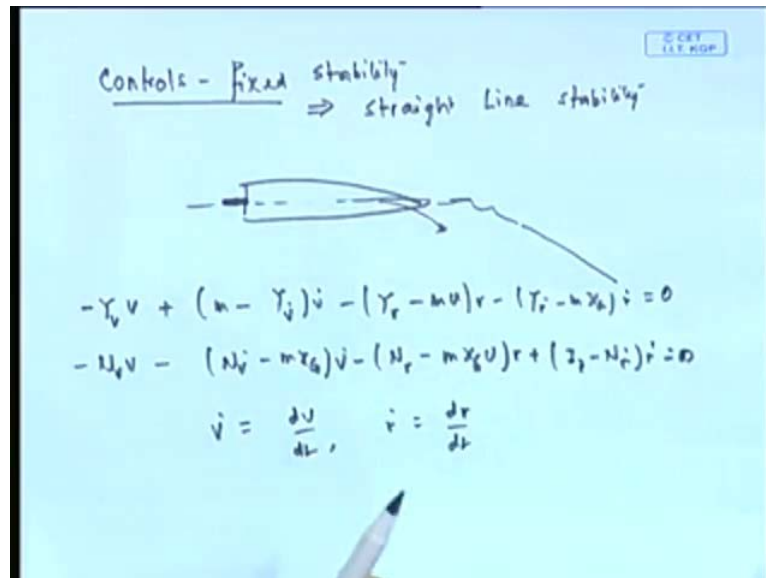


**Seakeeping and Manoeuvring**  
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**Module No. # 01**  
**Lecture No. # 27**  
**Controls-Fixed Stability**

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See in this lecture, this hour I am going to talk about control fixed stability, which is same as actually, straight line stability that you know, we have talked earlier. If you remember, we have talked about this that if a ship was discard, then after that it may take another straight line or may not take. Now, what is this control fixed? Let me also explain that see, I have this ship here right now by control fixed, what I meant is that we are not applying rudder, rudder is what is called control surface? You basically, are looking only at the hull. Now, the question is that, if I were to disturb the hull will it take another straight line or will it go here that was the question, we asked earlier. Now, what we want to study today is that based on those hydrodynamic derivatives and equation of motion, what is it? That is required to be satisfied to ensure that the hull has a control fixed stability.

You see, I tell you an analogy of that you know, you many of you coming by bicycle. Now, you know, you will know in some of the cycles, which the wheels is not properly aligned, you have to continuously kind of steer it in order to make on a straight course. Some of them, you will find out that it is much easier like, if you some of you try to ride cycle without your hands. You know like without putting, you will find that if a cycle is very shaky type like, you know wheel is not aligned very difficult to maintain straight line you fall. But there are some cycles, which you probably can more easily make a straight line. Basically, there is something that is there in the dynamics that characterizes this two like, if you have, you know your forward wheel is not aligned this kind of having a what you called is not exactly on a center plane etcetera. Then it will it is more tough you continuously apply, your hand.

So, similarly, in a ship sea now, what one you prefer obviously, most people will prefer a cycle which has it is inherent stability. Because otherwise that you continuously, monitor it may be effect on us. So, same thing applies for a ship, yes I could probably make a straight line by applying rudder continuously, but I do not want to apply, I want to make sure that. If I put the rudder straight, it goes on a straight line even the small waves etcetera, comes do not disturb it much, it maintain the straight line, which therefore, means that, I am looking for excuse me a ship, which probably has straight line stability, if not I want to study what is it how much it should have.

So, therefore, that is what we are talking and so unfortunately, we have to keep coming back to the equation of motion always, because that is what the starting point this. see now, let me just first write and then I will, I have this equation right minus  $Y \dot{v} \dot{v} I$  quickly write this, minus  $m Y \dot{v}$  dot there is a purpose of writing that also,  $Y r$  minus  $m$  into  $u$  was there, this be  $u$  anyhow  $r$  minus  $Y r$  dot minus  $m \times G r$  dot, this side is 0. And I have this equation minus  $N \dot{v} \dot{v}$  see this becomes. So, elementary that every time you end up repeating it right, but let me quickly writes it anyhow.

Now, here see I am going to what is this? You see  $\dot{v}$  is  $dv$  by  $dt$   $\dot{r}$  is  $dr$  by  $dt$ . So, if you look at that this equation is essentially, a setup linear differential equation, but with two unknowns in other words, I am going to rewrite that taking this.

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$$\left\{ (m - \gamma_v) \frac{d}{dt} - \gamma_v \right\} v + \left\{ (m x_g - \gamma_r) \frac{d}{dt} - (\gamma_r - \alpha v) \right\} r = 0$$

$$\left\{ (m x_g - N_v) \frac{d}{dt} - N_v \right\} v + \left\{ (I_v - N_r) \frac{d}{dt} - (N_r - \alpha x_g v) \right\} r = 0$$

$$(\ ) \dot{v} + (\ ) v + (\ ) \dot{r} + (\ ) r = 0$$

$$\begin{bmatrix} \ ] \end{bmatrix} \begin{Bmatrix} \dot{v} \\ v \end{Bmatrix} + \begin{bmatrix} \ ] \end{bmatrix} \begin{Bmatrix} \dot{r} \\ r \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$m \ddot{x} + b \dot{x} = 0 \quad x = x_0 e^{\sigma t}$$

And then, if I rewrite that the equation will look like this  $m \ddot{v} - \gamma_v \dot{v} - \gamma_v v + (m x_g - \gamma_r) \dot{r} - (\gamma_r - \alpha v) r = 0$  and.

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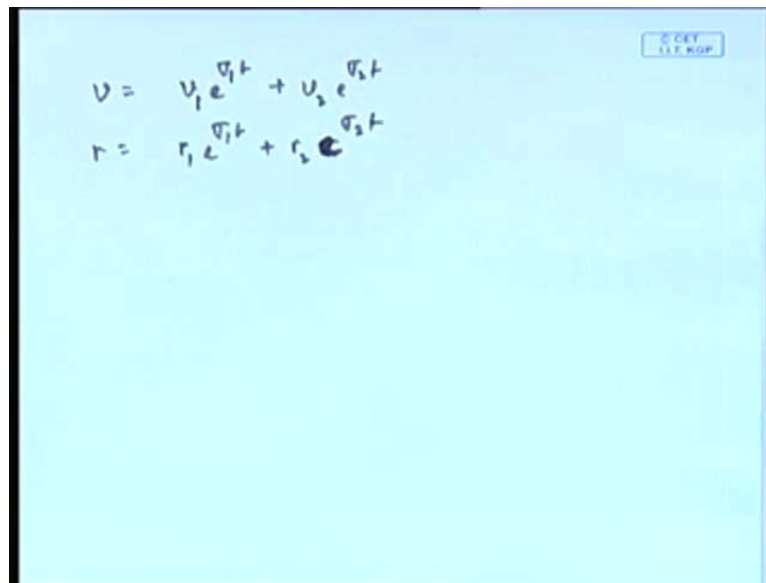
Actually, what is you know, if you look at that the equation part the for the front part also, what you will find out that this equation has a form something like that? You know something into  $v \dot{v}$  plus something into  $v$  plus something into  $r \dot{r}$  plus something into  $r$  equal to 0 the two equations. In other words, you the equation has a form like that, you agree with that this is how it looks like something into  $v \dot{v}$  plus something into  $v$  plus something  $r \dot{r}$  plus something  $r$  equal to 0. Essentially, it is seen if you look at this so this is nothing but 2 simultaneous linear differential equation, just like our this thing  $m \ddot{x} + b \dot{x} + kx = 0$  something like that just that it has no stiffness term right now here.

Essentially, it is two linear differential equation so, I am writing in this form this way. Now, you see why we are doing that, because you see when you do this also what you do you presume the solution to be something like  $x = x_0 e^{\sigma t}$ . And you substitute back and find out what is the value of  $\sigma$ , that is what we been doing it that is the way of solving it. So, you see when you, when I have this is a linear differential equation, this equation we will have a solution that will look like. See, I what to know from here, again

when I look back at this my purpose is to find out  $v$  and  $r$  with respect to time, I want to solve for  $v$  and  $r$ .

So, it is something  $v$  plus something  $r$  or something  $v$  plus something  $r$  you can say, but of course, there is differential equation. Because there is  $d$  by  $dt$  there you agree with that, this is with the  $d$  by  $dt$  here having been not there, it would be simple algebra, but this is a differential equation.

(Refer Slide Time: 08:23)



The image shows a slide with handwritten mathematical equations. The equations are:

$$v = v_1 e^{\sigma_1 t} + v_2 e^{\sigma_2 t}$$
$$r = r_1 e^{\sigma_1 t} + r_2 e^{\sigma_2 t}$$

In the top right corner of the slide, there is a small blue box containing the text "SECRET" and "S.T. KOP" below it.

So, how the solution will look like, your solution will look like  $v$  will obviously, it because it is 2 you might have 2 this thing say  $v_1 e^{\text{power of } \sigma_1 t}$  general equation, because it is 2 L 2 roots may be there and  $r$  will be  $r_1 e^{\text{power of } \sigma_1 t}$  plus  $r_2 e^{\text{power of } \sigma_2 t}$  sorry this is standard form of the solution. And I what I need to know I am not solving here or what I want to know is to find out the  $\sigma_1$   $\sigma_2$  values. Now, you see what is happening, **excuse me again** very important what do I want for stability? Suppose  $\sigma_1$  and  $\sigma_2$  any one of them is positive. Let say real positive what would happen  $v$  and  $r$  will keep growing with time, which means the path is going to be going farther and farther deviated without a control.

Suppose,  $\sigma_1$   $\sigma_2$  are imaginary number, if actually it is one imaginary both will be imaginary, because there be quadratic equation. What is meant by  $e^{\text{power of imaginary number}}$ ? It is sinusoidal number that means,  $v$  and  $r$  will show is going on oscillation, what will cause me to make sure  $v$  and  $r$  becomes 0 with time, only when

sigma 1 sigma 2 are real negative numbers therefore, I need to find out what is that make sure my sigma 1 sigma 2 are real negative numbers, that will be my condition for stability. Now, let us look at what is sigma 1 sigma 2 remember that this equation, I will go you know, you will have this notes later on though you can see the equation.

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$$\left\{ \underbrace{(M - \gamma_v)}_{A_1} \frac{d}{dt} - \underbrace{\gamma_v}_{A_2} \right\} v + \left\{ \underbrace{(m x_g - \gamma_r)}_{B_1} \frac{d}{dt} - \underbrace{(\gamma_r - m u)}_{B_2} \right\} r = 0$$

$$\left\{ \underbrace{(m x_g - N_v)}_{A_3} \frac{d}{dt} - \underbrace{N_v}_{A_4} \right\} v + \left\{ \underbrace{(I_v - N_r)}_{B_3} \frac{d}{dt} - \underbrace{(N_r - b x_g u)}_{B_4} \right\} r = 0$$

$$(\ ) \dot{v} + (\ ) v + (\ ) \dot{r} + (\ ) r = u$$

$$\begin{bmatrix} \ ] \end{bmatrix} \begin{Bmatrix} \dot{v} \\ \dot{r} \end{Bmatrix} + \begin{bmatrix} \ ] \end{bmatrix} \begin{Bmatrix} v \\ r \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\textcircled{m \ddot{x} + b \dot{x} = 0} \quad x = x_0 e^{\sigma t}$$

So, this is like, I can call it this way A 1 and I call it this way A 2 see, A 1 divide a t A 2 like that I just want to write it. So, that you will know, I can call it say B 1 I can call this with the minus sign right B 2. So, so that, I have this thing I can call this to be A 3 I can call this to be A 4 I can call this to B 3 I can call this to be B 4. Remember again, I have therefore, an equation of a form which looks like, if I were to just place it here and write it from here.

(Refer Slide Time: 11:16)

Handwritten mathematical derivation on a whiteboard:

$$v = v_1 e^{\sigma_1 t} + v_2 e^{\sigma_2 t}$$

$$r = r_1 e^{\sigma_1 t} + r_2 e^{\sigma_2 t}$$

$$(A_1 \frac{d}{dt} + A_2) v + (B_1 \frac{d}{dt} + B_2) r = 0$$

$$(A_3 \frac{d}{dt} + A_4) v + (B_3 \frac{d}{dt} + B_4) r = 0$$

$$\begin{pmatrix} (A_1 \frac{d}{dt} + A_2) & (B_1 \frac{d}{dt} + B_2) \\ (A_3 \frac{d}{dt} + A_4) & (B_3 \frac{d}{dt} + B_4) \end{pmatrix} \begin{pmatrix} v \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} A \sigma^2 + B \sigma + C &= 0 \\ A \frac{d^2}{dt^2} + B \frac{d}{dt} + C &= 0 \end{aligned}$$

I have  $A_1 \frac{d}{dt} + A_2$  plus  $B_1 \frac{d}{dt} + B_2$  equal to 0,  $A_3 \frac{d}{dt} + A_4$  plus  $B_3 \frac{d}{dt} + B_4$  or if I were to write it will be something like a  $A_1 \frac{d}{dt} + A_2$  plus  $B_1 \frac{d}{dt} + B_2$  plus  $A_3 \frac{d}{dt} + A_4$  plus  $B_3 \frac{d}{dt} + B_4$ . Now, the question is that see this  $\sigma_1 \sigma_2$  essentially, becomes the root you know actually, if you put this back here you will find out. And I am not going through that roots of a characteristic equation see given by A, when A B C are going to be nothing but if I were to multiply, that the terms essentially A B C will be, if I have this matrix no this matrix, then the term that will come as A into  $\frac{d^2}{dt^2}$  plus B  $\frac{d}{dt}$  plus C equal to 0.

In other words, what I mean if I were to take this is a I think you all know this is what is meant by the characteristic equation, that you will be finding out in any like you know, differential equation solution course. If I were to cross product is this one this matrix, if I take this determinant basically, this matrix and if I expand that I will have something into  $\frac{d^2}{dt^2}$  plus something into  $\frac{d}{dt}$  plus something equal to 0. So, that is having a form of  $\frac{d^2}{dt^2}$  plus B  $\frac{d}{dt}$  plus C, because you know that, when you took this part essentially  $\sigma^2$  comes out. So, this is a standard linear differential equation solution that is how you do you basically substitute back that here. And expand that and then get this, because of this e power you end up getting a quadratic equation in terms of sigma and then solve for that for sigma values.

So, what I mean is that, this is our standard we are not going to be discussing, how we solve it numerous way of solving it. But I am not looking at a solution remember, I am looking at the roots of the roots sigma and the condition of the sigma to be the L negative. Now, what is sigma, sigma is suppose to be the roots of this equation a sigma square plus B sigma Y, what is A B C? That is omega expand that, the term proportional to d by dt square is a etcetera. So, if I expand that for example, I end up getting things like.

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Handwritten mathematical derivation on a blue background. The derivation shows the expansion of a differential equation into a quadratic form, followed by the quadratic formula and conditions for real roots.

$$\underbrace{(A_1 B_3 - A_3 B_1)}_A \frac{d^2}{dt^2} + \underbrace{(A_1 B_4 + A_2 B_3 - A_3 B_2 - A_4 B_1)}_B \frac{d}{dt} + \underbrace{(A_2 B_4 - A_4 B_2)}_C$$

$$A\sigma^2 + B\sigma + C = 0$$

$$\sigma_{1,2} = \frac{-B/A \pm [(B/A)^2 - 4(C/A)]^{1/2}}{2}$$

For  $\sigma_{1,2}$  to be real -ve,

- (i)  $C/A > 0$
- (ii)  $B/A > 0$

See, if I will end up getting in fact, we can see that, what we would A 1 B 3 minus A 3 B 1 d square by dt square plus you end up getting A 1. Actually, if I were to look it is difficult to see here A 1 B 3 and A 3 B, 1 these are d by dt square term A 1 B 4 plus A 2 B 3 minus A 3 B 2 minus A 4 B 1 will be the d by dt term that is what, we will be writing here A 1 B 4 well, if you have A 1 B 4 plus A 2 B 3 minus A 2 B 3 minus A 3 B 2 minus A 4 B 1 right that into d by dt and finally, we have plus A 2 B 4 minus A 4 B 2 A 4 B 2. So, you see now this is my A, we will write down what is A B C later on in a minute, but what I want to show is this thing, what is sigma 1 sigma 2 length? So, a sigma 1 square plus B, so sigma 1 2 are minus B by A plus minus by 2. Let say, well we can put half here all by 2 this we all know.

Now, the question is in this relation first question, I want to make sure sigma 1 sigma 2 both are real negative, what are the condition? We will find out that 2 condition for

number 1 is  $C$  by  $A$  must be 0, why because you see suppose,  $C$  by  $A$  was less than 0 right. Let us assume,  $C$  by  $A$  is less than 0, then this term is going to be positive then this term is going to be more than  $B$  by  $A$ , because it is  $B$  by  $A$  plus something square root  $B$  by  $A$  plus something square root you agree. See this is  $B$  by  $A$  square plus some small some number square root of that is going to be more than  $B$  by  $A$ . So, now minus  $B$  by  $A$  plus  $B$  by  $A$  plus something small, so at least 1 root will become positive that is one. So, therefore, this you understand.

Now, number 2 is please find out that  $B$  by  $A$  of course, has to be positive because see  $B$  by  $A$  is suppose negative. So, this becomes positive etcetera, but this term will dominate here. So, what would happen is that this minus  $B$  by  $A$  plus minus something. So, if  $B$  by  $A$  was negative then minus  $B$  by  $A$  minus something, because after all this is a square root term. And, if this was less than 0 this would become imaginary term. So, what you end up getting is obviously, that I must have both these conditions satisfied that means, if I what to make sure my  $\sigma_1$   $\sigma_2$  both are real negative, I must have  $C$  by  $A$  more than 0  $B$  by  $A$  more than 0.

Now, comes the second question  $A$   $B$   $C$  now, we have to look back  $A$   $B$   $C$  and find out this is where my last lecture comes in picture.  $A$   $B$   $C$  are they positive are they negative is  $C$  by  $A$  positive negative etcetera, what is the criteria that we end up getting. So, we look at that. So, this part we understand right now, we will look at this part of a  $A$   $B$   $C$  part. Now, you see we have seen this  $A$  1 this terms all  $A$  and  $B$   $B$  and  $C$  lets look 1 by 1 that.



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$$A = \frac{L L}{A_1 B_3} - \frac{S S}{A_3 B_1} = + Va$$

$$A_1 = m - Yv \quad ; \quad +L \quad +ve$$

$$A_2 = -Yv$$

$$A_3 = (m x t - Nv) \quad ; \quad +/- \quad (S)$$

$$A_4 = -Nv$$

$$B_1 = (m x t - Yr) \quad +/- \quad (S)$$

$$B_2 = -(Yr - mu)$$

$$B_3 = (Iz - Nr) \quad ; \quad +L \quad +ve$$

$$B_4 = -(Nr - m x G u)$$

So, my A is A 1 B, what was this A was A 1 B 3 minus A 3 B 1 what is A 1? Now, we will have to look back term by term you know. A 1 is m minus Y v dot right just write this term A 1, then we can write all the term of course, here let me write all the terms on here A 2, A 2 was minus Y v, then we will find out A 1 A 2 A 3 m x t minus N v dot right A 4, A 4 was minus N v B 1 we will, we put this number then we will understand that m x G minus Y r dot B 2 minus Y r minus m u B 3 I z minus N r dot and B 4 minus N r minus m x G u.

So, this what we have got now, let us look at a I will actually, go to 1 and 2 and then I will leave it to you for the next part. Because you also have to do some now, you see this A 1 what did you find last time Y v dot is a large negative. So, minus Y v dot is large in fact, this same as m. So, it is very large in fact, is very large positive A 1, what is B 3? Minus N r dot what did, I say lat class very large positive remember N r dot was very large negative. So, minus N r dot was very large positive. So, this becomes very large positive.

What about B 3 no sorry A 3, A 3 remember x t can be plus minus because depend on which side, but x t is a small number N v dot what was it in last class? Cross couple term small value positive negative it is plus minus, but small right what is B 1 no sorry A 3 what is B 1 same thing Y I dot is actually same as this, because it is a same quantity. So, it is plus minus and small, what do I get large large small small never mind the plus

minus sign here, what do I get here always positive right it is going to be always positive. Now, I am going to leave it to you people to figure out exactly is in the same analysis, why B that quantity B which is given by what if I write the B is given by this.

(Refer Slide Time: 24:20)

The image shows handwritten mathematical work on a blue background. At the top right, there is a small logo that says 'CC BY-SA'. The work consists of several lines of equations:

$$B = (A_1 B_0 + A_2 B_3 - A_3 B_2 - A_4 B_1)$$

$$= \text{always } +ve$$

$$C = (A_2 B_4 - A_4 B_2) > 0$$

$$C = +Y_v (N_r - n^2 u) - N_v (Y_r - mu) > 0$$

$$= \left( \frac{N_r}{Y_r - mu} - \frac{N_v}{Y_v} \right) > 0$$

This is what it is no B and I am going to leave it to you as I said, to figure out from this analysis. You know you should do little bit of exercise as part, because this particular part of course, there is really very less numeric's possible it is more of a understanding. So, you work it out, you will find out that this is also, will turn out for a ship will be always positive, then what do I get, I have to have C by A always positive B by A always positive. Now, I find out that B is always positive a is always positive, so this criteria satisfied. So, it my they among the two now come to that a is always positive, what does it mean what it gives to me it says then that C.

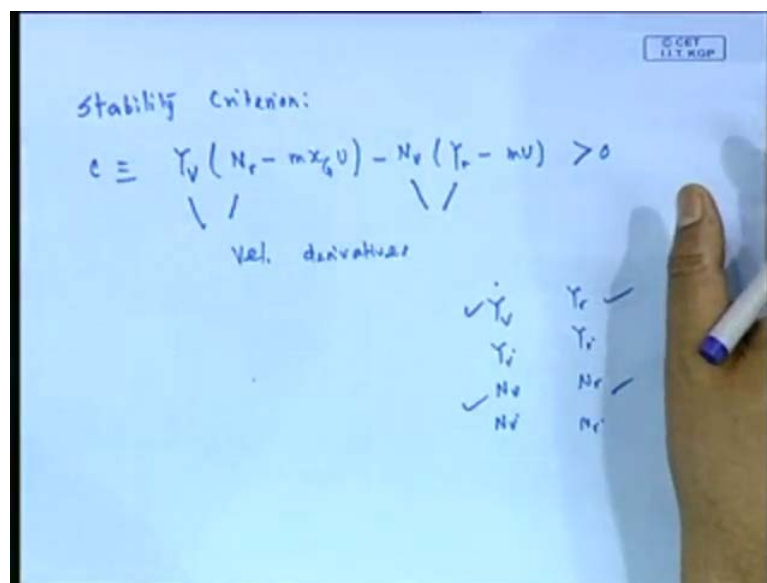
Now, the term C in fact, that also I leave it to you the C the term C. Let me write down the term C here, term C is what was the term C always going that is A 2 B 4 this term, this term if you work it out you will find out it is not very easy to say whether, it will be always positive or not, because it is the competing two terms, which are of similar magnitude. So, what it means, I will write it down this term this full term, this term will turn out to be A 2, we will come to that. Let us write it down this is basically, A 2 is minus Y v B 4 is what is B 4 we have to write it down A 2 B 4 from here let us look at this no A 2 is minus Y v B 4 is this thing. So, it is minus minus becomes plus, so Y v N r

minus  $m \times G_u$  this minus  $A_4 B_2 A_4$  is minus  $N_v$  again  $B_2$  is minus. So, this is minus minus plus minus  $N_v$  into  $Y_r$  minus  $m_u$  I think that is how it is.

Let us see this collectively, whether this is how it is just we just check this right this is just one second, we will be just write down one thing this is basically, this see we will come to that later on this is basic part right. It looks like that, what it basically means therefore, you know. By looking at the condition, what I find out that this therefore, the condition reduces to the fact that this must be more than 0,  $C$  must be more than 0. So, the condition reduces once again, if I were to look back at that, the condition reduces since my requirement for directional stability is that both  $C$  by  $A$  and  $B$  by  $A$  should be more than 0. But now, I found out that  $a$  and  $B$  are always positive for the hull, which means the condition the requirement therefore, becomes for me that is  $C$  must be more than 0.

Therefore my requirement is that  $C$ , which is given by this must be more than 0, that means, but well have I put that that means, I must have this more than 0, which basically means this more than 0. So, you see my condition of stability therefore, turns out to be this is the condition. So, you see here the interesting point that, we have now ended up getting is that, I must have my hull. So, with the values  $Y_v$   $N_v$   $Y_r$   $N_r$  in such a way I actually this is so important that, I am going to write it again in a big way.

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This is what? The condition is now this is now this minus 1. See that, now the most important thing that comes out. You see here I therefore, have a condition saying that my hydrodynamic derivatives  $Y_v$   $N_r$   $N_v$   $Y_r$  the velocity derivatives all are velocity derivatives. Interestingly, I do not have a term arising, because of added mass derivatives what, I said in last class among this  $4 Y_v$   $Y_v \dot{N}_v$   $N_v \dot{Y}_r$   $Y_r \dot{N}_r$   $N_r \dot{I}$  I have only this the velocity derivative coming here, accelerations are not coming. So, it must be such a way that this condition is satisfied that is very, very important or the criteria that has come out to be.

Now, you see this means that whether, the hull is directionally stable will depend therefore, on the values of  $Y_v$   $N_v$   $Y_r$   $N_r$  this is of course, with dimension, I will now, right now go to the usual form of because typically, we write this in a non dimensionless form I did not say that so far, because I thought I will it does not change the conclusion. So, this is what we call this C is what is called stability index in a call, stability criteria various terms are used that means, interestingly supposing, I want to tell I am designing a hull see take the case, I am designing a hull. I want to make sure the hull is stable then, I have to ensure that this side is positive. Again, there are design condition comes in I, if I make it very very positive, then it comes.

So, stable like your very stiff cycle, that turning we will require more effort. On the other hand, I also want to turn therefore, what I will do I want to make it positive, but not much positive. In other words essentially, this value this minus that, how much more than that, which is what I will come to the later on is a measure of stability and therefore, we find out the measure of stability is fully connected to the hydrodynamic derivatives. You cannot say I do not will not study hydrodynamics in order to become find out hardly stable or not, because stability is primarily decided by interplay of the high forces created all these are high forces created, because of the flow pass the hull.

If you look  $Y_v$  because of ship is going in the angle of attack, there is a  $Y$  force created.  $N_v$ , because you know of the same thing. So, you know you will understand this part now we will get back to that. Let me quickly go through a standard non dimensionalize form that people do it normally, that is I did not do that before, but normally what happen, all this coefficients you know, after all if you will write  $Y_v$   $N_v$  etcetera. Suppose, I took a model, so I have  $Y$  verses  $v$ , but if I twenty times bigger ship or hundred time bigger ship, then  $Y$  will be twenty times more  $v$  will be like that.

So,  $dY$  by  $dt$  will become scaled up difficult to understand say added mass. You know  $Y$   $v$  dot, if it is twenty thousand ton ship it may have minus eighteen thousand ton, whereas two ton model, minus 1.8 ton. So, obviously, you would like to represent them in a non dimensional way.

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Non-Dimensionalization of Derivatives / other parameters

$|U|$   $L, \rho$   $M, L, T$  parameters

$$m' = \frac{m}{\rho L^3}, \quad \dot{\theta}' = \frac{\dot{\theta}}{|U|}, \quad \dot{u}' = \frac{\dot{u} L}{|U|^2}$$

$$I_2' = \frac{I_2}{\rho L^5}, \quad r' = \frac{r L}{|U|}, \quad \dot{r}' = \frac{\dot{r} L^2}{|U|^2}$$

$$Y_v' = \frac{Y_v}{\rho L^2 U}; \quad Y_r' = \frac{Y_r}{\rho L^3 U}, \quad N_v' = \frac{N_v}{\rho L^3 U}, \quad N_r' = \frac{N_r}{\rho L^4 U}$$

$$Y_v' = \frac{Y_v}{\rho L^3}, \quad Y_r' = \frac{Y_r}{\rho L^4}; \quad N_v' = \frac{N_v}{\rho L^4}, \quad N_r' = \frac{N_r}{\rho L^5}$$

$$u = \frac{u}{|U|} = 1.$$

So, let us use a standard non dimensionalize of derivatives, then we will write  $C$  in a non dimensional form not only derivatives and others other quantities. Normally, what you do non dimension you know. Usually, you use forward speed  $U$  length  $L$  these are the one that is of course,  $\rho$  you will be using these are the terms basically, used in non dimensionalization like for example. And all the non dimensional terms are represented with the dash typically; in maneuvering studies the non dimensional terms are always represented. The nomenclature that is used standard nomenclature is to put a dash on the term to show that the term is non dimensional.

In fact, manoeuvring is all study of dash and dot dot will be the time derivative dash is non dimensionalization. So, this is written as  $m$  by  $\rho L$  cube normally, as I say  $L$  is used  $v$  dot is use as  $v$  by. In fact, I can call it  $u$  the forward speed, forward speed this is dash of course, sorry this is  $v$  no this is  $v$  dash  $v$  dot dash is going to be  $v$  dot  $L$  by remember, the  $T$  is taken with  $L$  and  $u$ . See normally, non dimensionalization what  $L M$  and  $T$  right non dimenalization, you will use the parameter fundamental thing of that fundamental 3 dimensions are  $L m$  and  $t$  that you agree right. Now,  $L$  is this  $L$  is taken

care of  $m$  comes from  $\rho$ , because  $\rho$  is after all mass by length cube. So, from  $\rho$  this  $m$  comes and  $T$  is comes from  $L u$  is  $L$  by  $T$ .

So, here the non dimensionalization, I usually done with  $u L p$ , which actually involves all the 3 fundamental dimensions, how you are combining them. So,  $v \cdot$  becomes  $v \cdot$   $L$  by  $u$  square  $I z$  dash will become  $L$   $5 r$  dash will become.

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Or  $v$  is actually this  $v$  is 0. Now, this  $v$  will not come here this is only dash etcetera, you know actually of course, then also  $u$  the forward speed  $u$  this is actually, as a let me call this not  $u$ . I call it  $v$  that  $u$  is the actual total velocity. So,  $v u$  the forward velocity becomes  $u$  by  $u$  bar will become 1, what I will tell you what is  $u$ ? See this  $u$  is the velocity vector  $u v$  this part is  $u$ . So, obviously, this for small value of  $v$  see this small  $u$  the forward velocity along  $x$  axis is this into  $\cos \alpha$ . And this is this into  $\sin \alpha$ , if  $\alpha$  is the angle of attack. If you say, if for small angle of attack means small value of  $v$ , which is what we assume obviously,  $\sin \alpha$  becomes  $\alpha$   $\cos \alpha$  becomes 1. So, that is why it becomes this.

So, that is why, when I non dimensionalize this terms you know let me look back at this equation. Again, if you non dimentionalize this term all that will happen, this will become dash  $S$ . And you will be able to find, I will look at  $Y v v$  you will find out that  $Y v v$  will have a unit of force, because after all this should have a unit of force  $Y v \cdot v \cdot$ , you will have unit of force well. Let me just explain that once from this expression.

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$$\begin{aligned} \gamma_v v &= \gamma_v' \cdot \frac{\rho}{2} L^2 v \cdot v' \cdot v \\ &= (\gamma_v' v') \frac{1}{2} \rho L^2 v^2 \\ \tau \cdot v &= \tau_v' v' \times \left( \frac{1}{2} \rho L^2 v^2 \right) \\ N_{r,r} &= N_r' r' \times \frac{1}{2} \rho L^3 v^2 \\ u' &= \frac{u}{|u|} = 1 \end{aligned}$$

See for example,  $\gamma_v v$  into  $v$ , you will find out it is  $\gamma_v v$  dash into  $\rho$  by  $2 L$  square  $v$  and  $v$  is going to be  $v$  dash into  $v$ , which will be  $\gamma_v v$  dash  $v$  dash into half  $\rho L$  square  $v$  square, you will find there is a unit of force. Similarly, I will leave it to you to find out that  $\gamma_v \cdot v \cdot v$  dot will be  $\gamma_v v$  dot dash  $v$  dot dash into this will have a unit of force. You can figure it out all, if you want to do it will obviously, half  $\rho L$  square  $v$  square, if you see  $N_v$  into  $v$  same thing you will find out, but if you find  $N_r$  into  $r$  you will find out this will become  $N_r$  dash  $r$  dash remember this moment half  $\rho L$  cube  $v$  square, because force into distance.

You know, you can figure it out yourself you can check it, but the important point here is that you will find out that this  $N \times G$  and especially, this term  $N \times G u$ ,  $u$  will become  $u$  dash is become 1, because you see  $u$  dash. So, as a result what happen, this term will become 1 this term when you dash it, you know when you dash it. When you, when I just want to show you here without having to write much, if I want to put dash it will become all that dashes only thing is that this term will become 1, because  $u$  dash is 1. So, this will become dash his will become one, that is all not much to like understand that. So, this is only just for the sake of telling, but let us looks back at this expression, here that is in terms of non dimensional coefficient.

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Stability criterion:

$$C \equiv Y_v (N_r - m x_G u) - N_v (Y_r - m v) > 0$$

Val. derivatives

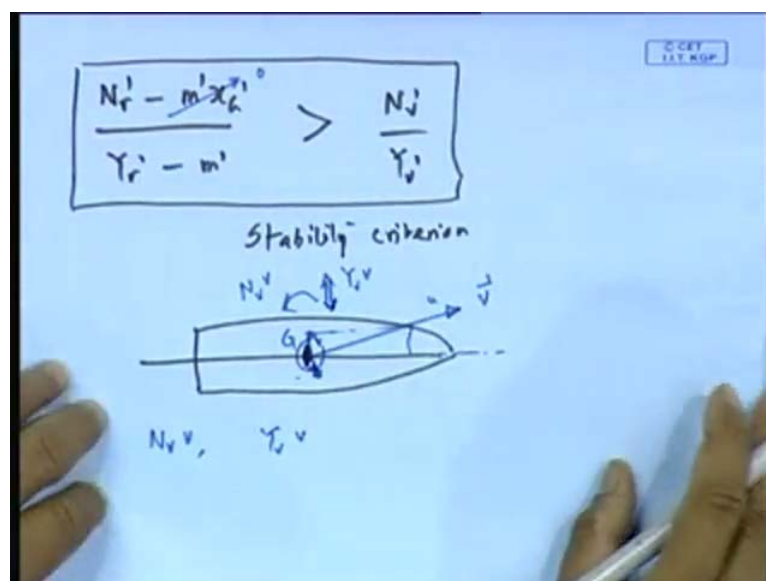
$$C = Y_v' (N_r' - m' x_G') - N_v' (Y_r' - m') > 0$$

$$\Rightarrow \left[ \frac{(N_r' - m' x_G')}{(Y_r' - m')} - \frac{N_v'}{Y_v'} \right] > 0$$

✓ $\dot{Y}_v$	$Y_r$ ✓
$Y_v$	$Y_r$
✓ $N_v$	$N_r$ ✓
$N_v$	$N_r$

So, if I were to term, in terms of non dimensional coefficient, what will happen in non dimensional form C will become  $Y_v$  dash  $N_r$  dash minus  $m$  dash  $x_G$  dash  $u$  become 1 remember minus  $N_v$  dash  $Y_r$  dash minus  $m$  dash must be equal to 0. This, if you see will turn out to be an expression like that this is same as saying this very important or rather, I will looking right this carry it back  $k h N_r$  dash minus  $m$  dash  $x_G$  dash divided by  $Y_r$  dash minus  $m$  dash minus  $N_v$  dash by  $Y_v$  dash more than 0, this will turn out to be same as this. So, this I will repeat next phase.

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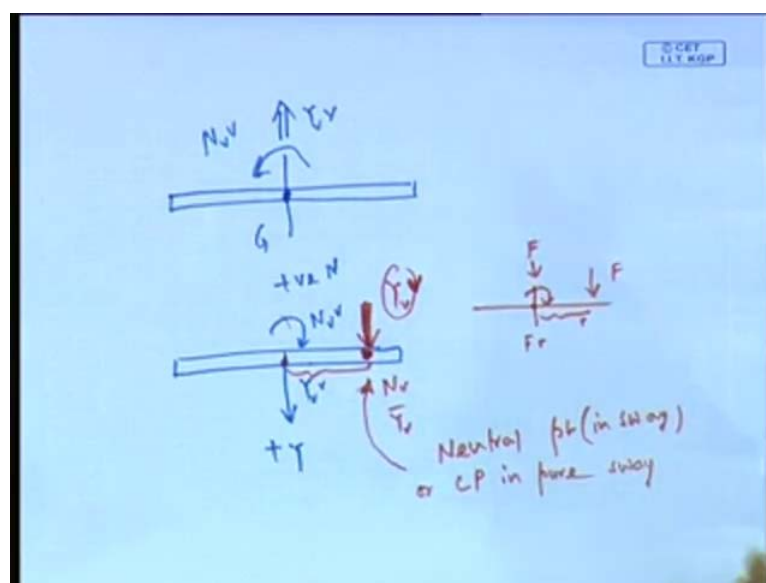




So, the criteria becomes  $N \cos \alpha - m \times g$  divided by  $Y \cos \alpha - m$  is more than  $N \sin \alpha$  by  $Y \sin \alpha$ , this is my criteria. Let us put some physics into it let us look at this term take this ship, what we will do is part where you know, we will put let us just put  $o$  at  $C.G.$ . So, that I do not have this term you know, because it is up to us you can always put that. So, if I put this  $x \cos \alpha$  let me put  $x \cos \alpha$  as  $0$  let me put  $x \cos \alpha$  as  $0$  means, this origin is at  $G$ . Now, let at this  $Y \sin \alpha$  by  $N \sin \alpha$ , you see what is happening understand this very very interesting. I am having a ship moving what is  $Y \sin \alpha$  force, because of  $v$  what is  $v$ , what is  $v$  dash?  $v$  dash is angle of attack that means, I am going along this direction. So, I have got  $u$  here, I have got  $v$  here rather sorry no sorry not this side  $u$  here. I have got  $v$  here this is my  $v$  right, when I go angle of attack I have  $v$  here.

Now, what happen, this is going to give me force some force is giving. So, it is going to give me see, when I go that this side the force is coming, this is also moment coming. So, the net force never mind the direction, the net force and moment is going to be acting at some point. Because what is happening see, I am getting a moment  $N \sin \alpha$  and a force  $Y \sin \alpha$  a moment here, I am getting some moment, never mind the direction here  $N \sin \alpha$  and I am getting a force here, which ever direction the force is actually, here may be other direction  $Y \sin \alpha$ . Now, you see here what is happening, this hull I am getting, if I give a  $N \sin \alpha$  and  $Y \sin \alpha$  tells me that, I get a moment  $N \sin \alpha$  and I get a force, yes it can be other direction it can be other direction whatever, the diagram is done.

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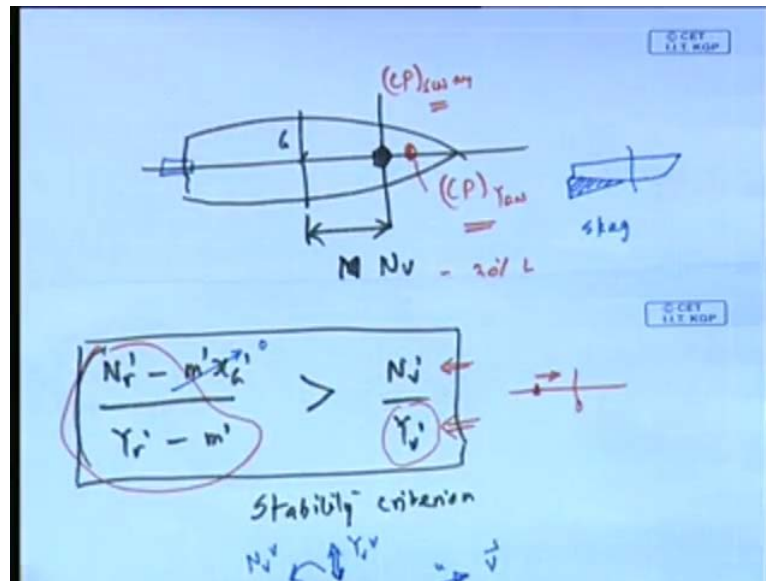


So, my point is that, if I were a stick like ship here there is a ship G here, I am getting  $N \cdot v$  here and I am getting. Let us say here,  $Y \cdot v$  which ever or you know, this the opposite side I wrote. In fact, because these are all negative sides in fact, I think I should discard that and write you this, because see remember this is my positive axis. So, I am getting here,  $N \cdot v$  and I am getting here a  $Y \cdot v$  what does it mean is essentially means, an equivalent force acting here at a distance of  $N \cdot v$  by  $Y \cdot v$ . Suppose, there was a distance force  $Y \cdot v$  force acting here, what would have happen this force is equivalent to this force  $Y \cdot v$  sorry not  $Y \cdot v$  And also, equal to this moment, because this into this distance, but whether this distance is  $N \cdot v$  by  $Y \cdot v$ .

See, it is like you is a basic carematix no a force here,  $f$  is equal to a force here into a moment of  $f$  into  $r$  right. You understand that see, if I have a force here acting this is nothing but a force here and a moment about this point. Now, what we here is opposite, I have here for this hull a moment here  $N \cdot v$  that is what I have described and a force here  $Y \cdot v$ , which is equivalent to a force acting at this point, just 1 force at a distance  $N \cdot v$  by  $Y \cdot v$ . So, this point of action you know is like, you can call a central of pressure into your yaw that means.

Supposing my ship was yaw is having a  $v$  motion them the hull would have experience the force at this point, which is equal to a force here and a moment here this distance, this is called a neutral point or in sway or neutral point. Actually, this is assumed or center of pressure in pure sway this particular point. So, you have to spend few minutes time.

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So, what is happening, if I were to now go back to this diagram this is 1 point. Now, look back at this equation, this equation I have here  $N_r$  minus well actually, I did probably made this another equation right here, I have  $N_r$  by  $Y_r$  minus  $m'$  must be more than this now  $N_r$  by  $Y_r$  minus  $m'$  dash this term. If you look at this term in a similar way or this full term becomes the center of pressure. So, called for pure yaw see here this comes only for yaw motion  $N_r$   $Y_r$  for yaw motion. So, this term, this represents to me this represents to me at the distance center of gravity, because I have discarded that distance from center of gravity of the point of action of force. If the ship was having a pure sway, just  $v$  this one it was only going in a angle of attack no rotation, then it will be at this distance from the CG as, I have explained here in the other diagram, that is it is at this distance from the CG, where my force acts the total force act.

Whereas this other term, this term would be that the distance where the force would act, if moment would act, if there was only yaw motion. So, what the criteria, tells you know is that the center of pressure of yaw that, if I call this to be  $CP_{sway}$  and if I call this to be  $CP_{yaw}$ , this must be ahead of this that is what it says. Now, you see now suppose, the ship is not stable, what means that this is less, how do I make it stable, I can make it stable by making, see  $Y_{\dot{v}}$  remember here again  $Y_{\dot{v}}$  is always negative, we have discuss that and of course. In this diagram typically for ship, you know this distance typically for ship is about you can say thirty percent of that this is fifty percent this about thirty percent of the length. Typically, for ships not always, what it tells me is that.

See remember this forward side that means, if this is negative this also negative typically, for ships you know. So, it is typically negative this negative and now of course, we know that this is always negative, that we have seen in my last lecture you can look back at that, what we are finding out therefore, is that typically for ship also this is negative. The negative implies, if you look back the forward part of the hull is dominating forward part of the hull is dominating. Now, supposing I want to make the ship stable, you know what I have to do, I have to make sure my aft becomes dominating basically, I have to make  $N_v$  dot suppose,  $N_v$  was negative somewhere here. So, this is 0 value somewhere here, I have to make it less negative, remember  $N_v$  I have to make less negative going towards positive side.

What it means go back to the last class, I gave to make the stand aft part more and more dominating, how do I do that add a skeg that is exactly, we keep doing there is a hull here, I want to make aft dominating what I do I add a skeg or I add the forward part or I can move the C G forward other way round. So, C G forward will do the same thing. Now, you understand why for a bow and arrow my C G is forward why feather by feather, I am adding steg dominating stern by moving my Eigen on the front, I am making my C G move forward. Essentially, I am trying to make this forward so stern dominating, because I want to make sure this moves back therefore, that this criteria satisfied.

So, you see there is a full reason now, we understand from physics. So, in design also you have a problem, you have to do that. Anyhow, I am going to stop it here time is up for today's class, we will get back on this little bit and then carry out from here on to other topics. So, with that I will. So, about this is very important, because now you have a understand me that directional stability essentially, is a playing round with hydro dynamic derivatives, which is nothing but the hydro dynamic characters for the hull and yaw and therefore, cannot avoid hydro dynamic forces, if you want to study you want to make sure the ship is stable. So, with that I will end.