Seakeeping and Manoeuvring
Dr. Debabrata Sen
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture No. # 24
Dynamic Equations of Motion – I

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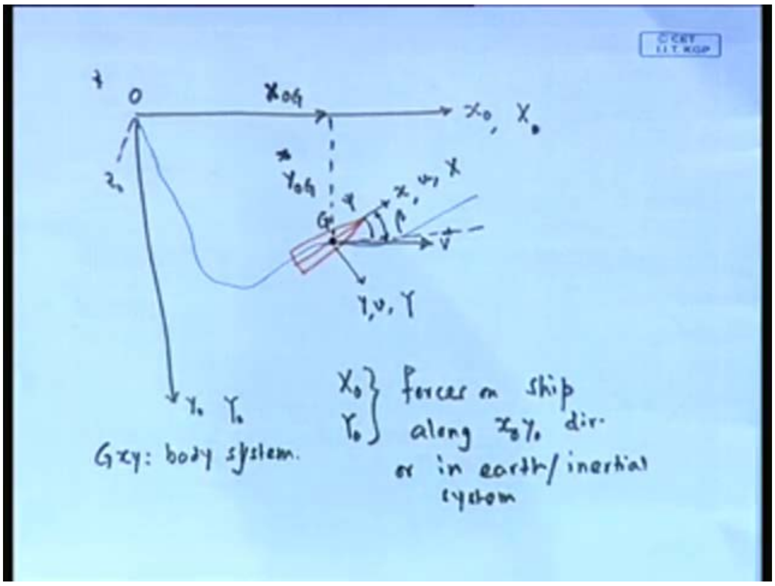

See we are discussing, now manoeuvring, which I mention is essentially motions in the horizontal plane as for as a ship is concerned quickly draw this definition diagram, that I had drawn last time that is essentially necessary just in order to you have this let me just quickly draw that.

(No audio 01:02 to 01:57)

What this is just quickly a recap of what we have done. See what is happening is remember we are discussing now the subject of a ship manoeuvring in a horizontal plane. And I have mentioned that in last class during introduction is that the way it will behave depends purely on the hydrodynamic forces that gets created on the body, because of the flow passed it because of the fact that it is not moving with 0 angle of attack. See here, this is the trace this blue line is the trace of it is centre of this gravity some point we are taking a C G the trace of that.

And the orientation is of course, given by this beta and psi angle. Now, I what I we need to know is to find out this trace along with the orientation. So, we have as we mentioned define, an inertial system with perhaps, an origin at o which is inertially fixed earth fixed o X 0 Y 0. We have a body fix system called G x y, the body has an velocity vector v which is somewhere in this side. Therefore, it is going with an angle of attack or drift angle beta because it is moving this side, but it is body you know like orientation is here. So, flow comes this side flow keeps forces and this forces must be equated to or used in a equation of motion.

Now, what we are going to do see that let us say this distance is my x no this is y sorry this is x 0 G and this is y 0 G. So, according and let me also define, certain quantities the forces on the hull along this direction that is along the direction inertial direction let me call it x 0 and y 0. That means X 0 Y 0 are forces on ship along X 0 Y 0 direction or in earth inertial system similarly, x and y are the forces along the body system which is defied with origin at G tan x axis here and y axis here is a body system; that means, G x y is body system or rather let me go back to this.

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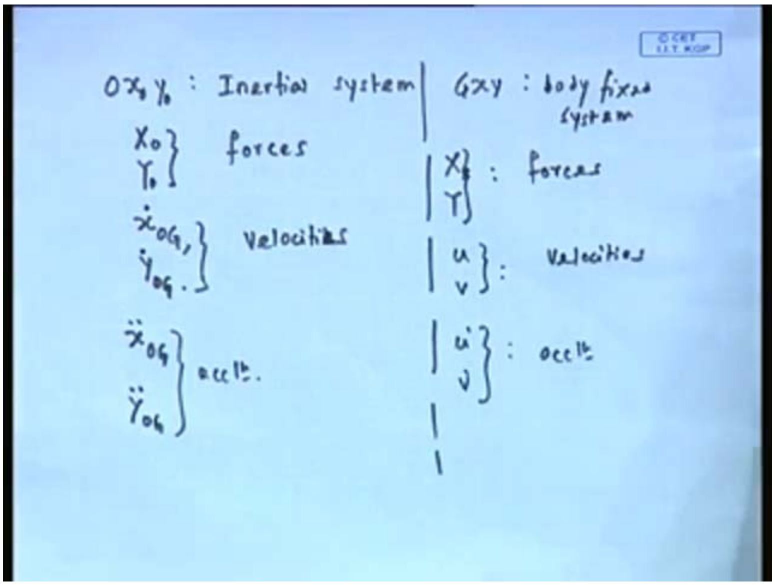

So, I have o X 0 Y 0 is inertial and in that system X 0 Y 0 are forces x 0 G dot Y 0 G dot are velocities acceleration. Now, G x y is body fixed system I define X and Y is forces u and v are velocities u dot v dot are see here once again, I will tell you inertial systems are all 0 x 0 Y 0. Therefore, capital X 0 capital Y 0 are forces along this direction N of course, is the moment, which of course, remains same if I call this to be moment N, N will remain same. Because in this diagram, the direction z body fixed direction z is downward to the paper that is if you go clockwise in a right handed system x to y downward, which is same as far as inertial system is concerned as far as body system is concerned.

So, as far as moment is concerned it is same it will be N whether m 0 and N are same now what we have written is here is that, if I take inertial system my forces is X 0 Y 0 X 0 Y 0 my velocities are x 0 dot dot y 0 dot dot along those direction here and acceleration is next derivative. But if I take a body system it is my force is x and y X and Y velocities u and v u and v and acceleration is the dot the question is that I have to write a develop and equation of motion Newton’s equation of motion, which is suitable for me to study and ultimately I have to write in the body frame.

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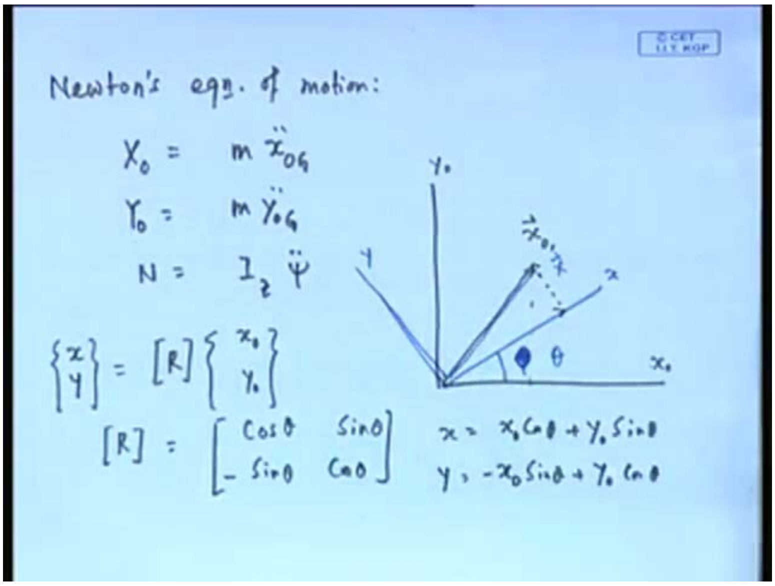

Now, let us look how we can do that see inertial frame of reference my Newton’s equation of motions, which will apply of course, in inertial frame why not in body frame is because body frame is rotating. Remember the body frame is rotating is not a fixed frame. So, we have to transfer that using some mechanism. So, I have X 0 equal to mass into x 0 G dot dot Y 0 equal to mass into Y 0 G dot dot and N of course, I can call it I let me call it I z dot psi dot dot going back to that. See, here force is mass into acceleration standard or if I have to look at that this side force is mass into this side acceleration inertial frame of reference.

Now, the point is that I need to transform that both this side I have to I want to rewrite this in terms of X Y and I want to rewrite this in terms of u v this equation right hand side and left hand X 0 Y 0 x o G Y o G that is what inertial reference. But I want to transform that and write them in terms of X and Y and in terms of this side u and v why is because remember that I what is this forces. There is a ship here the red one I want to find the force acting on that, because of it is motion and if you are sitting on the ship you are much better off to try to determine the forces along it is x axis and along it is y axis. Because along it is x axis is a resistance force along this y axis is a sway force. So, if I had to do along this x 0 axis you know it can be arbitrarily depending on the orientation.

Tomorrow the ship might been oriented in this way then my x 0 force becomes actually, sway force then next incident is this way my x 0 force become my search force. So, it becomes difficult to define that is why I need to transform them to this x and y axis. So, that is a thing that we will be doing now slowly that is transforming this to that axis how do I do that it goes actually using this transformation matrix Now, I will just tell about a transformation supposing I take two coordinate system these are the orientation between the two is this one is my X 0 Y 0 and one is my x Y this angle is basically my psi or theta whatever, you can let me call it theta.

Now, we can make theta psi later on now you see this everybody knows that now any vector define with respect to this that is in this coordinate system. And another one is define with respect to same one when I define; with respect to you know the coordinate transformation that is essentially what I am saying is that by coordinate transformation we know that x and y.

(No audio: 11:28 to 11:51)

If that if you look at that you will find out essentially x equal to x 0 Cos theta plus y 0 sin theta and y equal to minus x 0 this standard like you know, if you basically I do not have to go through all this term you know, if you know see this is my x. So, this actually this x 0 Cos theta plus y 0 sin theta etcetera etcetera this is a standard coordinate transformation that is transforming a quantity in coordinate system X 0 Y 0 to x y or x y to X 0 Y 0.

This x y to X 0 Y 0 getting x y from X 0 Y 0 and if you want to do the reverse you simply have R inverse, because R can be inverted and R inverse is R transpose that we all know.

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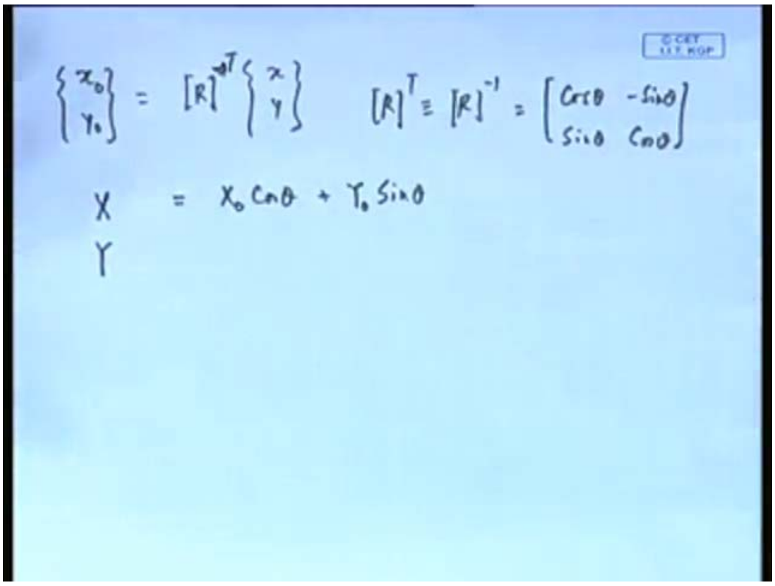

That means, if I were to do the other way round; that means, if I want to do X 0 Y 0 this becomes equal to R inverse, which becomes and this becomes I will just take my privilege of using this thing becomes same as R t. So, that R t, which is equal to R inverse this is actually simply the change Cos theta comes here minus sin theta comes here and sin theta comes here Cos theta comes here. So, this allows me to change one to other. So, then what happen, now we are now in a position write this way you see here. Now, x force x and force y x remember what is x x is the force along the body axis. So, x and y I can write in terms of this thing I can straight away write actually, here X 0 Cos theta plus Y 0 remember here X 0 Y 0 are well it applies to any vector.

So, in this case, I am applying the force vector because capital X is the force see it applies to any vector what it means is that if I define, a vector in a coordinate system one how do I get the comparison in the coordinate system two just by simple transformation that is all it says. So, x force in coordinate system body that is given by capital X is basically capital X 0 Cos 0 plus y 0 sin theta. Because this force vector essentially here what is happenings, remember body force vector I have a body force vector here somebody force vector here just like this.

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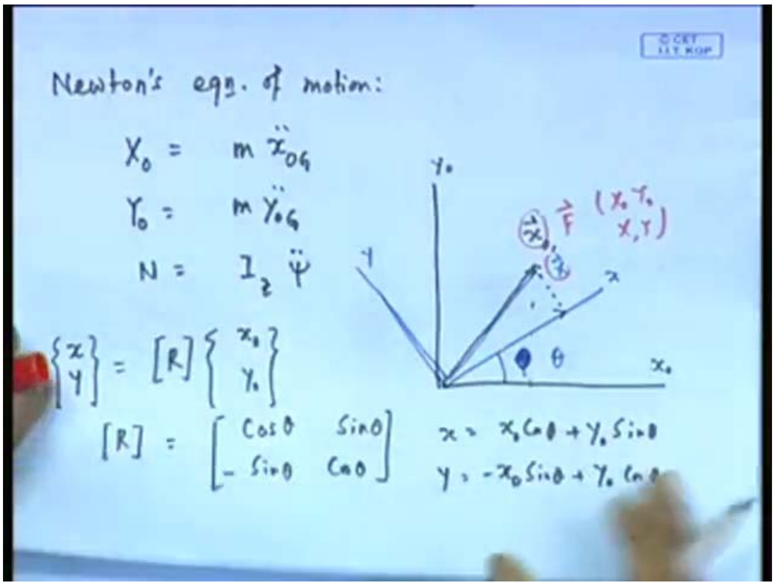

Instead of that instead I will just call it body force vector. So, this vector has components either X 0 Y 0 or X Y depending on how you are defining it. So, they basically are related to each other. If you call X 0 Y 0 or X 0 Y 0 or X Y they are you know related by this relations.

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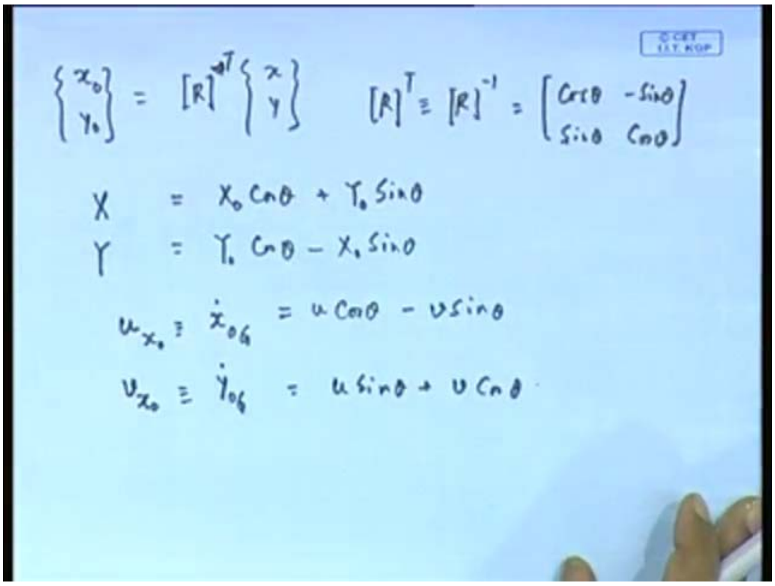

So, that is part we are writing y 0 becomes here y becomes here y 0 Cos Y in fact, we can write it at basically, y 0 Cos theta minus x 0 sin theta. Actually, you should write minus x 0 sin theta plus y 0 Cos theta same thing. Now, similarly, now the other way round u and this thing u v and x 0s. Now, u with respect to x 0 system which is actually, insignificant x 0 dot G I will write that that is equal to x dot o G and v in with respect to this well this x 0 basically, x 0 system inertial system, which is equivalent to y 0 dot G. Now, this remember here we are doing reverse the quantity in o we write in terms of u and v, which is a same thing of course, either way.

So, this will simply become u Cos theta minus v sin theta and this will become u sin theta plus v Cos theta. This becomes u sin theta plus v Cos theta I mean, this is opposite see no see here this u v u v u sin theta plus v Cos theta opposite here no this R t coming remember. See, here what I change is dot to this to this previous one in the force, what I did is this to this that is why it was R and this R 2 the same it is very trivial. Really, we need not like you know go through all that you know so much in detail.

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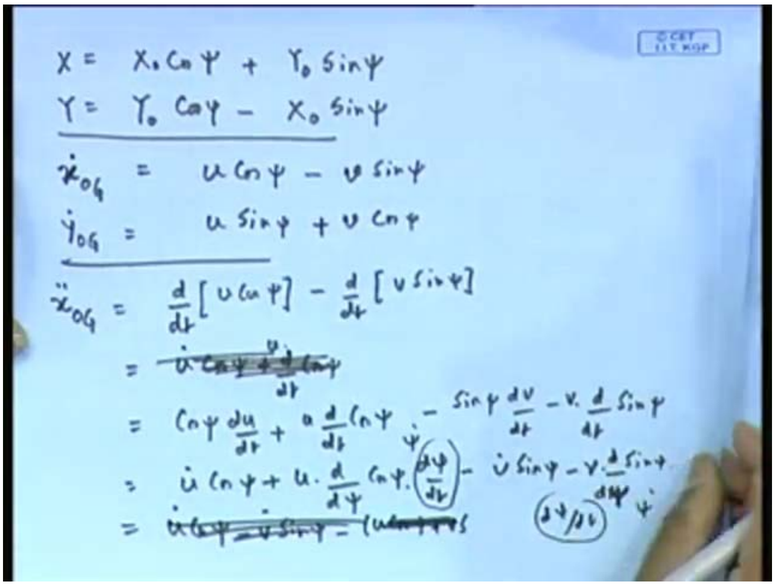

Now, I will just rewrite that because we need to do this thing so we enter the heading X equal to X 0. Now, let me put now straight way instead of theta psi, because actually ultimately what is happening theta is my remember this theta was a general coordinate transformation. But in our case, the this theta as you know from definition of this diagram psi, because psi was the angle between this horizontal line with respect to the heading not beta remember psi. Beta is the direction of the velocity vector with X axis, but direction of X 0 axis with x axis is psi heading angle.

So, I end up getting this plus now that there is a there is a reason that will be I am writing this for because it comes to this is one and now. Actually, these pens are all there double dot the problem will come and now comes the question of this is plus right absolutely. Now, come the question acceleration X double dot o G it is basically, what remember this is d by dt of that now what is this is see d by t dt of that in base etcetera. So, this is going to be u dot Cos psi first term plus d by dt of Cos sin into u actually, I have to write u here see or rather let me write it down separately this is basically see here Cos psi d u by dt plus u d by dt of Cos psi.

And similarly, this side is become sin psi dv by dt minus v into d by dt of sin psi now this see is u dot Cos psi now this term now d by dt of Cos psi is what d y d psi of Cos psi into d psi by d t. So, this is u d by d psi of Cos psi into d psi by dt like that we can write this becomes v dot into sin psi minus v into d by dt sorry d psi of sin psi into d psi by d t. So, if you just you can work it out and find it out this will this will turn out to be u dot Cos psi. I will write next line again, write that v dot that is coming from here this one see here this is psi dot remember this term is psi dot this term is psi dot.

So, this is psi dot d by d psi of Cos psi is how much minus sin psi so it is minus u psi dot sin psi. So, if you basically and this term is what d by dt of sin psi this becomes basically cos. So, if you do that this will become minus of maybe I should go to the next page u Cos psi plus v psi no sorry sorry u sin psi actually, let me write it down next page just following it otherwise, it becoming confusing here. Just keep that here and do it see here, I will just write here this one if this term is u minus u sin psi into psi dot, because you know d by this I have Cos psi minus sin psi or rather.

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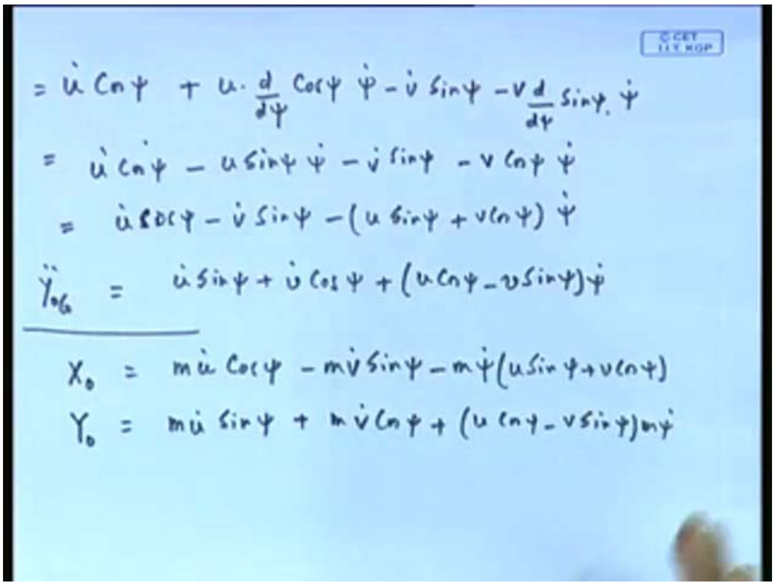

Let me write it again, otherwise, it is becoming confusing see u dot this is I will just repeat this line.

(No audio: 22:16 to 22:46)

Here, see this is minus sin psi u right this is minus v I am sorry Cos this two terms minus u this what we have got. So, similarly exactly similarly, if we did the other one Y 0 G dot dot I will not go through this full thing, but you will end up getting here u dot sin psi plus v dot Cos psi plus. (No audio: 23:51to 24:01) So, you have got this X 0 Y 0 expression now if I put this X 0 Y 0 expression in that equation of motion that first step, we wrote that is m x value z expression of the one that we wrote one second no now, Newton’s equation of motion see this equation X 0 equal to m x G dot dot.

So, if I put that here this because what I have done you see first step I have now represent that that in terms of u v. So, if I write that now this whatever, I have got x o G and Y o G if I write at that I end up getting this thing. I just write that straightforward I end up getting x 0 equal to see m into this thing no m into x o e rather than m into this part that is m u dot Cos psi minus m v dot sin psi m psi dot u and y 0 I end up getting.

(No audio: 25:20 to 25:43)

Or actually, we can write it this way. So, we have got this expression right this straightforward expression here. Now, next step is actually, I must rewrite this in terms of x y then see what happen, if I now change this in terms of x y then I have a relation that will be x equal to function of u v y equal to function of u v that is what we are exactly trying to do.

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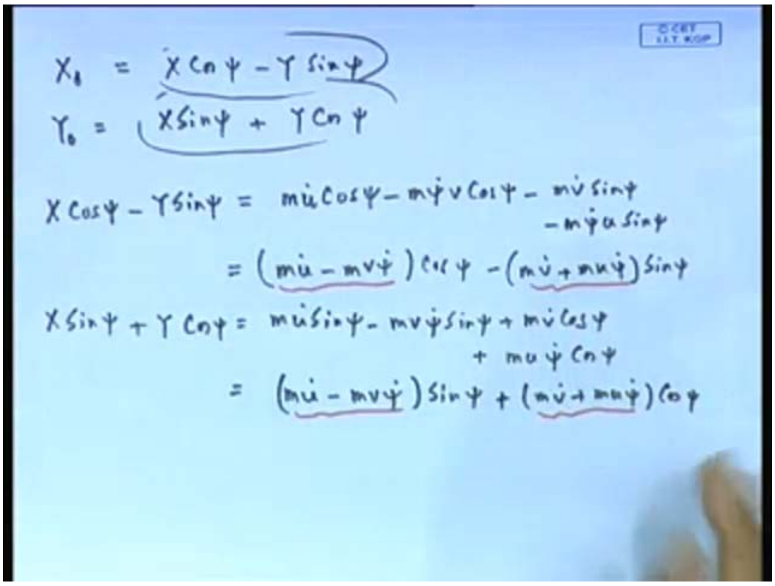

So, if you are going to do that now you see x 0 we already wrote that part of course, where is it or rather we will rewrite it. Because here we are trying to do the other way round see X X 0 is X see this is the transformation right now what we are doing this is equal to remember this is equal to this x 0 is equal to this side this Y 0 equal to this side see this X 0 Y 0 are basically given by this two. And this side of course, is the other side. So, when we write it up we end up getting something like this we I will have respond to write this. Because you will see this see X Cos psi minus y this become equal to m u dot psi minus m psi dot v there is a v there minus m v dot psi minus m psi dot u sin. So, this we can also write is as m u dot minus m v psi dot Cos psi minus m v dot plus m u psi dot sin psi. You know you just manipulation of that the other part is x sin psi plus y.

(No audio 28:36 to 29:19)

This is very interesting you know now I will rewrite this you will find out that see here why we do that we did not have to solve it see this part same as this part this is same as this. So, if you see X Cos psi minus Y sin is something Cos psi minus something sin psi X sin psi plus y Cos same thing into sin psi plus same thing Cos psi I do not have to solve it. You can you could have solve it and you eventually know X equal to this y equal to this in other words, if I were to write it this was the purpose of going through this many other ways. You can arrive at the same result many other ways, but this the interesting point here is that all that we have done is something like that see here we found out this.

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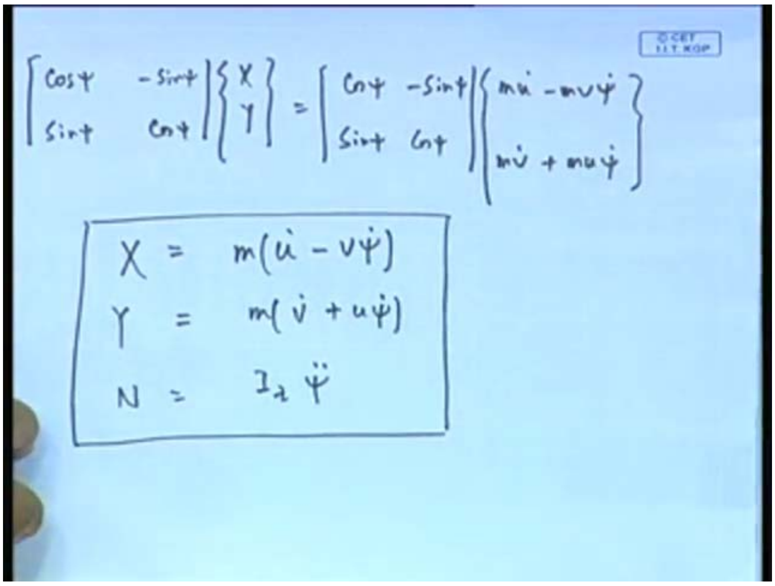

You can say Cos psi minus sin psi and sin psi whichever, way you can write actually, Cos psi this what I wrote this one? This side Cos psi minus sin psi Cos psi into X Y that is what you will you know and this side I will write the same way it is also Cos psi minus sin psi. I will just show it to you here it is you end up getting m u dot minus m v psi dot and m v dot plus m u psi dot. You see here Cos psi sin psi sin psi Cos psi what I wrote here and this term this term this term is same thing. So, if you wrote that immediately you know that X equal to this Y equal to this see so; that means, what I get is I will just write it down X equal to m u dot minus v psi dot Y equal to m v dot plus u psi dot of course, if I were to write N, N becomes I z.

So, this becomes my equation of motion in the rotating system. Of course, you see I was just in a minute tell you we have gone through this coordinate transformation etcetera etcetera, but you know this is a very very very standard procedure it is known for maybe one hundred year. Now, based on any rigid body kinematic, which you actually, have studied basic mechanics you would have seen in any statics and dynamics book the equation of motions are written can be written in a moving frame of reference. This is actually, moving as well as rotating frame of reference all I construct is only X Y, but I will show you the full form, if you consider x y z as well as, all rotations.

The point is that it looks like that what you find out force in the direction is mass into acceleration and direction. But there is an extra part coming, that is what is called lateral acceleration m v psi dot or m v r it is like your sentry like you know like coriolis force rotating frame of reference similar to that. This is the important point to recognize psi dot if psi dot was 0 of course, force is mass into acceleration they translate to each other Y is same N no question comes in some people will write psi dot is as R rotational velocity R psi dot is rotational velocity here.

So, you find x is mass into acceleration and an additional term additional force come m v psi dot m v R, m v R is m omega square by R. Because R is you know v by what the way you talk you know, if R or omega the way you take M G square by R is a same centrifugal for this is nothing but m G square by R this is same thing basically what is happening you are having this additional forces arising. Because of the fact that is rotating in other words if you I was to look at this body at the instant it is here it is also rotating as it rotates here additional force come in the direction X and Y arising because this is the rotation that is a point I was trying to now tell. Now, of course, having said that we will come to this G part later on, but let me just tell you just without anything else see what happens, this was very simple we have done it, but you know.

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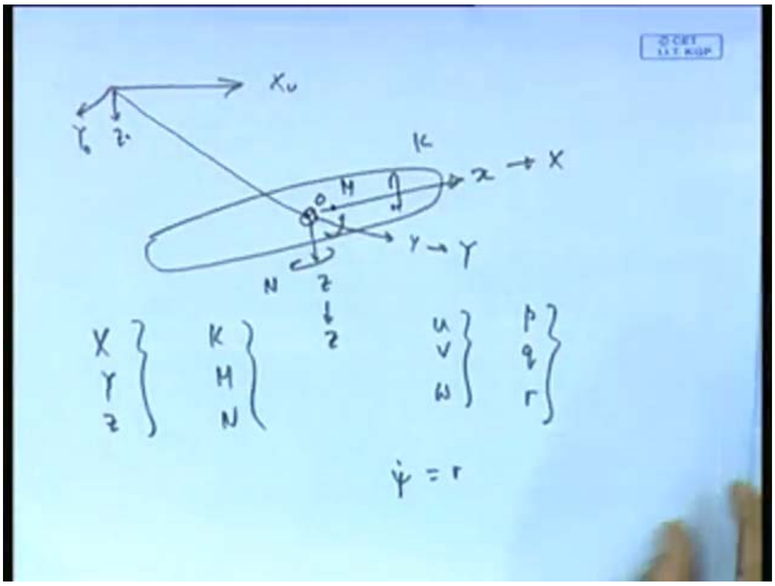

If you were to take a general body submerged body you call it X 0, you call it Y 0 we call it z 0 we have a point here this point maybe point o. And I have an axis system of X Y Z and I call the forces X Y Z K M and N that is X Y Z K M N the three forces along the body x y z axis three rotations about this X Y Z axis. If you were to define that, if were to go through the same equation of motion mass into acceleration is force you end up getting and also, if you had to take a general point o, which is not center of gravity necessarily some point o, which has a coordinate of with respect to G X G Y G Z G etcetera. Means, o is located at a point who from which the center of gravity is X G Y G Z G you end up getting the expressions, which I want to just show it to you, which is well known, which applies to any three dimensional body like a space craft missile torpedoes submarines aircraft anything they are well known.

So, this turns out to be the equation like, because we should probably see that u dot I will also tell you one thing is that u v w are linear motion and p q r they are called the rotational velocities. In our case, remember psi dot is r, but it is not necessary true for a general body, because here the orientation is not same z axis may not be same. So, there is again different transformation there. But anyhow, p q r are the velocities along it is own direction that is velocity over this is p pitch velocity is q. Your velocity is about is own axis is remember, your velocity is about it is own axis, if the body was like that velocity about this axis is what we call r.

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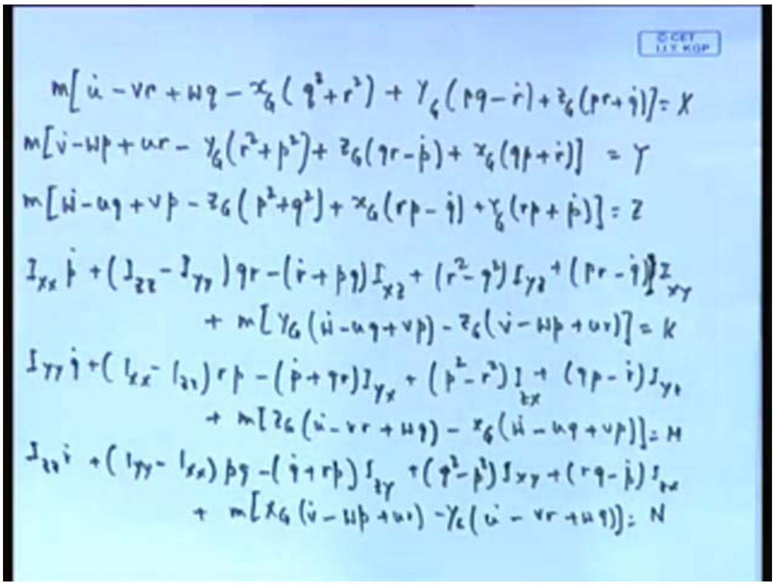

So, then what happened, it becomes v r x G y G z G are the coordinate of point o G with respect to o that is my o is here G may be here. So, x G y G z G are this coordinate of this point as measured. Why I want to show you is that see that we are dealing with a extremely simplified form of this equation although we know the entire part is x force let me write complete it with this side I have a reason for writing this why because there are forces originating from different.

(No audio 38:05 to 38:58)

Y G there is a three here then I X X p dot.

(No audio: 39:12 to 39:40)

Let me just write it, because I have a reason for writing this no sorry I X Y plus a m Y G w dot minus plus v p (No audio: 40:08 to 40:20) this is roll. Yes let me complete this Y 2 part quickly r p minus p dot, because we what I want to show you that from there first of all the origin of the terms second of all how it could be used to our ship form the one that. We have used and of course, we talk what we do with all this let me just almost there I Y Z plus m there is a nice cyclicity in this equation that is another thing that is why we want to write it the full form.

So, that if you write one you can get all of them without making a mistake last one z z r square plus I as I X X p q minus q dot r p I Z Y plus I X Y plus r r q minus p dot I z x plus m x G v dot minus yeah I am just completed v r see why I wrote all I will tell you know. You see it all whole lot of this expression what we do with all that etcetera the point is that in a 3 d case you see what is happening look at this we will look at this y and this equation more. M v dot we had m u r we had, but when you have a three dimension you know then you also have this w p another one, because it is on a three dimension.

The vertical remember w is heave force p is roll then, because Y G Z G X G are not necessarily 0 there additional force coming remember or rotation remember. See the body is something like here, but rotating about this point, because I am defining here. But I am defining the forces about this axis, but rotation is about this point. So, as it turns there is an additional component that comes p square plus q square into those distances that is kind of an inertia force that comes in these the rotation square into the distance. So, here this is so, like the additional term also come like that Y G p q r etcetera all this are coming because of this fact that there is a rotation at a different point where you are getting the force. Where you are taking a force here, but rotating about this point.

So, as it rotate here there is a component of force that comes, if you look at this rotational component you have p p that. Now, these terms are coming z z y I q r is again, because of the fact this is of course, will come always from the expression now these are coming x z y z etcetera. The reason is because you are defining the coordinate at o not at G had it been a G then I of course, I will have this only the principal moment of inertia, but because I am not at G and at o I have got this additional term coming similar way this come. The other thing I want to tell you cyclicity you know, if you have written this one you will find the first term u v w p q r u v w p q r v w u, if you see, if you make a cycle u v w you start from there you know like it will always follow v means, w v w u r p q you know all these terms follow.

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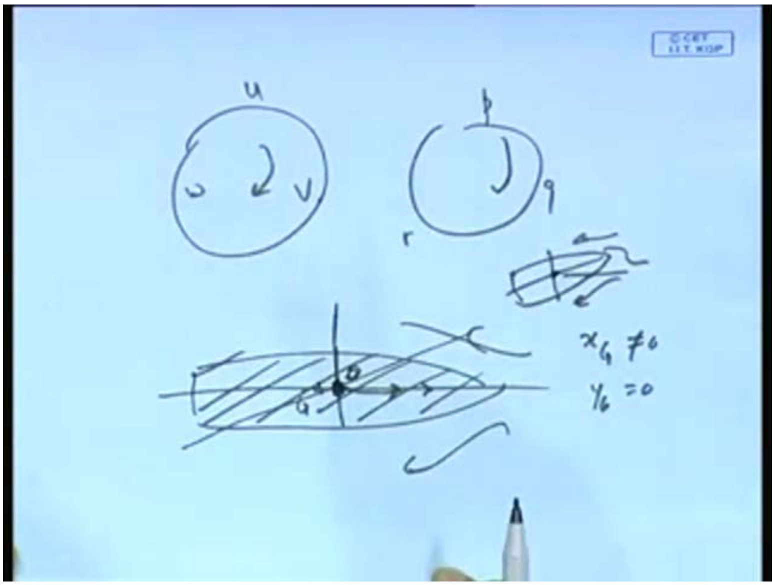

If you actually, write as you know like a circle like what I should say like make a circle like u v w and if you do another one p q r. You will see that all these term, if you starts from here next month will be this one like, if I have say here the rotation q the first term is q into r the next will be r into p and next will be p into q. So, you know, if you write one you can just see you know, if you write in a line you just have to see the first one rotation it will be always in this sequence, if the first one is this next line will be this next one will be this even, if in is I x z x y y z x same way x y z if you see that x y z y z x z x y that is the sequence.

So, there is a very nice cyclicity although it looks big it really you do not make much of error in that. Now, what we had in our equation only the two I had this one and I had this one actually, I had only this and this three I can say, because if I take x now, what did we get? You see, if I look at that I got here m u dot and v psi dot I have got this term and I have got this term this is not there because I had a 0 I had taken that remember I had expressed that in 0. Now, supposing I do not do that some now, this I will come to that supposing I do not do that what will happen to this then I will come to that in a minute.

So, I have got this term you can see from here straight forward and in here I have got; obviously, this v come here I do not have this I have this term and I none of have this term. Now, if I look at that none of them is there only this term there, because all are 0 I do not have p q and all that now here comes one modification. See quite often you know look at this expression quite often we do not I let me just draw another diagram, I have this ship here G is here, but I may not want to define, that to be the origin of the body system. Why because remember, all I have to measure the forces at some point of time all this x y z forces quite often it may be convenient to define the forces with respect to a point, which is fixed on a geometry not on centre of gravity.

Remember, if I take a point o which is somewhere like say one v p mid ship that is a geometric fixed point, but central gravity point depends on the kind of loading. Suppose, I take a particular hull I would have mid ship at the given location, but it is one c g can be anywhere, depending on the loading condition. Now, the fluid forces that comes around it that is on the right hand side on what they will depend. They will depend only on the external geometry. See the fluid forces that arises, because the flow is going passed it they are depending on the geometry of this hull does not matter where the G is located G can be located arbitrarily depending on the loading condition.

So, since at some point of time I have to be worried about the fluid forces it is sometimes convenient to find out the or decide the coordinate system at a geometrically fixed point. Because if I were to have a geometrically fixed point here I will be able to find out the see for example, I wrote it above this line I have this ship and then I want to find out the flow passed it. So, as a result quite often it is very common to apply this equations with o with a centre of with a with origin taken at a point, which is not a center of gravity. However, it will not be along this line, because remember ship is symmetric both starboard. So, normally you will take an o which is along the central line. Therefore, you are likely to have a non 0 x G, but Y G will be 0.

So, if that is the case, then I end up getting additional term minus x G into R square. Here this term minus x G into R square; that means, I will end up getting here, if I were to use another term here.

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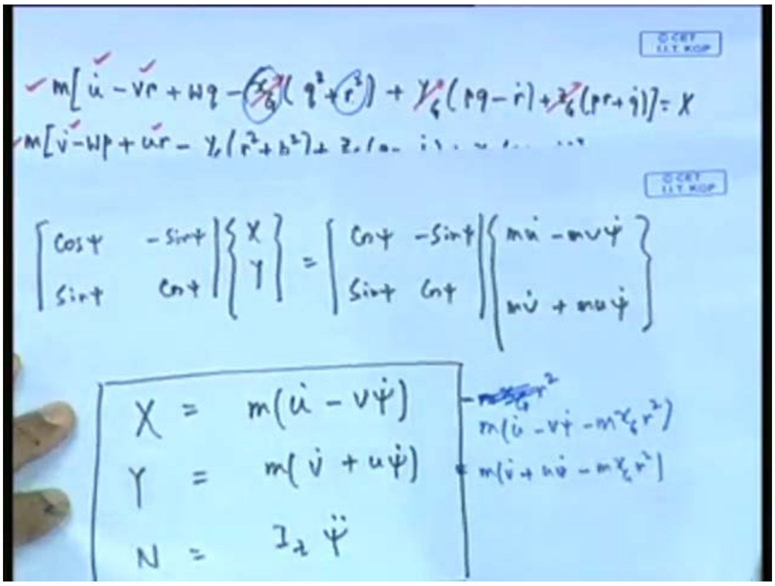

See, minus x G were m minus m x G into r square or if I were to put it inside it will be minus m x; that means, what I end up getting if were to write here m u dot minus v psi dot minus m x G r square. And in this I will end up getting an additional term, which is going to be here minus y G r square m v dot plus u r minus y G I square you see m v dot plus u psi dot minus m x G r square that is m x G r square comes in. So, I am not deriving this equation any further, but depending on that we may use this or we may use this depending on where you are taking the origin.

This will be more general because here you are taking the origin actually. In fact, there is one more point on this there may be also, me more may come, because what will happen, that I do not have p I do not have q, but I have r only. So, let us see whether there is any r term come m x G yeah here an additional term will come remember m x G u r m x G v dot plus u R this term will come, because v is also not 0. So, you will have I z z r dot plus m x G v dot plus u R this term will come extra. So, what I am trying to say that it is very possible for me to modify this equation we will be looking at that later on.

Now, what would happen is that in our case, we really would not look at this much, because it is a independent coordinate. Now, we will see that later on we will be able to use this equations this three set now what happen I have this equation. So, what the next question? This equation has nothing hydrodynamic about it simply says mass into acceleration is force that is all it does not tell what is the force how much is the force is the force going to be more less etcetera etcetera. So, obviously, our next job would be most important job would be to look at the hydrodynamics, where is the hydrodynamics purely in X Y and N it is this X Y and N, which are a representation of the fluid forces acting on the hull what we said here is that.

If I have a force x then x must be equal to m u dot minus v psi dot, where u dot u v and psi u dot v dot psi dot are the so called you know velocity is displays and etcetera. So, mass into acceleration equal to force is what we are trying to tell here. So, now, the now the next job, which is what we will be doing the most crucial part of the job is to now find out separately how I represent x y z. Remember, this expression and as I just mentioned these equation has nothing to do with hydrodynamics etcetera it is only to do with rigid body kinematics and dynamics any mechanics book.

You also would have done in your second year of mechanics courses that you know your truck is going with a crane there the crane rotating and some weight is coming down we use to do in our school days therefore, find out some force and acceleration. You are standing on a lift you are going. So, you throw a ball at what rate it will come down essentially you are looking at the rigid body kinematics and writing equation of motion or representing forces in a moving frame of reference the frame can be linearly moving as well as rotating etcetera etcetera that is all. So, this only represents this side only represent that.

This side only tells me, if I were to express forces in a moving frame of reference what are the additional forces come, because of the fact that the frame of reference moving nothing to do with the right hand side which is my actual forces. So, next class as we continue we are going to look at this side, because this is what is besides the behavior. So, with that we will end today’s class, but we will look at this and then we will see how we go on studying. All these, equation ultimately has to be applied to me to study how the ship would behave if I were to turn or if I had to make a z g zag with that I will end it today.