

**Seakeeping and Manoeuvring**  
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**Module No. # 01.**  
**Lecture No. # 25**

**Dynamic Equations of Motions – II.**

**See**, we are going to continue our talk on the dynamic equations of motion. Actually, last lecture we ended up getting this newton equation in this way, I will add those extra terms later on. **see this well of course**, the equation if I had the origin at center of gravity but, if my origin what located at some other distance and typically if I thought  $r_G$  to be  $X_G \neq 0$ . Because, normally you would always choose body coordinate at a distance with  $X_G$ . Normally, the center of gravity is on the center line and origin also you will put on the center line just  $X$  will be so. If I just to non 0  $X$  then the extra terms that comes out are. That is why I left it free you see etcetera.

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The image shows a slide with handwritten equations and a diagram. The equations are:

$$\begin{aligned} X &= m(\ddot{u} - \dot{\psi}v - x_G \dot{\psi}^2) \\ Y &= m(\ddot{v} + u\dot{\psi} + x_G \ddot{\psi}) \\ N &= I_2 \ddot{\psi} + m[m(\dot{v} + u\dot{\psi})] \end{aligned}$$

Below the equations, it says "NEWTON'S EGN." and "FORCE = mass x accel". To the right, there is a diagram of a ship's hull cross-section with a coordinate system  $(x_G, y_G, z_G)$  and a distance  $x_G$  from the center of gravity to the origin.

So, this is what was my newton's equation of motion that is what it says that is all. Essentially, this **this** is what it says but, now as I was mentioning today, which going to

be a rather important from the maneuvering point of view see now I need to figure out a way to represent the fluid forces that is these forces, X Y N are the fluid forces coming on the hull remember and all the forces external forces to which it is excited.

Now this force X Y and N. So, this I can divide in terms of let me call this to be generically as  $\bar{X}$  just **just** to show vectorially. This will consist of I can break it in components you know one will be what we can call hull forces, one will be because of rudder, so one will be because of propulsion and there can be other environmental forces. I will just explain what all this see there is this trajectory the ship is going this way velocity vector this it has u here it has v here etcetera.

Now, there is a rudder here there is a certain amount of force coming on the hull, why because? remember that this hull is going in this direction whereas, it is oriented here. So, the flow comes like that **flow is coming like that** to it. So, it is not really collinear like it is not center line. So, that will give you some force on the hull and moment. So, I call this to be the hull forces now of course, here in this see I am trying to figure out all force components that is arising here. Remember, this is a situation, where the body is undergoing a motion, it has certain u certain v may be certain  $\dot{\psi}$ .

So, all this thing if I look at that the **that that** are my inertia forces this side that should balance with my total force acting on the hull. What are the total forces? I have first of all a **se** a forces arising on the hull because of water is flowing this way see as an example suppose the ship was going on a straight line what would happen the net force that come is I am calling resistance force. Remember that is what I call resistance is not it? fluid forces acting on the hull in along X direction because the ship is going along X direction was resistance force it is a fluid force.

So, here it is non going along X direction but, with a **with a** some kind of an angle. So, there will be some forces created I am calling it hull forces now, obviously you have got this orientation because, you have actually cause rudder to some angle, which has given some force on that. In fact, that is what has started a hull ship to turn. So, that is rudder forces now, propulsion force because I have a propeller here, the propeller is going to give me a thrust so that gives me propulsion forces.

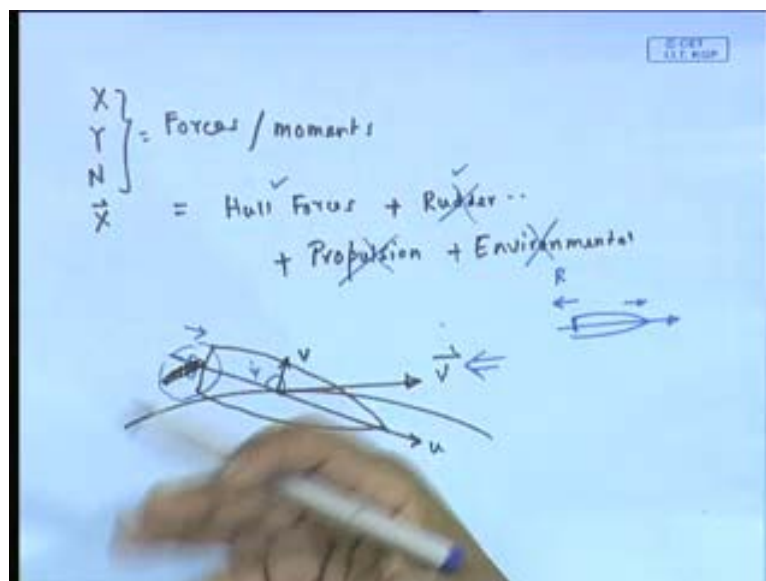
And if I have for example, waves, currents, wind etcetera. I we can call them environmental forces, now what we are going to consider only the hull forces why

because, consider this case I want to investigate first before the control surface that straight line stability remember, where my hull was going on a straight line and I just cause it to turn by a small angle disturb the hull environment because, calm water just like resistance I presume the study is in calm water point one.

So, this I remove it propulsion and resistance cancels out because, when it goes on a steady state at a given speed exactly **re** propulsion minus resistance will become 0 that is why it is going on a forward speed. So, I will not have a propulsion force here for my this consideration I only want to know this side rudder I do not have, because I am not applying rudder. Let us say actually, I we are trying to find out firstly hull forces we will model them eventually. But, rudder also when I want to study a just a bare hull with rudder fixed not apply, rudder is kept fixed. I do not it becomes the part of the hull I do not operate it.

Then what happens to the hull and its characteristic what are the forces getting created that is what I am trying to find out. Remember, that is most important because hull forces are created because of the flow around the hull, it was not there rudder force. I am actually imposing, I am basically giving a rudder angle. So, that there is a force coming propulsion I am having an engine operating environment I do not have a choice it was three waves current etcetera.

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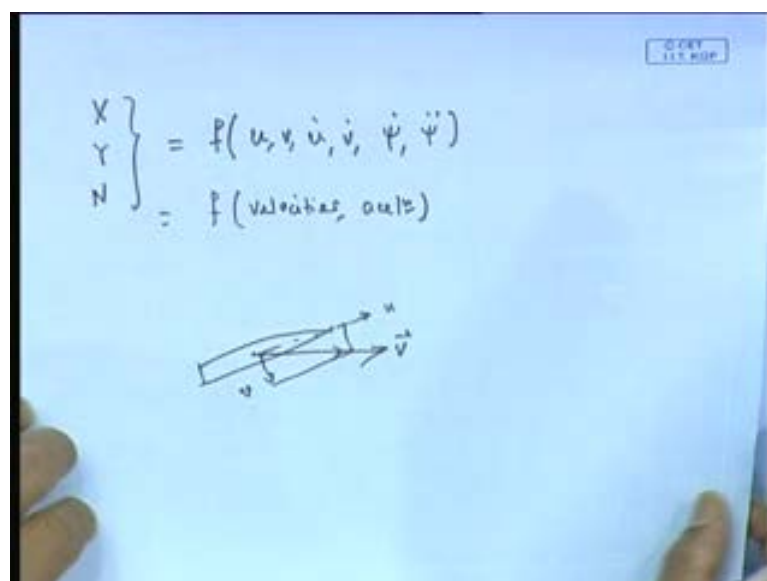


It is the hull forces that got created, because I am turning or I am having a non 0  $v$   $r$  etcetera means it is not going along a straight line that is what I want to look at first. So, we will be now trying to figure out how we can express estimate you know find out determine the hull forces and then write the equation of motion that is the most important part now. **this part is a most important part.**

Now, you see here we will write this way X Y N. Now, number one remember this forces are arising, why they are arising because of the fact that it is having some  $v$  and some  $r$  and some  $u$ . For example, supposing I and and of course, the acceleration obviously the forces must be function of  $u$   $v$   $u$  dot let me write it down. Velocities and accelerations basically function of that is obvious **right** because for example, if I have a forward velocity then only I am having some kind of a X force that is resistance force **right**.

So, if I have for example,  $v$  what is  $v$  angle of attack remember that if the ship is going along this side with this  $u$ , then of obviously I am having a small  $v$  because, remember this is my vector **right** see non 0  $v$  means what having an angle of attack because, only when it will go in this way  $u$  is along X axis  $v$  will be along this axis and this is the vector. So, obviously non 0  $v$  means it is going on an angle of attack and now of course, it may be having an acceleration on this side way.

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So, what I obviously what it means is that these forces will be necessarily a function of all the velocities and all the accelerations. They are functions of they will depend in some sense that is the most important point first. We understand so, this has to be understood that these forces, maneuvering forces and motions moments will be function of all possible velocities. In a general sense linear velocities rotational velocity and accelerations.

Now, this we keep in mind and now we will see how we can have an so called taylor expansion type thing. Now, what is happening now keeping it aside lets now look at this a function see here. By taylor expansion I can always expand you know that if there is a graph function of X again X and if I knew its value at some position X 0 that is F X 0 I knew then F at X become F X 0 plus X **sorry sorry** this is a taylor expansion everybody knows that **right** or if I call this to be delta X then it becomes or rather let me call it F X say rather you know that **right** this is a standard taylor expansion.

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The image shows a handwritten Taylor expansion formula on a blue background. The formula is:

$$f(x) = f(x_0) + (x-x_0) \frac{\partial f}{\partial x} + \frac{1}{2!} (x-x_0)^2 \frac{\partial^2 f}{\partial x^2} + \dots$$

Below the formula, it is simplified to:

$$= f(x_0) + \Delta x f'_x(x_0) + \frac{\Delta x^2}{2!} f''_{xx}(x_0) + \dots$$

To the right of the formula is a graph of a function  $f(x)$ . The x-axis has a point  $x_0$  and a point  $x$ . A tangent line is drawn at  $x_0$ , and a secant line is drawn between  $(x_0, f(x_0))$  and  $(x, f(x))$ . The horizontal distance between  $x_0$  and  $x$  is labeled  $\Delta x$ . The vertical distance between the tangent line at  $x_0$  and the point  $(x, f(x))$  is labeled  $f(x)$ .

What it means is that supposing I have a function F of X I know the value of the function at a neighboring point at X equal to X 0, then I can express the function at X by expanding in a polynomial form, the function about the neighborhood X 0 that means I can say that this value **this value** plus you know actually, if you see the first term there is nothing slope of this term etcetera **etcetera** this I think is very standard, everybody knows it and I we need not elaborate that this is in, if you are going to do that in one dimension.

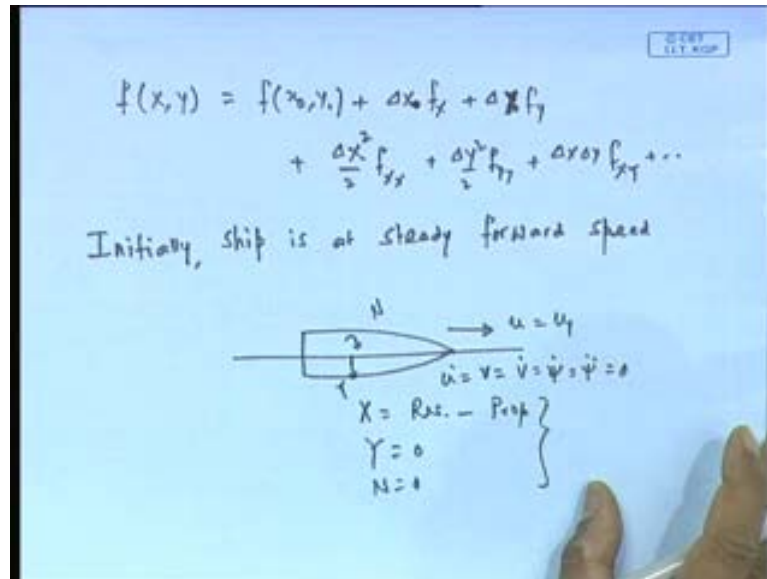
Now, supposing it was in a two dimension supposing the function was function of X and Y then of course, it will become function of  $X_0$   $Y_0$  plus  $\Delta X_0$  you know  $F_X$  I will just write plus  $\Delta Y_0$   $F_Y$  plus  $\Delta$  well  $\Delta X_0$  mean just  $Y$  call it  $\Delta Y$  only  $\Delta X$  square by  $2 F_X x$  plus  $\Delta Y$  square by  $2 F_Y y$  plus  $\Delta X \Delta Y F_{XY}$  etcetera **etcetera**, you know similar way that we all know that so this is a standard Taylor why I am saying that.

Now, you see the forces what we now what we go back to this equation relation, we know  $X$   $Y$  and  $N$  depends on this. Now, what we want to know remember now here this is very interesting this concept is important and interesting from all point of view. Initially, the ship is always going along a straight line. So, I have an initial condition, where my  $u$  equal to  $u_1$  let us say and everything else was 0. I will come back to that so, I know my initial state of a ship moving on a steady state line, it has got certain amount of force on that which also I know because that force is nothing but, a resistance force.

Now, what I have done is that I have slightly tilted it given disturbance to it I have slightly made a  $v$  and  $r$  and  $\dot{v}$  and  $\dot{r}$  and all that stuff, then my purpose would be to find out what is the additional force coming. Because of that, so this I will try to explain this see my initial. Then, in this case what happened that means  $v$  equal to well  $\dot{u}$   $\dot{v}$  equal to  $v$  equal to  $\dot{v}$  equal to  $\dot{\psi}$  equal to  $\ddot{\psi}$  equal to 0 all of them are 0 initially.

Also  $X$  may be resistance actually  $X$  will not be even resistance, even  $X$  will be 0 the reason is because, if you take propulsion  $X$  becomes 0. If you take propulsion because resistance minus propulsion is 0 but, never mind that let me write it propulsion  $Y$  is 0  $N$  is 0 that we agree because, obviously it cannot have any  $Y$  force this side it cannot have any  $N$  moment this side. So, when I am going initially I am just going on a straight line.

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Now, what we have done we have given a non 0 values of this parameters therefore, I want to find out what has happened to my Y and X and all that so, what would happen we will **we will** start with basically with Y. Because, X is less important we will **we will** just try to show in Y, then we will understand let me call this Y. so, Y is the force let me just take that is a function of again if I write  $u \dot{v} v \dot{\psi} \psi \ddot{\psi}$ .

So, now Y see here at u never mind all this parameter is equal to Y at initial value **initial value** is what u 1 because initially is going at u **u** equal to u 1 or u. I you may call it u I and everything as a 0 0 0 plus remember, this is my let me just write it down u minus u 1 d Y by d u plus v minus v 1 v 1 means 0 here. Actually, v minus rather let me other way **round let me other way** write let me **let me let me say** say initial etcetera.

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Handwritten mathematical derivation on a blue background:

$$Y = f(u, v, \dot{\psi}, \ddot{\psi})$$

$$Y(u) = Y(u_1, v, \dot{\psi}, \ddot{\psi}) + (u - u_1) \frac{\partial Y}{\partial u} + (v - v_1) \frac{\partial Y}{\partial v}$$

Say, initial values are  $u_1, v_1, \dots$

$$Y(u, v, \dots) = Y(u_1, v_1, \dots) + (u - u_1) \frac{\partial Y}{\partial u} + (v - v_1) \frac{\partial Y}{\partial v}$$

$$+ (\dot{\psi} - \dot{\psi}_1) \frac{\partial Y}{\partial \dot{\psi}} + (\ddot{\psi} - \ddot{\psi}_1) \frac{\partial Y}{\partial \ddot{\psi}}$$

$$+ (\dot{v} - \dot{v}_1) \frac{\partial Y}{\partial \dot{v}} + (\ddot{\psi} - \ddot{\psi}_1) \frac{\partial Y}{\partial \ddot{\psi}}$$

$$+ \dots$$

Then my Y at u v etcetera equal to Y at u I v I etcetera plus u minus u 1 d Y by d u plus v minus v 1 d Y by d v plus you know psi dot minus psi dot 1 d Y by d psi, these are 3 velocities, then I have got u dot minus u 1 dot. See here if I took only up to linear order if I were to take this function **this function** up to only this much that is only up to linear term that means I am presuming that it is too small and therefore, I can estimate that by simply this plus distance in the slope that is I am expanding the function by only a linear term or you can say up to a first order approximation.

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Handwritten mathematical derivation on a blue background:

$$f(x) = f(x_0) + \frac{(x - x_0) \frac{\partial f}{\partial x}}{1!} + \frac{1}{2!} (x - x_0)^2 \frac{\partial^2 f}{\partial x^2} + \dots$$

$$= f(x_0) + \Delta x \frac{f'(x_0)}{1!} + \frac{\Delta x^2}{2!} \frac{f''(x_0)}{2!} + \dots$$

Two diagrams illustrate the expansion. The top diagram shows a graph of a function  $f(x)$  with a point  $x_0$  on the x-axis. A tangent line is drawn at  $x_0$ , and a small interval  $\Delta x$  is marked on the x-axis. The corresponding change in the function value is  $f(x) - f(x_0)$ . The bottom diagram shows a similar graph with a point  $x_0$  and a small interval  $\Delta x$  on the x-axis. A tangent line is drawn at  $x_0$ , and the slope of the tangent is labeled  $f'_x \cdot \Delta x$ .



Which means value here is value here plus this distance onto the slope very simple value here plus slope into the distance give you this see this this part is  $F \Delta X$  if you take a slope I mean that is what is called linear approximation which essentially telling is that this graph has been assumed to be a straight line between the 2 that is what we are saying you know so if you take only obviously that is the first approximation if you want to go to the next level then I have to take this term etcetera etcetera this everybody knows it.

So here what I have done is that supposing I expand these forces only up to first order then I end up getting this but, now comes the fun you see now what is happening  $u_1 v_1 w_1$  as I told you initial values of which we have just now mentioned here this is initial value this basically  $u_1$  and this all initial values all the initial values are 0 that is initially the  $I$  values are all 0 that is 1 what is my  $Y$  value at initial also 0 because remember this value is the value in this direction in initial state that means when it is going on a straight line what is the  $Y$  force obviously 0 because there was no  $Y$  force.

So I end up getting for this particular condition that is if I were to expand  $Y$  about an initial condition which is the straight line motion steady motion of the ship then I have this 0 **right** I have all this 0 etcetera so what do I get I get here  $Y u v$  etcetera equal to  $u$  minus  $u_1 \frac{dY}{du}$  plus  $v \frac{dY}{dv}$  plus you you get also this  $\psi \dot{\psi}$  plus we will of course, this also we will now neglect later on let me write in the other page no because her this  $v \dot{\psi}$  let me i'lli'll go back to the next page the same thing.

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$$Y = f(u, \dot{u}, v, \dot{v}, \psi, \dot{\psi})$$

$$Y(u) = Y(u_1, 0, \dots) + (u - u_1) \frac{\partial Y}{\partial u} + (v - v_1) \frac{\partial Y}{\partial v} + \dots$$

Say, initial values are  $u_1, v_1, \dots$

$$Y(u, v, \dots) = Y(u_1, v_1, \dots) + (u - u_1) \frac{\partial Y}{\partial u} + (v - v_1) \frac{\partial Y}{\partial v} + (\dot{u} - \dot{u}_1) \frac{\partial Y}{\partial \dot{u}} + (\dot{v} - \dot{v}_1) \frac{\partial Y}{\partial \dot{v}} + (\psi - \psi_1) \frac{\partial Y}{\partial \psi} + (\dot{\psi} - \dot{\psi}_1) \frac{\partial Y}{\partial \dot{\psi}}$$

$$Y(u, v, \dots) = (u - u_1) \frac{\partial Y}{\partial u} + (v - v_1) \frac{\partial Y}{\partial v} + \dots$$

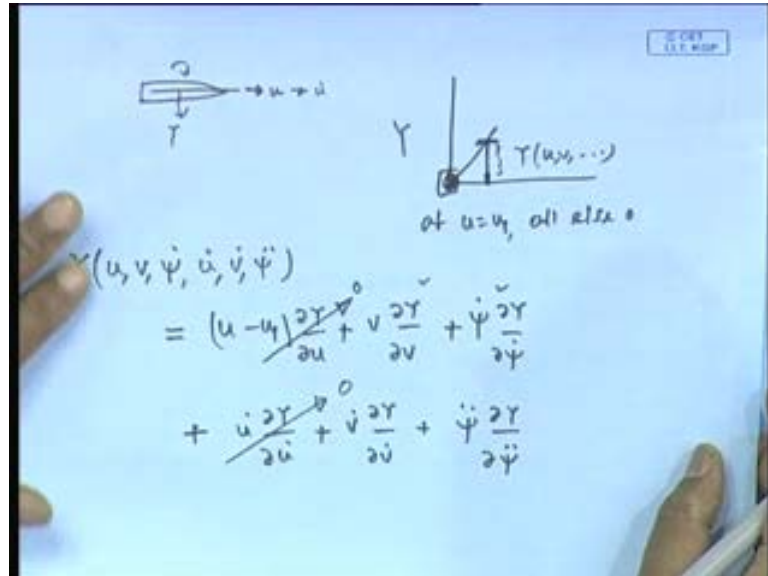
See the first term is 0 this term is Y because initially now this is this initially you see I am having this ship going on a straight line with only u equal to u 1 and everything else 0 now when the hull is going on a straight line what is the force coming in this side what is the moment coming this side 0 so the initial condition of Y is 0 what we are doing here you know is if you look at that this Y i'll before I write that i'll show this thing Y initially Y was 0 this point that is at u equal to u 1 and all other is 0 all else 0 my Y is here.

Now I am having some parameter here so Y actually goes like that initially 0 Y because there was no Y there **right** so what we I i now I **want** this is my Y this is my Y at u v w etcetera so what is happening therefore, i'll write it big way Y u v psi dot u dot v dot psi double dot this has become equal to u minus u 1 this we agree **now** now there more to it now we will see some picture what is d u by d Y essentially variation of Y against u how much Y changing how much is a sway force Y force see here this is my Y force I want to find out if I change u make changes here u **minus** d Y d Y d u d Y by d u d Y by d u is change of Y with respect to u what is the change.

If I change this side is there a change in the Y force no so it is because of symmetry so this goes to 0 similarly, if I were to but, this does not go to 0 of course, because there obviously is if I give a v here there will be Y here so this is not 0 if I give a rotation here

this psi dot is rotation also is not 0 because if I rotate there will be this thing but, about this again what is this I accelerate here what is my Y force 0.

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So you see I therefore, end up getting an expression which is only these 4 terms so we what I end up getting is essentially these 4 terms that means I will now write what do I get Y we will not write this value it becomes therefore, v into d Y by d v plus v dot into if I write now v dot plus psi dot into d Y by d psi dot plus psi double dot into d Y by d psi double and remember 1 thing these values v v dot psi dot psi double dot are very small values why small values because remember they are perturbation from a initial state.

So I have an initial value which was 0 then I have just make it small values so these are actually very small values these values are all small values that is that is to be basically kind of remembered **right** so I end up getting see remember here this this this this are necessarily very small values why small values remember that my initial value was 0 zerozerozero etcetera final values are v vdot psi dot psi double dot they are all just small change about the mean value.

In fact u minus u 1 if I were to take u minus u 1 is also small value because u was let us say 10 knot u 1 would be 10 plus minus something so what re, happen i'll come come back to this expression here so this I have got let me write also here this part of X here if I were to write X in a similar this thing notation you will find out you end up getting here

$\dot{u} \frac{dX}{du} + u \frac{d^2X}{du^2}$  plus  $u$  minus  $u$  I'll leave it for you to work this out actually we can keep all the term  $v \frac{dX}{dv}$  expression is not very important.

So actually the  $X$  expression all are kept here because you really do not know what is what may be the extra thrust coming because of those this thing but, if I were to keep  $N$  for the same reason of  $N$   $n$  is going to be simply  $v \frac{dN}{dv} + v \dot{v} \frac{d^2N}{dv^2}$  plus  $\dot{\psi} \frac{dN}{d\psi} + \dot{\psi}^2 \frac{d^2N}{d\psi^2}$ . Now, I will tell you what we are going to do we are not going to look so much on the  $X$  equation.

The reason is because  $X$  equation is my equation showing how it is having a force in this side. So, this force and which going to cause me motion on the  $X$  direction so what would happen instead of making a constant speed it might be perturbing on that say it is of 10 **knot** it might got 10 point 2 **knot** etcetera, which is of not much interest for us from the maneuvering point of view. In the maneuvering our interest is always this and this.

Because, we want to see from the straight line how it is turning  $X$  only causes go to on a straight line therefore, we will be looking at this and this more carefully. Because,  $Y$  is trying to make it go this way and  $N$  is going to turn it now coming to that now I will again.

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$$\begin{aligned} \checkmark Y &= v \frac{\partial Y}{\partial v} + \dot{v} \frac{\partial Y}{\partial v} + \dot{\psi} \frac{\partial Y}{\partial \psi} + \dot{\psi}^2 \frac{\partial^2 Y}{\partial \psi^2} \\ X &= \dot{u} \frac{\partial X}{\partial u} + (u - u_1) \frac{\partial X}{\partial u} + 2u \dot{u} \frac{\partial^2 X}{\partial u^2} + \dot{u}^2 \frac{\partial^2 X}{\partial u^2} \\ &\quad + \dot{\psi} \frac{\partial X}{\partial \psi} + \dot{\psi}^2 \frac{\partial^2 X}{\partial \psi^2} \\ \checkmark N &= v \frac{\partial N}{\partial v} + \dot{v} \frac{\partial N}{\partial v} + \dot{\psi} \frac{\partial N}{\partial \psi} + \dot{\psi}^2 \frac{\partial^2 N}{\partial \psi^2} \end{aligned}$$

Now, before I we finalize the equation of motion I want to discuss these terms therefore, in a minute and then we will. So, you see typically we write this way  $Y$  as once again I

will tell  $Y$  and I write  $Y v$  that is actually  $d Y$  by  $d v$  then, we will write this as  $v \dot{Y}$  by  $v \dot{Y}$  plus let me put  $r$  here  $r$  is  $\dot{\psi}$  velocity.

Basically, this term that is this say see here what are the term rate of change or slope of either a force or moment depending on whether a is  $Y$  or  $N$  against a motion variable, which is either a velocity or acceleration linear rotation. So, we have got you know basically alpha the 4 alphas  $v v \dot{r} r \dot{r}$  there 2 e in this case  $Y$  and  $N$  so they have got total of 8 such terms, these terms are called hydro dynamic derivatives of the hull linear hydro dynamic derivatives linear. Because, I have call express the forces only up assumed it to be a linearly varying function about the mean.

That means I presume  $Y N$  are straight line about this and what happen here I say this  $Y$  force is  $Y v$  into  $v$ , what is  $Y v$  it is a slope supposing I want to find out how does  $Y$  varies against  $v$  what did I do? I said  $Y$  at  $v$  equal to 0 is 0  $Y$  at  $v$  equal to  $v$  is equal to  $v$  into  $d Y$  by  $d v$  right that is a very straight forward thing. So, what is  $d Y$  by  $d v$  it is the rate of change of  $Y$  against  $v$  now how much it is we do not know. But, this becomes a characteristic of the hull that means I am saying that let me find out for my given hull by some means how much will be the  $Y$  for per unit  $v$ .

Then, I will calculate and keep it with me. So that, becomes just like our you know  $b m t k m t$  etcetera a property of the hull hydrodynamic property of the hull that becomes hydrodynamic derivate. So, these terms are called hydrodynamic derivatives and we will be discussing this at length because the study of maneuvering borders all cen the central idea that remains is for you to understand what is meant by hydrodynamics derivatives I will come back to that in a greater length again this part because it is very important.

But, now let me go back to the equation of motion part. Now, little bit that is also an important part see my  $X$  was  $m$  there is an interesting thing you will see now, which is I, I wrote earlier but, you see there is a problem, I think that is what we wrote earlier right obviously now we have to just equate both sides this is what same thing we wrote this is  $\dot{\psi}$ . Now, the problem see is very simple now of course, forget the  $X$  also we could expand this way.

Now, I have say  $Y$  this side  $Y$  in terms of hull forces this expression I want to ask you this question now, I have this from newton's equation, force is mass into acceleration but, now I find that force I can express in this way. So, I can say this can I say that

directly or under some qualification, let me first write and then I will try to tell you. So, I can say therefore, let me just write down these two cases or the case number one also if you want we will just write down these two cases, we will not bother about this first one as such.

So, I can say this  $v \cdot Y \cdot v$  plus  $v \cdot \dot{Y} \cdot v \cdot \dot{v}$  because this  $Y$  is this **right** remember  $\psi$  double dot is  $r$  basically. So, I am just writing  $N \cdot v \cdot N \cdot v$  but, there is a small  $r \cdot N \cdot r$  plus  $r \cdot \dot{N} \cdot r$ , now the question is see I wrote this but, I tell you it is not exactly consistent do you know why because this side when I wrote I presume my  $v \cdot v \cdot \dot{r} \cdot r \cdot \dot{v}$  are small values, when I wrote this side I did not have any such approximation this side that is if I were to use another pen this side do not have any approximation involve that my  $v \cdot u \cdot \psi \cdot \dot{v} \cdot \dot{\psi} \cdot \dot{v} \cdot \dot{\psi}$  double dot are small we did not say that.

This applies this side for any value see this side, this is what we need to go slowly you know force equal to mass into acceleration is what this side is **right** this part, so I have force equal to mass into acceleration. Once again I will write down this way mass into acceleration equal to force this I said this first part but, now I am expressing the force by this side this force is this side but, this expression is not unqualified this expression is for small values of  $v \cdot v \cdot \dot{r} \cdot r \cdot \dot{v}$ .

So therefore, I also have to modify this if I want to determine the equation for small value of  $v \cdot v \cdot \dot{r} \cdot r \cdot \dot{v}$  what changes happens to this if I have that assumption embedded do you understand that I mean I want to make it go slowly this side mass into acceleration equal to force this part of force do not have any approximation on the values or smallness or largeness of  $v \cdot v \cdot \dot{r} \cdot r \cdot \dot{v}$  it can be any value this is this is valid.

But this side that is stating that force is equal to this is based on the approximation that  $v \cdot v \cdot \dot{r} \cdot r \cdot \dot{v}$  are small values. Because, they have assumed only linear approximation of the forces therefore, to be consistent if I want to make this equation that way, I should modify this side of equation presuming that you the same approximation same level of approximation why now you will understand in a second. Actually, nothing happens to all of that but, it only happens to these two terms.

Let us take these two terms  $u$  and  $\psi \cdot \dot{v}$ . Now,  $u$  is  $u$  initial plus  $\delta u$  **right** because  $u$  initial was not small remember  $u$  initial is not small it is may be **10 knots fifteen knots**. So,  $u$  of course, perturbation of  $u$  initial that is  $u$  initial plus minus something so it is  $u$

initial plus and if I were to multiply that psi dot then this is psi dot just I am just giving an example. Now, you see this term this what would happen is u I i psi dot plus delta u into psi dot now you see this is small this is also small.

So, this term is second order term I said that to you people many times no that small into small is small square. So, call the 2 2 small basically actually I remember this form a teacher in Canada day. So, I am very fond of saying that you know he keep saying that this Chinese teacher it makes it 2 small. So, it 2 small you know so as a result what happened we are ignoring it therefore, why the point is that it is second order therefore, this term becomes u I into psi dot. So, this term will become u I into psi dot not u into psi dot.

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$$X = m(\dot{u} - \dot{\psi}v - x_g \dot{\psi}^2)$$

$$= m(\dot{v} + u\dot{\psi} + x_g \dot{\psi}^2) = \dot{v} + u\dot{\psi} + x_g \dot{\psi}^2$$

$$m x_g \dot{\psi}^2 + m x_g (\dot{v} + u\dot{\psi}) = \dot{v} + u\dot{\psi} + x_g \dot{\psi}^2$$

↑

m x accel = force =

$$\dot{\psi} u = (u_I + \Delta u) \dot{\psi}$$

$\dot{\psi} + \Delta u \dot{\psi}$   
small small  
 2nd order

And similarly, this term will become u I into psi dot u I being the initial velocity or the steady velocity of the ship. So, the only change is that happens in this equation is that this I can make this equal to this provided this u I take here is the initial u. So, what is happening therefore, I end up getting the relation now, I will write it down in the next page by combining these two that I end up getting a relation that is m into v dot plus this u here I call it capital u psi dot capital u is my initial u the steady velocity plus X g r dot let me call it r dot is equal to v Y v v dot Y v dot plus r Y r plus r dot Y r dot, you can we have to assemble this equation in some just 1 second you can later on assemble.

Actually, I without validating let me also tell you this part that X part we can X part becomes you know  $m \ddot{u} = X \dot{u} + u \dot{u} - u I$  that is capital u into X u what I mean in the X equation the other terms variation of X against v etcetera can be neglected. Because, same reason X force although I mention but, X force force in X direction would not depend on the v and Y motion so strongly. Therefore, we can ignore that.

So, we can simply say this in fact this becomes your surge equation in a steady state case because, this becomes my resistance essentially X u etcetera etcetera. So, we can still write that way the next one is the other equation is I well I z. I am just deleting because I z is implied you know, whenever we talk of I i about the z axis now, so let me presume that you know just for making things simpler.

Whenever I mention I i z because it is only 1 axis there rotation wise  $I \dot{r} + m X g v \dot{r} + u \dot{r} = v N v c \dot{N} v \dot{r} + r N r$  but, we can of course, then obviously we can manipulate this equation to bring it in 1 side 2 side etcetera etcetera. So, I can see that the equation turn out to be minus the first 1 X u u minus capital u plus m minus let me write it down  $\ddot{u} = 0$  here.

See, we will write this way first v then v dot then r then r dot term so v comes in this side. So, it becomes minus Y v into v plus v dot means here 1 comes plus N comes here m minus Y v dot into v dot then r r is here this minus it becomes simply [FL] this is actually r this is actually r psi dot is actually r right. So, if I bring that r here then it becomes m u r minus Y r dot right if I call it minus then I into r minus this and this Y r actually here the sorry this in coming first Y r dot minus m X g equal to zero.

Then next one becomes minus N v v plus sorry this is minus v dot. We want to do so this is these two terms  $N v \dot{v} - m X g v \dot{v} - r m r - m X g u \dot{r} + I \dot{r} - N r \dot{r} = 0$  in fact this becomes my classical linear dynamic. So, this is my classical linear dynamic equation just one second this is the surge equation we are not much concerned with because we are looking at the ship turning if it is going on a straight line well if it is making some acceleration we do not care is going same direction.

Maneuvering is essentially as it is turning making arbitrary trajectory. So, it is basically these two, what you see very interesting you will find there is a force in proportion to



velocity force in proportion to **acc** linear acceleration rotational velocity rotational **acc**  
 there is  $r \dot{}$  missing here same thing here  $v \dot{}$   $r \dot{}$ , what is the validity of that it is  
 valid only for small values because I have linearize expression.

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$$m\dot{u} = x_u \dot{u} + (u-U)x_u$$

$$m(\dot{v} + U\dot{\phi} + x_v \dot{r}) = vY_v + \dot{v}Y_v + rY_r + \dot{r}Y_r$$

$$I\dot{r} + m x_r (\dot{v} + U r) = vN_v + \dot{v}N_v + rN_r + \dot{r}N_r$$

$$\Rightarrow -x_u(u-U) + (m-x_u)\dot{u} = 0$$

$$-Y_v v + (m-Y_v)\dot{v} - (Y_r - mU)r - (Y_r - m x_r)\dot{r} = 0$$

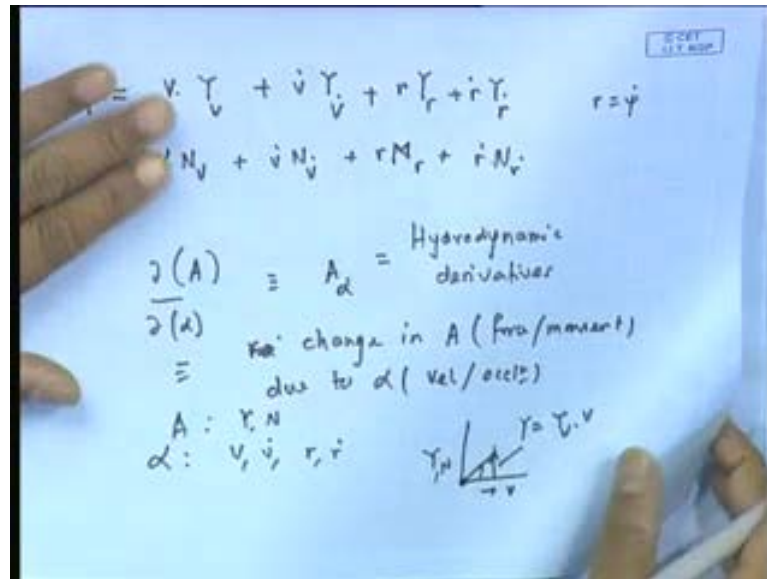
$$-N_v v + (N_v - m x_r)\dot{v} - (N_r - m x_r U)r + (I - N_r)\dot{r} = 0$$

Linear Dynamic Eqs of motion in Horizontal plane

Now of course, we have to find out the solution for this equation, which gives rise to my  
 $v \dot{}$   $r \dot{}$  would tell me how under the influence of the forces the you know like ship  
 behaves now. See, the interesting part is this, the concept comes like that lets take a  
 example  $Y v$  what is  $Y v$   $Y$  force arising because of  $v$  that means I have given a  $v$  that  
 means I will come to this at a much better length say I have given it a  $v$ .

Then, it creates a  $Y$  force may be it creates a force like, that now the question is see here  
 like that you will see just here the example look at this our example with a hill motion if  
 I were to hill it by external forces moment, what happen a static force comes if the  
 metacentric height is high. Say I end up getting what do I end up getting a delta or I like  
 to write this way writing moment this way or I can tell delta  $g v g m$  into theta.

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So, if theta is this side my force on the other side. So, I say it is stabilizing force bringing it back here what is happening just see the analogy I will come to this in detail. I give a v here if I give a v here force that got created you would get suppose in the force is on this side this is what it is this force is trying to resist and bring it back **right**. So, if I want to give a v here once again the the hull force that got created will try to bring it back because the force is on the other side.

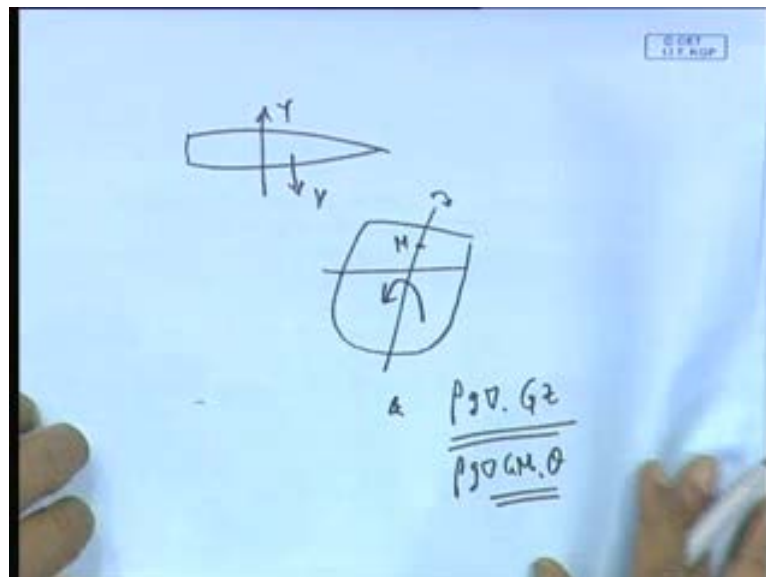
So, the tendency of the forces to make v as small and small like here tendency of the moment would be to make theta small. So, that theta becomes 0 the moment become 0 you know like as the ship turns what would happen theta will become progressively 0 and therefore, the moment will become also 0. So, we call it stabilizing supposing the moment was opposite side g m was negative what would have happened you give us point 1 degree theta. Immediately, the moment will try to make the theta a little more when it becomes little more the moment is even further cause the theta will even further it will keep on increasing **right**.

So therefore, you understand here that is hull forces now, this is only one. So, what are the hull forces here all these terms this Y v N v etcetera these terms this etcetera, these are the characteristic of the hull and these not by isolation if I were to solve this equation, which is what we will be doing eventually and see the characteristic you know that that characteristic of this solution then I would know that what happens to v v dot r r dot.

Initially, it was 0 now, I have given a small non 0 value left my hand left that effect do they come back to 0 do they increase what happen or do they stay as it is.

You see this is the question just like my theta question if I give an external hilling moment a wind I have got a theta created a hilling angle created I remove the external force. Thus the force generated in the hull, the moment generated in the hull because of hydrostatic pressure tries to make theta small and eventually 0 or tries to make theta big, this is the question we have to answer and this question obviously comes out from solution for this equation.

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And when I want to solve this equation, I have ended up knowing these quantities also, I will first discuss tomorrow next class but, first I will mention those quantities and then eventually go back to that. Now, just understand these quantities once again I will just want to see you one close that with a analogy see this first term  $m \dot{v} \cdot \dot{v} \cdot \dot{v}$  just see this term then I will end end this today's class.

Now, you see in equation of motion in sea keeping what did you find mass plus added mass into acceleration **right**. Here, I am finding mass plus minus  $\dot{v} \cdot \dot{v} \cdot \dot{v}$  into acceleration what is then  $\dot{v} \cdot \dot{v} \cdot \dot{v}$  it is nothing but, added mass with a minus sign because, you see what is this  $\dot{v} \cdot \dot{v} \cdot \dot{v}$  if I accelerate the body in the direction  $\dot{v} \cdot \dot{v}$  with a  $\dot{v} \cdot \dot{v}$  the amount of force it got created.

So, you know this way I will again discuss that you will understand that this part is nothing but, minus of added mass we are just writing them with a different equation in that I said  $m$  plus added mass into acceleration here I am saying  $m$  plus minus  $Y \dot{v}$  into acceleration obviously minus  $Y \dot{v}$  is nothing but, now added mass is a property of the hull for a given hull there is an added mass, what is added mass essentially rate of change of  $Y$  force or force in a direction certain direction for acceleration in some direction have added mass force in heave direction for unit acceleration heave direction same thing here.

But then, added mass is a property of the hull just like  $g$   $m$   $t$  not  $g$   $m$   $t$  I say  $k$   $m$   $t$   $b$   $m$   $t$   $a$   $w$   $p$   $m$   $c$   $t$   $p$   $c$  the intrinsic property of the hull. So, these are all property of the hull so, I will just end saying therefore, that see in hydrostatic we had all the hydrostatic parameter hull property based on hydrostatic pressure. Then, came resistance you have got resistance coefficient as property, some kind of property as a function of  $c$   $F$  etcetera function of velocity also of course.

Then comes sea keeping added mass damping now comes maneuvering, where hydrodynamic derivatives all these are geometric dependent all these are essentially fluid pressure integration on the hull just like the pressure is coming for different you know from different mechanism but, they are property of the hull, we will discuss we will end it here discuss this and interestingly, we will discuss what it takes for my ship what should this property be so that, my ship becomes stable.

This is the question, we will be answering in next few classes with that I will end today.