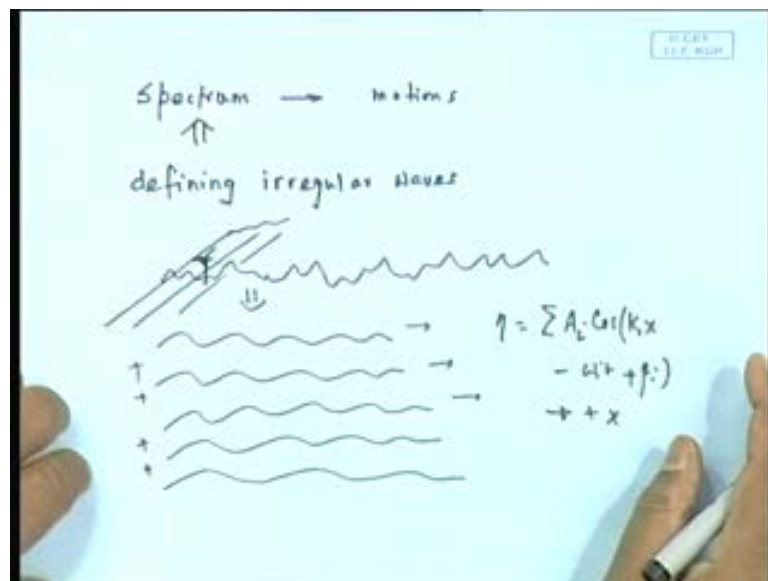


Seakeeping and Manoeuvring
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Lecture No. # 17
Description of Short-Crested Sea

Ok, see in the earlier lectures we have learnt about how to describe a spectrum.

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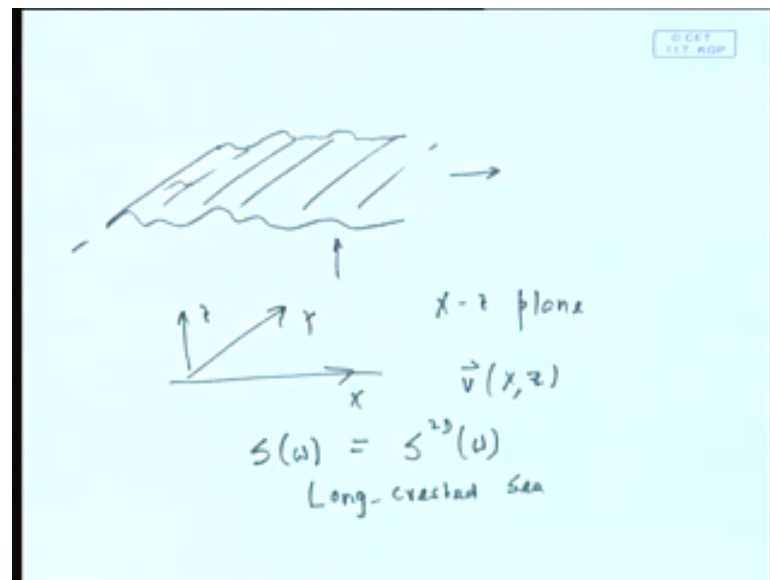


How to describe an irregular sea by means of a spectrum and how to get motions in the spectrum. So, I was having this for defining... ok, we are going to revisit this part in this lecture again because when the spectrum that we define earlier, had certain limitation certain approximation that we did not explicitly set.

What did I do for this spectrum? See I set this record is sum of... ok, what we said you see, in defining a irregular record we said that it can be composed of adding sine wave but, what we probably did not emphasize at that time is that all of them, we assumed to have been travelling in the same direction, which basically means if I add all of this thing see eta I obtain by making something like a you know $i \cos$ say $K_i X$ minus, $\omega_i t$ plus β_i , all of them are travelling along plus x direction.

Then what happen if you stand, supposing I was standing here you know I was standing here and I look my left and right side, what will I see? I will find the crest to be infinitely long. In other words this wave would have appeared like with the long lines you know like.

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If I were to draw the wave would appear all the crest lines are long, if I stand here because it is infinitely long, because I have assumed all of them are going to the X direction.

So, what happen take any Y value whatever the Y value is the eta is same, that means if I were to you know like used a X, Y, Z coordinate this was in an X Z plane, which means whatever Y equal to constant. I cut a plane my velocities are all in X Z plane which means if I were to see the particle the way it moves etcetera, the velocity vector you know V vector it is all having only a dimension of or dependence on X and Z. So, this is essentially a two dimensional fluid motion there is no motion on the Z direction. So this sea that what we have obtained this S omega was actually what was called a two dimensional spectrum because what we have presumed is that the waves are all going in one direction therefore, the motion is two dimensional, so this is what Y we have got or this is sometime call as long crested sea.

However, now why we are revisiting it is because suppose, you actually go on an ocean and suppose you actually you know like go on a ship or make an observation you are

stranding here you are not going to see your right hand, left hand side waves to be same. You will find out a randomness in not only in along X direction also along Y direction, so you are not going to find this crest line to be same both sides, so obviously somebody may question that the spectrum that I defined it is good it represents this signal by means of sin but, what you have said is the waves are long crested whereas, in reality they are not therefore, we need to now define what is called short crested sea? Now, I we will come back to this question again see what is happened.

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The image shows a whiteboard with handwritten mathematical equations. At the top, there is a sketch of a wave with an arrow pointing down to the first equation. The equations are:

$$\eta = A \cos(kx - \omega t)$$

$$\eta = A \cos(kx' - \omega t)$$

$$= A \cos\{k(x \cos \theta + y \sin \theta) - \omega t\}$$

Next to the equations is a diagram of two coordinate systems: a fixed system with axes x and y , and a rotated system with axes x' and y' . The angle between the x and x' axes is labeled θ . A hand is visible at the bottom right, pointing at the equations.

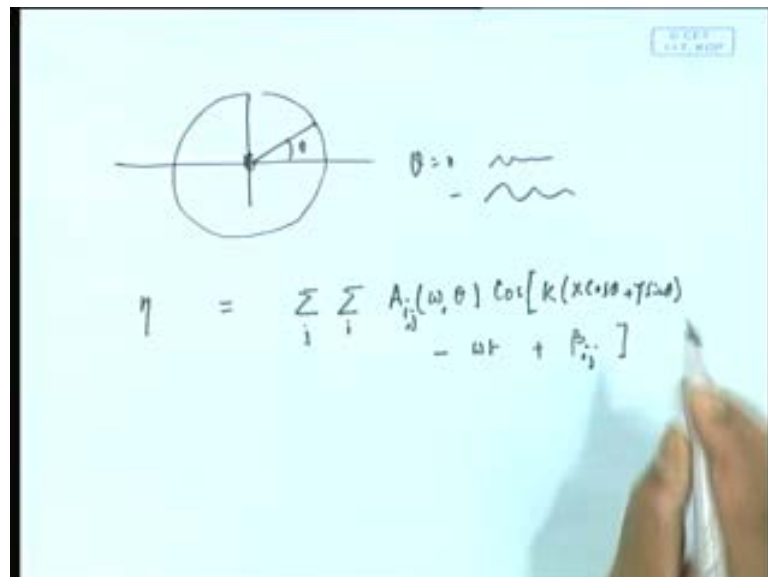
Let us take it this an example somebody went to an ocean there is a boy, there or some equipment you measure and the data as I say look something like that. Now it was our question we presumed, we kind of composed that by making them sine waves in one direction what if I were to add? We have to think that this is not only all the waves coming in one direction but, also one direction of different length but, also I think that this can be broken down to waves of all lengths in one direction and all directions.

For example, now if I look see now, I come slowly now, I eta looks like $A \cos K x$ minus ωt , now this is in X direction supposing the wave was going in X dash direction, what would be the formula? Formula would be $A \cos K x$ dash minus ωt . Suppose, see this is when the ship is going in along X direction and let me call this to be X dash, if not ship, a wave. This represents a wave travelling in

positive X direction suppose, I want to find out an equation of a wave travelling in positive X dash direction all I have to do is to write X as X dash.

Now, suppose this is theta, then what is X dash? X dash is it is Y, X Cos theta plus Y sin theta that is a standard transformation, so I can write this A Cos K. So, this would be this full thing I can put, so this would be a wave travelling along a direction making theta with positive X axis. So, this one now what I we will do is that, we can presume this wave to be composed of all waves coming from not only all frequencies that means omega or K but, also from all thetas.

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So, in other words what we are going to think that in this point say, I were to draw a circle waves here that I measure are not only coming from theta equal to 0 all but, also coming from theta equal to theta etcetera, etcetera. So, what happen then I can tell eta to be see earlier I had this, I will write it this way A i omega, let me write it this broad way K well x, I just write it blank here minus something like that see I did not fill up the gaps remember what we had earlier if all the waves are assumed to be coming along X direction I have A i omega cross K x minus omega t plus beta K x minus omega t this is.

But now what we are going to do, we are going to suppose I presume that look this particular thing can be broken down to all waves not only from all omegas but, also from all thetas therefore, I have to put one more this here. So, this let us see this i, this i index is for omega, so I have to put another index theta here and I can put j here, j here then

this is $\cos \theta$ plus $Y \sin \theta$ this becomes of course also $i j$. So, this basically represents therefore, as if I am saying that the waves are coming from all direction and all sum, so it is a, see it is a very interesting thing it is a question of only breaking it down remember that when you are sitting in a office a satellite has measured ocean height data it only gives you this data.

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$$\eta = A \cos(kx - \omega t)$$

$$\eta = A \cos(kx' - \omega t)$$

$$x' = x \cos \theta + y \sin \theta$$

$$= A \cos[k(x \cos \theta + y \sin \theta) - \omega t]$$

$$A = \underline{A(\omega, \theta)}$$

it varies η that is all. It is up to us to think this η is composed of what kind of summation of waves, now remember that we could sum all the sine waves because sine waves summing gives a realistic wave because waves are linear, if I put a black box I add all the sin then what I get is a sine wave but, here I did not say, I only added only those sine's which are along the X axis but, now I want to say that look I can break it down by adding all the sine's not only along same X but, also along all X's.

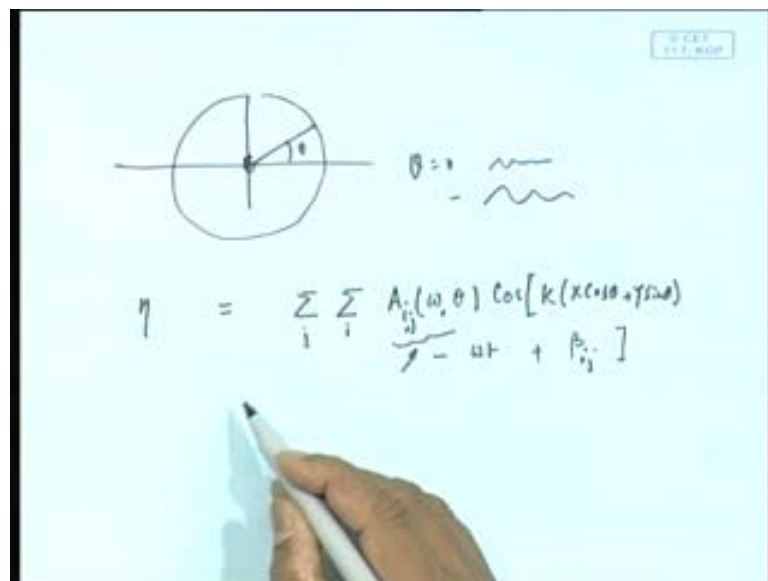
Obviously, that would give me a simulation of this picture is not going to look like a long crested anymore because obviously there is a dependence on Y in other words, if I were to find out η at sum X and sum Y there are not going to be same earlier there was no Y, so no matter what Y you to η was same but, now you are going to be different so it will have a spread three dimensional picture, so that will be a better model for a 3 D spectrum, 3 D waves.

3 D wave meaning again when I say the word 3 D waves which means the velocity of the particles would have both u v and w earlier had only u and w that means particles are

moving along X axis or Z axis. If you make a cut Y same but, here it is not going to be same, so this obviously is a three dimensional wave, so this is what is called short crested sea. You see that means if I were to break it down to this way and now I plot similarly, you know that process, I will come energy etcetera because you know the energy is the principle thing.

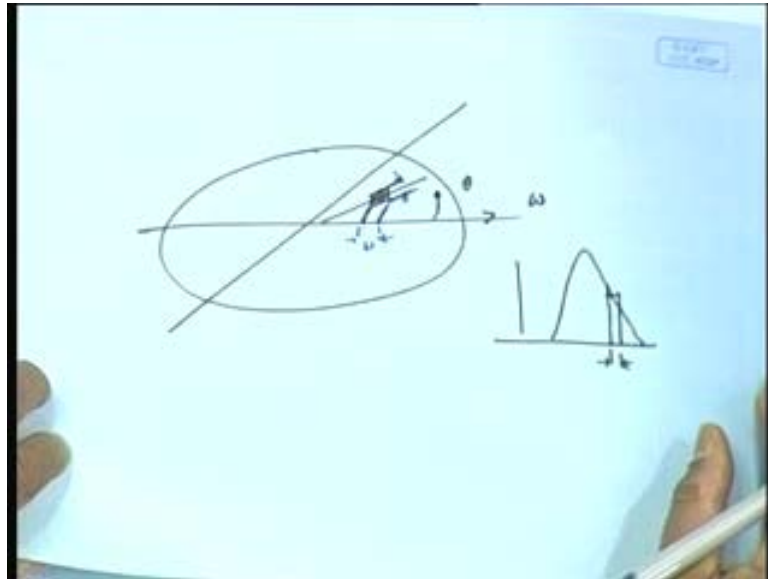
After all what I have to do is to plot this A, see here what would happening? What was happening is that, here is that earlier a was a function of only theta and now A is going to be a function of omega, now A is going to be a function of omega and theta that is where is this picture did I do that, we can just do it from here also see this A is going to be a function of remember earlier it was only function of omega in the 2 D case but, now it is a function of omega as well as theta.

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So, having said that so the interest here as I said this is already there here see omega and theta question is now I need a spectrum obviously, what would happen if I were to put a spectrum here, in this we will come to this spectrum shape here.

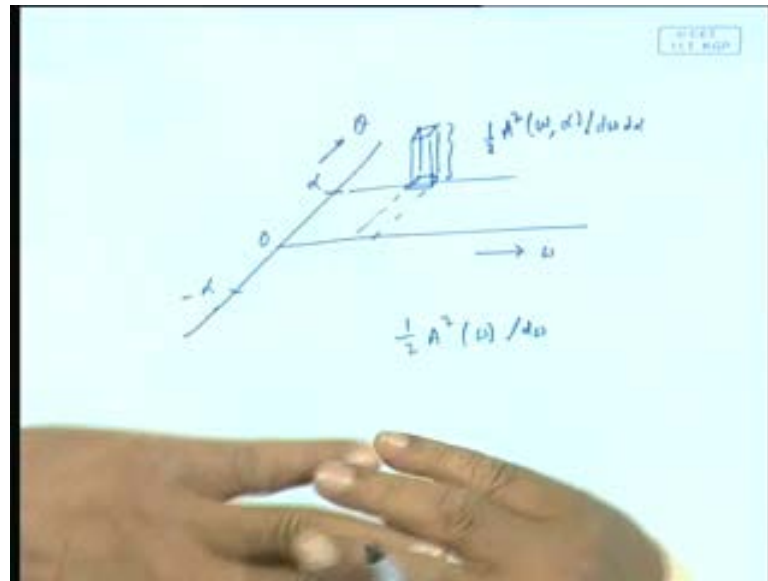
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Once again I have to keep drawing this, you know all the time say we will call it this way and this is theta. So, if I were to now plot all the waves of between this frequency and this length see now earlier what I did? In a spectrum, I plotted the energy of all the waves a quantity proportional to a quantity proportional to energy of all the waves coming between this frequency band.

Let us say all the waves of energy between 50 to 60 meter, I plotted one place 60 to 70 meter, I plotted another place like that I plotted but, now what I have to do is to plot a 2 D plot so it is going to be a bell shaped curve, it is going to be a shape not going to be a 2 D part. What I have to do is to plot see in this diagram, so if I call this to be say this is theta, this is omega then all the waves of length between this d omega that is this omega and angle between this is what I am going to plot here at the top. So, if I keep plotting like that then I'll end up getting a graph like that in fact instead of doing that.

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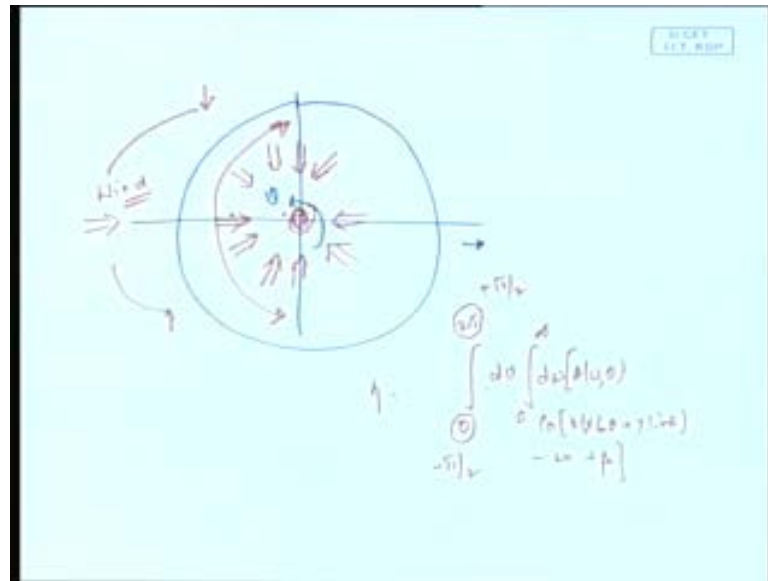


We can also this is a bell shape one I can also do more I will put omega here and I put theta here. So, what is happening say theta is 0, let just give an example this is alpha here this is a minus alpha here like that it goes.

So, what I am going to do? I am going to plot here this value, this block which is going to be a measure of half A squared omega, here I wrote alpha, alpha divided by d omega d alpha of course, remains this thing. You can also say d alpha, let me also put it this just for the sake of, so this is what I'm going to put see early I in have this what I earlier I did I put half a square omega divide by d omega, here I have put this because remember the overall volume, now the volume under this should become the energy, so this is what is what we mean by describing short crested sea.

Now, that means it has got a dependence on omega and on theta, now let us lo at some physical reality part, so this we understand this way of plotting it.

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Now, let us say from all which direction the wave comes in now I'm going to plot in this way theta again because we have to keep on plotting that, so I call this easier to see this is say omega, see this is omega axis and this angle is the theta.

Now, you see here what I'm saying I have all the waves coming from this, let put it this way I assume that at this point what I measured, is all the waves coming this side, then all the waves coming this side, then all the waves coming this side, then all the waves coming this side. So, what I am saying you see that the measured data consist of all the waves coming from all sides but, now there is a problem you see here take this opposite all the waves coming from this side and this side. What would happen? If there a same frequency, they would cause a standing wave but, however we have a progressive wave.

So, what happen is that, that is very important we have to say that all the waves that is coming are only over one half sector because the other half sector is exactly opposing and if I consider them, I end up getting a standing wave or if it is a different frequency see if I use this as A 1 and this as A 2, I end up getting a standing wave or a part progressing in one side. So, what happens therefore, from physical point you can easily understand from physical point of view, I did not have to take presume this integration this that means if I were to well **if I were to** call eta to be an integration over d theta, over d omega of a omega theta you know $\cos kx \cos \theta$, plus $Y \sin \theta$ minus ωt plus β something like that.

If the graph know you understand, this is only a integral form of the equation instead of writing there I said summation is integrate from a omega theta this but, over d omega d theta and here it was 0 to infinity and here it was 0 to 2 pi but, now we see that this should be actually one half. So, one half means 0 to pi or it is easier to write in terms of minus pi by 2 to plus pi by 2 because that gives you a dominant direction and spread both sides.

See zero to 2 pi is a question of opinion from which you call zero, zero to 2 pi means I called this we have all the way this much, now here I am saying the waves will be only over this one of the half sector. So, I can call this see if I call zero to 2 pi from here I am calling but, if I call this way then I have got basically minus pi by 2 to plus pi by 2. The question what I am saying is that whether you call zero to pie or pie to 2 pie or you know like minus pie by 2 to plus pie by 2 is the same thing because you are talking of waves coming over in a half single. So, it is a mere convention to use physically or few more I will tell you more easier to we should use minus pie by 2 to plus pie by 2 why? Because remember winds waves are created because of winds primarily these are wind driven waves, no wind generated waves.

So, wind is when to go in to blow in some direction wind and you would expect most waves to be in that direction but, we are saying that this wave can be actually from ninety degree plus minus, means one eighty degree. Obviously this will be the primary and you will think waves coming also from somewhere this direction and also up to this direction, so if I take this as base then the waves become minus pi by 2 to plus pi by 2 rather I would have a most dominant wave, very high kind of wave at the direction the wind blows there will be other waves coming both sides 90 degree spread that is more logical after all I only looking at the integration.

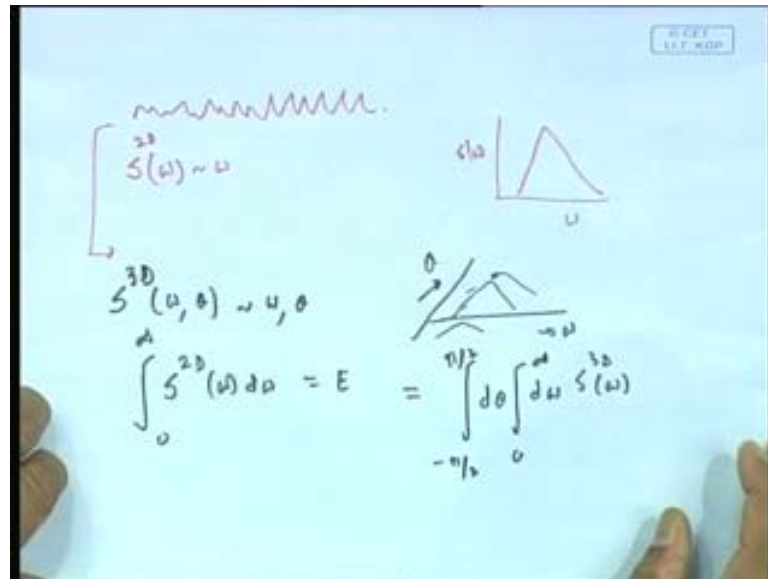
So, if I do for example, zero to minus pi by 2 and zero to plus pi by 2 with zero is that direction of the wind they need makes more physical sense and of course, it satisfies my property also, so we typically this is done. So, we therefore what we are saying, we say that I can model once again coming back to this irregular wave I can model this irregular wave by summing or taking waves from all direction but, now I find out that the directions I can take, I need to take only over one half of the full circle that I can write more easily from minus pi by 2 to plus pi by 2 in integral sense.

Sir but, this is assumption that the waves coming from different direction might of the same omega for standing waves.

No even otherwise, see even otherwise the point is that if no same omega because after all what is happening that when you are considering integral for each omega there is an omega, there is an omega for each omega. So, if you take each omega and each omega they will be a standing wave so therefore, I cannot have because we are looking at a propagating wave therefore, what we are doing is that, we do not have that or if you want **if you want** you can say you have that its amplitude zero. So, the dominant waves are all coming from basically within this half section that is what we are saying that makes more sense in the modeling part.

Now, comes the question of spectrum well we have seen that obviously the spectrum you have to plot this part, we need to plot, so called what I just now I showed in a diagram we showed that we are plotting this height right d omega, d theta etcetera but, how do I know model it? See now I have a modeling question, I really do not know how it has spread after all when I get a signal like this you know I can make a Fourier analysis and get Fourier analysis allows me only my frequency distribution but, I cannot have the spreading distribution, theta distribution I cannot get that information see if I get us, give you a signal I tell you to $f f t$, I can find a omega verses omega but, here I am making that there is also theta there that I cannot not get from $f f t$ signal processing you must, we must understand this, so then what happen I have to make a model of it?

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Now, comes the question of this model, now you see same wave it goes like that again we get back to the same wave now person A made it a 2 D model, so here the formula for S omega 2 D verses omega here this person A has made this and has a formula for that it looks like this, omega verses S omega now same signal is analyzed by person B and he wants to write this in terms of a kind of a you know like a omega theta omega, he wants to write it as 3 D omega theta verses omega and theta. You know if you want to plot you will obviously end up plotting that now he the same signal remember.

So, what is happening one interesting point is obviously, what is happening is that see energy under this wave is S 2 D omega d omega, this is my energy of the wave zero to infinity and energy under this wave, this signal, this model is going to be but, this two must be same because they represent the same energy, total energy after all energy cannot change it is a question of viewing it the same waves, I am thinking person A thought well I am going to model it by saying it is coming all from direction one, person B says no I am going to think it is coming from direction one, direction two, direction three etcetera, etcetera. Energy remains same area under that it will remain same at volume under this.

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$$\int S^{2D}(\omega) d\omega = \iint S^{3D}(\omega, \theta) d\omega d\theta$$

$$S^{3D}(\omega, \theta) = S^{2D}(\omega) f_2(\theta)$$

\Downarrow
 $S^{2D}(\omega) \cdot f(\theta)$
 Spreading function

So, I have this fundamental relation that is area under S 2 D, if I will to write it again that is if I were to write this in S 2 D ω d ω must be same as. So, here comes the question of modeling, you know what is done is that people model this S 3 D, it is this in us this is the very important assumption, this is assumed as first of all it is assumed that it can be broken down to two part f_1 this thing and f_2 theta, as if see this is an assumption you presume that I can break it down to this a part purely function of ω and a part purely function of theta and now this part which is taken to be S 2 D ω and we call it f theta.

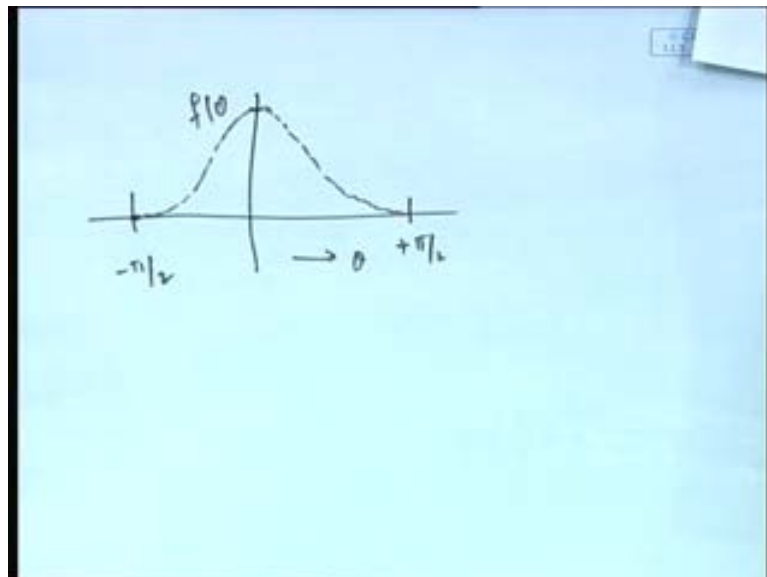
So, it is presumed therefore see this is very important that a 3 D spectrum can be written down as a 2 D spectrum multiplied by a function which is function of theta alone, this is known as spreading. You can easily know why it is spreading function because what we have done we have taken S 2 D, see if I were to now lo at this side this is my ω i have then S 2 D part here and I simply spread it this side, this is theta side by a function multiplied that with the function and go down, paper it down that is gives me spreading see for example, if f theta was one then it will be equally spread means there is a equal chance of all waves coming from all directions.

But we know from physics that wind is blowing one side and you expect most waves in that side and only a small waves on other side as you go ninety degree of the wind, you expect very less wind waves coming out. So, wind is blowing in suppose wind blows this

side you expect most waves are this side and very waves coming this side very less wave, I mean mostly seen I eighty percent here may be little less on coming from 45 degree, even less from 30 degree practically nothing from 90 degree 45, 60, 90 like that.

So, this obviously it is always spread, it will it is suppose to have this theta is suppose to have this if I were to plot zero to pi by 2 or here minus pi by 2 you expect this spreading to have a bell shape that's make sense isn't it? Because this tells me the strength of the wave coming in direction zero, wind direction and obviously the strength will reduce the percentage of waves coming from direction away from zero would be less as you go further from zero it is spread.

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This idea we must get in your mind, once again I will tell you the spreading function concept, so if I were to put theta here, if I were to put f theta you would expect it to first of all f theta will not be, will be zero beyond minus pi by 2 or plus pi by 2. So, if I call this plus pi by 2 minus pi by 2 you will expect it to be maximum here, zero here and it will be something like something like that, that is your that is a realistic model.

Of course, you can all say why not that well we can always do that people have also analyze this spreading function somewhat by taking the wave slope you know actually although I say you cannot measure but, you could measure nowadays with sophisticated instrumentation from a random signal of slope also, after all it is all going X direction I have a slope in one direction, if it is coming from this direction I have slope in different

direction, so if I measure the slope I get a feel the content of **content of** directional you know waves, so people have been doing obviously it is found most waves are in the direction of the wind so this is what we do.

Now, comes the question of how do I model it so we look that to this equation again and we go back to this now I find out this let us write in another paper.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the equation is $\iint S^{3D}(\omega, \theta) d\omega d\theta = \int_0^\omega S^{2D}(\omega) d\omega \int_{-\pi/2}^{\pi/2} f(\theta) d\theta$. Below this, there are arrows pointing to the components: a downward arrow from $d\omega d\theta$ to $\int_0^\omega S^{2D}(\omega) d\omega$, and an upward arrow from $\int_{-\pi/2}^{\pi/2} f(\theta) d\theta$. The second equation is $\int_{-\pi/2}^{\pi/2} f(\theta) d\theta = 1$, with a wavy line under $f(\theta)$ and the text "spreading fn." written below it.

So, I have S^{3D} ω θ is equal to S^{2D} ω f θ , the other next come the question, well I am going to put it here now integration over that $d\omega d\theta$ is same as integration about that I mean the integration about that this.

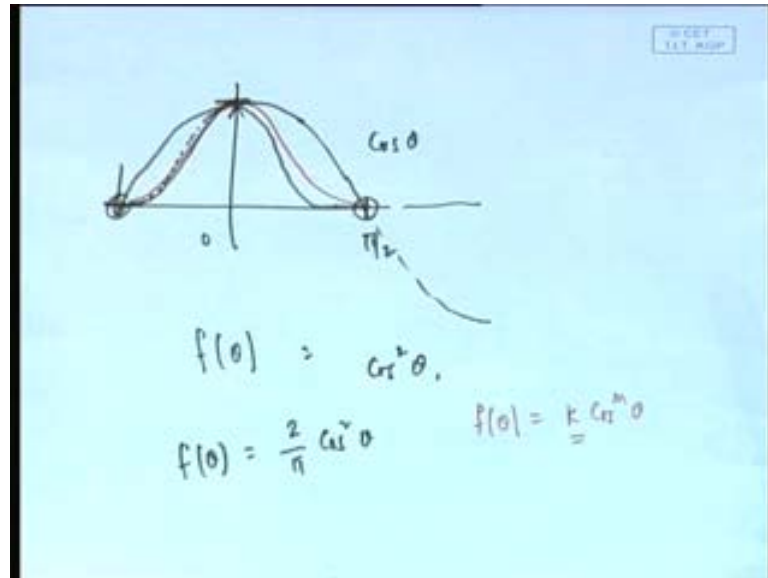
Now, this see carefully follow area under that is S^{3D} this $d\omega d\theta$ but, S^{3D} is written as S^{2D} ω and f θ $d\theta$, f θ , so when you integrate that it will be between the ω and between this θ this is just from identity from the approximation we used because we said this is, this into this.

Now, however understand that this side is also same as S^{2D} ω $d\omega$, this side same as this because we found the area is same, so if this is same as this what do I get? I say I get that $-\pi/2$ to $\pi/2$ must be equal to one, what we end up getting is that we say f θ is a model, I can use any formula but, do not you must use a formula for f θ , so that this condition is satisfied this is very important for us to understand.

We must have this condition satisfied if θ and $d\theta$ must be equal to one over this, so this is a spreading function and we must have this relation.

Now, what can be possible spreading function well as you understand since it is not fully clear people have model the spreading function many ways.

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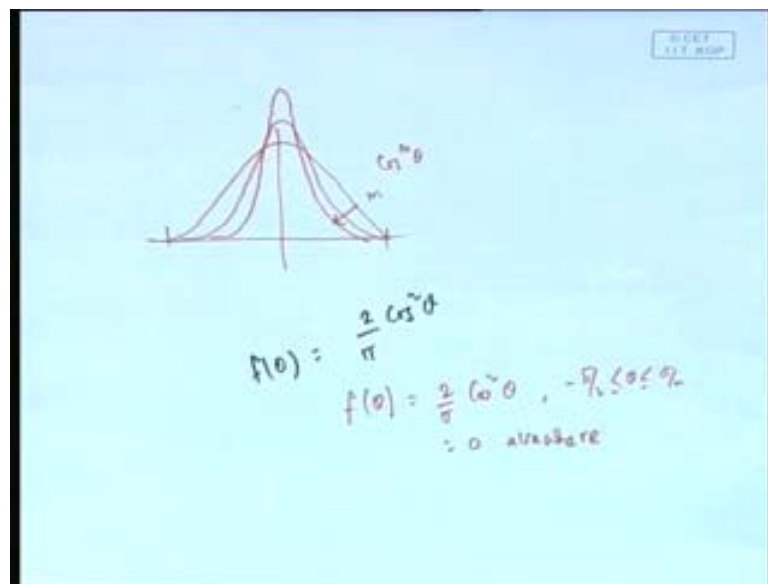
Most usual we have modeling is to use a Cos theta graph, why Cos theta graph you see if you use Cos theta, a Cos theta here it is pi by 2 here is zero Cos theta basically goes like that Cos theta has this look know that it is zero maximum at zero and at pi by 2 maximum value at theta equal to zero and at pi theta pi by 2 it is zero, so it has got this kind of a spread.

So, essentially $f(\theta)$ it would obviously you cannot use sin theta as a model because sin theta would be exactly opposite, you would want to have cos theta $f(\theta)$ remember I wanted to have $f(\theta)$ which is maximum here and as you go down, lowering down this is one possible way but, people do not model that there is a problem of using cos theta alone why because you see here you know it is like this the graph or maybe it is like this but, what you want is actually like this because, so I comes down papering and this can be done by using cos square theta or power of cos theta.

So, typically you will find out now that $2/\pi$ is a possible function that will actually look like this and it also has an interesting property that the slope becomes zero here, the

function maybe we should lo pertinent it becomes like that it goes to zero, if I use this forget this just cos square theta you take, if I take cos square theta it will go to actually maximum here go down to zero here as well as the slope is zero here but, then of course,, if we use cos square theta the integration is not one. So, therefore you have to constant coming 2 pi by, now from this analogy we can find out that there can be f theta of the form which is which will lo like cos.

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Well you can have define m you can find out of course, you will find out that larger the m, more no peaky it is A as m goes up, see as what I mean if you use cos square theta that of course, there is a constant 2 by pi. If we use cos four theta, it will be some other constant because obviously the area under that has to be one as I said it may be, in fact it may be something like eight by three pi that can be worked out but, my point of view of saying is that as you use more power then you have got peakier. What does a peakier means? Obviously, it means this less and less spread.

So, you know if you use now here is the question of you know modeling let us say that you went to north Atlantic you find that well there is a three dimensional wave all right but, majority of the wave is much more in one direction much less in spread. So, you would use a function, choose a function from literature which is the higher part of cos more focused in fact larger the air, we call more focused the waves because it more in

one direction smaller the m more spread \cos^2 the lowest one that is used is typically $2 \text{ by } \pi \cos^2$ normally you we do not use anything lower than that.

Once again the spread is something like spread you know like how much the directional content is there and it is very illogical to think that waves from all direction are of same probability. Cos say it again? No, you do not have to see minus $\pi/2$ to plus $\pi/2$ we are taking because we have actually chosen see what we are doing is, I will tell you what I we will write this way it is this I will answer your question for minus $\pi/2$ equal to zero else.

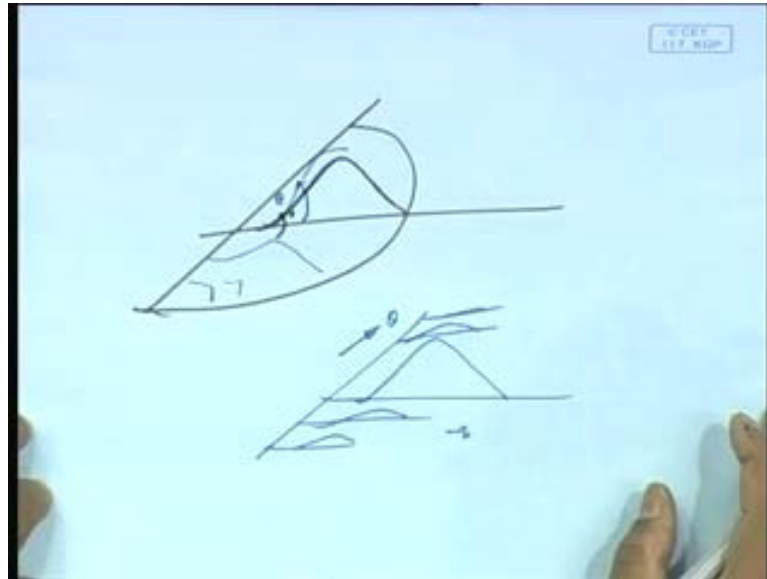
Now, the question is actually why we to minus it is not question of functions value of theta what we chose is that we say from physical ground I can use from any zero to 2π as long as I use what you say is correct as long as I use from zero to π sum one you need it will become same thing question is what to take? So, it satisfies us or easier to visualize I if I to minus $\pi/2$ to plus $\pi/2$ because what would happen then I can think that zero is my dominant wave direction and I am spreading.

If I take 0 to π you know like π then, I have to think my dominant wave direction is $\pi/2$ I can always take from in fact $\alpha - \pi/2$ to $\alpha + \pi/2$ and make α anything you want 360 degree, then you take 182 you know like 4 whatever but, we do not want to do it because one thing you know that we should not unnecessarily complicated things, why we have chose always X wave direction because it is easier to think that way, so that is the reason why you can choose anywhere does not matter.

What yours question is only with the reference point, if I choose the reference point is ninety degree dominant then I have zero to 90 to 180 degree, if I use 270 degree of dominant well I will have 180 to 360 degree. So, should you use 270 or 180 as dominant direction, rather it is better to use a zero degree of dominant direction that is what we are doing so this is how it is?

So, this how the spreading goes here and now what is happening, is that if I were to draw this you know like spectrum there several people drawn in separate ways.

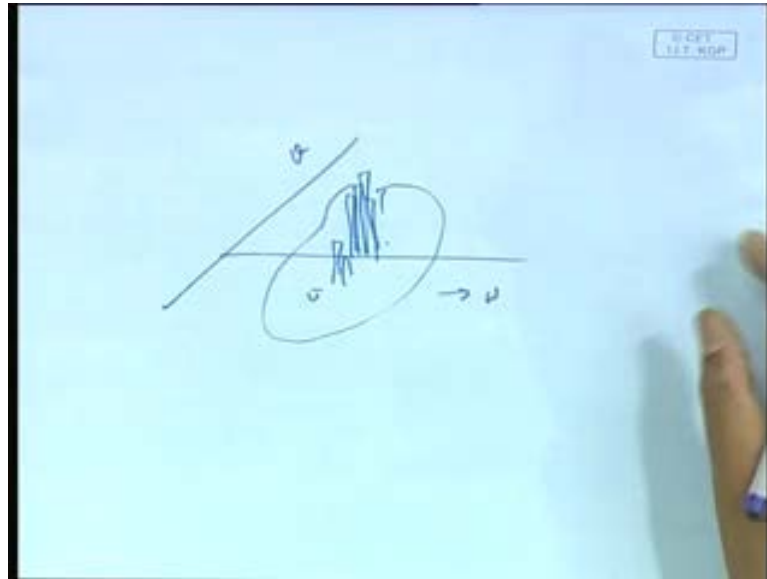
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If I were to draw in a theta direction rather it is difficult for me to plot this say, so this angle is theta so what would happen it is something like your this graph you are kind of rotating this graph rotating, this graph both sides and pulling it down that is what you are doing right because theta is this see this is theta.

So, we are essentially this graph you know this graph you are rotating it this and pulling it down see if we, if I were to look at this way rotating it and pulling it down both sides as you go ninety become zero. So, this amount it is coming down is f theta, so this is one we have. So, if you look at that you are going to get some picture you will find picture will looking like that you know a kind of a shape will come like that not really it does not look like a conical shape because question of the way you are looking and if you were to draw in terms of this as theta, this as omega you will find out this thing coming down to zero as you go less.

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In fact lot of plots have been made in actual literature of directional content they will look something like this you know, I will tell you the picture will look something like you know there is some bar charts there it is difficult to plot here but, it will, it is going to with lot of spikes over this domain, certain domain because not all waves of all comes certain omega and certain theta if you were to measure it comes out like a cluster like a like a bell shape.

People have done it measure as I said nowadays it is becoming fairly sophisticated to also be able to determine directional content from what is known as wave slope. We have say ocean satellites they are collecting whole lot of ocean data, they collect besides wave heights. Nowadays they are also collecting slopes similarly, the data voice are they are they keep collecting slopes, so long back it was only wave height but, now you also have slopes coming in and from there directional content has been worked out many times so they look like that, so this is what is a 3 D spectrum. Now of course, having said 3 D spectrum now suppose I have going to spectrum now so you see opposite now you see how do I reconstruct the signal?

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The image shows a whiteboard with handwritten mathematical equations and a diagram. At the top, the spectrum is labeled $S^{3D}(\omega, \theta)$. Below it, the wave function $\eta(t)$ is expressed as a double sum over modes i and j from 1 to N :

$$\eta(t) = ? = \sum_{i=1}^N \sum_{j=1}^N A(\omega_i, \theta_j) \cos[k_i x \cos \theta_j + k_i y \sin \theta_j - \omega_i t + \phi_{ij}]$$
 The amplitude $A(\omega_i, \theta_j)$ is then defined as:

$$A(\omega_i, \theta_j) = \sqrt{2 S^{3D}(\omega_i, \theta_j) \Delta\omega_i \Delta\theta_j}$$
 At the bottom, a diagram shows a triangular spectrum on the left with a checkmark, an arrow pointing to a complex, oscillatory wave function on the right, which is also marked with a checkmark. A hand is visible at the bottom right, pointing at the wave function.

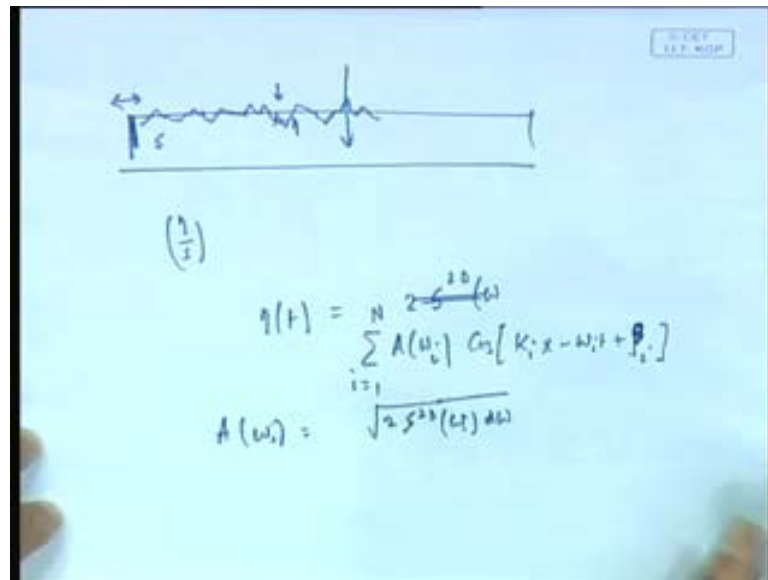
So, I have this spectrum $S^{3D}(\omega, \theta)$, so I want to find out what is $\eta(t)$ so there the like some people want to find out this $\eta(t)$ what we can do of course, is that this is given by straightforward a let me put it K here i , ω_i this will be a random value. So, what you have to do is that you will obtain this by i equal to something one to n , so what is happening, you can reconstruct this way and where a $\omega_i \theta_j$ will be given by $2 S^{3D}$.

See earlier what is happening, if you remember the this is why I am, why I say it this is that here the question is like that I have this formula given after all this formula is known to me now. So, I have this formula given I can find out this reconstruct a realistic realization of the ocean wave, reconstruct what should be the $\eta(t)$ means reverse problem I have a spectrum here see in a 2D case, I have the spectrum here find out what would be go from here to there where do we use it? What is the use of this kind of information very useful because think of this.

I have a ship; I have to find out what is this realistic like realization. Now, somebody tells me use c spectrum by this formula and I go to use the c spectrum, he says take c state four, so I know this formula so I have this given to me this is given to me but, I want to know this means physically what kind of waves let us talk of tank wave tank I want to make a model study I cannot fit this, what do I fit? I have to fit this, after all in a tank I have to create waves which will be like this, what is this? Is this should be such

that it is statistically equivalent to this, so I must have to create a real time signal remember this is a real time signal in real time. So, if I want a 2 D spectrum I have to create real time spectrum, let's say you are doing a model test for a ship or for an offshore structure in sea state 3, so I have to send the waves which is same as sea state 3.

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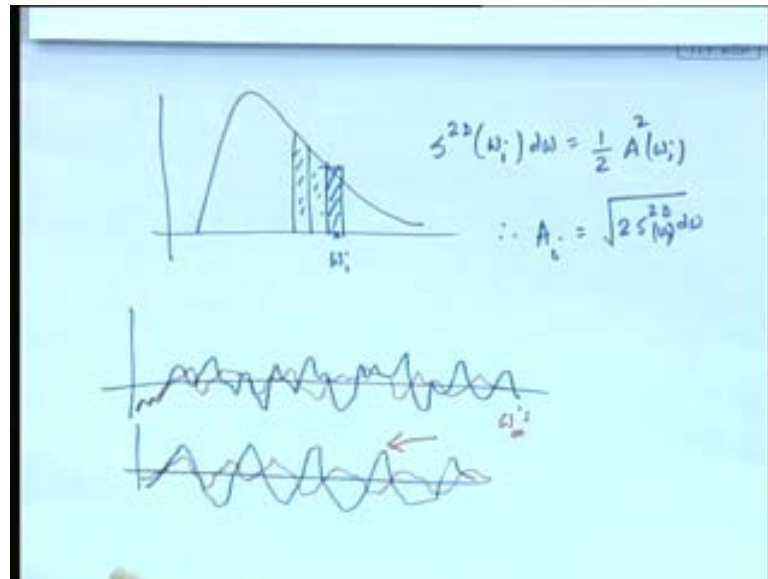


What do I send? Well I have a tank here, I have a wave maker here which oscillates this way as it oscillates it creates waves so this is S and this is eta, so I have actually eta by S defined you know typically to push it to centimeter and this frequency you are going to get this height etcetera.

So, the question is that I want to create this, this is my place where I want to put my structure, so I want to create a realistic wave I want to test this structure in sea state three, so I go to formula, I end up getting this spectrum but, this spectrum I cannot fit I have to break it down to this signal and then only get this signal fit and of course, I get back a signal out similar which I have to reconstruct back to this after all reality is in real time there is no imaginary, no frequency domain etcetera.

So, what is happening I require to create this? So, I said for 2 D spectrum I created that by simply saying eta t was equal to two S 2 D, well let me put it the other way round A omega i, omega i Cos K i x minus omega i t plus say some beta or theta, there is a beta we call it beta i. So, you see here an A the omega i was given as square root of 2 S 2 D omega i d omega this how we have got it.

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If you have a doubt what we have done is well here what we have done is something like this you see this one what is this $S^{2D}(\omega_i)$, this ω_i let us say $d\omega$ equal to half $A(\omega_i)^2$, A well we can call it $A(\omega_i)$ as A_i equal to square root of two $S^{2D}(\omega_i) d\omega$ at this ω_i you know therefore, I end up getting this A_i by summing this all. So, what I mean for each one, I find out what is for? I break it down to several parts for each one, I find out what is the amplitude of the wave of this frequency add them up with a random phase.

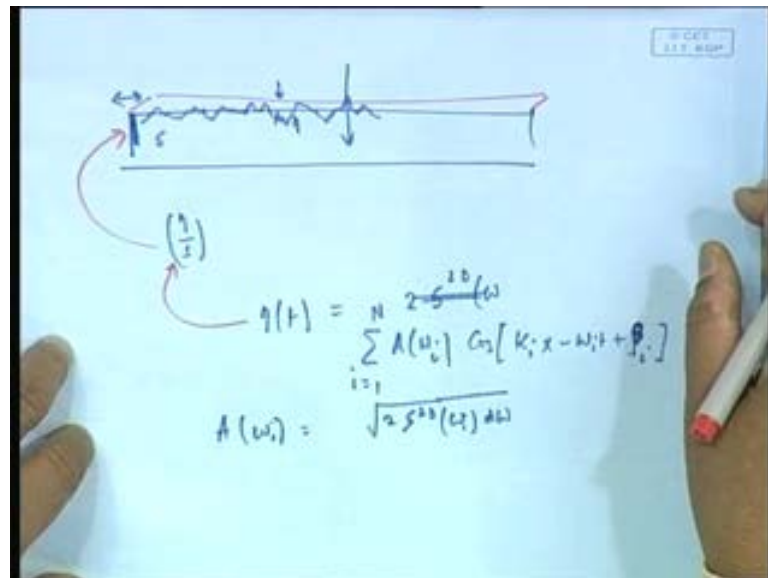
Here what I have to do in a 3 D case I have to do the same thing but, I have to do the larger breaking, I have to break not only over the i axis but, also over the j axis. So, what would what I'm saying therefore, is that see it is, this is you will understand this, so if I take the same 2 D spectrum of say height let us say height equal to five meter I will end up getting a may be wave like that.

Now, if I use a 3 D spectrum what I will probably find out is a slightly more randomness in the sense that this might have much larger number of waves of one particular frequency like typical or rather if I were to use a extreme example a 2 D might give something like say more like this, in a very special case 3 D is going to give me slightly more spread some somewhere small somewhere bigger etcetera.

In other words this will have much more focus ω because all are in one direction this one will be a little more spread you see, so anyhow this is only a qualitative

assessment the question therefore, you see here that when I talk about experiment now I have this interesting point, I have this tank I have to create this wave, so what we do you know we break it down to this signal then I break it down to the feed I have to give to this wave maker and I give that see

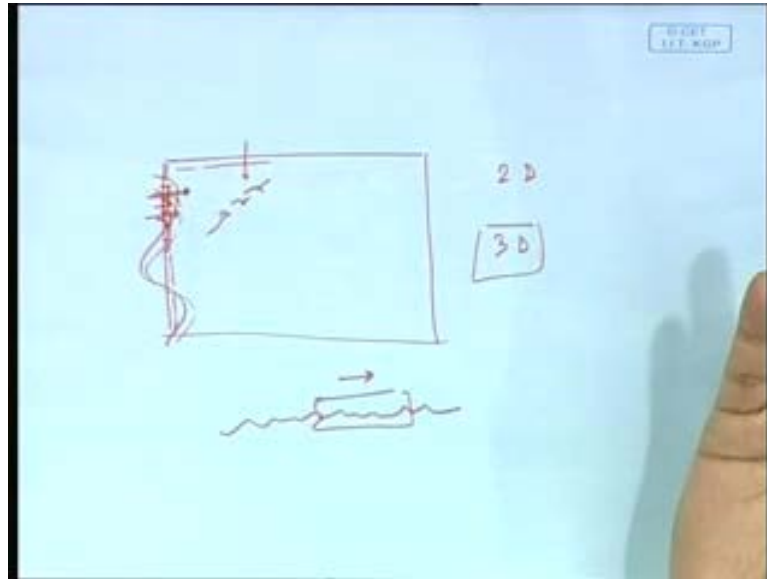
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Now, I have to get this I have to find out what is the corresponding this S value and that I give it to here I push it that way. So, now see if I want to make a 2 D spectrum I simply give this signal if I want to give a 3 D spectrum I will give this signal so as well as the tank is concerned I can make actually of course, in the 3 D spectrum in a tank I cannot really make in a long tank, that is not correct because in a long because the point is that there, we are making all the wave in a one direction in fact that is I have to correct that.

Well we can create this way the spectrum time signal but, however I really cannot create waves in a long tank use a 3 D spectrum because in this case I am actually sending waves in all one direction actually for this purpose only I need a so called you know like wave basin, which looks like that I will just spend this little time.

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You you'll have a wave like IIT madras has got wave basin like that we are, we have got this wave maker this side earlier days people use to send wave this side and wave this side and then combining you could get nowadays very interesting phenomena is there you can actually have multiple segment of wave maker, if you move them like a snake type if you move this one here, this one here, this one here etcetera you will find out you can make directional wave how you know it is the I also tell you this suppose I moved this mode the crest would have come here, this is slightly the crest have come here, this is slightly as the crest, so you end up getting a wave like that.

So, what happen it is very complicated theory but, you can move this wave maker like a snake type like when it is here next one is here, next one is here, like that and then you can make directional wave very complicated but, you can break but, having said that, so what we understood I want to just spend this time on this class on 3 D spectrum only.

See what I understood is that I have 2 D spectrum, I have 3 D spectrum. 3 D seems to be more realistic after all when you go to ocean you state 3 D why the hell do we use 2 D spectrum? What is the use of 2 D spectrum then? Somebody may ask the question why I spend so many times how to find out the response in 2 D spectrum now there is a good reason for it.

You see as I said in 2 D spectrum, I'm presuming all the waves coming from one direction therefore, you would expect it is much more all are focused in one direction

and suppose I have a ship which responds much heavily for wave coming in one direction then it is going to go up and down much more, now same wave if I same ship for same wave height if I assume come from all direction this is going to be slightly mellowed in this direction see then what will happen extreme example single wave suppose this full thing is modeled by one wave of 5 meter height. So, in 2 D case there is a wave coming of 5 meter height this side, 3 D case I will say there is a wave of 4 meter height this side one meter height from other side.

Now, this body has seems to respond mostly for wave in this direction so in the case one the 2 D case, I end up getting usually a larger response larger heave for example, or larger in the case of being larger role therefore, if a ship is designed which can survive 2 D spectrum, it would normally be lesser in 3 D spectrum therefore, if you could actually after all you are going to design the ship for 2 D spectrum coming from all direction.

So, if I could make the ship survive a 2 D spectrum from all direction then I would more or less think that that would be more than sufficient for it to survive a 3 D spectrum, this is one of the reason why 2 D spectrum despite its assumption is still valid, is still used for all my you know actual calculating otherwise we would not have done it we would not have used at all 2 D spectrum but, even now it is used.

Now, tomorrow's class we will find out the response in 3 D spectrum which is you are going to find out it is nothing but, one more level of algebra see here I have added all the omegas and responses in omega direction there I have to repeat that in theta direction. So, here I have one piece of paper with omega and this thing here, I have to do each piece of paper for one theta ten pieces and add them together it is just one level of algebra, so there is nothing thinking involving that it is only a level of algebra involved but, still having said 2 D is useful because 2 D gives me usually a large usually but, not always a larger response a more conservative kind of a design comes out of that.