

**Seakeeping and Manoeuvring**  
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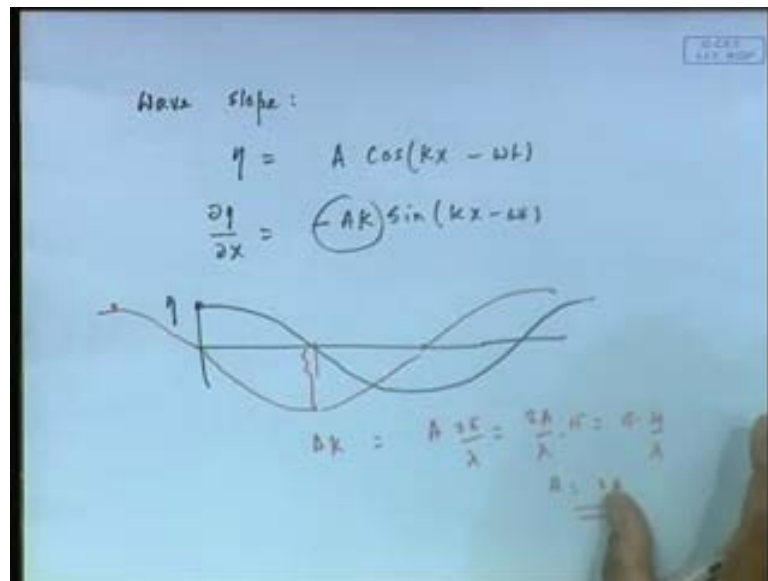
**Module No. # 01**

**Lecture No. # 03**

**Definition of Ship Motions and Encounter Frequency**

As you can see on the board today's topic of the lecture is, Definition of Ship Motions Encounter Frequency etcetera, but before I go I just wanted to have a small part on waves which I missed out earlier, which is on wave slope.

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Wave slope:

$$\eta = A \cos(kx - \omega t)$$
$$\frac{\partial \eta}{\partial x} = -Ak \sin(kx - \omega t)$$

The diagram shows a sinusoidal wave  $\eta$  on a horizontal axis. A vertical line is drawn at a point on the wave, and a small right-angled triangle is drawn below it to illustrate the slope. Below the diagram, the following equation is written:

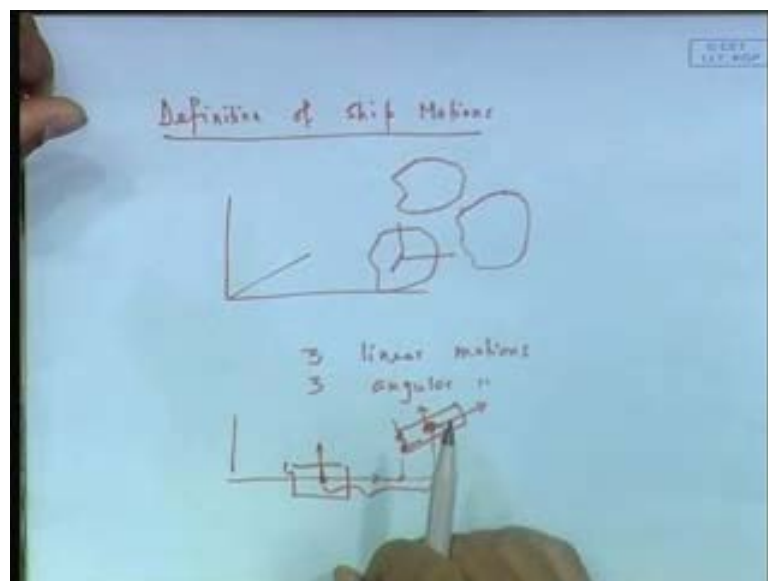
$$Ak = A \frac{2\pi}{\lambda} = \frac{2A}{\lambda} \cdot \pi = \frac{H}{\lambda} \cdot \pi$$

where  $H = 2A$  is the wave height.

See, if I had this given by A, then the slope it would have been, if I just take a derivative minus A k, so its maximum value becomes A k and it is very simple to see, if I had this wave here, eta then its slope would have become etcetera. So, this is also important because, some time a ship might follow the wave slope and the amplitude of wave maximum value of wave slope would be A k which is A into 2 pi by lambda or you can say it is 2 A by lambda into pi or pi into H by lambda; when H equal to wave height of 2 A.

So, this is just to kind of end the wave slope concept that we basically have something called wave slope and it **it** will have a phase gap with respect to wave, actually if you can see when the peak of this occurs here, the peak of this occurs other way round we can see, **the** this peak is occurring here, where this peak is occurring here. So, again one can see, the phase gap between the two for example, there is a motion that follows this and a motion that follows this we would know that they actually do not occur maximum the same time; when the maximum of this occur this is 0. So, with that this is about wave slope, now I will come to definition of ship motion.

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Now, you might wonder, what is there to define (No audio from 02:40 to 02:50) one might actually wonder what is there to define is that, now **now** here I will start this from a very elementary point, you see in an spatial axis system, let say something like that, I have an object here, somewhere what is my objective to basically trace, where this object goes with time, it is how it is revolving in space, that is what I want to find out.

Now, how do I find it out, now supposing I fix some coordinate axis on the body, then essentially my objective would be to define or to have some system where, if I knew what it occupies, I would be able to trace how it occupies next second etcetera. Now, **we** as we know any point can be defined or any location can be defined by 3 linear motions and 3 angular motion, now here comes the question of axis thing.

Let us take one more example in a two case, let say there is an object a ship looks like this instant one, and it looks like this instant two; now it is a 2 d plain, let me just talk of 2 d plain, so what I can do if I want to find out where this object is first of all see, the geometry is better specified with a hull fixed coordinate, because if I were to fix a coordinate on the hull with respect to the any point, let us say point 10 meter forward will always remain 10 meter forward.

So, the hull is defined by a set of points which is invariant which does not change with time, if I had a coordinate fixed somewhere on the hull that is important, where on the hull we have not specified anywhere on the hul. As I said this particular point, this say 10 meter (O) you know like (O) say this point is 10 meter after the ship I am say 2 meter above it will always remain so that point is this point here say p point p point here.

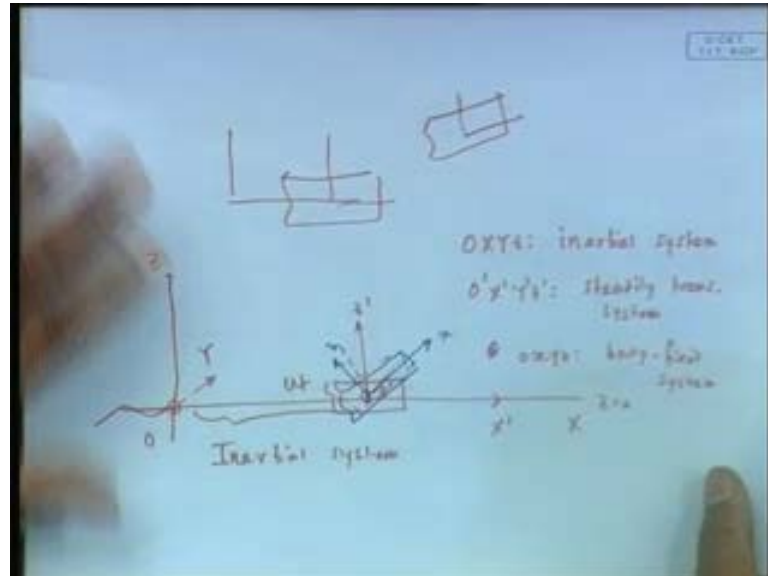
Now, so therefore, what I need, I need to find out this location that means, the (O) the location of this coordinate system and the orientation, how do I do, so I can go from here to here by basically shifting the ship up to this much, then raising it by this much, then rotating it. So, you see here, I have two translation one rotation that define fully this, but now it is a general 6 degree of freedom equation means, I would need equally 3 degrees, you know like three translation and three rotation, but even here I will try to now tell you where this coordinate comes in.

Now, you tell suppose I took this coordinate here, so what happen? I shifted this body from here to here, raised it and then tilted it, so I have got this much, this much that is this much is x displacement, this is x z displacement and rotation. Now, supposing I took the point here, fix point coordinate here, I can always take the coordinate here, because its fix point, then my displacement would have been this much, this would have been this much and rotation may have been something else also, depends on which point I take (Refer Slide Time: 05:53).

So, you see, if I want to go from here to here trace it, first of all I must define a fix coordinate system at some point on the body, which can be anywhere and how much thus displacements are depends on where the coordinate is, see that is very important therefore, if I say the ship has roll so much, it is really not complete unless you said about which axis, so this is the point I am trying to say. So, the definition of ship motion therefore, basically since my objective is to find out here to here, all I need to first define

a fixed coordinate system and then tell how much they are displaced and rotated, again there is a problem, let us take up this rotation business.

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So, again I will draw a picture, so there is another object here, now it is rotated to this point eventually, now you see here, now you can follow my also hand, let us talk about rotation like an structure, supposing first I rotate that in a roll axis, suppose this or rather in a in a different object, I think I I need some kind of this explanation.

Suppose, this is an object somewhere here, now I can first rotate it this much, then translate about this axis, this way and then this way, so it would have occupied one space, but if I did the reverse, first I did that then I did that, then I did that it will occupy another state, if I have the same phi the 3 angles, so therefore, again what it occupies depends on the order of rotation which order you are rotating (Refer Slide Time: 07:35).

So, you know since my objective therefore, is to essentially find out where this is by means of 3 linear translation, linear rotation I first of all need to define a coordinate system, second of all need to define the the order of rotation, if I want to specifically tell. So, all this becomes important therefore, if I just say again roll again you may say your roll is 10 degree mine is 11 degree, because I have define, but the objective is in the same location. So, remember that there is an ambiguity this is the reason, why I wanted to talk on definition of ship motion.

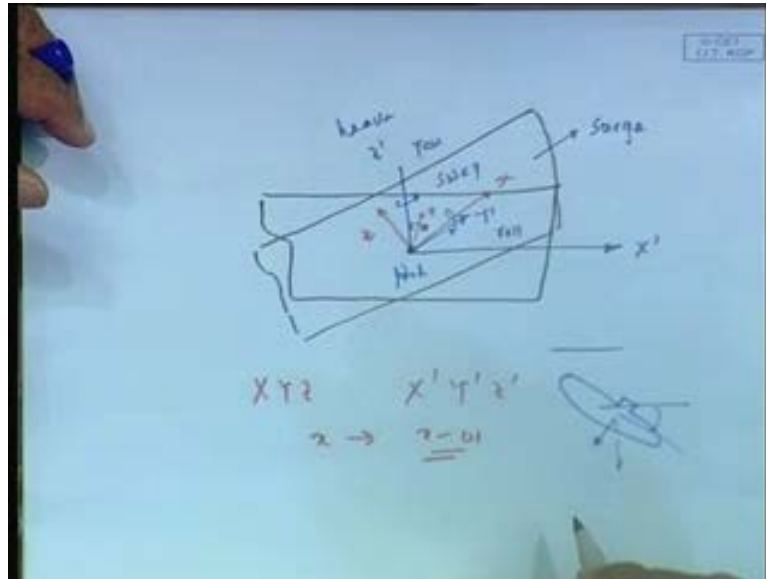
So, before therefore, when we go I need to define coordinate system, so what we do now I draw it here **you know**, so this is one coordinate system I have to fix say we always normally fix on z is equal 0 9, because this is where the waves come, say this is z is equal to 0 9, so my wave is something like that it is always easier, there is no fixed reason why you have to do it here, you can do anywhere else, but normally we do it.

We call this let us say, z x **I am** we are using let us say consistent, right handed coordinate system x, y, z, now this we I will call it an inertial system, now the ship basically moves with a steady forward speed of u. So, I have another system which is parallel to this, let us say I may call it x bar, z bar and this distance is becomes u t **you know** the translation.

I will call this to be a translating system with a coordinate fix on the hull, somewhere on the hull, but not rotating, but parallel to the O X system that means, it is a steadily transmitting system; that means, if I call O X Y Z as inertial system I am going to call this, let me call this O dash, **O dash**, X dash, Y dash, Z dash as steadily translating system fixed with the body that means, the point O dash is fix on the body.

Then comes a final system, which is the point O is fixed on the body, but now it is, but although point O is fixed on the body, remember O X dash, Z dash is not fixed on the body, it is parallel to the original system, now I have got a body system which is you can say small, well let me call it small o here, small x, small y, this is my body fix system. So, body fix system is the colour will help, a system like that, this is my small x, this is my small z and small y will come, may be I will just draw this two in a next one in a bigger way that will be easier.

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So, I have now here one system use colours X dash, Y dash, Z dash some point O here and this is hypothetical shape, but actually the shape would have been somewhere here, and in this case body fix system would look like well may be this is x, this is y, this is z, you understand that means, small x y z is fixed on the body with respect to that you can define your lines plan, your hull surface, etcetera.

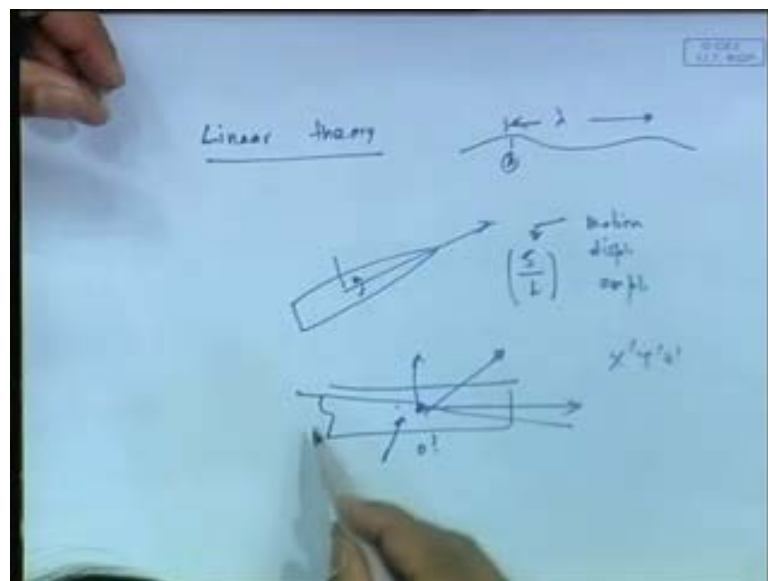
The body is invariant with respect to the small x y z system, it is body fix system whereas, X dash, Y dash, Z dash is actually parallel to my inertial system, but see if I kept it inertial system that the shape would have moved forward. So, the only thing is that the difference between the two, this basically x y z and X dash, Y dash, Z dash is at X coordinate travels with  $x - ut$ , we are just going forward that is all.

Now, come the question definition, now everybody knows that motion along x axis we call it surge, now let us, now here come the question which is surge this or this, that **that** question is comes in, that is why the debate let us say, now I call this let me define this strictly speaking since, I want to know this orientation I would like to know the displacements along this three and the rotation above the three, we could do that, we could also do this X Y Z both of them **you know** are possible. Let me call it here to be along X axis, which X open question this or this, but let me call this and motion along x axis we call it surge that everybody knows, y axis we call it sway, z axis we call it heave.

Rotation above this we call it roll about y axis we call it pitch, along z axis we call it yaw this we know, but here comes the question you think of an aircraft or a submarine suppose, the body is oriented this way which is heave this displacement or this displacement according to my definition, this is heave what is pitch, rotation, pitch angle is this angle now that means, aircraft can have a pitch of 90 degree, because sometime it can come straight down, but **if I** if you say somebody the ship is pitching 90 degree then he laugh at you stream hot pitch.

So, you understand the difference this is very important, a body which is making arbitrary motions in 3 d space can have roll pitch yaw etcetera, which can be in fact, if the if the aircraft is making a completely opposite turn, you can have a pitch of 180 degree for that matter, but it will make no sense for a ship a **(())**. So, therefore, what is happening is that for ship, now this comes at the **the** issue, having said that for ship what happens, the displacements are always small, at least the pitch and yaw, yaw first of all we do not bother, because you can always orient the hull along the y axis.

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See, the x axis need not be x axis, I will come to this your point for example, at any point when the ship is here, I will choose this as x axis, I do not have to choose this x axis the line and obviously, this rotation that it undergo this small rotation oscillatory wise, so what happen is that, the two things, number one remember we have been using linear theory, what it said, that the waves look as small, wave height is small compare to **(())**.

Now, wave is like excitation which causes the hull to move, now if my waves are small compare to slope is small means this is small compare to this  $a$ ,  $a$  by  $\lambda$  is small you would expect to be consistent that the motion displacements with respect to the dimension the hull, if I were to call displacement  $x_i$  by  $l$  they will be equally small, when this is motion amplitude 0.38

And if it is small it turns out that there will be no difference in the definition whether you tick this small  $x$ ,  $y$ ,  $z$  or this  $X$  dash,  $Y$  dash,  $Z$  dash, see this is the most crucial point for us, that if my displacements are small which is what would be the case, because we have started with the presumption that we are studying in small amplitude motions (Refer Slide Time: 16:38).

And as **you know** for ship even if I did not do that, yaw and pitch will not be large remember the ambiguity is with the rotation not with the linear translation; now have the three rotations, your roll pitch and yaw, now pitch we have done in trim it is never more than 3, 4 degrees, yaw I can always fix it, because I always fix my  $x$  axis along the base line.

Now, if two are small then third can be big does not matter it can get happen, so only roll or hill in the static case it can be big, but which is the practical case we have also done large amplitude hill etcetera in hydrostatics. But, as since pitch is small or trim is small we can dig up a bit and if that is the case we can always remember roll is above the  $x$  axis as a result this can be large, but since this angle is small if this small  $x$  and this  $x$  becomes in a same direction.

So, what happen therefore, in my ship definition we end up getting the **the** displacement equivalent to the linear displacement along that  $X$  dash,  $Y$  dash,  $Z$  dash system and its rotation about that, that means, about that **this** system which is steadily transmitting, this is very important **you know**, because are studying ship motion, because we are always often using the word roll pitch yaw etcetera **etcetera**.

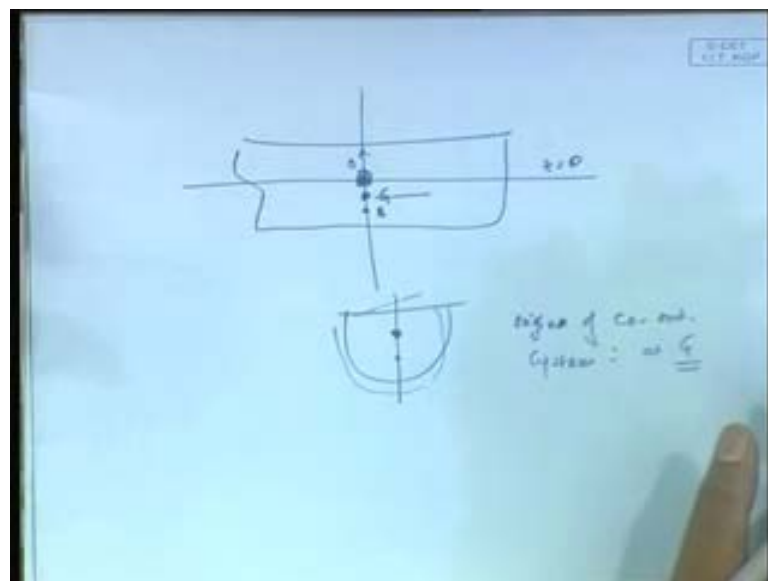
So, now we understand that we can do that, but for example, if you have to study a aircraft or a submarine or a spacecraft you could not do it just like that, unless you define properly, but here we could do that, because ship is primarily on a two-dimensional space that is **on the** on the free surface and it does not trim very much more than 2, 3 degree may be 4 degree and 4 degree also it might look bigger from draft point of view,



but angle is very small like **you know** the draft may look like that, but as you we all know the angle is on a 2, 3 degrees may be 4 degree.

So, we could do that and this is exactly very important, so when I am saying roll heave pitch and all these six parameters we are actually using equivalently a coordinate system which is the steadily translating system, but it still gives me one more question, the other question is what about this coordinate, where is o where do you fix it, you fix it here, you fix it here, do you fix it here.

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Now, again here, I will tell you, this that in our one earlier day literature some time people use to use, this coordinate see here, my remember from **from** hydrostatic balance it might central gravity somewhere here, my central LCD must be LCG otherwise, it look for a **for**, so called **you know** like equilibrium condition.

Now, let us say G was here, B is here **and you know** and we draw a line vertical and this point I call it O which is on the free surface  $z$  equal to 0, at one time in earlier day literature people were using this to be the coordinate system; that is the point O which lies on the free surface mean free surface vertically on same line at LCG and LCD, GMT, but today more of a convention is to use G to be the coordinate system.

So, once again you understand that there are again two definition, older they program for example, the program that we have to compute **you know** ship motion in US Navy steep

theory etcetera, old one they used this point O nobody can say that it is wrong or right. But, remember that strictly speaking your ship motions are not fully defined, if you did not tell O the once again I will tell you that it is something like this, if I were to rotate the body about this point, I will end up getting some configuration and if I wrote about this point and getting other configuration.

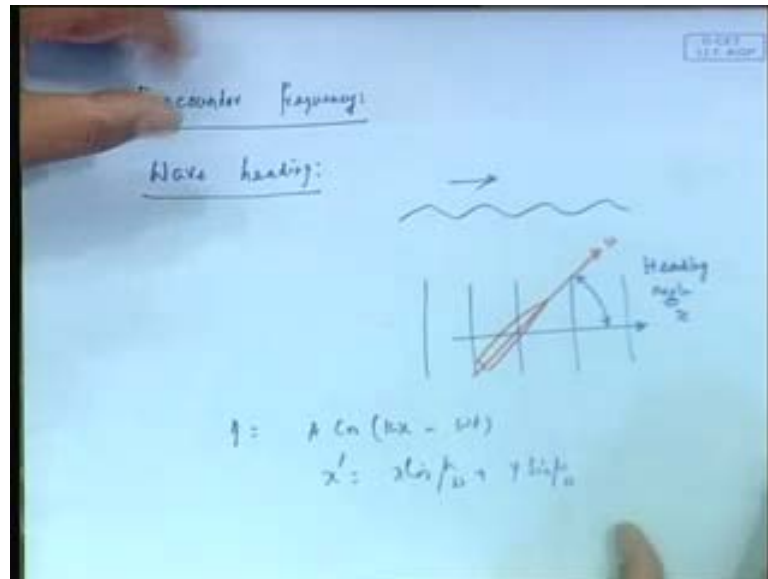
So, therefore, the exact evaluation of what is space it occupies depends on about which axis you have used, so therefore, you have to first tell the axis and then define the motion and then under body is undergone so much of motions.

And for us our purpose, now we are finding out that the definition of the six names are well known everybody knows **you know**, surge sway, heave roll, pitch yaw we all know this, only question is that if somebody asks you question, what do you mean by roll, then strictly one has to say that well I am defining a coordinate system with origin at C G **etcetera** etcetera, then I will tell the rotation about in the body about its own the G x axis, I am calling it roll, this is the strict definition.

We over look this, we very casually mention this, because we do not mention that my definition is based on this assumption of smallness etcetera **etcetera**, that is why I try to tell you, because subsequent to that we are not discuss anymore, subsequently we assume my trim is only small etcetera **etcetera**, so this is becomes important for our point of view. So, now as well as ship motions are concerned we are going to use this definition, we **we** use the origin **origin** of G there is an advantage of doing that though, because we will find out that when we do any moment of inertial etcetera **etcetera**, if you have a axis through G you have least moment of mass moment of in asia, if you have any other then that is additional term that comes in makes it more complicated although the results are same.

So, therefore, we are going to for our class we will define in this once again I emphasis that, this is what is convention, but one cannot say that this is absolute, so this is about the definition of ship motion, so I **I** hope this clear to you this idea of ship motion definition is clear to you. Now, I will switch to this other part call encounter frequency (No audio from 23:26 to 23:42) perhaps, because before that, let me also talk about something called wave heading, this is a part of that let me talk of wave heading.

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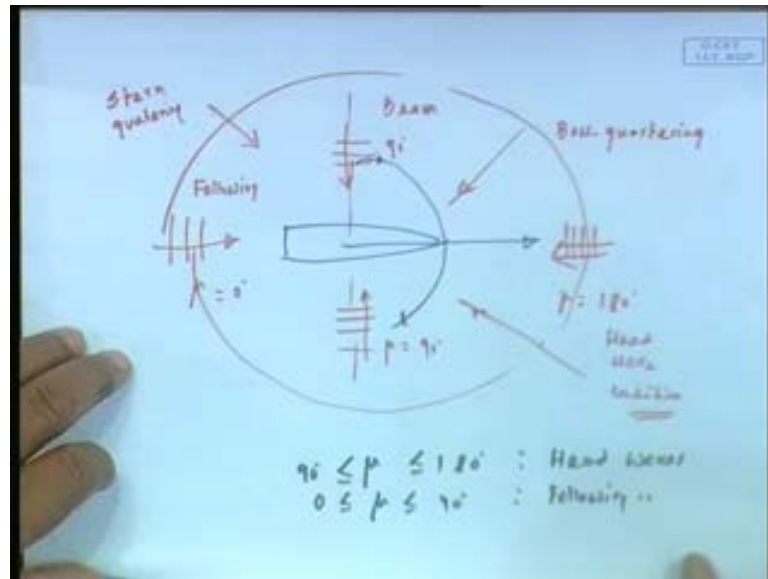


See, remember we are now putting our ship in a regular wave field not a random wave field, we have a single sin wave coming, let say a wave which is going like that, in this direction, so if I were to draw the quest line, the quest line will look like that, going this side x axis, but the ship need not be travelling this line it can always travel along this, it **it** can travel any direction.

So, this is my direction of ships for steady forward motion and this is the direction of wave propagation, remember the wave propagation can always be arbitrary, actually speaking when we use this formula eta equal to A cos k x etcetera, remember this formula applies when the wave propagation of x direction, but if you want to make it x dash you can always make a substitution of **x** x equal to x dash whichever way it is, if I call it x dash or x dash is x cos say some mu wave plus y sin mu etcetera **etcetera**.

It is very simple coordinate transformation, we are not talking about it you can always transfer the coordinate **it**, see it is always good to fix the x axis to one of it, so we can fix the ships forward axis or you can fix the wave sea normally it is easier ship forward axis for ship motion perhaps. Now, the question is that this angle that is the direction, now this is again very important to understand, we are defining the angle between the direction of wave propagation and the direction of ship propagation as heading angle, some people do opposite that is why I am telling, some people do with a minus x axis this I am calling heading angle.

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Now, we will see this from the point of ship's point of view, so I have the ship here, travelling this side, so if now my waves come this side, my heading angle let me call this, let us call this  $\mu$  this becomes equal to 0 degree; if it came this way or this way for that matter this is 90 degree, 90 degree, if it came this way then  $\mu$  equal to 180 degree.

And if I were to draw a circle for example, the wave can come from any direction, now two things we will understand one thing is that, mainly one thing is that, because of symmetry, because you see ships are all geometries are symmetric about the **the** geometry symmetry was center plane, geometry in side things will not be symmetric and I keep saying that you normally develop a geometric symmetric. So, that when impose forward there is no motion in force in this side otherwise, you would have to have something else correct it, **you know** whether its an automobile or aircraft or whatever the external geometry on which fleet force acts you want to make it symmetric, along the plane in **in** which it moves.

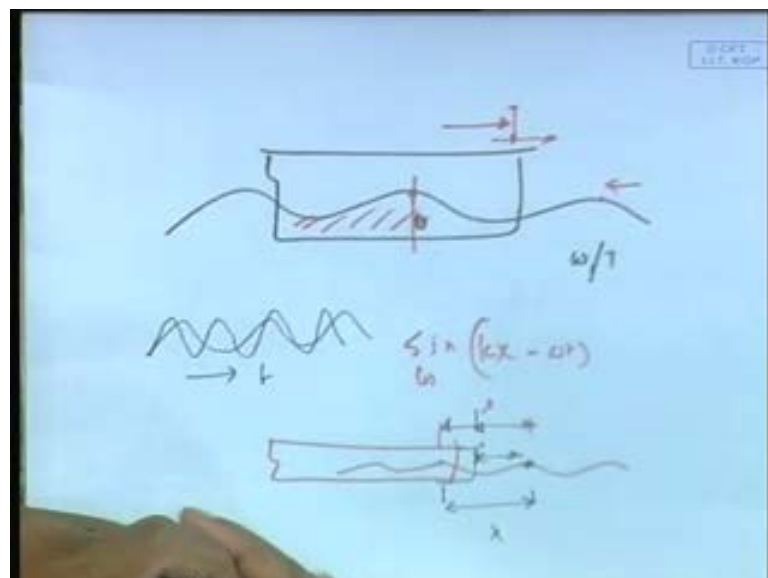
So, what happen as far as my study is concerned, if I take only half structure is good enough, because **you know** if the waves comes from this side the effect that will have exactly same as it came from this side; question looking at. Now, this 180 degree is known as head wave or head wave condition, which makes sense you simply heading into the waves, this is called beam wave condition; I just write beam here, this is called following, this is strictly when you have  $\mu$  equal to 0 as following waves.

Now, this is known as bow-quartering wave, these two and this is stern quartering, in general this half that is if I were to take this half, wave coming from this half will be head wave condition, that is  $\mu$  between 90 degree and 180 degree this is my generally head, because in this condition my wave is actually travelling in to the head in some sense, and if it is 0 to we call it following wave.

What I am saying here is that, when I say head wave condition it could strictly there is no again very formal definition, it could basically be anything when the waves are coming in this sector **you know**, it is coming this sector and the ship is going in to it of which specifically you call head wave to be 180 degree this is beam wave. These two are that is, 135 degree you call it bow-quartering wave and again similar thing which is coming from back side that is this half we are calling it following waves of which this one exactly we call strictly the following wave and this is stern following waves in **in** other words, this is falling wave condition you can see.

So, this is my heading angle business, now the question is this **you know**, now you see let us **let** take this part or I will just show another diagram the wave is coming through this and ship is moving that, we have to talk about the encounter frequency.

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Let us now again you look at this way, there is a ship here or any body and I have this wave, we know this wave everything of the wave has a periodicity of  $\omega$  and  $T$  or  $\omega$  or  $T$  means we have found it out, in last two lectures that whether it is velocity,

pressure, acceleration anything that the wave has or the property of the wave is sinusoidal with a period  $T$ . Now, you for example, take a point here and the ship is not moving right now, let me take a small point here, there is the pressure acting on that, what would be the nature of the pressure it will be oscillatory with a period  $T$ .

Because, everything is having a period  $T$ , so if I not take all the points, all of them have a period  $T$ , so if I add them all up I get my force that will also be period  $T$ , so what would happen exciting force which is pushing it, which is what is causing the ship to oscillate, would have the same period  $T$ . You see because, everything on the wave is having a period  $T$  with that velocity how whichever way you differentiate, whichever way you **you know** like integrate regular sin wave, because it is  $\sin$  or  $\cos kx - \omega t$  is perfectly sinusoidal no matter, how you manipulate it how many times you add for a single wave.

So, therefore, it is effect on the wave on the ship which is nothing but, integration of pressures with interaction whatever would also have a example, say there is a peak here, so when the peak comes let say here some effect is here, some force a obviously, that will repeat again when the next peak comes in. So, it will keep on repeating at that, because the same scenery will repeat just after  $T$  seconds. So, therefore, what would happen the ship would experience, the environment at period  $T$  provided it is not moving or **at what you know** at what it the oscillation comes.

Now, comes the question, that the ship begins to move let say this wave is actually moving this side, what would happen now you are standing here, say you are standing observing here, now this crest if the you are not moving the crest would have passed by you every  $t$  seconds. But, what is happening, now you are also moving to it, what would happen you would going to feel it at a time lower than  $T$  second obviously, because if were to draw this picture in this way, and if I were to draw this picture of, if this was my  $\lambda$  and you was standing here by the time this peak has come here you actually have moved to this location.

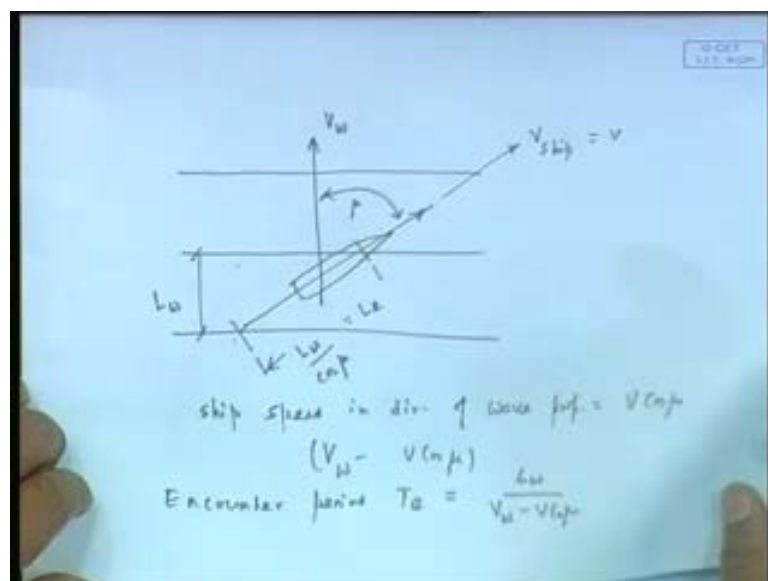
So, by the time this has come here you have moved to this location, if this distance which this  $\lambda$  then if you would feel this peak, because by the time **you know** like the, so what happen this peak you think to have come to you when this much of time is enough. In other words, when the wave has travelled only this distance you would **(0)**, so this is

now what is happening is that obviously, this pressure that I set the pulse that is coming initially it was like that at time if I do, but now I am feeling it faster, because remember whenever my crest pass by I got the pulse a particular force, now if I did not move at every  $t$  second my wave crest pass by. But, now I am moving into it, so the crest pass by the same point at some other time and that is my encounter period and therefore, my excitation that I feel on a moving body would be at the encounter period. So, the signal that I get, the ship case is at that particular oscillation period, this is what is encounter period.

In fact, this is exactly same and we will we will derive the formula for the  $(\omega)$ , it is exactly same as the beating frequency, not beating **sorry**, at doppler shift frequency that all of you would be knowing. I think **that in** that in high school physics you all learned that why does an approaching train, whistle **you know** sounds high pitch and residing becomes low pitch same thing, because when the object emitting noise approaches you the relative speed is different and therefore, the frequency goes up and it goes up what do you call doppler shift.

In fact, I can go the extend to say that even the fact, one of the fact that the universe is expanding was seen from rate shift of the spectrum rate means lower frequency, so if object only goes up, so light coming would actually shift towards the lower frequency, so, this is the same thing here.

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Now, let us look at this the **you know** derivation of that, let us take a wave moving this way with speed  $V_w$  and I have here the ship moving  $V_{ship}$ , say we call it  $V$  and which is angle  $\mu$ , remember, this is my angle between the direction of motion of the ship and the direction in which the wave propagates, remember these are crest lines wave is moving this side, ship is moving this side.

Now, remember this is wave length  $L_w$ , so how much is **this** this is  $L_w$  by  $\cos \mu$  or we can call it  $L_e$  encounter wave length, now the direction of the ship we can do in many ways, direction of the ship speed  $V$  in the direction of this is basically  $V \cos \mu$ , the **the** ship this thing.

So, (No audio from 37:15 to 37:32) **so** to an observer therefore, the wave speed is relative wave speed is going to be  $V_w$  minus  $V \cos \mu$  actually this is a very straight forward thing we do not this thing, now what is happening the by definition, therefore, the period of encounter is  $L_w$  divided by  $V_w$  minus  $V \cos \mu$  we, you can define another derive that in very many **many many** ways, now this is the only one that is all we need, in fact.

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The image shows handwritten mathematical derivations on a whiteboard. The equations are as follows:

$$T_e = \frac{L_w}{V_w - V \cos \mu}$$

$$T_w = \frac{L_w}{V_w} \quad ; \quad L_w = V_w T_w$$

$$T_e = \frac{V_w T_w}{V_w - V \cos \mu} = \frac{T_w}{1 - \frac{V}{V_w} \cos \mu}$$

$$\left\{ \begin{array}{l} \omega_e = \frac{2\pi}{T_e} \\ V_{rel} = \frac{V_w}{1 - \frac{V}{V_w} \cos \mu} \end{array} \right.$$

$$\rightarrow \omega_e = \omega \left( 1 - \frac{V \cos \mu}{V_w} \right)$$

$$\omega = \omega \left( 1 - \frac{V \cos \mu}{V_w} \right)$$

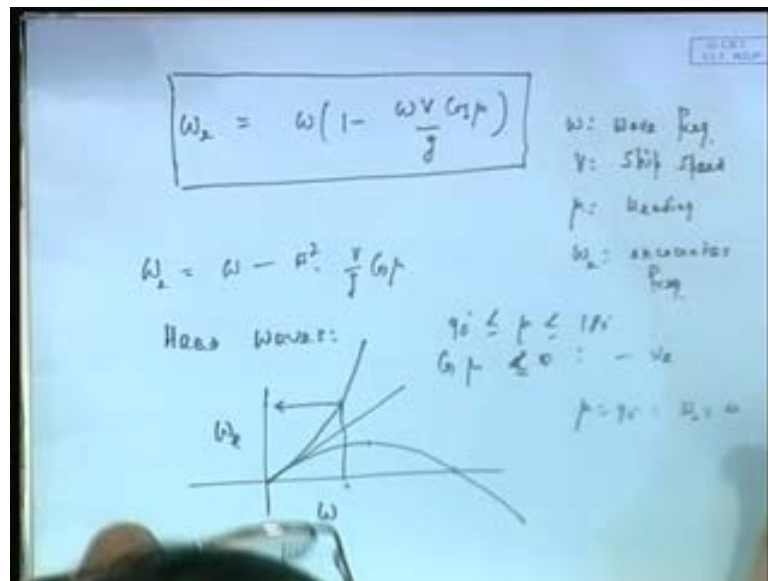
Then what happens from there we can easily find out that, since  $T_e$  is  $L_w$  by  $V_w$  minus  $V \cos \mu$ , but  $T_w$ ,  $T_w$  is my you see here, the word  $e$  is encounter, the word  $w$  is original wave, I am specifying with  $A_w$ , so that there is no confusion. So, this is nothing but,  $L_w$  by  $V_w$  by definition obviously, the wave length by phase speed of the wave, so therefore,



The becomes  $\omega$  w T w, I will just write it down, because **L** see L w that means, L w is V w T w this is really trivial, so there is no point of spending too much time on that.

But, then  $\omega_e$  by definition is going to be  $2\pi$  by T e and we will use the relation V w equal to g by  $\omega$  w. So, we call that I am end of getting, I leave it to you to work out the more detail  $\omega_e$  becomes  $\omega_1$  minus  $\omega$  V by and writing just  $\omega$  it turns out to be, I repeat this, this is very important, so **sorry** this is  $\omega_e$ .

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We end up getting  $\omega_e$  as  $\omega_1$  minus  $\omega$  V cos  $\mu$  by g, when I am writing this  $\omega$  is wave frequency V is ship speed  $\mu$  is heading, and so this is the form now, but look at this equation, this equation is quadratic equation in  $\omega$  in other words, basically  $\omega$  is becomes  $\omega$  minus  $\omega$  square into V by g cos  $\mu$ .

So, this is my encounter frequency equation, very **very** important equation encounter frequency equation; we can work out very simply many things on that, but there are very interesting corollaries on this equation that we will talk about. You see, suppose I am in a head sea condition or head waves **sorry** head waves, what is the value of  $\mu$  it is 90 degree 180 degree, what is value of cos  $\mu$ , cos  $\mu$  becomes negative **yeah, less than** less than 0, less than equal to 0 you can say, so basically negative.

Now, if cos  $\mu$  is negative, let us say this term become positive **(0)**  $\omega_e$   $\omega$  into 1 plus  $\omega$  square V by g cos  $\mu$ , now if I were to do a plot here,  $\omega_e$  versus

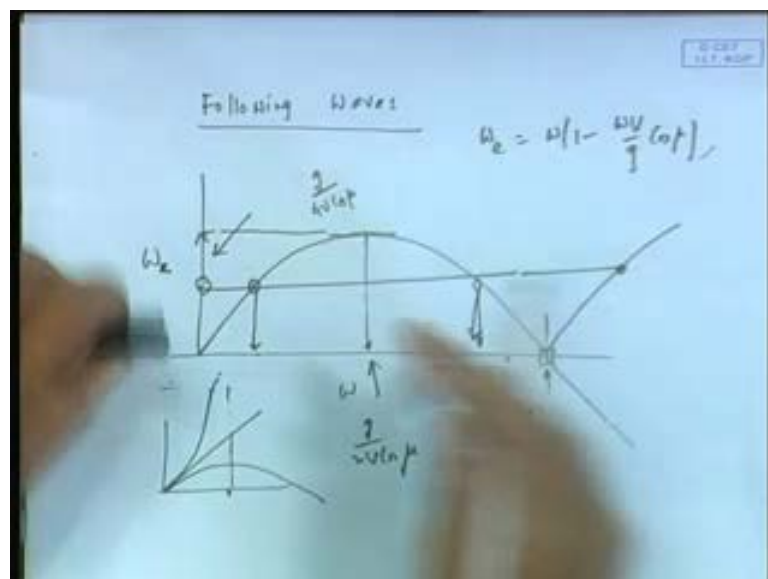
omega, how will it look like, it it will look like this. **If this was**, if my cos mu was less than 0 or negative, then this will become positive and I would actually the graph will look like that. So, what would happen is that, you are going to get for a positive omega a positive omega a one to one corresponding for anything you are going to get 1 here or 1 omega equal 1 omega, so you are going to get that.

Now, suppose it is exactly beam sea condition, what happen the limiting case in the in between case cos mu is equal to 0, 90 degree, when mu is 90 degree, so this relation becomes straight line, so the omega is equal to omega.

So, in this case omega e equal to omega, but the problem comes when it becomes less than mu becomes 0 to 90 degree or cos may become more than 1, then what happen this equation looks like this, now this is something that we need to talk little bit. Because, what happen you find here, that there is a 0, you also find that there is a maximum and then it becomes negative what does it physically mean, what does physically omega e equal to 0 mean, what does omega negative means, this is what we need to also talk, because we have to have an understanding of the physics.

See, we all understand that omega is oscillation frequency that is something pushing the hull at that every 10 second, what is meaning by pushing it at every minus 10 second, you have a difficulty in understanding that, so we need to talk about this the following wave condition and cases (No audio from 44:11 to 44:23).

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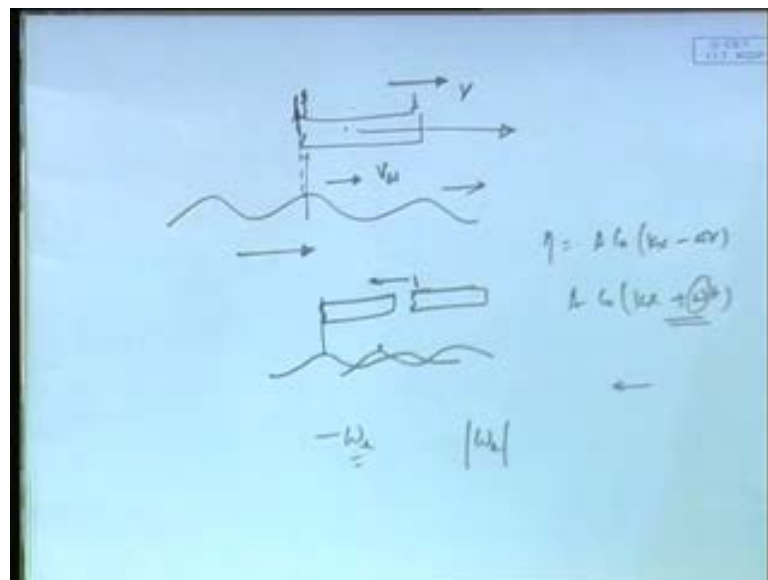


So, here we are finding out the corresponding, now we will omega e versus omega, coming like this, going like this, first of all let us find out **this** this location **this** this is going to be you see here, the formula is again if I write omega e equal to omega 1 minus omega V by g cos mu. In fact, this is when this is 0 that means, when omega equal to g by V cos mu that means, this value is g by V cos mu that means, at omega equal to g by V cos mu I have theoretically a 0 encounter frequency.

Now, I will leave it to you as exercise and you should do the exercise, what is this one which is can be easily done, because maximum of that will occur at d omega e by d omega equal to 0, if you set it up, you will find it out. And you will end up finding that this value is exactly half of that, g by 2 V cos mu, that is this is half this frequency, you will find out **you know**, and you will find out that it is connected to the groups speed which is what I was telling before hand etcetera, this particular one.

And not only that you will find out that this value, this omega e becomes actually g by this is g by 4 V cos mu this that means, **in** in following wave it actually equals to maximum of g by 4 V cos mu, then goes to 0 it becomes negative.

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Now, let us talk about this, the concept of negative etcetera **etceteara**, physically the concept, firstly from the 0 and then negative and then the other values, see what it means, now here again see, now here there **there** two cases, the ship is moving like that there is a wave also moving like that, you was standing here let say somewhere here or here.

Now, what is happening is that, supposing you are moving at a speed  $V$  and the phase is also moving at a speed  $V$   $w$  are exactly same, what would happen as you stand here you would think that, you are not moving with respect to water your **and the** and the period it takes for the crest to pass by you is infinity, because it is not passing by **you** you are moving at a same speed; this is what is zero encounter frequency. Which means, that both the component of the ships speed and the wave speed **in** in the direction is exactly same.

Now, here comes **you know** confusion in many of the students mind that if **if** I stand and **I** if I find water is not moving speed is same, then can I study the following wave condition same as hydrostatics, we have nothing moves **right**, because hydrostatic is easy.

See, in hydrostatic what has happen, I was not moving, water is not moving **you know** everything is calm, here I am moving water is moving, but with respect to me water does move at least the wave its not the water does move, the wave from does not move can we therefore, make it same the answer is emphatic no. Because, what happen it is only the wave form that does move, but there are whole lot of dynamics that rose up indeed, in this condition becomes much more dangerous with respect to losing stability.

It is analogically same as you are trying to walk on a very smooth **you know** like frictionless road on a **on a on a** ice **you know** rink, trying to drive a car there is no grip, as there is no grip there things trying to fall, I mean I give you my personal example of driving in a ice covered parking lot in Canada. And you try to break you see there is no grip at all, because nothing moves, so **you know** you do those stability same thing happens therefore, it is not same it actually represents the, represents much more dangerous situation.

So, this is the following wave, now what is happening see here we know now you see, we know that longer before one more thing I will tell you here, see there is another interesting thing, you see here there can be two omega  $e$ 's for same omega  $e$  there can be two omegas which means I can have the same encounter frequency for two kind of **you know** like two conditions (Refer Slide Time: 48:44). So, if I sit on a ship and if I were observing the, **its** its response the oscillation period that means, omega  $e$  I really cannot

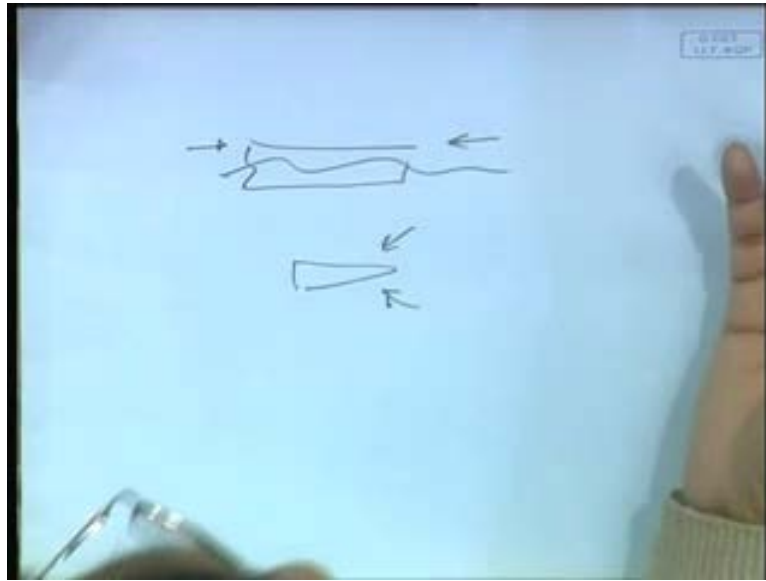
tell often what is my incoming wave, because it could be this one or it could be this one, so there are two  $\omega$  all these things, why happen is essentially connected with this thing.

See, when the ship becomes for example, longer it travels faster, so let us say it is at certain speed and this is longer, so it will traveling faster you will find out the oscillation is going on the other side, but similar periodicity may come when it is traveling slower. Because, what would happen is that actually is negative frequency concept that is, that see you are standing here, now time  $T$  you are here, wave crest is here, but after a while you have moved to this point and the wave crest has moved only to this point.

So, the wave crest, if you stand here you will think it is moving backward actually you will think it is going to move backward, so what would happen is that, but the periodicity is same. So, what happen negative frequency means nothing but, it is like  $\eta$  equal to  $A \cos kx - \omega t$  and if you take  $A \cos kx + \omega t$ , what it means, it basically means a wave travelling in negative  $x$  direction. But, frequency still remain  $\omega$ , so in our case, because we have defined all as positive  $x$  direction my negative  $\omega$  is actually implies in encounter period of  $\omega$ , but the wave is trying to receive from you, like you are leaving it behind this is the idea of negative frequency.

So, what we do quite often for calculation purpose is that, it does not since, it does not extend to go this way, you can reverse this, if you take an absolute value it reverse here, and if you reverse it you get one more point as far as frequency is concerned possible solution at times. So, this the negative encounter frequency can be little complicated, because there is no uniqueness of solution same negative same value of that frequency may occur for different wave conditions that is one, but we always have to use the absolute value for our excitation purpose, because the excitation that comes is  $\omega$  that  $\omega$  value, so this is very important for us to understand.

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Now, a good I may say a fortunate circumstance is that most ships actually respond or behave much strongly on head condition when the wave or other, when the waves come this way that way etcetera. Ship motions are much more pronounced when **it** it hits into wave, but that does not mean their should not be studied for following wave, because in following wave condition the importance come with respect to as I said laws of stability, in fact, there are cases where ship begins to **(())**, roll heavily, lose stability and the ship is lost.

So, following condition is also extremely important, but from a different perspective, in our course since, we are doing for the first time sea keeping, our primary focus is to figure out how much ship moves, what is this **you know** acceleration etcetera motion, it is much more pronounced in head see, so we will mostly concentrated on head sea condition, but following sea is also there as far as frequency encounter is concerned.

So, that is very important following sea concept is like sometime not clear to people, but this is the situation it is nothing but, you are moving faster, so the wave crest appears that actually you are going on the other direction to you, although wave is moving in the x direction, but to you it will appear as if it is going in the minus x direction and therefore, you think.

And obviously, as the name implies your oscillation period will come down see 0, then it will come down it will become much low, and as you can see here in following wave

also there, is a maximum you cannot go beyond that this maximum is also quite small. So, periodicity becomes much lower in that, in the same graph if I had to plot it, it is like this, like this, like this for a same omega you can find out that depending on condition omega is much more here, same here, lower here. Another point I will just end it, that head sea condition, we are finding it omega is more than omega beam sea it is same and following it is typically less.

So, therefore, if there is a 10 second wave, if you are heading into it 8 second, 7 second you are encountering beam sea means 10 seconds, following mean it will appear to it 12 second, 13 second, 15 second may be lower. So, this is important, with that I am going to stop this part of **you know** the encounter frequency and then we will go to the next lecture thank you.