only you know like for trajectory simulation I talked about only Y and N; this is important Y and N that is sway and yaw is only important for the initial stability type initial directional stability.

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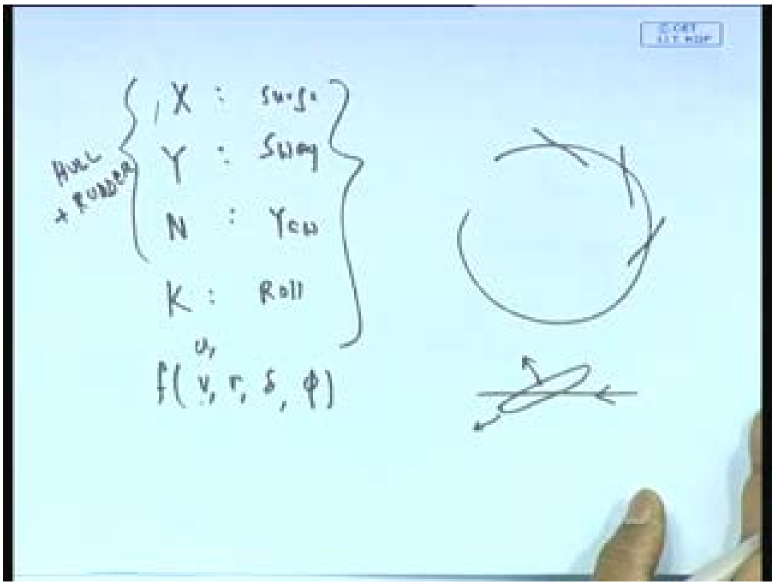

But if I want a trajectory stimulation, remember I also slow down as I turn, I did not mention that but I was slow down obviously I have turned here ship is going like that going like that, going like that drag is much more on that even if I take it is lifting surface the ship's lifting surface the flow comes like that lift is here but, it is also force this side force will this will also slow down.

So, if I want an actual trajectory during the turn in fact the ship can slow down by 30, 40, 50 percent of speed depending on the speed that it is going, so I also need X equation then sometime I also need K equation because it is also rolling during turn even zig zag turn it will be rolling heeling rather than rolling, so this is what roll.

So, you know that more modern trajectory simulation type of equation will consider all the 4 degree of turn you may consider three but, two is only meant for stability studies small amplitude studies when you turn and or X has to be there because it slows down, speed will come down, so then what happen I have to expand this forces hull forces to a very long term and that I am going to write down and all of them even the functions they are v, r I am, not take that v dot, r dot but, I told you that the added masses are fully known v r and delta and phi connect with the force see v r delta phi and then remember here I have hull forces plus rudder forces.

These forces are hull plus rudder because, when I obviously doing this manoeuvers; I have to apply rudder, so this expansion a typical example, I will give you is in fact there is there is also suppose to be u there because I am taking the surge since I am taking surge I have got u, v, r, phi and delta all of them are there.

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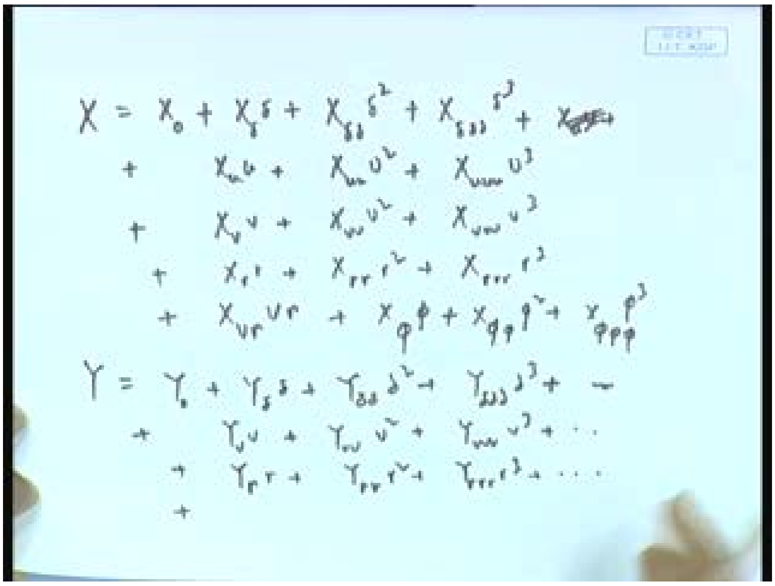

So, you can imagine if I expand them how it becomes but, typically what would happen that x would become, I just write down one typical part the way it is did it X 0 there is always the means that the value that are taken then X delta del, plus X now this model is known as combined quadratic cubic model where both the terms are written.

Actually, there is a choice whether I can only take this much in the mod of course, in this case mod does not arise for x delta part then I have got x u u plus, let me write it here then plus this is only one term taken v and r here x phi phi, plus x phi phi phi square plus see here what you have you will see I mean similar thing will happen with Y normally that is the expansion used for delta you have gone up to cube, for u gone up to cubem for v gone up to cube, for r gone up to cube, for v r only v r phi up to cube but, you may need to Y for example, I am just give the Y part then we will stop it here then normally Y one is there is a delta of course, delta will of course, go up to cube.

Actually, here maybe I should, I should continue in the next page I will just continue from here looking at this because otherwise this things will just becomes longer and longer see why I want to do that see p, p actually is another one that is introduced that is roll motion, roll velocity see phi was roll angle.

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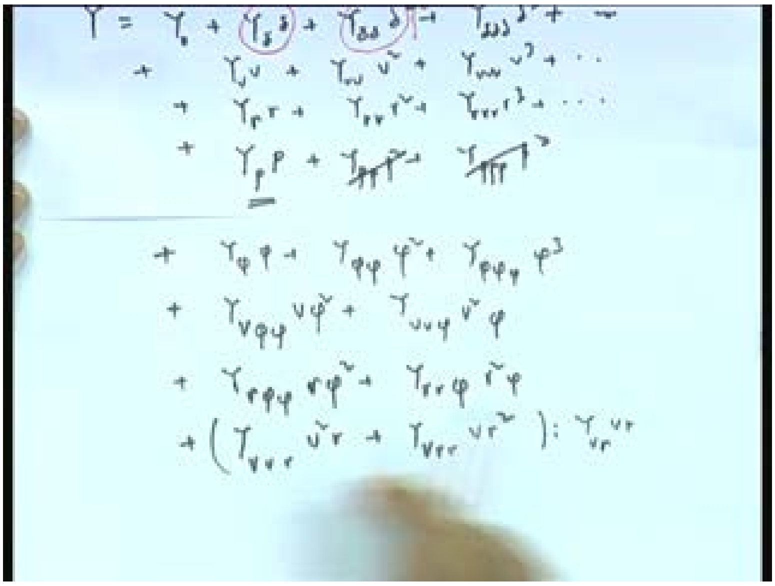

But since it is I assuming is rolling and rolling also can be time dependent as in turning because it as it goes there see as it zigzag it, roll is not fixed you know it is something though its time dependent if phi depends on time d phi by dt is p.

So, you also have this p terms coming then comes I will just continue this actually normally this is not taken, just up to one probably model see then I have phi, no here now comes the question is coupled terms. This couple normally it is taken as in this case let me see v phi phi v phi square, plus Y v v phi v square phi, plus Y here r phi phi r phi square, plus Y here r r phi r square phi I have not made this v v r.

See, here what I am trying to say, what I am trying to say if you look carefully I have Y against delta actually, delta let me see that we have not taken here see Y against delta, Y against v, Y against r, Y against v are combination v v r v square r v r r v square r these two has not taken some people will replace this line by simply Y v r v r this one term sometime then I have got term with respect to p, I have got with respect to roll motion phi then I have got v and phi r and phi.

So, my point is that you can write like that as whole lot of expansion people write that but, the most important term that are written here and I will show that in this case normally you can go up to this much, this is also very small because what is happening. Now, let us look at this physics of that what is Y delta it is rudder angle we will get back to this I have to, I get back to this a part just look at Y delta there is a rudder here.

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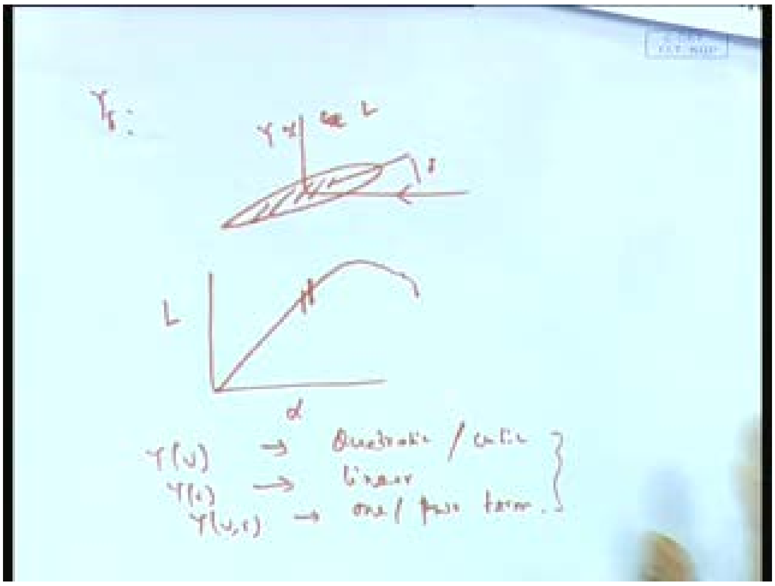

What is a rudder? Rudder is the hydro foil surface flow comes this side the ruder angle is delta now you have done this all of them you have done this Y force is proportional to c l lift or rather to lift force is in a same very close to this thing. Tell me how does lift verses alpha varies? Mostly you have done that it becomes a straight line and then there is a stall lift against alpha for a lifting surface how does it vary? So, it is L verses alpha like the straight line therefore, mostly for actual surface up to the angle that you use lift is actually your straight line therefore, up to Y delta it is adequate, up to Y delta is adequate but, sometime if you want to go up to very large angle you may use Y delta delta, but there is no point of going beyond.

It is seen from experiment that is Y verses v has a large variation so normally one retains this line, this line normally there is no need to retain this, because this is normally good enough it is found out that variation against r is not much, it is more or less straight line against r r I s not also very high for a ship r is a rate of turn you are not turning at a very high rate like 90 degree per second like this, but only torpedo zonal may term v is may be neglected in a two dimensional model because we are not taking p, then comes phi most cases of usual day to day study phi can be neglected and phi can be neglected or you can take only up to this much if at all you want to take roll this one either this you can neglect v phi is very small coupling, because if I do not take phi there is no question of v verses phi, this one you take this no this also you neglect this one, this one you take this and this. So, what happen you take v variation up to quadratic r variation basically I will just come in an end here because we are running out of time.

So, v variation etcetera in fact next class, I will pick up from this point again because we are running out of time like that if you do you will end up getting what is known as a more or less non-linear conventional non-linear equation, once again I want to tell you before I end what I today mention is the concept of non-linearity essentially it is comes down to be the fact that the forces can be non-linear etcetera. So, I need to ad equally represent thing how much is adequate? Nobody seems to know, so you can have if you take one thousand terms nobody stops you, but should we take it does not make sense physically we see and find out that many of them are actually straight line.

So, physically what happen from experiment and all we find out some of them as a non-linear variation, some of them do not have. So, you take only up to that point and you end up getting what is known as a working model there is no fixed model Americans use one model, European use one model we may use another model, but all of them have some similarity, so I will discuss this final model at least one or two forms in next class so we will know this non-linear coefficient.

Then we will talk about experimentally determining this all these coefficients because when you do experiment you do not to do only for linear, when you do experiment you end up doing linear and non-linear this is the reason why I wanted to introduce the non-linear coefficients, before I talked about experiments; with that I end today’s class, and next class we will pick up beginning talk about this equation, and then go for the you know like experimental determination thank you.