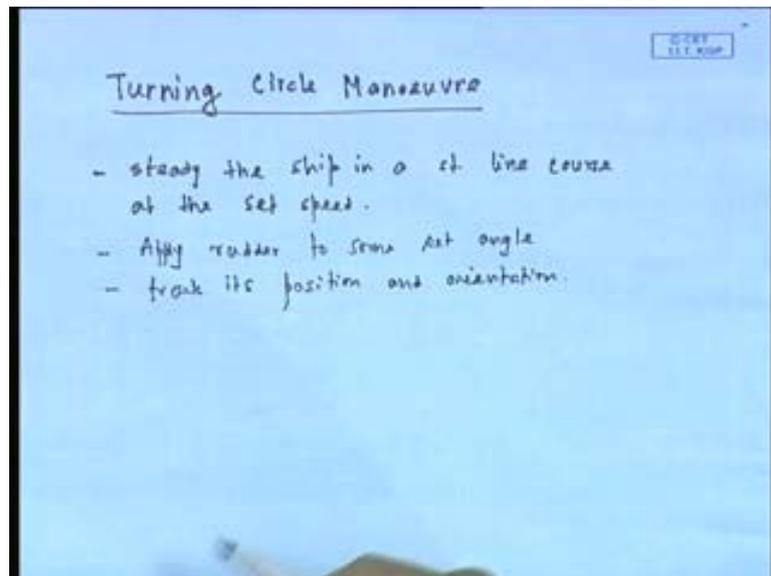


**Seakeeping and Manoeuvring**  
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**Lecture No. # 30**  
**Definitive Manoeuvres - II**

See this lecture, I am going to basically talk about the other most important Definitive Manoeuvre called, turning circle manoeuvre.

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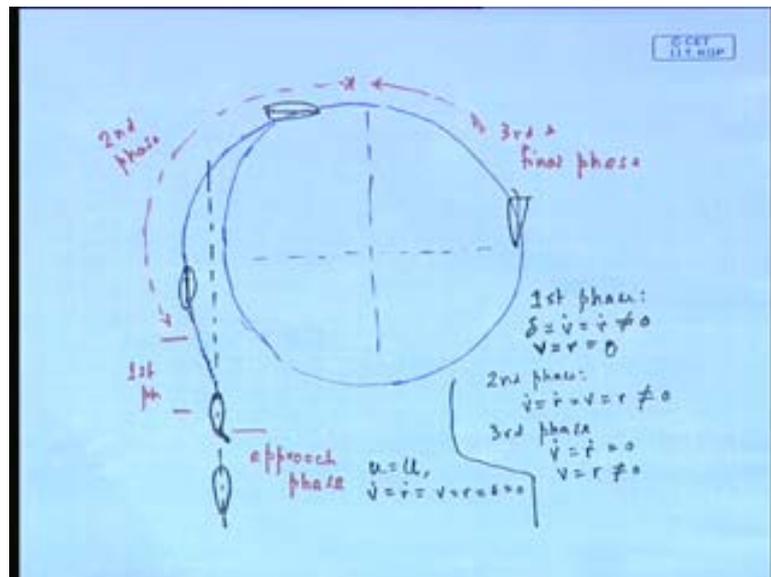
See **excuse me**, in the last classes, we discussed about pull out manoeuvre, spiral manoeuvre, both of which were used for stability, finding out whether the vessel is stable or not; then we talk of zigzag manoeuvre which basically assessed, how quickly it responded to rudder.

Today, we will talk about this manoeuvre which is one of the most important manoeuvre, because remember most ships must have a turning ability, it should be able to turn, u turn whatever make a circle. So, as the name implies this manoeuvre is essentially a manoeuvre to assess its turning ability, how did it do that? It is a very simple thing, I mean I will first write down the steps of doing which is very really nothing.

All that you have to do is to steady the ship in a course (No audio from 01:45 to 02:05) set speed, when it says **you know** for all manoeuvres, once you do initial condition fixation; then the engine is just not change any more it is just set, all that we do is that apply rudder (No audio from 02:25 to 00:35). And that is it, and track (No audio from 02:37 to 00:50) track its position (O), I mean so there is really as far as the procedure to carry out this manoeuvre is concerned, there is nothing.

It is like in an automobile or something you give a stirring and hold and see, how it responds, what is important is to figure out, how it responds. And here you will find out the response of a ship is quite **quite** different than the response of any other vehicular system.

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What would happen? Let me just draw this diagram, so the ship is coming here initially you may apply the rudder here, at this point you probably apply rudder, this is my rudder we apply. What would happen **you know**, you follow the trajectory it turns out the trajectory, may be I should do some other colour, it will go (No audio from 03:52 to 04:11) like that (Refer Slide Time: 03:54) (No audio from 04:12 to 05:03).

See, these are very simple diagram **right**, but the thing is that what is important here to notice would be that, this is my approach phase, let me call up to this much is approach phase, (O) of course, I simply steadied the hull. Now, as soon as I apply to rudder, **you**

**know** this rudder we have applied is to make a turn on right hand side, you may say star board side, I apply the rudder this way, I will show the coordinate system afterwards.

Now, what happens you see this track, then the track we followed track turns out to be like this, where it is actually over shot slightly on this side, on the other side. So, I want to turn this ((O)) over shot in the other side. So, basically, normally what **people do**, people like divide that into certain phases, we call up to some point here as 1st phase, up to some point here may be somewhere with people this is approximate, I will tell you about this(Refer Slide Time: 06:19).

Then the rest part is 3rd and final phase (No audio from 06:28 to 06:37), this is just a very, you may say rough way of dividing this trajectory in some phases, just for the purpose of study. This approach phase is of course, the phase where  $\dot{u} = u$  and everything else is 0 **you know**,  $\dot{v} = r$  equal to  $\dot{r} = v$  equal to  $r = \Delta$  also is 0, all are 0.

Here of course, **the** this 1st phase, let me **took at the** look at the last phase. The way it is divided you see, rather may be you can look all this thing, 1st phase (No audio from 07:31) actually we call 1st phase to be the one, where as you apply  $\Delta$ ; immediately what happen there is an acceleration that is created. So, basically  $\Delta \dot{v} \dot{r}$  is not 0, I mean we call that, because initially what happen when the rudder is applied? The vehicle remember the velocity is very small, but it begins to **it** it see, this  $\dot{v} \dot{r}$  was 0, initially at this phase, but now the moment I apply external force given by  $\Delta$  there is going to be a change in velocity.

Now, even if the velocity was very small 0 to some other very small value, the acceleration is not very small, so what we call is that this phase is this thing whereas,  $v$  and  $r$  is taken as 0 more or less. See, **see** I will come to that, **let** let me first write it down **then** then I will tell you the 2nd phase forget the  $\Delta$  now,  $\dot{v}$  (No audio from 08:48 to 09:08) see now, let me explain to **this** this part in the turn, this is actually important for us to understand **you know**.

See, we understand that, let us talk reverse way approach phase of course, there is nothing given, **fine** final phase when it reaches a steady state it is steadily turning, so when it is steadily turning there is no acceleration, there is nothing changing with respect to time. So,  $d$  by  $dt$  of the forces are 0, because if there was a force acting  $d$  by  $dt$  of

velocities would not be 0 that means, the vehicle would still accelerate; but however, as **you know** if you hold it afterwards the initial transients will dry down and you will reach a steady state you keep on turning.

So, that is what we are calling the final stage, and **that is this which** this is the one is of importance to us, eventually that is how it turns, this has to turn at a fixed speed, so that is the what we call final stage, but before that is reached you see, there are it goes to two intermediate stages approximately, you really cannot divide where one ends, where one starts.

The point is that, initially when you just give rudder, I will show this diagram in terms of the time plots moment you give the rudder, what would happen you will immediately the body will immediately have acceleration, but the velocity is still very small, acceleration is rate of change of velocity. So, even if see velocity, which was see  $v$  was 0, then it becomes 0.001, then it become 0.005, let us say obviously there is acceleration, but the numerical value of  $v$  itself is small,  $r$  itself is very small.

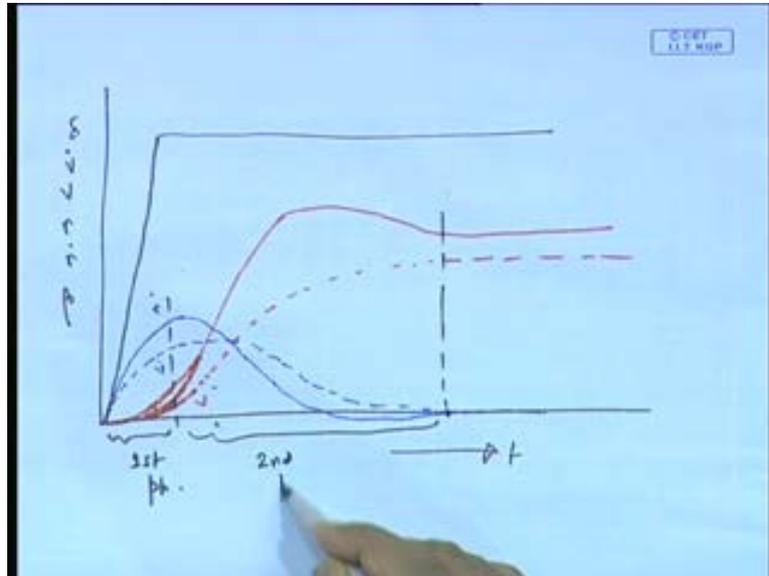
So, that is small time, that we call 1st phase, you cannot really delineate, because there is really no point where it ends, in 2nd phase is the more transient phase where all the forces arising, because of  $v$   $\dot{v}$  **r well**  $\dot{r}$  that means, this motion parameters all of them exist. But, eventually the forces associated with the acceleration that means, the velocity body's acceleration will diminish, because it has to reach a steady state.

Because, obviously, **you we** we need to understand this way, see I will show that from equation afterwards you give a force, external force, rudder force, so in a system I imposed a force, so this force is going to accelerate the body first, because I have an extra force coming, so the body begins to accelerate. Then what would happen, eventually the flow around the hull will be kept created, and the flow around the hull will give a force on the hull and ultimately it will reach a case where my net force or net moment is 0, because if it was not, it will keep on accelerating, so that will happen a 3rd phase.

Now, but it has to reach to 3rd phase, so here I give an **external** external force by rudder, here I **it the** the, at this last phase the hull force and the rudder force must balance, but hull force must get created, so **it is** this time where the hull forces are getting created. So, first you have an acceleration force, eventually there will be velocities **on the** on the hull

which is quite substantial and eventually the acceleration forces will diminish, because they will balance.

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So, what will happen, if I were to plot a time plot little, this will be more interesting, if I were to plot, a time plot say with respect to time, here I am plotting say, **I am** this is without **without** anything  $v \dot{v} r \dot{r}$  and you can say beta, beta I will come later on, which is basically beta **(0)** drift angle beta, that is this angle. This is another interesting thing I have to tell you, no ship can ever turn, no air craft can ever land, where the drift angle or angle of attack is 0, you cannot have a ship **(0)**, I will come to that is interesting you cannot have a ship just, which is tangential to the path, circular path you cannot have, it has to have a drift.

Air planes do not land this way, they land this way, if you see you will find while landing it is like that, I mean like this, we will come to this, there is a interesting fluid mechanics explanation for that. So, here what we have done let us put a delta, so we have given rudder of course, we always have to plot this way, because you cannot give rudder instantly, **you know** it there is a rate of turn typically 4 degree per second or something.

And you would have given rudder, normally you give this rudder to the maximum which is 30 to 35 degree for a typical ship rudder angle, maximum is normally set a 35 degree most **you know** common. And you obviously, want to know the turning ability at the maximum value normally, now having said that, you now what would happen initially

you see, now let us look at  $v \dot{}$  and  $r \dot{}$ ,  $v \dot{}$  will immediately try to go like that, that is  $v \dot{}$ .

Because, there is a force there and  $r \dot{}$  may be also I will put this here, this is  $v \dot{}$  increase, but the velocities  $v$  and  $r$  are small. So,  $v$  and  $r$  are still growing, this may be  $v$  and this may be  $(\dot{0})$  I made the mistake I should have plotted it brought it, say this is  $v$  and say this is  $r$  or rather this is also very small say this  $r$ .

What I am saying, let me first draw the full thing then it will be easier, then what would happen this will come down and eventually it may overshoot or something whatever it will ultimately go to 0, this is also come down and ultimately go to 0, this thing of course, will it may shoot, but ultimately at this location (No audio from 16:12 to 16:25). So, here say somewhere up to here or something, you call this 1st phase why because, **you know** what is happening remember, I give a force, hull force, rudder force, externally I have this body, I apply rudder this is a rudder here, I turn rudder rudder force came.

So, I imposed a force obviously, immediately there will be an acceleration coming was mass into acceleration equals force, I have given a  $y$  force, so I end up getting immediately an acceleration, but remember acceleration does not beat large numerical value of velocity. Because, I can have velocity 0.001, 0.002, 0.007 velocities are still small, but acceleration can be high, that is what has happened.

Ship is always, I always say this ships are always sluggish system you give an force today **it it** it goes with brain after a sometime, then it begins to respond, so it is very sluggish it takes time for it to understand and begin responding always, which is of course, the classical case of titanic, **you know that you know** it does not respond on the spot. So, same thing happen the velocity and the response basically means as will response it understood that **yes**, there is a force it begins to move essentially, so this  $v$  takes a little time, then of course, **yes** it has got the message that **yes**, there is a force there.

So,  $v$  and  $r$  begins to grow up, but eventually what happen rudder force and hull force in the final stage it has to equate, because **you know** the if there is in the final stage when you are turning steadily obviously, when you are turning steadily, the very word steady means,  $d v \text{ by } d t$  is 0 because, otherwise it is not steady turning  $d r \text{ by } d t$  is 0. See,

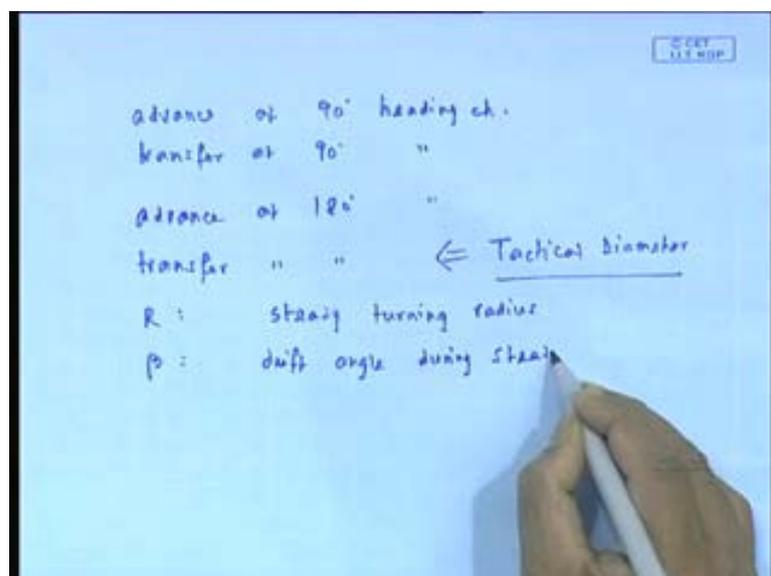
steady turning imply the word steady means,  $d$  by  $d t$  is of velocities are 0, the very meaning of the word steady means that, time dependence is not there, so obviously, this stage I do not have  $v$  dot and  $r$  dot.

So, what would happen, **the** that is the initial transient this  $v$  dot  $r$  dot have gone to 0 and here and as **as** a result this  $v$  and  $r$  would have reached a steady value, see from here, so this is my 2nd phase, this is my 3rd phase, you may call steady turning phase it is this, that is of our interest essentially, but we cannot only go by that, we also need to need something else.

Now, here you see now, comes the question of measure, now what we do we need to measure things, how do we have to quantify, we have to quantify the quality of this turning quality, so the measures are always like that. Let me put it this way, this **this** side this my  $x$  axis in a global system, **you know** if I presume that initially it is the direction of the initial approach, this is my  $y$  you can say, so **this is** this side is known as advance and this side is known in our this thing as transfer.

So, what happen is that this distance see from approach from approach point, let us say the  $c g$  point I take to where I my heading has turned to 90 degree, heading has turned to 90 degree, this is not the maximum heading remember, because maximum will come here, this is not the maximum advance, this is what is called advance at 90 degree and of course, this distance would be called transfer at 90 degree.

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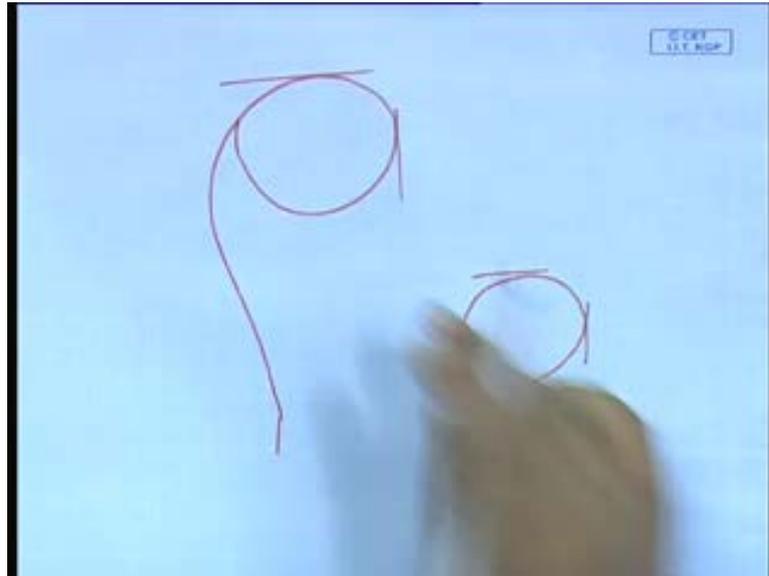


So, I mean, I am I will write in other piece of paper this this two terms, because otherwise it becomes you know confusing, so we have measured (No audio from 20:54 to 21:12) similarly, obviously, come to this one here to here, where it has undergone 180 degree heading change. So, this distance you will call transfer at 180 degree, this is very important term and of course, this distance you may call advance as 180 degree. Actually the transfer at 180 degree is also called tactical diameter (No audio from 22:53 to 22:08), this is known as tactical diameter.

And this of course, the radius that you have got standard this  $r$  if I call it, this is steady turning radius  $r$  is my steady turning radius, when I am turning steadily (No audio from 22:39 to 22:53), this angle  $\beta$  is my drift angle during steady turn (No audio from 23:57 to 23:11). In this diagram actually  $\beta$ ,  $\beta$  and  $v$  I will show that later on this basically  $v$  and  $\beta$  are synonymous, this is sorry just one second this is  $v$  if you, you know it is synonymous to  $\beta$ , I will just write that, but actually later on we will find out that  $v$  dash is minus  $\beta$ ,  $v$  dash will minus  $\beta$  we will find it out afterwards through equation that we will see.

But, what I mean that drift angle of  $v$  are essentially same, essentially the synonymous quantities, so we have got this, so these are the measure, now what are the most important one if you see you will obviously, see that (O) the important ones are this distance and this distance normally, you do not do at maximum. Now, that is one thing that you do not do at maximum, you do at when 90 degree heading has has taken place it is easier to measure or whatever in a, I mean in a in a in a in a trial you do this distance where the hull has become 90 degree not remember, it will gone slightly further.

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So, maximum advance would be slightly more than advance at 90 degree, but normally advance as 90 degree is a measure it actually tells you safety bound is it not, because suppose there is a shore there and you want to turn, you might hit that, remember that this diagram is not necessarily connected with circle; I will show you why, because it could happen that a ship is very slow, so a ship might actually start turning and it may actually go and turn here.

So, you see here, the advance is much larger, but someone may actually its turn like that, so here advance is much smaller for the same tactical diameter. So, that is why why I am saying this is, because by itself this is a important measure, because it tells me how much of land I mean like water should be there similarly, this side obviously, this tells me the base in area where I am turning.

So, essentially I have this sort of situation, and so I have this this is advance this is advance, this is what we can call at 90 degree this advance and this is basically transfer or tactical diameter,  $r$  is this thing, my radius of steady steady turning radius. Now, let us look at to at some of this equation form, try to find out whether I can predict steady turning radius, let us see how at different phases the equations are. See, for this I need to go back to the hull remember here, when I look at the equations now, I have now an additional force coming, because of the rudder.

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$$m(\dot{u} - rv - x_G \dot{r}^2) = X$$

$$m(\dot{v} + ur + x_G \dot{r}) = Y = Y_{HULL} = Y_v v + Y_r r + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r}$$

$$I \dot{r} + m x_G (\dot{v} + ur) = N = N_{HULL} = N_v v + N_r r + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r}$$

$$Y = Y_{HULL} + Y_{RUDDER}$$

$$N = N_{HULL} + N_{RUDDER}$$

Now, see my equation of motion look like earlier, if I were to look  $m$  into  $\dot{u}$ , I am just writing this  $r$  into  $v$  minus  $x_G$  actually this of course I can, let me ignore this part, I will just write this  $y$  and  $m$  (No audio from 26:47 to 27:00), and here (No audio from 27:01 to 27:17), **now** now why I wrote this **you know** there is an important. This is **what what** what we have done forget this, we will just ignore that this was my  $Y_{HULL}$  which I wrote as  $Y_v v$  plus  $Y_r r$  plus  $Y_{\dot{v}} \dot{v}$  plus  $Y_{\dot{r}} \dot{r}$  this is my  $N_{HULL}$ , this is what we have done earlier **right** (No audio from 27:41 to 27:49).

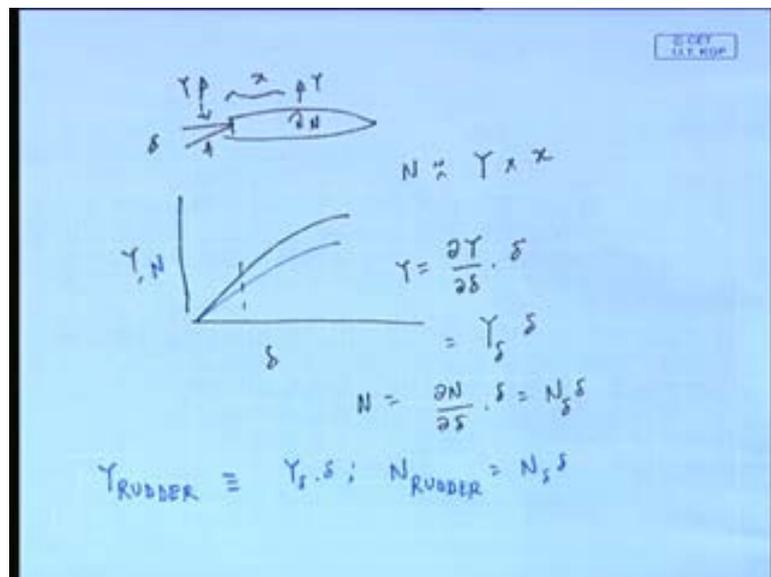
But now, what we have to do remember this our now  $Y$  will become  $Y_{HULL}$  plus  $Y_{RUDDER}$ , because I am applying the rudder and  $N$  becomes similarly,  $N_{HULL}$  plus  $N_{RUDDER}$  that we agree, because obviously, now I have a calling rudder. What we have done, see this hull, this is rudder here, I apply the rudder, what is rudder do creates a force here, **this force** this particular force say  $Y$ , some  $Y$  force let me call this,  $e Y$  or say  $Y_R$  I call it, this  $Y_R$  is for my equation of motion taken to be a  $Y_R$  here, and  $N_R$  **right**.

Because, you see what is happening, when I give this, it gives you a lift force, some kind of a lift force obviously, so that means what, **my** this  $Y_{RUDDER}$  force,  $N_{RUDDER}$  force or basically this two, remember that when I say it is the  $Y$  force coming on the, because of the rudder on the hull, why I am made this diagram because, some of you who would have studied the control surface by itself. Say, **you know** like rudder is the control hydro foil, rudder is like a hydro foil surface, you would find out that as the flow comes here, I

have got **you know** propeller and all we have found out, I got a lift force, I have got a force here etcetera.

So, this force is acting on the rudder, what here we call  $Y$  rudder is the force that, because of the rudder acting on the hull C G which of course,  $Y$  part is same as this part, but this part is gives me moment of this into the distance. So, I am saying that this force is equal to this force and this, remember these two are the one these two, this is this, this is this (Refer Slide Time: 30:03). Now, how do I model it you see here, if I were to plot this angle delta rudder angle versus  $Y$  it is going to be some graph like that or rather let me put it in a another diagram.

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Supposing (No audio from 30:27 to 30:35), so if I were to plot **sorry**  $Y$  versus delta, it will become something like that similarly,  $N$  if I take this  $Y$  here, if I take  $N$ , after all  $N$  you will find out is proportional to  $Y$  into **some distance** some distance  $x$ , if the  $Y$  force comes here **right**. And this is more or less this distance, so what we can do it, I can call it to be  $d Y$  by  $d \delta$  into delta, so this is written as  $Y \delta$  into delta, if for small angle, if I were assuming to a linear similarly, if I make it  $N$  and I call this a  $N$  something like that.

Then (No audio from 31:31 to 31:45), so **I** what I am saying therefore, rudder force (No audio from 31:51 to 32:09) up to at least linear order, we can say that and of course, we are presuming all the study at small angle. So, what we are going to do, you see this side

we are going to add that, so you are going to add here obviously, on this side plus  $Y \delta$  delta here, plus  $N \delta$  delta **right**.

Now, if you do that what is happening, see initially I have an equation by bringing this on this side and right hand side was 0, remember I brought this **this** side and right hand side was 0, my right hand side is going to be not 0, but  $y \delta$  delta that is what will happen, that is what I was trying to get at all this time.

So, that it will look like, this equation will look like, you understand that see, in earlier equations I had taken this blue, this much blue, what I did I brought it in this side, and made the right hand side 0, that is all I wrote something into  $v \dot{\phantom{v}}$ , something  $r \dot{\phantom{r}}$ , something  $r$  equal to 0. Now, what will happen I will still do that, but leave this red rudder side on right hand side, its easier I can bring it you may say why not well normally we bring it this side.

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$$-\gamma_v v + (m - \gamma_v) \dot{v} - (\gamma_r - mU)r - (\gamma_r - m\alpha_v) \dot{r} = \gamma_v \delta$$

$$-N_v v - (N_v - m\alpha_v) \dot{v} - (N_r - m\alpha_v U)r + (I - N_r) \dot{r} = N_v \delta$$

$$\text{Ph 1: } \frac{(m - \gamma_v) \dot{v} - (\gamma_r - m\alpha_v) \dot{r} = \gamma_v \delta}{-(N_v - m\alpha_v) \dot{v} + (I - N_r) \dot{r} = N_v \delta}$$

$$\text{Solve for } \dot{v}, \dot{r}$$

$$\text{Ph 2: All terms present}$$

So, if I do that my equation of motion will now, turn out to be I will just write it down coughing, it will turn out to be minus  $Y v$  plus  $m Y v \dot{\phantom{v}}$  dot, this is minus  $r$ , this is going to be, this is the other one is going to come out to be, this is minus is it (No audio from 34:05 to 34:14), this signs we can check, what are the signs are correct,  $c \dot{\phantom{c}}$  dot (No audio from 34:22 to 34:38), **sorry** here,  $r \dot{\phantom{r}}$  dot is there. See now, why I am saying is that, see its very very interesting, if I want to study the equation for different phases, it is very simple, because phase 1, I had  $v$  and  $r$  not present I have only  $v \dot{\phantom{v}}$  dot and  $r \dot{\phantom{r}}$  dot.

So, I simply have for phase 1 (No audio from 35:05 to 35:24) and here (O) this one (No audio from 35:27 to 35:39), what are the unknowns here, two  $v \cdot r$  dot right it is like something  $v$  dot, something  $r$  dot, something something  $v$  dot something,  $r$  dot something, so you can easily solve for (O) linear equation no no problem at all, this is phase 1. Phase 2 of course, we cannot do it, because phase 2 will involve all, so we really cannot do phase 2 as such, so we we are not doing it phase 2 as such, but important part is, so I will this is phase 2 itself.

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Phase 3 or steady turning phase:

$$- \cancel{Y_v} + \cancel{(\rightarrow Y)}$$

$$- Y_v - (Y_r - mU)r = Y_S \delta$$

$$- N_v - (N_r - m'x'_g U)r = N_S \delta$$


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$$- Y'_v v' - (Y'_r - m')r' = Y'_S \delta$$

$$- N'_v v' - (N'_r - m'x'_g) r' = N'_S \delta$$

So, I will go to phase 3, phase 3 well let me well rather, (O) (No audio from 36:23 to 36:32), let me go to the next page for phase 3, phase 3 is most important to us, so in the phase 3 what do I get, let me write it down from here (No audio from 36:48 to 37:14) sorry, sorry I make a mistake again I have to take this and this, this and this, so I just made a mistake here (no audio from 37:27 to 37:57), this is agreed right, but then I will tell you (Refer Slide Time: 37:23).

This is in a dimensional form if I were to write non-dimensional form then I basically will have this will become 1, so it will become something like minus  $Y v$  dash  $v$  dash minus  $Y r$  dash minus  $m$  remember  $Y \delta$  also becomes dash, here non-dimensional (No audio from 38:28 to 38:40)  $m$  dash right. I can very easily solve a  $v$  dash and  $r$  dash, without any question I can solve a  $v$  dash and  $r$  dash, I can easily writ actually I will before writing I next some more modification.

Now, you see this is something  $\dot{v}$ , something  $\dot{r}$ , something something, so, straight forward I can write  $\dot{r}$  is something,  $\dot{v}$  is something, but you know we do not want  $\dot{r}$  by itself, what we want is  $r$ , that is the steady turning radius, we probably may not want  $\dot{v}$ , but we want  $\beta$ .

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Turning Radius:  $R = \frac{L}{r}$ ,  $|v| = rR$

$$r = \dot{\psi} \quad ; \quad r = \frac{v r'}{L}$$

$$r' = \dot{\psi}' = \frac{rL}{v} \quad ; \quad r = \frac{v r'}{L}$$

$$R = \frac{v}{r} = \frac{v}{\frac{v r'}{L}} = \frac{L}{r'}$$

i.e.  $r' = L/R$ ,  $R = \frac{L}{r'}$

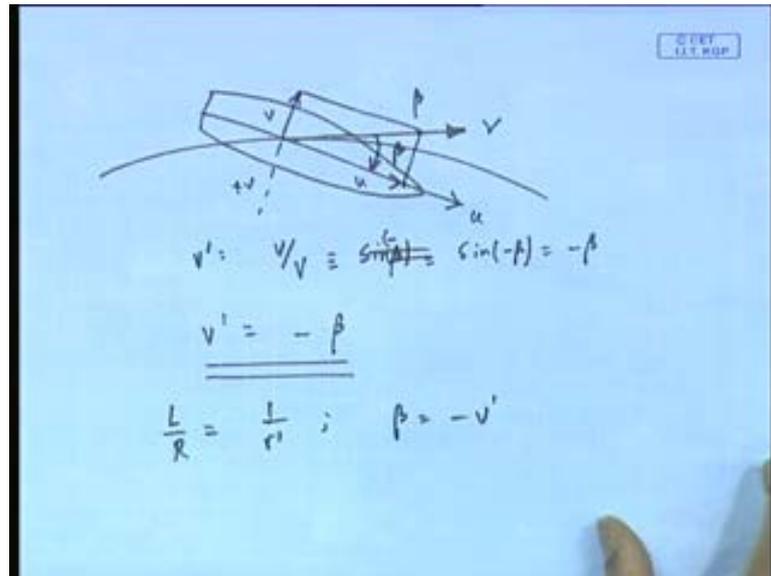
$r' = \dots$   
 $R = \frac{L}{r'}$

So, let us transform this to well we can solve for them  $\dot{r}$  equal to something,  $\dot{v}$  equal to something you can substitute the expression for  $\dot{r}$  in terms of capital  $R$ . So, first let us do that I will just do in one shot, see here, see turning radius  $R$  equal to  $v$  by  $r$  that is why this thing right or rather  $v$  equal to  $rR$ .

Now,  $r$  is of course,  $\dot{\psi}$  now  $\dot{r}$  is this by definition was  $rL$  by  $v$  that is why by definition, that the way we define  $\dot{r}$ , if you recall is by taking  $rL$  by  $v$  that means, what we are saying is  $r$  equal to  $v \dot{r}$  by  $L$  straight straight forward. Now, I am just writing this expression just  $v$  by  $r$  of course, again the same thing I am writing, because  $v$  this expression which is equal to  $v$  by  $v \dot{r}$  by  $L$  equal to  $L$  by  $\dot{r}$ .

See here,  $v$  by  $r$  is this, if I do  $r$  equal to  $L$  by  $\dot{r}$ , that is  $\dot{r}$  equal to  $L$  by  $R$  (No audio from 41:09 to 41:19), why I am writing this, because you see here, we will be solving for  $\dot{r}$ ,  $\dot{r}$  is something. Then  $R$  becomes simply  $L$  by  $\dot{r}$  actually form here we can write other way round also or  $R$  by  $L$  equal to  $1$  by  $\dot{r}$ , normally you like to know  $R$  by  $L$  that is turning radius per ship length this part, so I find out that this is nothing but, inverse of the non-dimensional yaw velocity, straight forward.

(Refer Slide Time: 42:01)



Then comes the question of beta, see here the ship is like that, goes like this, this is my v, this is my u, this is u, this is v (No audio from 42:21 to 42:34) you can easily from here (No audio from 42:36 to 43:00) why minus because, the reason **we** we put minus, because remember this is my plus v side it is the question of convention this side, so **when** when I have got v that side, I get plus beta, beta is **(0)**, so therefore, beta and v are in opposite direction, so what I find out v dash is equal to minus beta.

So, **we** I have got these two things, L by R equal to 1 by r dash and beta equal to minus v dash all that I need to do is to solve for v dash equal to r dash equal to and write down in terms of L by R, **if I** if I do that I will leave to you to detail (Refer Slide Time: 43:37).

(Refer Slide Time: 43:57)

$$\frac{R}{L} = \frac{1}{r'} = -\frac{1}{\delta} \left[ \frac{Y_v'(N_r' - m'x_v') - N_v'(Y_r' - m')}{Y_v'N_s' - N_v'Y_r'} \right]$$

$$v' = -\beta = -\delta \left[ \frac{N_s'(Y_r' - m') - Y_v'(N_r' - m'x_v')}{Y_v'(N_r' - m'x_v') - N_v'(Y_r' - m')} \right]$$

$$N_v'Y_v' - (Y_r' - m')N_v' > 0 \quad \frac{N_v'}{Y_r' - m'} > \frac{N_v'}{Y_v'}$$

$$(N_s' - m'x_v')Y_v' - N_v'(Y_r' - m') > 0$$

But, if I do that what I do end up getting is something like this (No audio from 43:53 to 44:44) (O), let me write it down then I will say the correct method (No audio from 44:51 to 45:30), look at this here you will actually find out here v dash is going to be you know like not not not this one sorry this one 3rd stage (Refer Slide Time: 45:30).

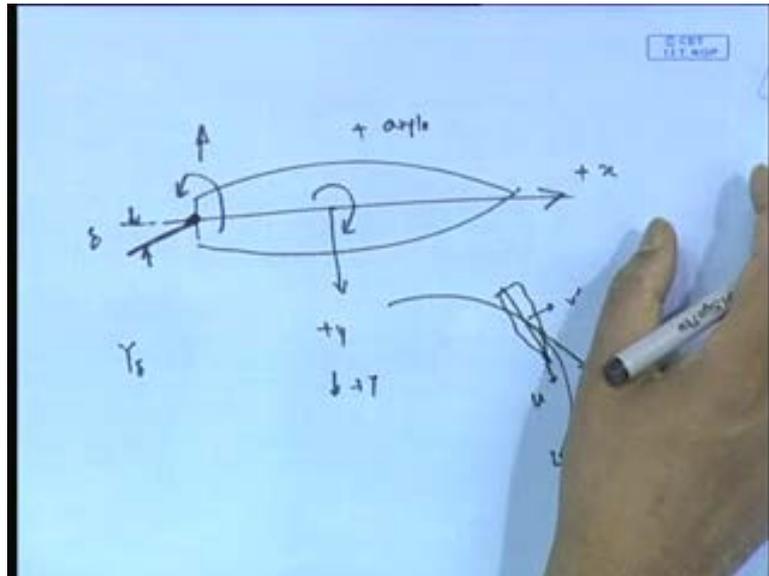
If you write it down there you see, let say r dash inverse it is going to be basically you know like this into that, this in that by that like the way it was simple solution, you have to check that you have to check that I will leave to you for checking, but it will come out come out to be like that R by L will turn out to be here Y v dash N r dash minus m dash x t dash minus N v dash Y r dash minus m dash by Y v N delta N v y delta.

This is going to be N delta Y r minus m Y delta N r minus m Y v N r minus m x g that is this one N v this one comes here and of course, here this is reverse, because n delta into other term etcetera, how it comes actually, this is very interesting I tell you many ways. First of all what is the stability criteria for a ship that we found out, if you recall you know the stability criteria that we found out was, it says I must have N r into Y v well of course, let me write N r Y v minus Y r minus m all this dash I will put into into N r, this equation was N r by Y r minus m more than N v by Y v, N v should be equal to 0, this is what the criteria right if you put a dash here.

You of course, actually this is this, now if you look at this here in fact, essentially this is, in fact this term if you had to use x G non 0, actually the term would have been

essentially  $N_r$  dash no sorry sorry (No audio from 47:38 to 47:57) this was the criteria. In fact, here actually there is a term  $N_r$  minus, this  $N_r$  minus that was the criteria, the criteria that the ship possess straight line stability was this this term means, what it is this term this term is this term, this term is also this term basically the same term.

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So, what you find out  $Y_{\delta}$  you know is you know, I will leave this to you for an exercise this  $Y_{\delta}$  part, you see here leaving a side let me look at this,  $N_{\delta}$  the the sign convention remember, if I had this ship here, once again I will tell you this is my plus x, this is my plus y, this is my plus angle. So, when I put it this side a rudder, tell me delta is positive or negative, delta is here this delta positive or negative, it is negative because, I turned it this way  $\delta$ , so delta is negative.

Now, the ship is turning you will find out that this will cause the ship to turn this side, with a drift angle, some angle drift angle beta, we do not know the drift angle, but it will basically have a  $v$  here, small  $v$  dash here,  $u$  here. Now, I will leave to you to find out that, you will find out that I will have a positive beta, if I give a negative delta, now remember, now there is a question coming  $Y_{\delta}$   $N_{\delta}$  you may want to know, whether  $Y_{\delta}$  positive  $N_{\delta}$  positive again I will tell you.

If I give a negative delta which side the force comes, this side the force comes, so you will know what is  $Y_{\delta}$ , negative delta give me see  $Y$ , plus  $Y$  is this side negative  $Y$ . So,  $Y_{\delta}$  is going to be positive what about  $N$ , negative delta gives me positive  $N$ , so  $n$

delta is negative, so if you take this that  $\delta$  is positive always,  $N$  delta is negative always then you will find out that the direction, I will actually probably come to the direction little later, but you will find the direction part, I will leave that to you.

But, what is interesting you will find out that see,  $R$  by  $L$  first of all  $T$  by  $L$  is constant, if these values are constant that means, I have the same rudder same this thing number one, is there a velocity coming in picture, no velocity comes in picture, because for a particular ship, you see dash values are always constant it is a property of the hull.

So, whether it going at 10 knot or 20 knot or 30 knot, my  $R$  by  $L$  is constant inversely proportional to delta that is of course, is very important  $R$  by  $L$  is inversely proportional to delta number one means, larger the rudder angle smaller the turning radius **right**, is it 30 degree it will have a turning and quicker turn smaller means, smaller turn.

$R$  by  $L$  is **(0)** constant,  $R$  by  $L$  is constant, we will probably pick it up, we I will bring back to this the beginning of the next hour also, because it we need to discuss this quite a lot to find out that this tells me for a stable ship very nice behavior, and also this tells me for a unstable ship that I have no proper relation between  $R$  by  $L$ .

So, **this will** by the equation I will know which side the drift is, why the drift has to be there, without  $v$  dash I do not have these forces coming at all  $Y$   $v$  etcetera, if **I do not have** I do not have  $r$ , see if I have this 0,  $Y$  dash  $v$  dash 0 basically means there is a these are forces coming, because of  $v$  if I do not have it then my  $r$  is 0 it does not turn. So, if I have to turn, I must have this which means I must have beta.

I will end it at that and we will pick it up from this point, I will begin writing this equation next class and carry on and then also we will talk very interesting, which is not really in your intuition about rolling or heeling during turn. You will find that the ship turn outward not inward, **we will** we will leave it at that for this class, thank you.