Seakeeping and Manoeuvring.

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Lecture No. # 26

Hydrodynamic Derivatives

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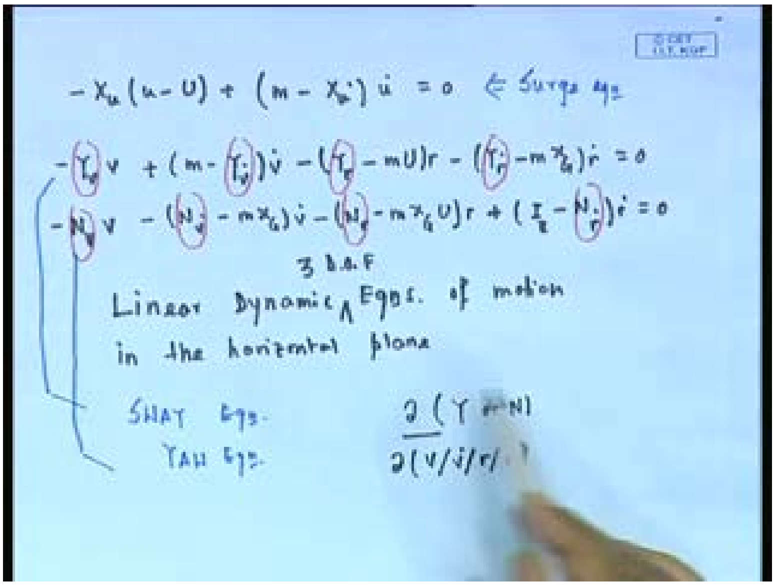

See in the last class we talked about the equation of motion that we have to write I will just write it down just this way around or maybe because to start it we always have to write see this were what we wrote yesterday in last class as the linear dynamic equations of motion in the horizontal plane. So, this is my first one is. So, called surge equation.

Next one sway equation and this one is my yaw equation. So, we had got here three modes of motion x y z. So, sometime you may call this to be linear dynamic equation of motion in the horizontal plane. I would write three degree of freedom by meaning, here I have got only three modes where I am allowing the ship to move. Why I am saying you know this is the simplest one that you have to have you could also have little more complicated one for example, if I allow the ship also to roll because, when it turns it may also roll.

Then I would have one more equation here would have been roll equation of motion then, I would have four d o f equation of motion etcetera. So, I could have more complicated equation of motion.

Now, what I mention yesterday if I if you look at that the important part in this equation of motion the part that consist of hydrodynamics of the hull arises in, I will only look at sway and your equation primarily because this equation is of relatively less importance for us.

See here it is terms like this terms like this terms like this terms like this and it is this. So, we have this terms and I mention that what does this terms mean they basically means the rate of changes of forces or moments y or n that is d of y or n that is d of y or n against d of v v dot r r dot rate of change of the sway force or yaw moment with respect to sway velocity or sway acceleration or yaw velocity or yaw acceleration.

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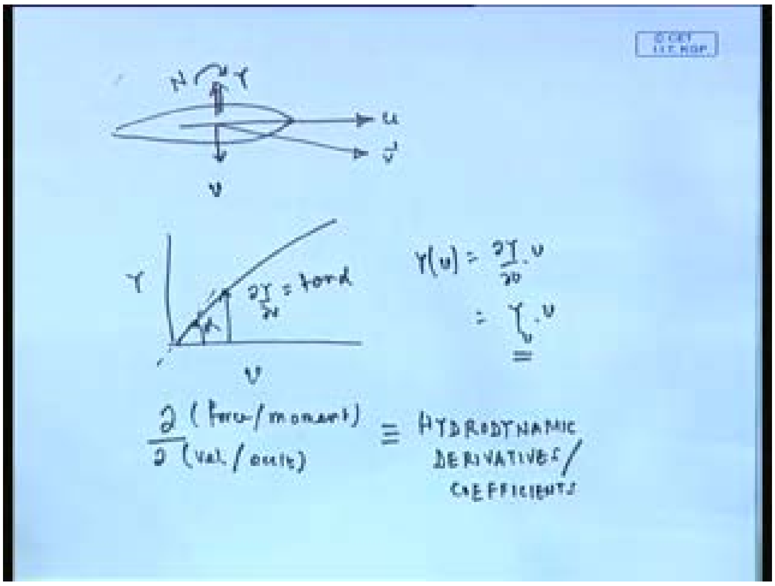

What it means is that I have hull now I am trying to leave it a velocity here v what we want to look is that supposing the ship is going see what means; that means, that the ship is actually going with this velocity vector here. So, the question is; that means, it has got the component of velocity this side

What we want to find out if suppose there is. So, much v what is the corresponding y what the corresponding n. So, suppose I could measure say for a given v for whatever v this is y. So, if I could measure v verses y arising only because of v let us say some kind of a graph then this slope would be my d y by d v.

What does it give me if i call this that is; that means, if I want to find out some value y at v then it is going to be d y by d v into v or Y v into v this is the approximation with the linear approximation of the force. So, this terms you see this terms which are hydrodynamics forces rate of change of hydrodynamic forces or moment rate of change of hydrodynamic forces of moment against any of the velocity parameters this terms are known as hydrodynamic derivatives because they are hydrodynamics derivatives derivative of the hydrodynamic forces and moments.

So, you see this terms that is d of d some people call them also coefficient some people sometime call them also hydrodynamic coefficients see what happens this is a very important concept you know; that means, I have a given geometry.

Now, i am trying to give it some kind of acceleration in some direction I will come to this one by one we will see this magnitude etcetera then it; obviously, then certain hydrodynamic forces created; obviously, that will depend on geometry suppose you take a ship a particular geometry if I give a v regardless of it is whether today tomorrow day after it will give. So, much y So, d y by d v will become in centric property of the hull that is the characteristic of the hull.

Once again I will come to that suppose I have a given geometry now in that given geometry say design number one say design number one I give it certain velocity v it will give you certain y now d y by d v that is power unit y the amount of v bar the amount of y that comes out will remain same for the hull that is the characteristic of the hull that is for example, if you if you have a hull.

Now, if it was having a draft of say five meter it is going to have certain k b it is going to have certain k b center of buoyancy now that k b is a fixed for the hull if it is having five meter hull it will have. So, much of k b or. So, much of b m t regardless of you know what you say. So, that becomes a characteristic of the hull.

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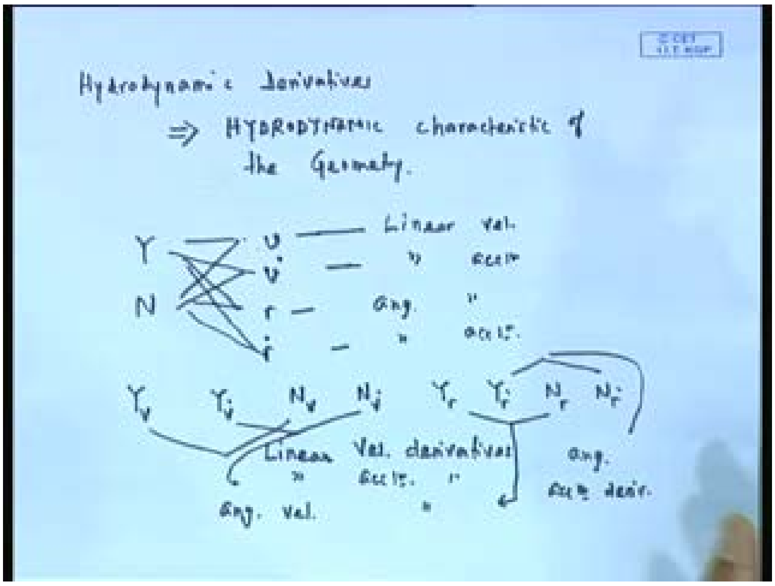

So, hydrodynamic derivative is therefore, why I am saying is not because just like certain parameters like hydrostatic parameter for a given geometry.

Remember when I give the example of k b at five meter remember at five meter draft the underwater hull geometry is fixed. So, for that geometry there is a fixed location of center of buoyancy similarly there is a given geometry you; obviously, if that given geometry if that has got a certain amount of v then it will have certain amount of y; that means, its rate of change of y against v will remain constant

So, these things therefore, characterized is the hull added mass we will come to this added mass etcetera in a minute they are all characteristic of the hull. So, this is what call the hydrodynamic derivation now how many we have got remember if this two I have got I have got n this side I have got v v dot r r dot this is velocity acceleration acceleration right.

So, I have got if you look at that i will come back to that I have got Y v Y v dot N v N v dot y r y r dot N r N r dot you know these are the eight linear derivatives that we have if you look at this first here you will see is that eight there one two three four five six seven eight very important eight derivatives that is obvious because I am talking of two motion forces y and n and four velocity. So, you know to into four gives you eight of which you know just for definition purpose this is this two are linear velocity derivatives.

Linear because v you know straight line velocity and r is rotational velocity. So, you can say then these two you can say are linear acceleration derivative right this two I can say r and you will have velocity derivatives and this two are going to be angular acceleration derivative what I means in therefore,.

See I have got linear velocity this one. So, Y v N v this is linear acceleration this is angular velocity angular acceleration what I why I am wrote is that this is only for your convenience you know you can therefore, find out the if I want to call more generally velocity derivative then of course, this are velocity derivatives and if you take acceleration derivatives this are acceleration derivative.

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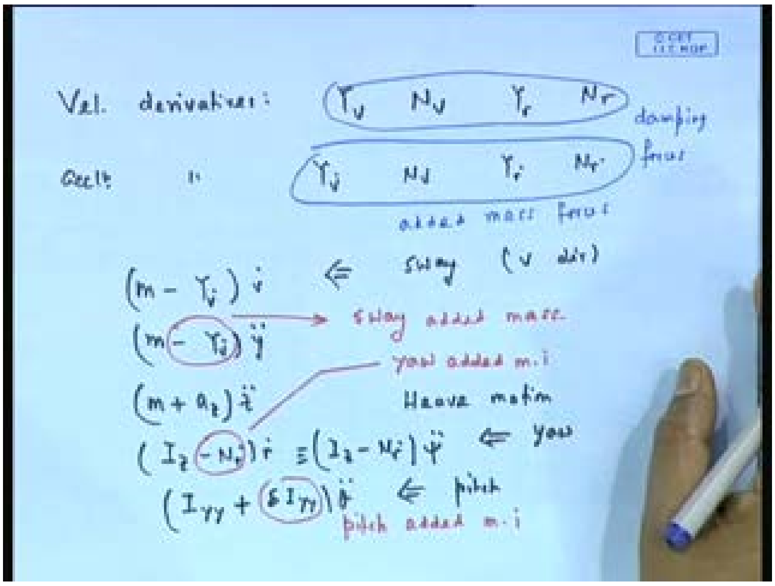

So, I have got basically I again if I want to say that I have velocity derivatives I have got acceleration derivatives the which these are linear velocity derivative these are angular velocity derivative etcetera now I will like to discuss this term turn by turn about the magnitude are they positive are they negative etcetera, but remember why it is important because see once again we go back to the equation of motion; obviously, it depends on the magnitude.

We need to know whether they are positive negative etcetera because this has the bearing on how the equation of motion will have a solution just like your met centric height should be positive there is a condition that comes out for servility purpose

So, we of course, need to study this because the unknown part in this red marks this derivatives the rest of the terms are only mass property see this m m center of gravity location mass distribution moment of inertia. So, rest is basically mass and moment of inertia and location. So, the hydrodynamics comes only in this; that means, the unknown hydrodynamic forces that has got created on the hull are fully represented by this eight terms in this equation of motion of course, if you take more complex equation and it will be more, but.

To start with the basic minimum unknown becomes this eight provided there is no rather action I have not taken a rather action rather is not working it is only a bare hull now let us look at the terms, but before that let us also look at this physics see here I have this term n minus Y v into v dot I will just look back at this term this m minus Y v dot into v dot.

So, I have this acceleration term becomes n minus Y v dot into v dot compare that with what we did in sea keeping and saying heave this is an sway mode right. So, essentially this is m minus Y v dot into this is y double dot you know like because if I call x y z to a displacement analogy with that c in heave my inertial force as mass plus added mass into z dot.

So, if you compare that what you find you will; obviously, find that this term this is this is nothing, but sway added mass with a negative sign now similarly if you look at this i z minus N r dot r dot this term I will write it here which of course, is now you see in the pitch or roll.

Say in pitch equation I have I Y Y plus delta I Y Y into theta dot what do you find what was this was my pitch added moment of inertia. So, obviously what happen to this.

This is equivalent to added moment of inertia right. So, you see what I mean is that some of these terms are actually same as added mass added moment of inertia that we have seen again added mass added moment of inertia are also intrinsic property of the hull. So, we now understand this now why I am saying is therefore, what happens that this terms this acceleration derivatives.

This essentially because what we call added mass forces and of course I can call then to be damping forces but this damping is not same as the damping that we had in free surface. So, but these added mass is same as in the case of sea keeping I can call this to be damping forces but I am very kind of hesitant to call that. So, straight forward because you have to understand that in sea keeping I had added mass and damping that damping arose because of free surface waves because the ship created waves but in this case they are not creating waves and that phenomena of this force is not same as that damping.

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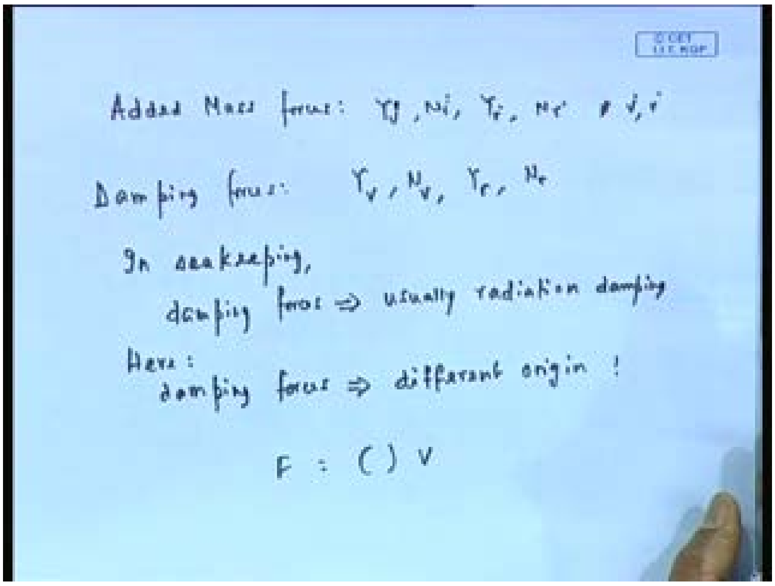

In other words they are damping was called radiation damping forces means that that is what we call them in sea keeping let me write it down here one otherwise you will have confusion. So, we have got see they are not forces they are basically this multiply multiplied by the respective velocities v dot r dot that what you get added mass forces and moment

Damping, see these are all dot here; the only thing I want to tell you is that in sea keeping what we studied damping forces I just writing that the different origin you know the why I am saying why I am want to tell you is that essential in fluid mechanics this is another thing that you should understand essentially in fluid mechanics any force that is proportion to velocity; that means, any force that you write something into velocity you normally call it damping fore that word damping is the very generic term you know if it is proportion to inertia.

Where acceleration you call inertia force see in any equation of motion mass into any force in proportion to acceleration is called as inertial force any force is in proportional to velocity is called damping force and any force in proportion to displacement we call stiffness and statistic force.

So, here because of the nature we are calling damping force, but this damping force should not be consist to the sea keeping damping force we have done in the first part of that course why I am saying because the added mass force remain same, but the damping force does not remain same that one must recognized. So, this we can keep going on, but.

Let me now go to one interesting point let us taken term by term to find out their magnitude where they are positive where they are negative what happen etcetera this a very interesting now let us take v.

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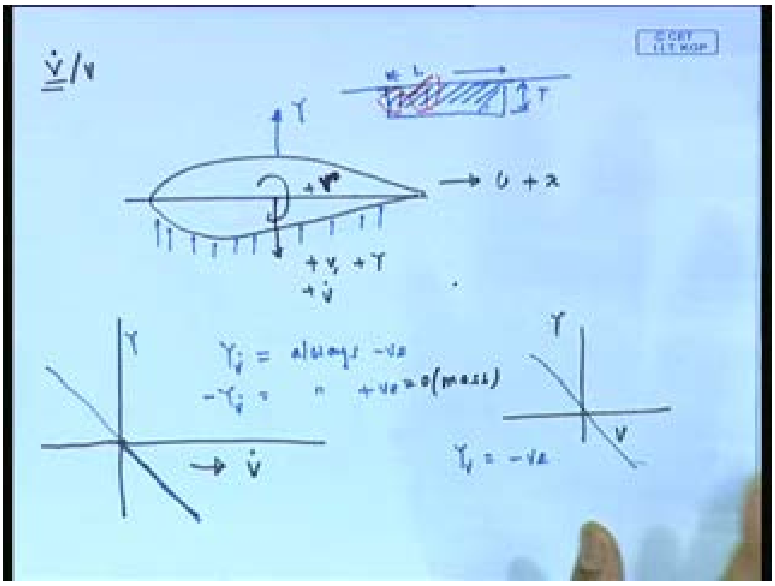

So, I have this body here now I give a see remember this is my direction right now let us take one by one suppose I accelerate the body this side plus side remember this is plus side this is a of course, minus side see positive axis no this is plus x this is plus y and this what we are calling this side plus psi or theta whatever plus r let me rather put it plus r x two y axis system matures

Now, you see what we are doing we have to pay attention to this now you know now I am giving I want to find out there is a graph I am putting let us take first Y v dot v v dot v dot and v say I am giving a v dot plus v dot means I am accelerating the volume this side I want to find out which direction and about how much y will come.

Now, you tell me if I were to push it accelerate this side what would happen the resistive force if now remember accelerating this side the forces resisting will all come in this side right if I then that means, ultimately what I will get is y will be this side see if I want to push the body this side which side the force will come fluid force on the other side.

So, if I give the plus v dot say I am accelerating here I will get a minus y. So, if I get a plus v dot I am going to get a minus y that means, I am going to get evaluate some area that mean the graph will look something like that or rather let me put it in different color and if I get a minus v dot; obviously, I will get a plus value. So, what the then what happen this y.

So, what happen to Y v dot is always negative. So, that actually minus Y v dot is always positive. In fact, that make sense because minus Y v dot is the added mass as I have just found out here earlier minus v Y v dot is added mass this added mass is of course, is positive. So, you understood. So, interestingly what you will find out Y v dot is always negative that is one how much is the magnitude now added mass.

Added mass is in this mode that is you are pushing the hull this side into a hull. So, if I were to look the cross section of the hull and we put that here somewhere here this is my draft t this is my if I swayed it then basically it is added mass it is essentially same as the mass of the flea being accelerate.

So, this mass is going to be almost same as the mass of the hull itself this we have talked see now how do I explain that see if I took this pen here in heave you remember when I say the way it push down the entire bottom part of the particle get push down. So, that the mass of that added mass becomes almost like semicircle added to this length.

So, this if you do that it will become almost same as the mass of the hull itself. So, if you do the same thing this side here we are doing this side same result here it will be the bottom surface pushing the water there will be the side surface there is a the surface that l by t that as if this much is in pushing. So, it is going to be essentially similar to some kind of a you know I will tell that like water mass of this semicircle or whatever like the semicircle you know almost like this mass; that means, added mass in sway is almost same as the mass or rather this is more or less order of mass.

Order of mass why I am saying this because; that means, it is very large quantity after all mass of hull is very large quantity. So, this is a large negative quantity not Y v dot actually Y v dot into v dot will be actually this is not yeah this is added mass sorry. So, Y v dot itself is a very large negative quantity.

So, this we have done now let us look at Y v v verses y. So, I the same thing will apply if I were to give it here a v not in acceleration velocity what will happen the fluid is going to again going to give a push on this side right. So, what will happen you are going to get again a graph which will look for a positive v i get a negative y for a negative v i get a positive y.

So, I am going to get something like that again Y v will become it is also fairly large one may not be large as this we do not know but it will be negative quantity moderately large I can say moderately large. So, you see; that means, now always. So, this is universal; that means, Y v and n Y v dot are always negative.

Now, we will come to N v and N v dot for the same v we will try to see this N v and N v dot in few minute before that n N v let is look at this y r then I mean like we will take one by see we have got eight terms we need to go slowly one by one to find out how they are. So, this is with Y v and all let us look at this.

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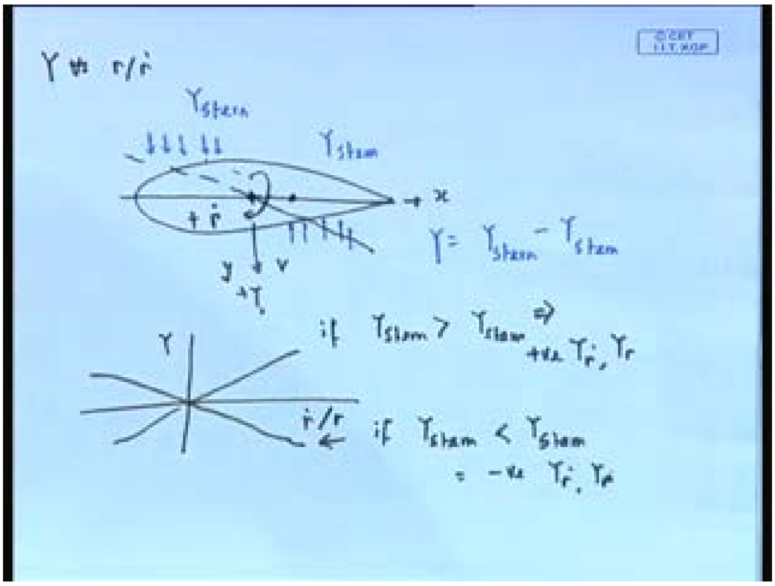

Now, here actually let me write this down this Y verses that is what we have done this Y verses v v dot now we will do Y verses r r dot we will try to see that is d y by d r d y by d r dot let us see how they look like again. So, I am basically turning it basically I give a plus r remember this is plus r.

This is plus y all right v is not there. So, we do not care remember when we do Y v v r etcetera which means I am trying to evaluate for only that particular motion only for v what is y only for r what is y that is what we are doing. So, here I have to give it only r what is r turning it. So, I turn it what happens here see here if I turn it fluid pressure acts on this side and here fluid pressure acts on this side right. So, let me call this y stern.

This is y stem. So, what happens to my y y is y stern plus y minus y stem because remember this is my plus y direction once again let me see this is my plus y direction y stern is plus y stem is minus.

Now, or rather well let us first talk r dot part right r dot part instead of r same thing r dot r will remain same if I talk r dot part now you see if y stern is larger than y stem then what would happen it will become positive. So, this will become see here I put in this r r dot it will become right but if opposite happens it will become like that.

So, in this will give me. In fact, I can make it r r dot both because you know now the same thing will happen whether this is y what now the point therefore, is that see as far as y against r or r dot is concerned that is another interesting thing y and r this I will come afterwards when it comes to y and r or r dot is concerned half the hull produces y in one direction other half of the hull or half means half part on the other direction now. So, therefore, it is like large number minus another large number

The final number is the small number. So, first of all the values of these are small number one number two is that whether they are positive or whether they are negative that depends on whether stern is more influential that means, stern is dominating or stem is dominating for example, this is why I will tell you an interesting point suppose this y i will tell you an interesting point suppose this c g point if you make it move forward what would happen stern will become more dominating and therefore, it will become more and more positive.

You agree with that see the physics you see if I were to shift this point the rotation here if I were to shift the point the rotation here what happens the stern part becomes larger. So, that stern becomes more and more larger number. So, therefore, this will become more and more positive or even it is negative there suppose there is a hull I had my y r y r dot negative.

If I shift my c v forward it will be less negative going towards positive. So, what our y r y r dot positive or negative depending on the which is better it can be both, but small relatively small value relatively small value compare to the Y v Y v dot value we have none. So, this we have done now.

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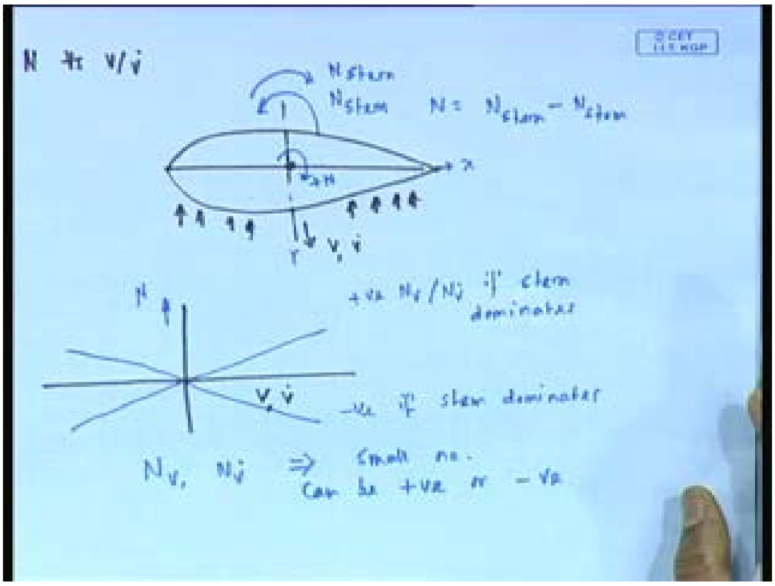

Now, we are going to look at the n part that is now we will see N verses v v dot now remember here I am giving a v v dot same thing. So, what happen I have got here force is acting this side what will this forces do.

This forces are going to give me this side forces are going to give me this side n see I am pushing this body this side. So, the fluid forces are acting along this line right now I am trying to find out n the moment remember this is my plus n by axis because this is my x this is my y right now you see this part of the hull is going to give me moment this side this force is moment this side.

This force is going to give me n stern. So, the next moment n is again n stern minus n stem agreed. So, this will apply to you give a v or v dot. So, then exactly the same thing is going to happen. So, if I were to now plot v v dot I mean I am taking both of them together one could take only separately now what happens see if n stern is more than n stem the I am going to get a positive value if this is again dominating n stern dominates n stem then it will this n or it is going to be like that.

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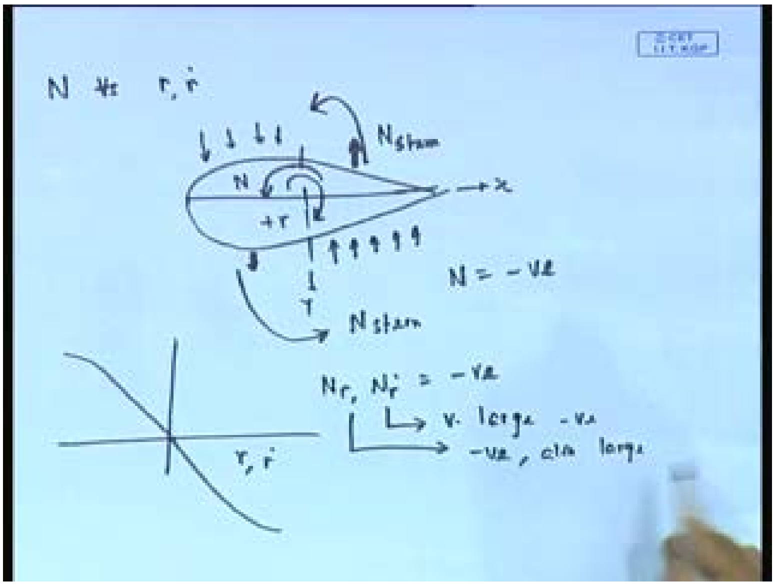

So, the conclusion again N v N v dot can be positive or can be negative now it is agreed right. So, we end of having also this conclusion now comes the final one that is N verses r r dot this is little more again I have this hull now I am remember giving the rotation of r plus r this is plus r.

What happen if I do that this side my fluid forces acting this way and this side fluid forces is acting this way because it is time to rotate right. So, what this is going to do this is going to give me this side force is going to give me a moment this side that is n stem and this is going to give me a moment this side that is true

So, therefore, my net n is going to be negative because it is going to be this side total n agreed see if I were to rotate it this way this giving force this side this giving force this side this giving moment this side this giving moment this side both of them are going to give moment this side that means, when I try to turn it here I get a moment this side when I want to turn it this way I get a moment this way.

This part again gives me see you can imagine there is a net force here that gives moment this side and there is a net force here that will give you moment this side. So, I am going to get a merge because both forward and outside gives me moment of the same direction they give added. So, therefore, what I will get if I get r r dot then I am going to get always. In fact, N r dot we have seen in earlier I just explain here minus N r dot is your added moment of inertia just like each added moment of inertia. In fact, it will have a magnitude for this thing of almost mass into square which is something like point two five l very large negative number.

So, we end up getting the same thing but this may not be very large maybe because it is a velocity derivative it may be moderately large. So, you see we end up getting this confusion that is very interesting now I am going to summarize this conclusion now. So, what I end up getting here is now see why I am taking time because this is very important because this magnitude actually will decide the characteristic of stability of the hull that we will talk in the next hour.

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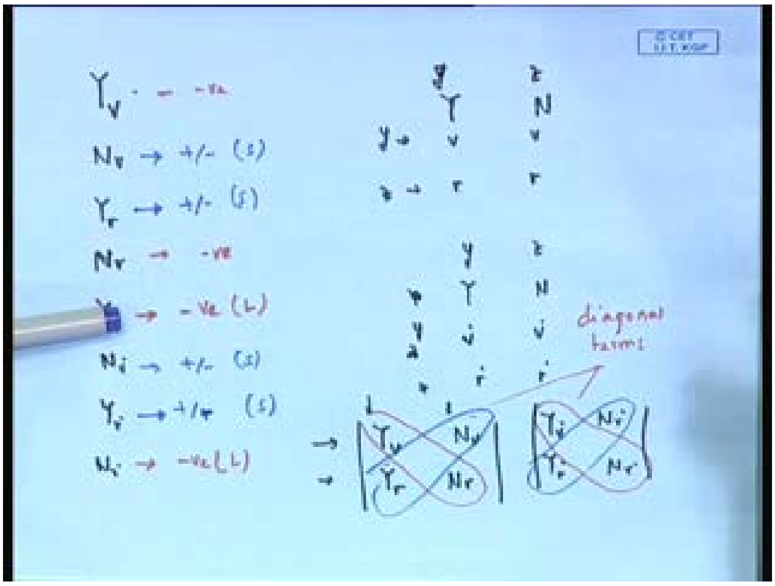

See Y v N v y r N r Y v dot N v dot y r dot N r dot now I will also write this thing see here remember direction x forces here is y and n that is x sorry y direction and z direction moment z direction now in Y velocity here is v and z direction velocity is r. So, this is basically v v and r r if I write why I am saying you know basically you will see.

Similar with happen y z y z if Io like write acceleration this y n and sorry this y n is here if I write y and z here this v dot v dot r dot r dot you see Y v dot and therefore, this term therefore, what I mean that that the term becomes like that Y v N v y r N r it is like a diagonal term here because this is actually y axis is this and here Y v dot N v dot y r dot N r dot.

I will tell you what I meant by that see y force is the force is in direction y y in which the velocity is v. So, Y v is the. So, called diagonal term means you are trying to find out force in the same direction in which velocity is taken.

Once again let me explain that Y v is force in the direction y with respect to or gradient of that with respect to motion in direction y that is d y by d v once again Y v represents force in direction y d of force in direction y divide by d of velocity in direction y both are in y direction. So, d y by d v.

Whereas d y by d r would be force in direction y divided by you know rate of change against motion about direction z because r is that. So, that is y and z this one N v moment in direction z because or due to velocity in direction y d n by d v this is motion moment in direction z because of motion moment in direction because of motion in direction z. So, that is why this two are the diagonal terms of

If I write here y z y z this side the force is this sides are motion this two are diagonal terms this two are cross couple terms because they are represent force of moment in direction one one direction due to motions in another direction.

Do you understand that it is force in one direction because of motion in another direction. So, this two are my diagonal terms and this two are my cross diagonal term cross couple terms. So, they are these waves are diagonal terms of which these two are added mass terms this is actually sway added mass with the minus sign minus of sway added mass minus of yaw added moment of inertia

Why I am saying that remember this diagonal term turns out to be if I look at this these two diagonal terms Y v dot N r dot large negative even this two are negative and fairly large I am not putting the large part a Y v and N r that is what we are found out in all these part that we have gone through if you if you look back now going step by step see Y v all dot is always negative Y v also Y v dot is always negative Y v is also negative.

Y v dot is always negative large Y v is negative we have done that then look at these y r y r dot can be positive or can be negative, but small values. So, y r let me put a different colour and y r dot then we had N v N v dot again it can be positive or negative, but small value and finally, we had got N r N r dot negative which I of course, wrote N r dot negative large and N r negative large.

What do you find interesting what you will find this cross diagonal term are always positive or negative or small value the diagonal terms are always large value it also makes sense because diagonal are negative diagonal terms are always negative what it means by the diagonal terms that is force coming in at particular direction because of the motion in that direction.

So, of I want to push it in this direction I will always get a force opposite direction if I want to rotate in plus r i will get always in moment in opposite direction right it is like you know stability if I want to give a theta this side I get a restoring moment opposite side coming because of diagonal term unfortunately here we have also cross that diagonal terms. So, anyhow. So, this is what we should remember. So, this is a very important and interesting point of which as I told you this terms are added mass terms these are damping terms.

Added mass terms are inertia forces these are forces because of velocity we do not yet know necessarily or rather I can say that understanding of this forces are more known; however, we will find out in the next class that importance of this is much more as well as maneuvering is concerned. In fact, maneuvering characteristics stability characteristics will be decided by basically this quantities.

So, with that I am going to close this hour of lecture in the next hour we are going to talk about the directional stability concept that is what it take for the ship to be directional stable what should be the criteria the same question that we ask that when I throw a particular arrow it goes on a straight line provided I put a mass on the forward side and a feather at the end which is what every tribal person knows and when they are making bow and arrow the same thing we will discuss find out formally. So, with that I will end today this hour of lecture.